Data Structure and Algorithms [CO2003]

Chapter 6 - Tree

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Contents

Outcomes

- **L.O.3.1** - Depict the following concepts: binary tree, complete binary tree, balanced binary tree, AVL tree, multi-way tree, etc.
- **L.O.3.2** - Describe the strorage structure for tree structures using pseudocode.
- **L.O.3.3** - List necessary methods supplied for tree structures, and describe them using pseudocode.
- **L.O.3.4** - Identify the importance of "blanced" feature in tree structures and give examples to demonstate it.
- **L.O.3.5** - Identiy cases in which AVL tree and B-tree are unblanced, and demonstrate methods to resolve all the cases step-by-step using figures.

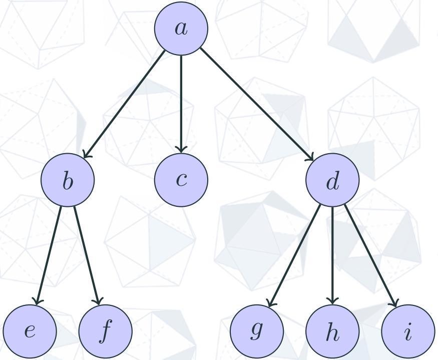
Outcomes

- **L.O.3.6** - Implement binary tree and AVL tree using C/C++.
- **L.O.3.7** - Use binary tree and AVL tree to solve problems in real-life, especially related to searching techniques.
- **L.O.3.8** - Analyze the complexity and develop experiment (program) to evaluate methods supplied for tree structures.
- **L.O.8.4** - Develop recursive implementations for methods supplied for the following structures: list, tree, heap, searching, and graphs.
- **L.O.1.2** - Analyze algorithms and use Big-O notation to characterize the computational complexity of algorithms composed by using the following control structures: sequence, branching, and iteration (not recursion).

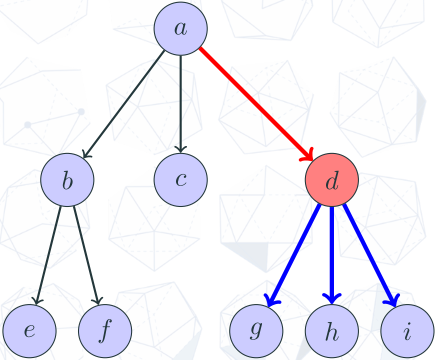
Basic Tree Concepts

Definition

A tree consists of a finite set of elements, called nodes, and a finite set of directed lines, called branches, that connect the nodes.



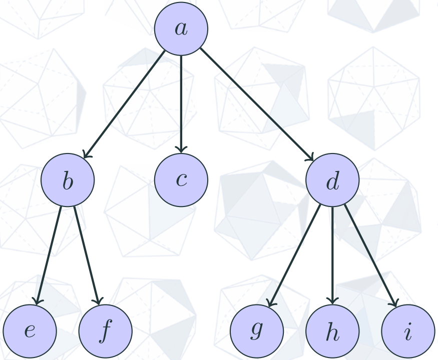
- Degree of a node: the number of branches associated with the node.
- Indegree branch: directed branch toward the node.
- Outdegree branch: directed branch away from the node.



For the node d:

- **Degree** $= 4$
- **Indegree branches**: ad
  $\rightarrow$ indegree $= 1$
- **Outdegree branches**: dg, dh, di
  $\rightarrow$ outdegree $= 3$

- The first node is called the root.
- indegree of the root = 0
- Except the root, the indegree of a node = 1
- outdegree of a node = 0 or 1 or more.

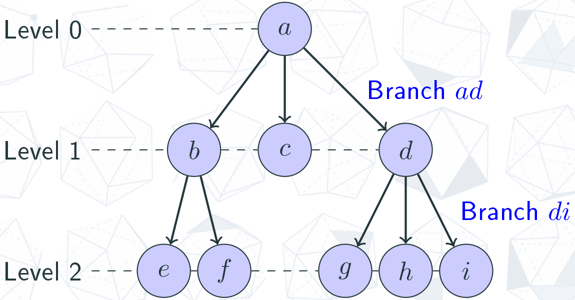


Terms

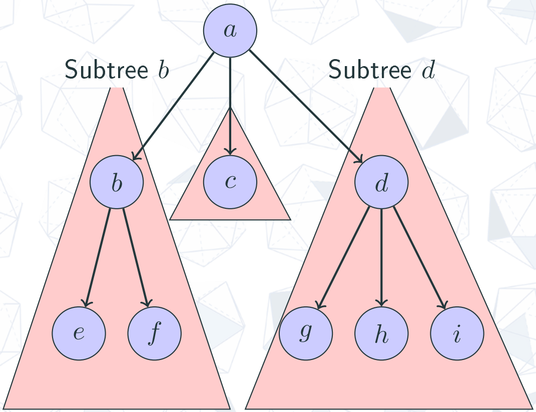
- A root is the first node with an indegree of zero.
- A leaf is any node with an outdegree of zero.
- A internal node is not a root or a leaf.
- A parent has an outdegree greater than zero.
- A child has an indegree of one.
  $\rightarrow$ a internal node is both a parent of a node and a child of another one.
- Siblings are two or more nodes with the same parent.
- For a given node, an ancestor is any node in the path from the root to the node.
- For a given node, an descendent is any node in the paths from the node to a leaf.

Terms

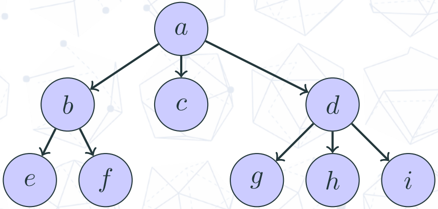
- A path is a sequence of nodes in which each node is adjacent to the next one.
- The level of a node is its distance from the root.
  $\rightarrow$ Siblings are always at the same level.
- The height of a tree is the level of the leaf in the longest path from the root plus 1.
- A subtree is any connected structure below the root.



- Parents: a, b, d
- Children: b, c, d, e, f, g, h, i
- Leaves: c, e, f, g, h, i
- Internal nodes: b, d
- Siblings: $\{b, c, d\}, \{e, f\}, \{g, h, i\}$
- Height = 3


Subtree b
Subtree d
a
b
c
d
e
f
g
h
i

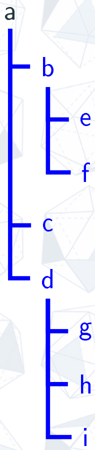
- organization chart



- parenthetical listing

  $a\ (b\ (e\ f)\ c\ d\ (g\ h\ i))$

- indented list

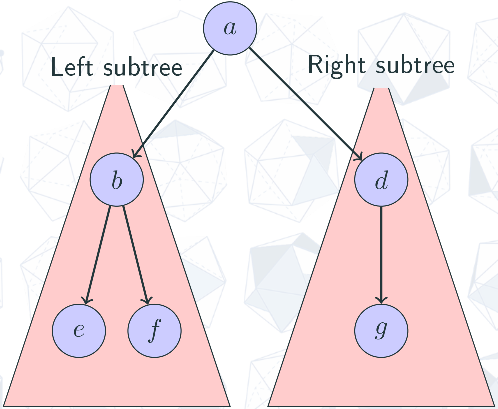


Applications of Trees

- Representing hierarchical data
- Storing data in a way that makes it easily searchable (ex: binary search tree)
- Representing sorted lists of data
- Network routing algorithms

Binary Trees

A binary tree node cannot have more than two subtrees.



- To store N nodes in a binary tree:
  - The minimum height: $H_{min} = \lfloor \log_2 N \rfloor + 1$
  - The maximum height: $H_{max} = N$
- Given a height of the binary tree, H:
  - The minimum number of nodes: $N_{min} = H$
  - The maximum number of nodes: $N_{max} = 2^H - 1$

Balance
The balance factor of a binary tree is the difference in height between its left and right subtrees.

$$B = H_L - H_R$$

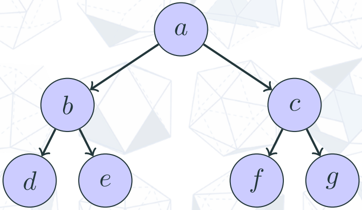
Balanced tree:

- balance factor is 0, -1, or 1
- subtrees are balanced

Complete tree

$N = N_{max} = 2^H - 1$

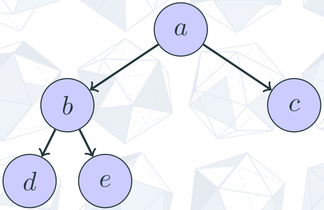
The last level is full.



Nearly complete tree

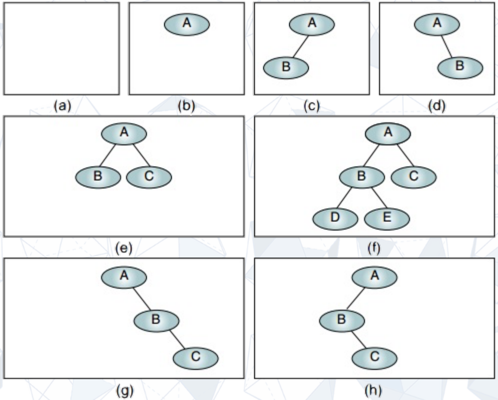
$H = H_{min} = \lfloor \log_2 N \rfloor + 1$

Nodes in the last level are on the left.



Definition

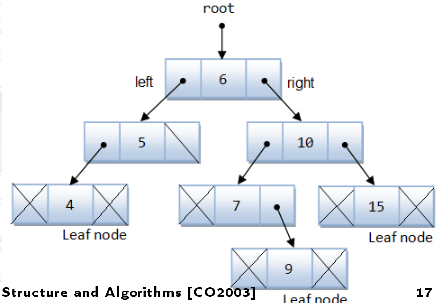
A binary tree is either empty, or it consists of a node called root together with two binary trees called the left and the right subtree of the root.



```
node
   data <dataType>
   left <pointer>
   right <pointer>
end node
```

```
binaryTree
   root <pointer>
end binaryTree
```

```
// General dataTye:
dataType
   key <keyType>
   field1 <...>
   field2 <...>
   ...
   fieldn <...>
end dataType
```



Suitable for complete tree, nearly complete tree.

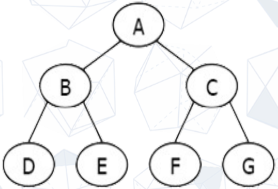


Figure 1: Conceptual

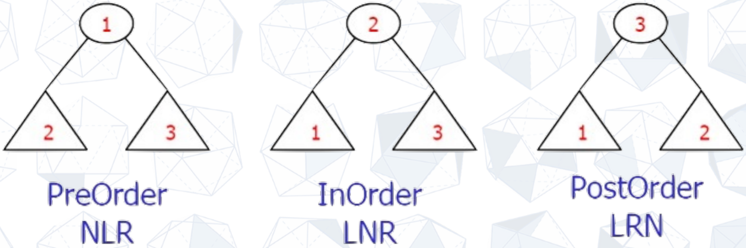
```
binaryTree
  data <array of dataType>
end binaryTree
```

0	1	2	3	4	5	6
A	B	C	D	E	F	G

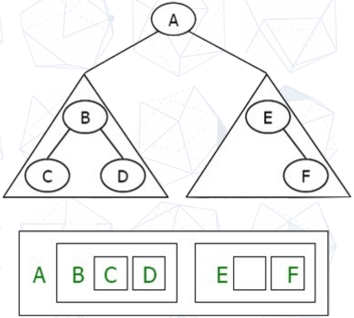
Figure 2: Physical

- Depth-first traversal: the processing proceeds along a path from the root through one child to the most distant descendent of that first child before processing a second child, i.e. processes all of the descendents of a child before going on to the next child.
- Breadth-first traversal: the processing proceeds horizontally from the root to all of its children, then to its children's children, i.e. each level is completely processed before the next level is started.

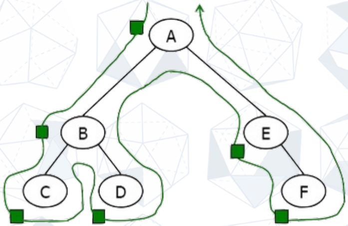
- Preorder traversal
- Inorder traversal
- Postorder traversal



In the preorder traversal, the root is processed first, before the left and right subtrees.



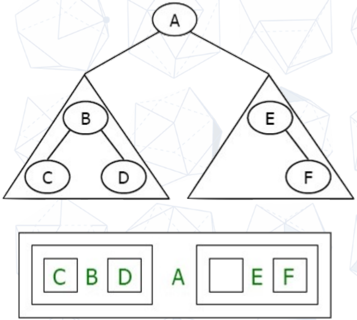
Processing order



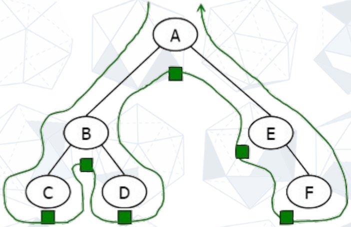
Walking order

```
Algorithm preOrder(val root <pointer>)
Traverse a binary tree in node-left-right sequence.
Pre: root is the entry node of a tree or subtree
Post: each node has been processed in order
if root is not null then
    process(root)
    preOrder(root->left)
    preOrder(root->right)
end
Return
End preOrder
```

In the inorder traversal, the root is processed between its subtrees.



Processing order

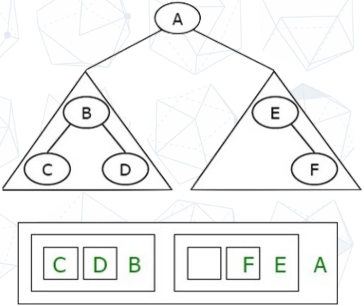


Walking order

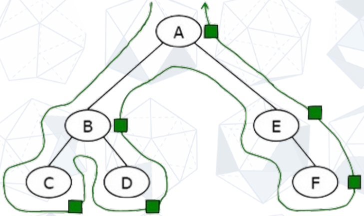
Algorithm inOrder(val root <pointer>)
Traverse a binary tree in left-node-right sequence.
Pre: `root` is the entry node of a tree or subtree
Post: each node has been processed in order
if *root is not null* **then**
    inOrder(root->left)
    process(root)
    inOrder(root->right)
end
Return
End inOrder

Postorder traversal (LRN)

In the postorder traversal, the root is processed after its subtrees.



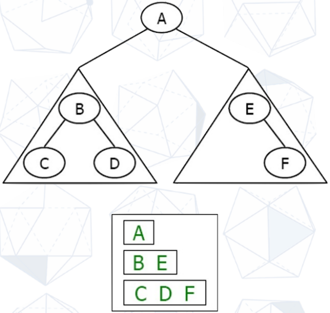
Processing order



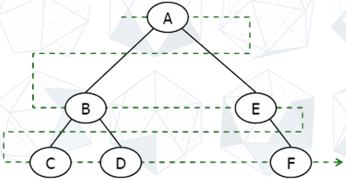
Walking order

Algorithm postOrder(val root <pointer>)
Traverse a binary tree in left-right-node sequence.
Pre: `root` is the entry node of a tree or subtree
Post: each node has been processed in order
if *root is not null* **then**
    postOrder(root->left)
    postOrder(root->right)
    process(root)
end
Return
End postOrder

In the breadth-first traversal of a binary tree, we process all of the children of a node before proceeding with the next level.



Processing order



Walking order

Algorithm breadthFirst(val root <pointer>)
Process tree using breadth-first traversal.

Pre: `root` is node to be processed
Post: tree has been processed

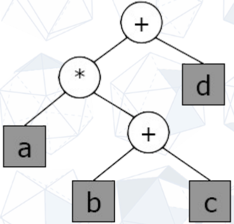
currentNode = root
bfQueue = createQueue()

```
while currentNode not null do
    process(currentNode)
    if currentNode->left not null then
        enqueue(bfQueue, currentNode->left)
    end
    if currentNode->right not nul then
        enqueue(bfQueue, currentNode->right)
    end
    if not emptyQueue(bfQueue) then
        currentNode = dequeue(bfQueue)
    else
        currentNode = NULL
    end
end
```

Expression Trees

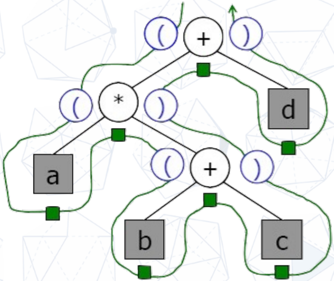
Expression Trees

- Each leaf is an operand
- The root and internal nodes are operators
- Sub-trees are sub-expressions



a * (b + c) + d

Infix Expression Tree Traversal



((a * (b + c)) + d)

Algorithm infix(val tree <pointer>)
Print the infix expression for an expression tree.
Pre: `tree` is a pointer to an expression tree
Post: the infix expression has been printed

```
if tree not empty then
    if tree->data is an operand then
        print (tree->data)
    else
        print (open parenthesis)
        infix (tree->left)
        print (tree->data)
        infix (tree->right)
        print (close parenthesis)
    end
```

Algorithm postfix(val tree <pointer>)

Print the postfix expression for an expression tree.

Pre: `tree` is a pointer to an expression tree

Post: the postfix expression has been printed

if *tree not empty* **then**

    postfix (tree->left)

    postfix (tree->right)

    print (tree->data)

end

End postfix

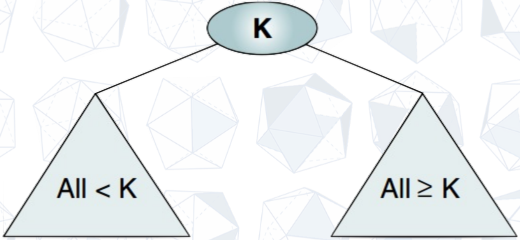
Algorithm prefix(val tree <pointer>)
Print the prefix expression for an expression tree.
Pre: `tree` is a pointer to an expression tree
Post: the prefix expression has been printed
if *tree not empty* **then**
    print (tree->data)
    prefix (tree->left)
    prefix (tree->right)
end
End prefix

Binary Search Trees

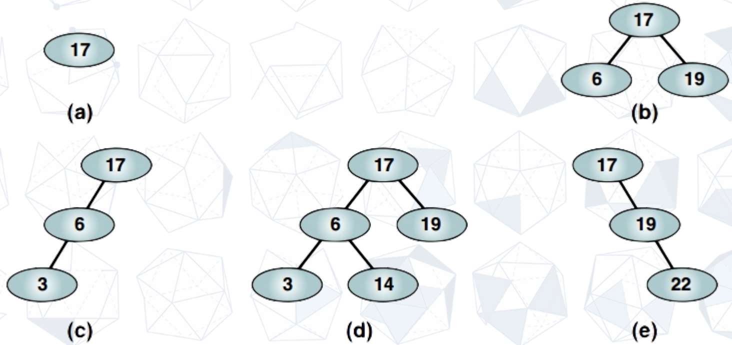
Definition

A binary search tree is a binary tree with the following properties:

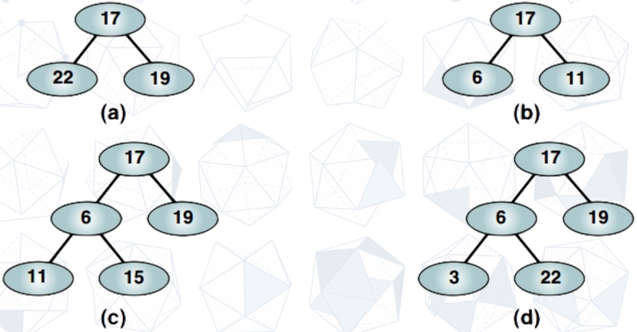
1. All items in the left subtree are less than the root.
2. All items in the right subtree are greater than or equal to the root.
3. Each subtree is itself a binary search tree.



Valid Binary Search Trees

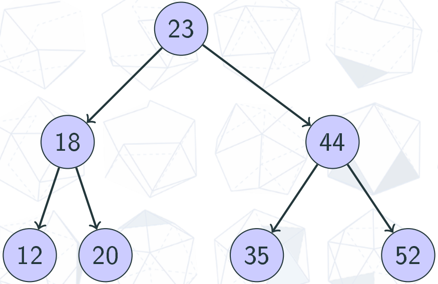


Invalid Binary Search Trees



Binary Search Tree (BST)

- BST is one of implementations for ordered list.
- In BST we can search quickly (as with binary search on a contiguous list).
- In BST we can make insertions and deletions quickly (as with a linked list).



- Preorder traversal: 23, 18, 12, 20, 44, 35, 52
- Postorder traversal: 12, 20, 18, 35, 52, 44, 23
- Inorder traversal: **12, 18, 20, 23, 35, 44, 52**

The inorder traversal of a binary search tree produces an ordered list.

Binary Search Tree Search

Find Smallest Node

Algorithm findSmallestBST(val root <pointer>)

This algorithm finds the smallest node in a BST.

Pre: `root` is a pointer to a nonempty BST or subtree

Return address of smallest node

if *root->left null* **then**

  return root

end

return findSmallestBST(root->left)

End findSmallestBST

Find Largest Node

Algorithm findLargestBST(val root <pointer>)

This algorithm finds the largest node in a BST.

Pre: `root` is a pointer to a nonempty BST or subtree

Return address of largest node returned

if *root->right null* **then**
  return root
end

return findLargestBST(root->right)

End findLargestBST

Recursive Search

Algorithm searchBST(val root <pointer>, val target <keyType>)

Search a binary search tree for a given value.

Pre: `root` is the root to a binary tree or subtree

`target` is the key value requested

Return the node address if the value is found

`null` if the node is not in the tree

Recursive Search

if *root is null* **then**
  return null
end

if *target < root->data.key* **then**
  return searchBST(root->left, target)
else if *target > root->data.key* **then**
  return searchBST(root->right, target)
else
  return root
end

End searchBST

Iterative Search

Algorithm iterativeSearchBST(val root <pointer>, val target <keyType>)

Search a binary search tree for a given value using a loop.

Pre: `root` is the root to a binary tree or subtree

`target` is the key value requested

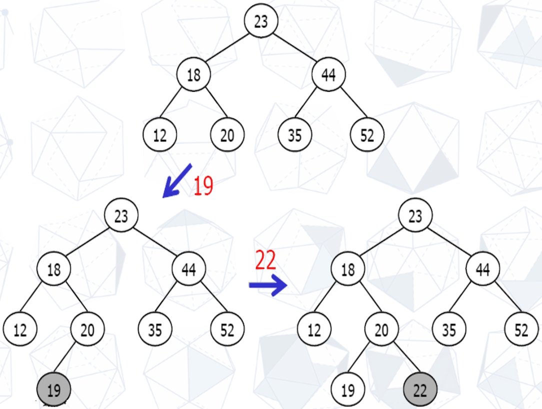
Return the node address if the value is found

`null` if the node is not in the tree

Iterative Search

```
while (root is not NULL) AND (root->data.key <> target) do
    if target < root->data.key then
        root = root->left
    else
        root = root->right
    end
end
return root
End iterativeSearchBST
```

Insert Node into BST



All BST insertions take place at a leaf or a leaflike node (a node that has only one null branch).

Insert Node into BST: Iterative Insert

Algorithm iterativeInsertBST(ref root <pointer>, val new <pointer>)
Insert node containing new data into BST using iteration.

Pre: `root` is address of first node in a BST
`new` is address of node containing data to be inserted

Post: new node inserted into the tree

```
if root is null then
    root = new
else
    pWalk = root
    while pWalk not null do
        parent = pWalk
        if new->data.key < pWalk->data.key then
            pWalk = pWalk->left
        else
            pWalk = pWalk->right
        end
    end
    if new->data.key < parent->data.key then
        parent->left = new
    else
        parent->right = new
    end
end
End iterativeInsertBST
```

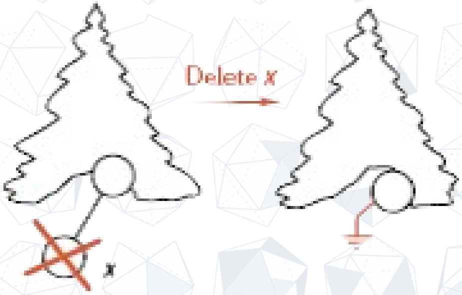
Algorithm recursiveInsertBST(ref root <pointer>, val new <pointer>)
Insert node containing new data into BST using recursion.

Pre: `root` is address of current node in a BST
`new` is address of node containing data to be inserted

Post: new node inserted into the tree

Insert Node into BST: Recursive Insert

```
if root is null then
    root = new
else
    if new->data.key < root->data.key then
        recursiveInsertBST(root->left, new)
    else
        recursiveInsertBST(root->right, new)
    end
end
Return
End recursiveInsertBST
```



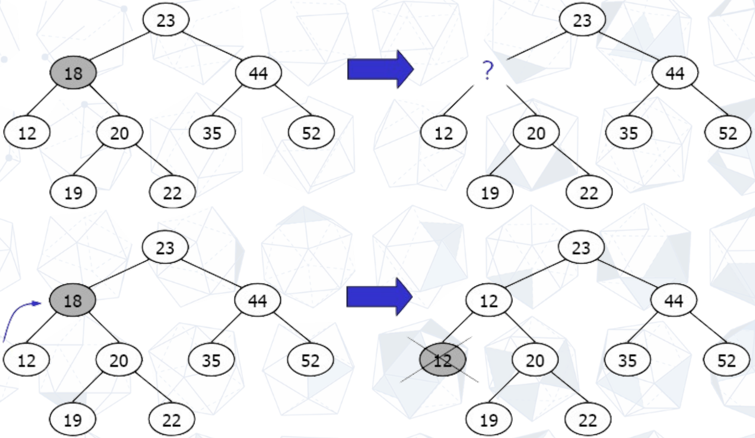
Deletion of a leaf: Set the deleted node's parent link to NULL.

Delete node from BST

Deletion of a node having only right subtree or left subtree: Attach the subtree to the deleted node's parent.

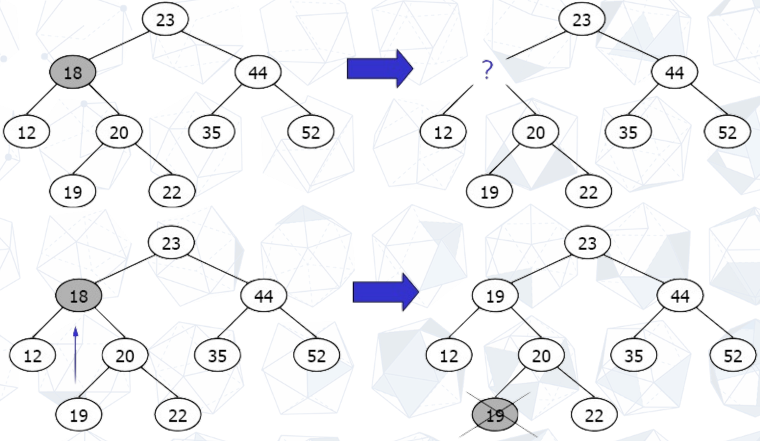
Deletion of a node having both subtrees:
Replace the deleted node by its predecessor or by its successor, recycle this node instead.

Delete node from BST



Using largest node in the left subtree

Delete node from BST



Using smallest node in the right subtree

Algorithm deleteBST(ref root <pointer>, val dltKey <keyType>)
Deletes a node from a BST.

Pre: `root` is pointer to tree containing data to be deleted
`dltKey` is key of node to be deleted

Post: node deleted and memory recycled
if `dltKey` not found, `root` unchanged

Return `true` if node deleted, `false` if not found

Delete node from BST

if *root is null* **then**
    return false
end
if *dltKey < root->data.key* **then**
    return deleteBST(root->left, dltKey)
else if *dltKey > root->data.key* **then**
    return deleteBST(root->right, dltKey)

Delete node from BST

```
else
    // Deleted node found – Test for leaf node
    if root->left is null then
        dltPtr = root
        root = root->right
        recycle(dltPtr)
        return true
    else if root->right is null then
        dltPtr = root
        root = root->left
        recycle(dltPtr)
        return true
```

```
else
    // ...
    else
        // Deleted node is not a leaf.
        // Find largest node on left subtree
        dltPtr = root->left
        while dltPtr->right not null do
            dltPtr = dltPtr->right
        end
        // Node found. Move data and delete leaf node
        root->data = dltPtr->data
        return deleteBST(root->left, dltPtr->data.key)
    end
end
End deleteBST
```