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The Dark Side of Circuit Breakers

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ABSTRACT

Market-wide circuit breakers are trading halts aimed at stabilizing the market during dramatic price declines. Using an intertemporal equilibrium model, we show that a circuit breaker significantly alters market dynamics and affects investor welfare. As the market approaches the circuit breaker, price volatility rises drastically, accelerating the chance of triggering the circuit breaker—the so-called "magnet effect," returns exhibit increasing negative skewness, and trading activity spikes up. Our empirical analysis supports the model's predictions. Circuit breakers can affect overall welfare negatively or positively, depending on the relative significance of investors' trading motives for risk sharing versus irrational speculation.

STOCK MARKET CRASHES IN THE absence of clear macroeconomic causes raise questions about the extent of confidence in the financial market from market participants, policymakers, and the general public alike. While the mechanisms behind these sudden price drops are still not well understood, various measures have been adopted to intervene in the trading process in the hope of stabilizing prices and restoring market order. These measures, which are sometimes referred to as "throwing sand in the gears," range from market-wide trading halts and price limits to restrictions on order flows, positions, margins, and even transaction taxes.[1] They have become an important part of

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[1] Contingent trading halts and price limits are part of the normal trading process for individual stocks and futures contracts. The motivations behind them, however, vary. For example, the trading halt of an individual stock prior to major corporate announcements is motivated by the desire for fair information disclosure, while daily price limits on futures are motivated by the desire to guarantee the proper implementation of the mark-to-market mechanism and to deter market

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Table I
Market-Wide Circuit Breaker Adoption

For the leading stock markets by market capitalization in 2020 (source: World Bank), the table indicates those markets that have implemented market-wide circuit breakers (MWCB) and those markets that have implemented individual stock price limits. Y indicates adoption, N indicates no adoption, and Y/N for China denotes its adoption and then abandonment of a circuit breaker.

	Market Cap (Tn $)	Market Cap Rank	Circuit Breaker	Price Limit
Developed markets				
United States	40.7	1	Y	Y
Japan	6.7	3	N	Y
Hong Kong	6.1	4	N	N
United Kingdom	3.6	5	Y	Y
France	2.8	6	Y	N
Canada	2.6	7	Y	N
Developing markets				
China (mainland)	12.2	2	Y/N	Y
India	2.6	8	Y	Y

the overall market architecture. Yet the merits of these measures, either theoretically or empirically, remain largely unclear (see, e.g., Grossman (1990)).

Arguably, the most prominent among these measures is the market-wide circuit breaker (MWCB), which was first introduced in the United States in 1988 after the 1987 Black Monday stock market crash.[2] An MWCB temporarily halts trading in all stocks and related derivatives when a designated market index drops by a prespecified amount during a trading session. Since the introduction of the U.S. MWCB, circuit breakers of various forms have been adopted around the globe.[3] Table I shows the leading developed and developing economies as of 2020 which markets have adopted MWCB and which have adopted price limits. The U.S. MWCB was first triggered on October 27, 1997, which led to its redesign. It then stayed untouched (including during the "Flash Crash" of 2010) until March 2020, when it was triggered four times in a span of two weeks at the onset of the COVID-19 pandemic. Following the turbulent stock market declines in 2015, China introduced its MWCB in January 2016. After being triggered on the first day of its installment and again later the same week, it was immediately abolished.

These recent events have revived the debate about the role of circuit breakers and market interventions in general. Questions that remain unanswered include what are the goals of circuit breakers and other types of trading restrictions, how do they actually affect the market, and how is their

manipulation. In this paper, we focus on market-wide trading halts in the underlying markets such as stocks and their derivatives, which have very different motivations and implications.

[2] See the Internet Appendix (Section I) for a brief history of the MWCB mechanism in the United States. The Internet Appendix may be found in the online version of this article.

[3] According to a 2016 report, "Global Circuit Breaker Guide" by ITG, over 30 countries around the world have rules on trading halts in the form of circuit breakers, price limits, and volatility auctions.

effectiveness impacted by market participants, mechanism design, and market conditions.

In this paper, we develop an intertemporal equilibrium model in which investors trade either to share risk or to speculate on their own beliefs. We first examine how the introduction of a circuit breaker changes investors' trading and equilibrium price behavior. We show that, in general, a circuit breaker lowers the overall price level. More importantly, it substantially alters price dynamics. In particular, we show that as the market approaches the circuit breaker trigger, conditional price volatility rises sharply. In addition, returns exhibit increasingly negative skewness. Both effects reflect the fact that a circuit breaker tends to destabilize the price during large market declines. Consequently, as the market falls closer to the circuit breaker, the likelihood of a circuit breaker being triggered rises at an increasing pace. This is the so-called "magnet effect" often suspected by market participants, but yet to be formalized. Our model also predicts that expected returns and trading volume tend to increase as the market approaches the circuit breaker. We find supporting evidence for these predictions using transaction-level data from E-mini S&P 500 futures.

We next use the model to examine the welfare implications of a circuit breaker, which depend critically on investors' trading motives. If investors trade primarily for risk-sharing reasons, the introduction of a circuit breaker reduces overall welfare. If, however, they trade mainly to speculate on their irrational beliefs (e.g., due to panic at times of market turmoil), a circuit breaker can improve overall welfare under the objective probability measure. When both trading motives are present, there will be a trade-off between the two effects.

In our model, the financial market consists of a stock and a bond. There are two (classes of) heterogeneous investors in the market. The heterogeneity can take two different forms: it can be in the investors' beliefs about stock fundamentals (payoffs) or in their utility function, which can be state-dependent. Under our formulation, these two forms of heterogeneity are mathematically equivalent in that they yield the equilibrium market behavior despite their different economic interpretations and welfare implications. For simplicity, we adopt the heterogeneous beliefs interpretation in our main analysis. (We return to the alternative interpretation in the welfare analysis.) In particular, we assume that the two investors have heterogeneous beliefs about the stock's future payoff. One investor's belief is set to be the objective belief, while the other investor's belief is different. We refer to the later investor as irrational investor. The two investors trade competitively in the financial market.

Without the circuit breaker, the market is dynamically complete, and both investors trade continuously to achieve the efficient allocation under their own beliefs. Given the possibility of continuous portfolio rebalancing, the relatively more optimistic investor is willing to take on much larger stock positions relative to the more pessimistic investor.

The introduction of a circuit breaker fundamentally changes investors' trading behavior and the equilibrium price. We start by considering the market equilibrium at the triggering point of the circuit breaker, which occurs after a series of negative shocks. The trading halt forces the more optimistic investor to hold her stock position for an extended period without an opportunity to rebalance in response to new shocks. Such extreme illiquidity substantially reduces her willingness to hold the stock upon market closure. As a result, the stock price has to drop significantly at market closure to induce the pessimistic investor to absorb more shares of the stock, disproportionately raising the pessimistic investor's importance in determining the stock's price.

We next consider price behavior near the circuit breaker. No-arbitrage requires that the stock price be continuous over time. Thus, the low price level at market closure and its continuity imply that the stock price will decrease significantly as the market moves closer to the circuit breaker threshold. The sharp price decline in response to decreasing fundamentals corresponds to heightened price sensitivity to fundamental shocks, which causes the conditional price volatility to rise significantly.

The mechanism above leads to the following model predictions. First, the presence of a circuit breaker lowers the stock price relative to its level without the circuit breaker. Second, (conditional and realized) price volatility increases *at an increasing rate* as the market moves closer the circuit breaker. Third, due to the negative correlation between volatility and price, realized return skewness also turns negative as the market moves closer the circuit breaker. Fourth, the conditional expected return rises as the market approaches the circuit breaker and the price level drops; this is due to the price drop being driven more by the increasing influence of the more pessimistic investor than by changes in fundamentals (e.g., expected future payoffs). Fifth, the unwinding of the more optimistic investor's stock position when approaching the circuit breaker results in a rapid increase in trading volume. These price and volume behaviors are in sharp contrast to those without the circuit breaker, which are relatively stable around the same price level.

We also show that the changes in price dynamics in the presence of the circuit breaker gives rise to the so-called magnet effect. Under this effect, when the price approaches the circuit breaker threshold, the likelihood of hitting the circuit breaker increases substantially compared to under normal conditions. This is because the increasing price volatility in the neighborhood of the circuit breaker greatly accelerates the likelihood of actually reaching it.

We further explore the model's predictions empirically. By both their purpose and their design, circuit breakers are rarely hit. It is difficult to assess their impact by relying purely on the actual triggering events themselves. We, instead, exploit the dynamic nature of our model and examine its unique predictions for the behavior of prices (including realized volatility, skewness, and average return) and volume as the market approaches the circuit breaker (second through fifth predictions above), without necessarily hitting it. Specifically, we use transaction-level data for the E-mini S&P 500 futures from 2013 to 2020 to construct volatility, skewness, return, and trading volume measures and then run piecewise linear regressions on a measure of the distance to circuit breaker (DTCB) while controlling for the leverage effect, a time trend, intraday seasonality, as well as lagged dependence of the dependent variables.

The empirical results are overall consistent with our model's predictions. In particular, for return volatility, the unconditional regression coefficient on DTCB is −17.5, implying that a decrease in DTCB of 1% (7% is the Level-1 trigger threshold) leads to an increase in volatility of 17.5 basis points (bps) normalized to daily scale. However, when the DTCB is within the range of 2% and below, the regression coefficient nearly doubles to −30.0, reflecting an accelerating rise in return volatility as the DTCB drops closer to zero. These results reflect the destabilizing influence of the circuit breaker on price in its neighborhood and the magnet effect.

We note that while our model focuses on circuit breakers, it can be extended to study other types of trading interventions such as price limits, trading restrictions, and other forms of market freezes and slowdowns. The underlying mechanism driving the impact of a circuit breaker applies to these situations as well.

Related Literature. Prior theoretical work on circuit breakers focuses on their role in restoring orderly trading and reducing "excess" volatility in a market with various microstructure imperfections, such as limited market participation, nonsynchronized trading, and asymmetric information. This is motivated in part by the apparent breakdown in the trading process during the 1987 market crash. For example, Greenwald and Stein (1991) argue that, in a market with limited participation and the resulting execution risk, circuit breakers can help synchronize trading among market participants and improve the efficiency of allocations (see also Greenwald and Stein (1988)).[4] Subrahmanyam (1994) shows that in the presence of partial participation/optimization and asymmetric information, circuit breakers can increase ex ante price volatility when investors with fixed orders shift their trades to earlier periods with lower liquidity supply (see also Subrahmanyam (1995)). Using a setting similar to Greenwald and Stein (1991), Kodres and O'Brien (1994) show that circuit breakers can reduce the welfare loss from the initially imperfect trading process, at least for some market participants. A common starting point in this work is a trading process with major imperfections in a noisy rational expectations setting. Leaving noise trades outside the model, these models do capture important aspects of the market, but they are partial equilibrium in nature.

By developing a general equilibrium model in an intertemporal setting, we make several contributions to the literature. First, we properly capture investors' most basic trading motivations, risk sharing and speculation, and their

[4] In Greenwald and Stein (1991), limited participation takes several forms. In particular, value traders, who act as price stabilizers, enter the market at different times with uncertainty. This uncertainty in their participation, which is assumed to be exogenous, gives rise to the additional risk in execution prices. Also, these value traders can rely only on market orders or simple limit orders, rather than limit order schedules, in their trading.

resulting trading behavior, with and without circuit breakers, and thus the impact of a circuit breaker is a full equilibrium outcome. Second, a general equilibrium model captures the welfare of all market participants, which allows us to examine the full extent of a circuit breaker's welfare impact. Third, our intertemporal setting yields unique predictions on how circuit breakers affect price dynamics, in particular, how they destabilize the market as prices approach triggering levels, giving rise to the magnet effect. Notably these predictions are testable even without a circuit breaker being triggered.[5] In addition, our model provides a basis to further include other forms of market imperfection beyond irrational speculation, such as participation costs, coordination failure, asymmetric information, and strategic behavior, which may be relevant to capture and quantify more fully the cost and benefit of circuit breakers. Nonetheless, focusing solely on their marginal influence may understate the fundamental merits of the market mechanism itself.[6]

Empirical work on MWCB is scarce due to the fact that their likelihood to be triggered is very small by design. Goldstein and Kavajecz (2004) provide a detailed analysis of the behavior of market participants in the period around October 27, 1997, the only time the U.S. circuit breaker had been triggered since its introduction until very recently. They find that leading up to the trading halt, market participants accelerated their trades. In addition, they show that sellers' behavior was less influenced when approaching the circuit breaker than was that of buyers', who were withdrawing from the market by canceling their buy limit orders. These patterns are consistent with what our model predicts: buyers (more optimistic investors) pull back from the market and sellers (more pessimistic investors) become the marginal traders when a circuit breaker is near.[7]

Our model shows that the presence of a circuit breaker leads to unique price dynamics in its neighborhood. In particular, the model produces testable predictions about the dynamic behavior of return moments for a wide range of prices without a circuit breaker being triggered. Such an approach clearly

[5] The predictions of our model on price volatility differ from those of Subrahmanyam (1994) in nature. Apart from the differences in modeling choices, such as general equilibrium versus noisy rational expectations equilibrium and mild imperfections versus more severe imperfections, Subrahmanyam (1994)'s results are about the ex ante price volatility at the trading halt point while ours are about the dynamics of volatility when the market approaches the circuit breaker. While there are many channels through which a market intervention can affect the overall price volatility, our predictions on volatility dynamics are distinct in the presence of circuit breakers and directly testable, which we further confirm in the data.

[6] In this spirit, our paper is closely related to Hong and Wang (2000), who study the effects of periodic market closures in the presence of asymmetric information. The liquidity effect caused by market closures that we find here is qualitatively similar to what they find. By modeling the stochastic nature of a circuit breaker, we are able to fully capture its impact on market dynamics, such as volatility and skewness.

[7] Ackert, Church, and Jayaraman (2001) study the impact of MWCBs using experiments. They find that circuit breakers do not impact prices significantly but do impact market participants' trading behavior substantially by accelerating trading when the market price approaches the circuit breaker.

demonstrates the power of an effective model in guiding our empirical analysis. It is also confirmed by our empirical results.

A related empirical literature studies the impact of conditional trading restrictions on individual securities including futures. For example, Bertero and Mayer (1990) and Roll (1988, 1989) study the effects of trading halts based on price limits imposed on individual stocks around the 1987 stock market crash and find different results.[8] Although the focus of our paper is on MWCBs, our results are broadly compatible with the empirical findings on the impact of trading halts for individual assets.

The rest of the paper is organized as follows. Section I describes our basic model. Section II provides the model solution. In Section III, we examine how a circuit breaker affects investor behavior and equilibrium price dynamics, and we derive testable predictions on a circuit breaker's unique impact on price and volume dynamics. Section IV considers circuit breakers' welfare implications. In Section V, we empirically examine the model's predictions for price and volume dynamics. Section VI discusses robustness and possible extensions. Section VII concludes. The Appendix outlines the key steps of the proofs. An Internet Appendix provides additional institutional details on circuit breakers, additional details for the proofs, additional results from the model, and the robustness of the empirical results.

I. The Model

In this section, we first present a simple model of circuit breakers as the basis of our analysis. We then provide additional discussion on the model's assumptions.

We consider a continuous-time endowment economy over a finite time interval $[0, T]$. Uncertainty is described by a one-dimensional standard Brownian motion Z, defined on a filtered complete probability space $(\Omega, F, \{F_t\}, \mathbb{P})$, where $\{F_t\}$ is the augmented filtration generated by Z.

Financial Market. There is a single share of an aggregate stock, which pays a terminal dividend of D_T at time T. The process for D is exogenous and publicly observable, given by

$$dD_t = \mu D_t dt + \sigma D_t dZ_t, \quad D_0 = 1, \tag{1}$$

[8] Chen (1993), Lauterbach and Ben-Zion (1993), Santoni and Liu (1993), Lee, Ready, and Seguin (1994), Kim and Rhee (1997), Corwin and Lipson (2000), Christie, Corwin, and Harris (2002), Jiang, McInish, and Upson (2009), and Gomber et al. (2012), among others, study the effects of trading halts and price limits on the market behavior of individual stocks. Chen et al. (2019) examine the impact of daily price limits on trading patterns and price dynamics in the Chinese stock market. Brennan (1986), Kuserk, Locke, and Sayers (1992), Berkman and Steenbeek (1998), Coursey and Dyl (1990), Ma, Rao, and Sears (1989a, 1989b), and Chen and Jeng (1996) study the effects of trading restrictions related to price fluctuations in futures markets.

where $t \in [0, T]$ and where μ and $\sigma > 0$ are the expected growth rate and volatility of D_t, respectively.[9] In addition to the stock, there is a riskless bond with zero net supply. Each unit of the bond yields a terminal payoff of one at T.

Both the stock and the bond are traded competitively in a financial market. Since there is no intermediate payoff/consumption, we use the riskless bond as the numeraire. Thus, the price of the bond is always one.[10] Let S_t denote the price of the stock at t (cum-dividend).

Agents. There are two agents, A and B, who are initially endowed with zero units of the bond and ω and $1-\omega$ shares of the stock, respectively, with $0 \leq \omega \leq 1$ determining the initial wealth distribution between the two agents.

Both agents can trade in the market. Let θ_t^i and ϕ_t^i denote the stock and bond holdings of agent i at t, $i = A, B$, respectively. We impose the usual restrictions on trading strategies to rule out arbitrage. Agent i's wealth is then given by $W_t^i = \phi_t^i + \theta_t^i S_t$, and

$$dW_t^i = \theta_t^i dS_t, \tag{2}$$

with W_T^i being agent i's terminal wealth.

We assume that the agents have preferences in the form of expected utility over their terminal wealth. They choose trading strategies to maximize their own expected utilities. For tractability, we further assume that both agents' utility functions take the logarithmic form

$$u^i(W_T^i) = \ln(W_T^i), \quad i = A, B. \tag{3}$$

The two agents have different beliefs about the terminal dividend D_T and "agree to disagree" (i.e., they do not learn from each other or from prices).[11] Agent A has the objective belief in the sense that her belief measure is consistent with $\mathbb{P}$, the physical probability measure. In particular, $\mu^A = \mu$. Agent B's belief measure, denoted by $\mathbb{P}^B$, on contrast, is different from but equivalent to $\mathbb{P}$.[12] In particular, he believes that the dividend growth rate at time t is

[9] For simplicity, throughout the paper we refer to D_t as "dividend" and S_t/D_t as the "price-dividend ratio"; even though the dividend will be paid only at T. More generally, D_t can be interpreted as the expectation of D_T at t.

[10] Since there is no intermediate consumption in the model, it is natural to use the bond price as the numeraire. This modeling choice is also motivated by the problem at hand, which is about trading halt during an otherwise continuous trading day. Given that a trading day is a rather short horizon, we would expect the bond price or the interest rate to stay mostly constant. In this case, using the bond price as the numeraire is a reasonable approximation. Under this numeraire, the bond price stays at one.

[11] The formulation here follows the earlier work of Detemple and Murthy (1994) and Zapatero (1998), among others.

[12] More precisely, $\mathbb{P}$ and $\mathbb{P}^B$ are equivalent when restricted to any σ-field $\mathcal{F}_T = \sigma(\{D_t\}_{0 \leq t \leq T})$. Two probability measures are equivalent if they agree on zero probability events. Agents' beliefs should be equivalent to prevent arbitrage opportunities under any agents' beliefs.

given by

$$\mu_t^B = \mu + \delta_t, \tag{4}$$

where δ follows Ornstein-Uhlenbeck process,

$$d\delta_t = -\kappa\left(\delta_t - \bar{\delta}\right)dt + \nu\, dZ_t, \tag{5}$$

with $\kappa \geq 0$ and $\nu \geq 0$. Equation (5) thus describes the dynamics of the gap between agent B's belief and the physical probability measure, which is the same as agent A's belief.

Notice that δ_t is driven by the same Brownian motion as the dividend. With $\nu > 0$, agent B becomes more optimistic (pessimistic) following positive (negative) shocks to the dividend, and the impact of these shocks on his belief decays exponentially at the rate κ. Thus, the parameter ν controls the sensitivity of B's conditional belief to realized dividend shocks, while κ determines the relative importance of shocks from the recent past versus the distant past. The average long-run disagreement between the two agents is $\bar{\delta}$. In the special case in which $\nu = 0$ and $\delta_0 = \bar{\delta}$, the disagreement between the two agents is constant over time. In another special case in which $\kappa = 0$, δ_t follows a random walk.

Given the two agents' beliefs, $\mathbb{P}$ and $\mathbb{P}^B$, let η be the Radon-Nikodym derivative of $\mathbb{P}^B$ with respect to $\mathbb{P}$. From Girsanov's theorem, we then have

$$\eta_t = \exp\left(\frac{1}{\sigma}\int_0^t \delta_s dZ_s - \frac{1}{2}\frac{1}{\sigma^2}\int_0^t \delta_s^2 ds\right). \tag{6}$$

Intuitively, agent B will be more pessimistic than A when $\delta_t < 0$. In that case, paths with high realized values for $\int_0^t \delta_s dZ_s$, which appear after a sequence of negative shocks to Z, will be assigned higher probabilities under $\mathbb{P}^B$ than under $\mathbb{P}$. Similarly, paths with positive shocks to Z will be assigned higher probabilities under $\mathbb{P}^B$ when $\delta_t > 0$.

A difference in beliefs is a simple way to introduce heterogeneity among agents, which generates trading. The heterogeneity in beliefs can also be interpreted as heterogeneity in utility, with state dependence. In particular, we have

$$\mathbb{E}^B[u(W_T^B)] = \mathbb{E}\,[\eta_T u(W_T^B)] = \mathbb{E}\,[\tilde{u}^B(W_T^B, \eta_T)], \quad \tilde{u}^B(W_T^B, \eta_T) \equiv \eta_T \ln(W_T^B). \tag{7}$$

Thus, we can reinterpret the two heterogeneous agents as having the same objective belief but different utility functions. While agent A has the simple logarithmic utility function over her terminal wealth as given in (3), agent B has a state-dependent utility function, $\tilde{u}^B(\cdot)$, as given in (7). We discuss the economic meaning behind such a state-dependent utility later in this section. Although these two different interpretations give rise to the same market behavior, they can lead to different welfare and policy implications for circuit breakers, as we discuss in Section IV.

Circuit Breaker. To capture the essence of an MWCB, we assume that the market will be closed whenever the stock price S_t hits a threshold $(1-\alpha)S_0$, where S_0 is the endogenous initial price of the stock and $\alpha \in [0, 1]$ is a constant parameter determining the floor of downside price fluctuations in the interval $[0, T]$. For simplicity, we assume that the market will remain closed until T after the circuit breaker is triggered.

In practice, the circuit breaker threshold is typically based on the closing price of the previous trading day instead of the opening price of the current trading session. However, the distinction between today's opening price and the prior day's closing price is not crucial for our analysis. The circuit breaker not only depends on but also endogenously affects the initial stock price, just as it does for the prior day's closing price in practice. We provide more details on the actual operation of circuit breakers toward the end of this section.

Market Equilibrium. We now define the market equilibrium in the presence of a circuit breaker. No-arbitrage requires that the stock price process is continuous. Let τ denote the time when the circuit breaker is triggered, that is, when S_t first hits $(1-\alpha)S_0$. It follows that τ is given by

$$\tau = \inf\{t \geq 0 : S_t \leq (1-\alpha)S_0\}, \quad \alpha \in [0, 1]. \tag{8}$$

Let $\tau \wedge T$ denote $\min\{\tau, T\}$. We then have the following definition.

DEFINITION 1 (Equilibrium with Circuit Breaker): In the presence of a circuit breaker, the market equilibrium is defined by F_t-stopping time τ, trading strategies $\{\theta_t^i, \phi_t^i\}$, $i = A, B$, and continuous stock price process S_t, all defined on $[0, \tau \wedge T]$, such that:

- (i) Taking stock price process S_t as given, the two agents' trading strategies maximize their expected utilities under their respective beliefs and budget constraints.
- (ii) For all $t \in [0, T]$, both the stock and bond markets clear:

$$\theta_t^A + \theta_t^B = 1, \quad \phi_t^A + \phi_t^B = 0. \tag{9}$$

- (iii) The stopping time τ is consistent with the circuit breaker rule in (8).

Discussion. The model described above takes a parsimonious form, mainly for clarity and tractability. Here, we elaborate a bit more on the model's specification.

In the model, heterogeneity between agents, which gives rise to their trading needs, takes the form of different beliefs. However, as we discussed above, this specification can have other interpretations such as different utility functions. In this case, agent B's utility function becomes state-dependent and takes the form in (7). If η_t is positively (negatively) related to dividend shocks, which is the case in (5) with a positive (negative) ν, agent B's utility increases (decreases) with aggregate dividend/consumption, holding constant his wealth/consumption. The former case (positive relationship between η and D) is reminiscent of behavioral biases such as "representativeness," while the latter case (negative relationship between η and D) is reminiscent of behavior such as "catching up with the Joneses" (see, e.g., Abel (1990)). We adopt the interpretation of different beliefs in most of the paper, for expositional convenience, but return to both interpretations in the welfare analysis in Section IV.

We note that one can generate trading by introducing heterogeneity of other forms, such as different endowment shocks (see, e.g., Wang (1994)). We choose a formulation using different beliefs/preferences as it is not only simple to specify and interpret, but also helpful in our welfare analysis.

In the model, we consider the possibility of only one circuit breaker such that, once triggered, the market is closed until T, the end of the trading session.[13] In reality, there can be multiple circuit breakers during a trading session, with different triggers. For example, currently in the United States, a market-wide trading halt can be triggered by price drops of 7% (Level 1), 13% (Level 2), and 20% (Level 3) from the prior day's closing price of the S&P 500 Index (SPX). In addition, for Level 1 and 2 circuit breakers, the market closes for only 15 minutes and then reopens until the end of the day, unless a Level 3 halt is triggered.[14] Although our analysis focuses on the single circuit-breaker case, for the sake of clarity, our setup can be extended to allow for multiple circuit breakers and market reopenings.

II. Solution to Equilibrium

We now present the solution to the market equilibrium. We first consider the case without a circuit breaker; we then present the case with the circuit breaker. To distinguish the two cases, we use the symbol "$\widehat{\ }$" to denote variables in the no-circuit-breaker case.

A. The Benchmark Case: Without Circuit Breaker

Without any circuit breaker, the market is dynamically complete. The equilibrium allocation in this case can be characterized as the solution to the social planner's problem given by

$$\max_{\widehat{W}_T^A,\,\widehat{W}_T^B} \mathbb{E}_0\Big[\lambda \ln\left(\widehat{W}_T^A\right) + (1-\lambda)\,\eta_T \ln\left(\widehat{W}_T^B\right)\Big] \tag{10}$$

$$\text{s.t.} \quad \widehat{W}_T^A + \widehat{W}_T^B = D_T.$$

[13] The fact that the price of the stock reverts back to the fundamental value D_T at T resembles the rationale of a circuit breaker to "restore order" in the market. In this sense, instead of viewing T as the end of the economy, it can be viewed as the reopening of the market.

[14] More details on the history of the MWCBs in the United States and their current form are provided in Section I of the Internet Appendix.

From the agents' first-order conditions and budget constraints, $\lambda = \omega$. The following proposition summarizes the market equilibrium including the stock price and individual agents' portfolio holdings.

PROPOSITION 1: *Without the circuit breaker, the equilibrium stock price is*

$$\widehat{S}_t = \frac{\omega + (1-\omega)\,\eta_t}{\omega + (1-\omega)\,\eta_t\, e^{a(t,T)+b(t,T)\delta_t}} e^{(\mu-\sigma^2)(T-t)} D_t, \quad t \in [0, T], \tag{11}$$

where

$$a(t,T) = \left[\frac{\kappa\bar{\delta} - \sigma\nu}{\nu/\sigma - \kappa} + \frac{1}{2}\frac{\nu^2}{(\nu/\sigma-\kappa)^2}\right](T-t) - \frac{1}{4}\frac{\nu^2}{(\nu/\sigma-\kappa)^3}\left[1 - e^{2(\nu/\sigma-\kappa)(T-t)}\right]$$
$$+ \left[\frac{\kappa\bar{\delta} - \sigma\nu}{(\nu/\sigma-\kappa)^2} + \frac{\nu^2}{(\nu/\sigma-\kappa)^3}\right]\left[1 - e^{(\nu/\sigma-\kappa)(T-t)}\right], \tag{12a}$$

$$b(t,T) = \frac{1}{\nu/\sigma - \kappa}\left[1 - e^{(\nu/\sigma-\kappa)(T-t)}\right]. \tag{12b}$$

The two agents' shares of total wealth at time t are

$$\widehat{\omega}_t^A = \frac{\omega}{\omega + (1-\omega)\eta_t}, \quad \widehat{\omega}_t^B = 1 - \widehat{\omega}_t^A, \tag{13}$$

and their stock and bond holdings are

$$\widehat{\theta}_t^A = \widehat{\omega}_t^A \left(1 - \widehat{\omega}_t^B \frac{\delta_t}{\sigma\sigma_{\widehat{S},t}}\right), \quad \widehat{\theta}_t^B = 1 - \widehat{\theta}_t^A; \quad \widehat{\phi}_t^A = \widehat{\omega}_t^A\, \widehat{\omega}_t^B \frac{\delta_t}{\sigma\sigma_{\widehat{S},t}} \widehat{S}_t, \quad \widehat{\phi}_t^B = -\widehat{\phi}_t^A. \tag{14}$$

The conditional volatility of $\widehat{S}_t$, $\widehat{\sigma}_{S,t}$, can be computed in closed form from equation (11), which is given in the Appendix.

PROOF: See Appendix Section A. □

As (14) shows, several forces affect the agents' portfolio positions. First, all else equal, agent A owns more shares of the stock when B has more pessimistic beliefs (smaller $\delta_t < 0$). This effect becomes weaker when the volatility of stock return $\widehat{\sigma}_{S,t}$ is high. Second, the wealth distribution, as given in (13), also affects the agents' portfolio holdings, as the richer agent tends to hold more shares of the stock.

We can gain more intuition on the equilibrium stock price given in (11). Let $\widehat{S}_t^A$ and $\widehat{S}_t^B$ be the stock prices in the two single-agent economies, with only agent A and only agent B, respectively, as the representative agent, which are given by

$$\widehat{S}_t^A = \frac{1}{\mathbb{E}_t[D_T^{-1}]} = e^{(\mu-\sigma^2)(T-t)} D_t, \quad \widehat{S}_t^B = \frac{1}{\mathbb{E}_t^B[D_T^{-1}]} = e^{(\mu-\sigma^2)(T-t) - a(t,T) - b(t,T)\delta_t} D_t. \tag{15}$$

We can then rewrite the equilibrium stock price as

$$\widehat{S}_t = \left(\frac{\omega}{\omega + (1-\omega)\eta_t}\mathbb{E}_t[D_T^{-1}] + \frac{(1-\omega)\eta_t}{\omega + (1-\omega)\eta_t}\mathbb{E}_t^B[D_T^{-1}]\right)^{-1} = \left(\widehat{\omega}_t^A \frac{1}{\widehat{S}_t^A} + \widehat{\omega}_t^B \frac{1}{\widehat{S}_t^B}\right)^{-1}. \tag{16}$$

That is, the stock price with both agents is simply a harmonic average of the stock prices in the two single-agent economies, with the weights $\widehat{\omega}_t^A$ and $\widehat{\omega}_t^B$ being the agents' shares of total wealth. Therefore, controlling for the wealth distribution, the equilibrium stock price is higher (lower) when agent B's belief is more (less) optimistic, that is, when δ_t is larger (smaller).

We consider two special cases to gain more intuition about the model. One special case is when the disagreement between the two agents is equal to zero, that is, $\delta_t = 0$ for all $t \in [0,\ T]$. The stock price then becomes

$$\widehat{S}_t = \widehat{S}_t^A = e^{(\mu-\sigma^2)(T-t)} D_t. \tag{17}$$

This is a version of the Gordon growth formula, with σ^2 being the risk premium for the stock. The instantaneous volatility of stock returns becomes the same as the volatility of dividend growth, $\widehat{\sigma}_{S,t} = \sigma$. The shares of the stock held by the two agents are constant and equal to their initial endowments, $\widehat{\theta}_t^A = \omega$ and $\widehat{\theta}_t^B = 1 - \omega$.

Another special case is when the disagreement is constant over time ($\delta_t = \delta$ for all t). The results for this case are obtained by setting $\nu = 0$ and $\delta_0 = \bar{\delta} = \delta$ in Proposition 1. Equation (11) then simplifies to

$$\widehat{S}_t = \frac{\omega + (1-\omega)\,\eta_t}{\omega + (1-\omega)\,\eta_t\, e^{-\delta(T-t)}} e^{(\mu-\sigma^2)(T-t)} D_t. \tag{18}$$

As expected, $\widehat{S}_t$ increases with δ, which reflects agent B's optimism on dividend growth. Its volatility now becomes stochastic.

B. With Circuit Breaker

We now solve for the market equilibrium in the presence of the circuit breaker. In this case, the market is no longer complete over the entire time span $[0,\ T]$, especially if the circuit breaker is triggered. However, a crucial feature of the model is that the market remains dynamically complete until the circuit breaker is triggered, at which point, the market enters a closing equilibrium in which market participants trade one last time before the market closes and then hold their positions until T. We first characterize the equilibrium at and before market closure under an arbitrary trigger rule for the circuit breaker. Using these characterizations, we then construct a market equilibrium under the circuit breaker trigger rule of interest, that is (8), thereby proving the existence of market equilibrium. Finally, we establish sufficient conditions for the uniqueness of market equilibrium.

Equilibrium Characterization. Consider an arbitrary trigger rule for the circuit breaker, which is not necessarily the threshold hitting rule in (8). Suppose that the circuit breaker is triggered before the end of the trading session, that is, $\tau < T$, where τ is the trigger time, which could depend on the full history of the economy. We start by characterizing the closing equilibrium at τ. Since the two agents behave competitively, they take the stock price S_τ as given and choose their stock and bond holdings to maximize their expected utility over terminal wealth, subject to the budget constraint:

$$V^i(W^i_\tau, \tau) = \max_{\theta^i_\tau, \phi^i_\tau} \mathbb{E}^i_\tau\left[\ln(\theta^i_\tau D_T + \phi^i_\tau)\right], \quad i = A, B \tag{19}$$
$$\text{s.t.} \quad \theta^i_\tau S_\tau + \phi^i_\tau = W^i_\tau.$$

Market clearing at time τ requires

$$\theta^A_\tau + \theta^B_\tau = 1, \quad \phi^A_\tau + \phi^B_\tau = 0, \tag{20}$$

which determines the closing price S_τ as a function of W^i_τ, $i = A, B$, and the other state variables at τ.

For any $\tau < T$, the Inada condition implies that terminal wealth for both agents needs to remain nonnegative, which implies that $\theta^i_\tau \geq 0$ and $\phi^i_\tau \geq 0$. That is, neither agent will take short or levered positions in the stock at τ. This is a direct result of the inability to rebalance one's portfolio after market closure, which is an extreme version of illiquidity.

Next, before the trigger time, both agents trade continuously to maximize their expected indirect utilities at $\tau \wedge T$, that is, $V^i(W^i_{\tau\wedge T}, \tau \wedge T)$. Given that the market is dynamically complete before $\tau \wedge T$, the equilibrium is equivalent to the following planner problem:

$$\max_{W^A_{\tau\wedge T}, W^B_{\tau\wedge T}} \mathbb{E}_0\left[\lambda V^A(W^A_{\tau\wedge T}, \tau \wedge T) + (1-\lambda)\,\eta_{\tau\wedge T} V^B(W^B_{\tau\wedge T}, \tau \wedge T)\right] \tag{21}$$

subject to the resource constraint

$$W^A_{\tau\wedge T} + W^B_{\tau\wedge T} = S_{\tau\wedge T} = \begin{cases} D_T, & \text{if} \quad \tau \geq T, \\ S_\tau, & \text{if} \quad \tau < T. \end{cases} \tag{22}$$

The resulting wealth allocation $W^i_{\tau\wedge T}$ is used in the closing equilibrium (19). Meanwhile, it also helps to specify the market-clearing price before τ like the complete market counterpart.

The following result characterizes the equilibrium price before and at the circuit breaker trigger time.

PROPOSITION 2: *Let*

$$\underline{\delta}(t) = -\frac{a(t,T)}{b(t,T)}, \tag{23}$$

where $a(t,T)$ and $b(t,T)$ are given by (12) in Proposition 1. Suppose that the market closes at time $\tau < T$. Then:

(i) At τ both agents hold all of their wealth in the stock, $\theta^i_\tau = \frac{W^i_\tau}{S_\tau}$, and nothing in the bond, $\phi^i_\tau = 0$, $i = A, B$. The market-clearing price is

$$S_\tau = \min\left\{\widehat{S}^A_\tau, \widehat{S}^B_\tau\right\} = \begin{cases} e^{(\mu-\sigma^2)(T-\tau)}D_\tau, & \text{if} \quad \delta_\tau > \underline{\delta}(\tau), \\ e^{(\mu-\sigma^2)(T-\tau)-a(\tau,T)-b(\tau,T)\delta_\tau}D_\tau, & \text{if} \quad \delta_\tau \le \underline{\delta}(\tau), \end{cases} \tag{24}$$

where $\widehat{S}^i_\tau$ denotes the stock price in a single-agent economy populated by agent i, as given in (15).

(ii) For $t \le \tau \wedge T$, the market-clearing price is

$$S_t = \left(\omega^A_t \mathbb{E}_t\left[S^{-1}_{\tau\wedge T}\right] + \omega^B_t \mathbb{E}^B_t\left[S^{-1}_{\tau\wedge T}\right]\right)^{-1}, \tag{25}$$

where ω^i_t is the share of total wealth owned by agent i before market closure and is identical to $\widehat{\omega}^i_t$ in (13). Moreover, the market-clearing price S is continuous on $[0, \tau\wedge T]$.

PROOF: See Appendix Section B. □

Because neither agent takes levered or short positions during market closure, with zero net bond supply, there is no lending or borrowing over this period. The two agents therefore invest all of their respective wealth in the stock at the market closure. This implies that at τ, the relatively optimistic investor faces the leverage constraint, while the relatively pessimistic investor becomes the marginal investor. Consequently, the stock price at τ, S_τ, cannot be lower than the pessimistic investor's evaluation. Otherwise, continuity requires that the stock price would also be lower than the pessimistic investor's evaluation just before τ, in which case both investors would want to take on leverage to invest in the undervalued stock, which contradicts with the unconstrained equilibrium before τ. Thus, the market-clearing price at the trigger time must be the pessimistic investor's evaluation.

The result that S_τ depends only on the belief of the relatively pessimistic agent is qualitatively different from the complete market case, where the stock price is a wealth-weighted average of the prices under the two agents' beliefs. This is a crucial result: the lower stock valuation upon market closure affects both the level and dynamics of the stock price before market closure, which we analyze in Section III. Notice that having the lower expectation of the growth rate at the current instant is not sufficient to make the agent marginal. One also needs to take into account agents' future beliefs and the risk premium associated with future fluctuations in the beliefs, which are summarized by $\underset{\sim}{\delta}(t)$.

Before the circuit breaker trigger time, the market-clearing price in (25) is reminiscent of its complete market counterpart (16). Unlike in the case of complete markets, the expectations in (25) are no longer the inverse of the stock prices from the respective representative-agent economies.

Equilibrium Existence and Uniqueness. Proposition 2 characterizes the market equilibrium under an arbitrary trigger rule for the circuit breaker. This allows us to obtain the equilibrium under the specific trigger rule given in (8), which we achieve in two steps. First, we consider the equilibrium under a trigger rule based on a given constant lower bound for the equilibrium stock price, denoted by $\underset{\sim}{S}$. From Proposition 2, this fully specifies the stock price on $[0,\ \tau \wedge T]$, including at $t = 0$. Next, we require that the triggering price satisfy $\underset{\sim}{S} = (1-\alpha)S_0$ and show that an equilibrium exists. This then gives an equilibrium under circuit breaker trigger rule (8).

Consider a trigger rule defined by a constant lower bound for the equilibrium stock price $\underset{\sim}{S}$, that is,

$$\tau = \inf\{t \geq 0 : S_t \leq \underset{\sim}{S}\}. \tag{26}$$

In this case, (24) implies that

$$D_\tau = \underset{\sim}{D}(\tau, \delta_\tau; \underset{\sim}{S}) \equiv \begin{cases} \underset{\sim}{S} e^{-(\mu-\sigma^2)(T-\tau)}, & \text{if} \quad \delta_\tau > \underset{\sim}{\delta}(\tau), \\ \underset{\sim}{S} e^{-(\mu-\sigma^2)(T-\tau)+a(\tau,T)+b(\tau,T)\delta_\tau}, & \text{if} \quad \delta_\tau \leq \underset{\sim}{\delta}(\tau). \end{cases} \tag{27}$$

The above property of the dividend process at τ for triggering rule (26) allows us to construct an equilibrium under trigger rule (8) and hence establish its existence. To this end, we separate our construction into three steps:

(i) For a given $\underset{\sim}{S}$, define a stopping time:

$$\tau' = \inf\{t \geq 0 : D_t \leq \underset{\sim}{D}(t, \delta_t; \underset{\sim}{S})\}, \tag{28}$$

where $\underset{\sim}{D}(t, \delta_t; \underset{\sim}{S})$ is given in (27).

(ii) Define

$$S'_t = \left(\omega^A_t \, \mathbb{E}_t\left[(\widehat{S}^{min}_{\tau'\wedge T})^{-1}\right] + \omega^B_t \, \mathbb{E}^B_t\left[(\widehat{S}^{min}_{\tau'\wedge T})^{-1}\right]\right)^{-1}, \quad t \leq \tau', \tag{29}$$

where $\widehat{S}^{min}_t = \min\{\widehat{S}^A_t, \widehat{S}^B_t\}$.

(iii) Evaluating S'_t at $t = 0$, we have the map F: $F(\underset{\sim}{S}/(1-\alpha)) = S'_0$. We then look for a fixed point for the map F: $F(S_0) = S_0$.

In the construction described above, the definition of τ' is motivated by (27) and is exogenously specified. It is given by a trigger rule specified by the dividend process with parameter $\underset{\sim}{S}$. We prove in the next result that τ' is exactly the circuit breaker trigger time τ in (8) for the equilibrium constructed. The process S' in (29) is motivated by (25) and parameterized by $\underline{S}$ in the construction. Once we identify a fixed point S_0 for the map F, S' becomes the market-clearing price starting from the initial price S_0 and τ' is the circuit breaker trigger time in (8), and hence a corresponding equilibrium is obtained. The following result formally presents the existence of equilibrium.

PROPOSITION 3: *Under circuit breaker rule (8), there exists a market equilibrium. In particular, τ' defined in (28) is the circuit breaker trigger time, that is, $\tau' = \tau$ in (8).*

PROOF: See Appendix Section C. □

For the uniqueness of equilibrium, we separately consider constant and stochastic δ. In the case of stochastic δ, given by (5), we are interested in the case in which $\bar{\delta} = 0$, that is, there is no persistent disagreement between the two agents. The case with persistent disagreement is captured by the constant-disagreement case. In the former case, we also set the initial disagreement to zero for simplicity, that is, $\delta_0 = 0$. The following result identifies the marginal agent at the circuit breaker trigger time in the case of stochastic δ.

LEMMA 1: *Suppose that $\bar{\delta} = \delta_0 = 0$ in (5). Then agent B is the marginal agent when the circuit breaker is triggered at τ for $0 \le \tau < T$.*

PROOF: See the Internet Appendix Section II.C. □

It is intuitive that agent B would be the marginal agent at τ. Since the circuit breaker will be triggered when the stock price has fallen sufficiently from the initial value, it will be on paths with more negative dividend shocks. Thus, starting with $\delta_0 = 0$, these shocks will make δ_t fall below zero at τ and in turn make agent B more pessimistic relative to agent A. Lemma 1 states that this is indeed the case for general parameters.

With the results above, we have the following sufficient conditions for the uniqueness of equilibrium.

PROPOSITION 4: *The market equilibrium is unique when*

(i) δ *is constant, or*
(ii) $\bar{\delta} = \delta_0 = 0$ *and* $\omega < 1 - \alpha$.

PROOF: See Appendix Section D. □

In the case of stochastic δ, $\omega < 1 - \alpha$ implies that the weight of agent B's initial wealth is at least α. This is a technical condition and is not necessary. Our extensive numerical analysis shows that the equilibrium remains unique even when this condition is violated.

III. Impact of Circuit Breaker on Market Behavior

We now examine how the circuit breaker affects market dynamics, including the behavior of the stock price and agents' stock holdings. We first consider the case of constant disagreement. This case allows us to obtain analytical results about how the circuit breaker impacts price dynamics and to illustrate the underlying mechanism. We next consider the general case with time-varying disagreements, which allows us to gain more general and quantitative results on the impact of the circuit breaker. Throughout this section, we consider $\alpha \in (0, 1]$ to allow for more trading and price evolution since the case of $\alpha = 0$ corresponds to trading only at $t = 0$.

A. Constant Disagreement

In the case of constant disagreement, we have $\delta_t \equiv \delta$. To fix ideas, we work with the case of $\delta < 0$, under which B is the more pessimistic agent. The results hold for the case of $\delta > 0$, under which A becomes the more pessimistic agent, by relabeling.

Price Level, Volatility, and the Magnet Effect. By comparing stock price and volatility in the equilibria with and without the circuit breaker, we can analyze the impact of the circuit breaker.

Price Level. We first consider the equilibrium price level. Because agent B is relatively more pessimistic, Proposition 2 implies that the stock price at τ, when $\tau < T$ and the circuit breaker is triggered, equals agent B's evaluation. That is, $S_{\tau\wedge T} = \widehat{S}^B_{\tau\wedge T} = (\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}])^{-1}$. The stock price prior to τ in (25) can then be written as

$$S_t = \left(\omega_t^A\,\mathbb{E}_t\big[\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}]\big] + (1-\omega_t^A)\,\mathbb{E}^B_t[D_T^{-1}]\right)^{-1}, \tag{30}$$

where we use the property of conditional expectations: $\mathbb{E}_t[S^{-1}_{\tau\wedge T}] = \mathbb{E}_t[\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}]]$ and $\mathbb{E}^B_t[S^{-1}_{\tau\wedge T}] = \mathbb{E}^B_t[\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}]] = \mathbb{E}^B_t[D_T^{-1}]$. Here, the stock price is still the harmonic average of their respective prices in the two single-agent economies. However, the terminal value is no longer the dividend at T, as in the case without the circuit breaker, but rather the price at trigger time $\tau\wedge T$, $S_{\tau\wedge T}$, which is determined by the valuation of agent B, the marginal investor at that time. Equation (30) helps us obtain the following comparison between stock prices with or without circuit breakers.[15]

PROPOSITION 5: *The stock price is always lower with the circuit breaker than without, that is, for all $t \in [0, T]$ and $D_t \geq \underline{D}_t = \underline{D}(t, \delta)$,*

$$S_t < \widehat{S}_t. \tag{31}$$

PROOF: See Appendix Section E. □

[15] Our proof in the Appendix further shows that Proposition 5 also holds for stochastic δ with $\bar{\delta} = \delta_0 = 0$.

This proposition simply states that, given the fundamental (D_t), the equilibrium stock price is always lower with the circuit breaker than without. The reason is that, with or without the circuit breaker, the inverse stock price is in general a weighted average between optimistic and pessimistic evaluations of the two agents (see (16)). However, at the circuit breaker trigger time, the stock price is determined by the pessimistic evaluation, which leads to a lower price than that without the circuit breaker, even before the trigger time.

Price Dynamics and Volatility. We next examine the impact of the circuit breaker on price dynamics and volatility, especially in its neighborhood. The following proposition summarizes the main results.

PROPOSITION 6: *Comparing the stock price with and without the circuit breaker, we have:*

(i) The price without the circuit breaker, $\widehat{S}_t$, and the price with circuit breaker, S_t, satisfy

$$\lim_{D_t \downarrow \underline{D}_t} \widehat{S}_t > \lim_{D_t \downarrow \underline{D}_t} S_t = (1-\alpha) S_0, \quad \textit{for all } t < T.$$

(ii) With the circuit breaker and $\delta \leq -\sigma^2$, the stock price volatility σ_S satisfies

$$\lim_{D_t \downarrow \underline{D}_t} \sigma_{S,t} > \lim_{D_t \downarrow \underline{D}_t} \sigma_{\widehat{S},t} > \sigma, \quad \textit{for all } t < T.$$

(iii) In addition, when ω is close to one and $\delta < -\sigma^2$, there exists a neighborhood around $\underline{D}_t$ such that in this neighborhood, as $D_t \downarrow \underline{D}_t$, $\widehat{S}_t/S_t$ increases and converges to a constant greater than one, and $\sigma_t^S - \sigma_t^{\widehat{S}}$ increases and converges to a constant greater than zero.

PROOF: See Appendix Section E. □

The first result simply states that when the fundamental approaches the threshold level $\underline{D}_t = \underline{D}(t, \delta)$, the probability of triggering the circuit breaker approaches one so that the stock price with the circuit breaker converges to the trigger level $(1-\alpha)S_0$, while the price without the circuit breaker approaches a level strictly higher than $(1-\alpha)S_0$ as Proposition 5 states.

The second result shows that when the market approaches the circuit breaker threshold, price volatility is higher than that without the circuit breaker, which is higher than the fundamental volatility. The intuition is as follows. Following negative fundamental shocks, agent B, being the more pessimistic agent, exerts more influence on the stock price. This is true even without the circuit breaker because the more pessimistic agent will gain wealth share following negative fundamental shocks. However, since the presence of a circuit breaker reduces the willingness of the optimistic agent A to hold stocks, agent B becomes even more important (more so than implied by his wealth share) in driving prices and eventually becomes the sole marginal investor upon market closure. As a result, any negative shock to the stock price gets amplified endogenously as it adds to agent B's pricing influence. This is the reason for the higher price volatility as the market approaches the circuit breaker threshold.

We further illustrate the mechanism driving the price dynamics around the circuit breaker by considering the case when ω is close to one (but not equal to one). In this case, agent A dominates the market in terms of wealth share. Nonetheless, B, the pessimistic agent, even though infinitesimal in wealth share, is still the marginal investor at the circuit breaker triggering point. Without the circuit breaker, the equilibrium stock price is close to

$$\widehat{S}_t = \left(\mathbb{E}_t[D_T^{-1}]\right)^{-1},$$

which is simply the valuation of agent A, who dominates the market in wealth share based on her view of the fundamental, which is D_T. With the circuit breaker, however, the equilibrium stock price becomes

$$S_t = \left(\mathbb{E}_t\left[\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}]\right]\right)^{-1}.$$

Now agent A evaluates the stock based on its future valuation at the circuit breaker trigger time, which is determined by agent B. Part (iii) of the proposition shows that in the neighborhood of $\underline{D}_t$, as the stock price approaches the circuit breaker, the ratio $\widehat{S}_t/S_t$ increases to a limit greater than one and the gap between σ^S and $\sigma^{\widehat{S}}$ widens to a limit greater than zero.

Magnet Effect. The magnet effect is a popular term used by practitioners to refer to changes in price dynamics that accelerate the process of reaching the trading halt trigger as prices move closer to it. While there is no established definition of this effect, we provide a more formal notion here.

Let $p_h(S_t) \equiv \mathbb{P}(\tau \leq t + h|\mathcal{F}_t)$ denote the conditional probability that the stock price, currently at S_t, will hit the circuit breaker threshold $(1-\alpha)S_0$ within a given time interval h, which we refer to as the conditional hitting probability, and let $\widehat{p}_h(S_t)$ denote the conditional hitting probability in the absence of the circuit breaker. The difference between these two hitting probabilities, given by $\Delta p_h(S_t) \equiv p_h(S_t) - \widehat{p}_h(S_t)$, then gives the effect of the circuit breaker on the conditional hitting probability. We define the magnet effect as the scenario in which $\Delta p_h(S_t)$ becomes increasingly positive when S_t approaches the threshold.

Some studies of circuit breakers associate the magnet effect with the unconditional probability of reaching the threshold. We find this association unsatisfactory. In general, the introduction of a circuit breaker shifts the overall equilibrium, including the price distribution, conditional or unconditional. If, for example, the unconditional volatility of price distribution increases, the probability of it reaching any threshold also increases. The magnet effect, however, really corresponds to the dynamic behavior of the stock price, especially as it approaches the circuit breaker. The conditional hitting probability defined above is one way to capture the effect of the circuit breaker on price dynamics near its vicinity. In other words, by its very nature, the magnet effect is a dynamic effect, not a static one.

According to our version of the magnet effect, the very presence of a circuit breaker increases the probability of the stock price hitting the threshold as the stock price moves closer to the threshold. The next proposition compares $p_h(S_t)$ and $\hat{p}_h(S_t)$ in our model. To ensure a nontrivial comparison, we reduce the horizon h as S_t approaches $\underline{S}$.[16]

PROPOSITION 7: *Suppose $S_t > \underline{S}$ and $h = (S_t - \underline{S})^2$. There exists a neighborhood around $\underline{S}$ such that when S_t is in this neighborhood:*

(i) *$p_h(S_t)$ increases as $S_t \downarrow \underline{S}$.*
(ii) *For $\delta < -\sigma^2$, $p_h(S_t) > \hat{p}_h(S_t)$. Moreover, if ω is also close to one, then as $S_t \downarrow \underline{S}$, $p_h(S_t) - \hat{p}_h(S_t)$ increases and converges to a constant greater than zero.*

PROOF: See Appendix Section E. □

Here, the magnet effect is driven mainly by the significant increase in conditional return volatility in the presence of a circuit breaker. As the stock price becomes increasingly more volatile when approaching the circuit breaker, the likelihood of hitting the circuit breaker increases as well.

Quantitative Analysis of Market Behavior. In addition to the analytical results presented above about the level of the stock price and its dynamics near the circuit breaker, we can explore quantitatively the effect of the circuit breaker on price and trading behavior over the whole state space. Given that the market behavior in this case is qualitatively similar to that in the general case with time-varying disagreement, we next discuss it in the context of the general case. The results in the special case of constant disagreement are provided in the Internet Appendix (Section III) for comparison.

B. Time-Varying Disagreement

We now turn to the general case with time-varying disagreement, which better captures the possibility of large intraday market swings.[17] To fix ideas, we focus on the case in which the difference in beliefs δ_t follows a random walk.

[16] If h is fixed, when S_t approaches $\underline{S}$, we have $\lim_{S_t \downarrow \underline{S}} p_h(S_t) = \lim_{S_t \downarrow \underline{S}} \hat{p}_h(S_t) = 1$, which gives the trivial result: with or without the circuit breaker, the price will hit the threshold surely.

[17] An important rationale for policymakers to introduce a circuit breaker is to curb disorderly trading, in particular "panic selling," under extreme market conditions. This is what motivates our modeling of the subjective belief deviation δ_t as time-varying, which is initially zero and becomes significant only when the market moves substantially within a day. Under such an interpretation, a paternalistic planner/regulator may view the trading motive of agent B as speculation driven by irrational beliefs, and see potential benefits from introducing a circuit breaker to curb such trading activities (see Section IV for more discussion on this). Moreover, while it might be difficult for the

We do so by setting $\kappa = 0$, $\nu = \sigma$, and $\delta_0 = \bar{\delta} = 0$. Thus, there is neither initial nor long-term bias in agent B's belief. The more general case with $\kappa > 0$ yields qualitatively similar results.

Under this specification, agent B extrapolates his belief about future dividend growth from realized dividends. In particular, $\delta_t = \sigma Z_t$. As a result, he becomes overly optimistic following large positive dividend shocks and overly pessimistic following large negative dividend shocks. More importantly, the circuit breaker is approached after substantial drops in D_t (from initial value one) or Z_t (from initial value zero), which corresponds to substantially negative δ_t. As a result, in this case agent B is always the more pessimistic agent near the circuit breaker.[18]

For calibration, we normalize $T = 1$ to denote one trading day. We set the expected value of dividend growth to $\mu = 10\%/250 = 0.04\%$ (implying an annual dividend growth rate of 10%) and its (daily) volatility to $\sigma = 3\%$. The downside circuit breaker threshold is set at $\alpha = 5\%$. For the initial wealth distribution, we assume that agent A (with objective belief) owns 90% of total wealth ($\omega = 0.9$) at $t = 0$.

Given the two agents' beliefs, we now examine how the circuit breaker changes market behavior. In particular, we focus on the price-dividend ratio S_t/D_t, conditional return volatility $\sigma_{S,t}$, expected return under agent A's belief, which is also the objective probability, $\mu^A_{S,t}$, and agent A's stock holding θ^A_t. Agent B's stock holding is given by $\theta^B_t = 1 - \theta^A_t$. Since the impact of the circuit breaker changes throughout the day, we set the time to $t = 0.1$ to fix ideas. We discuss time variation in the circuit breaker's impact in the Internet Appendix (Section IV).

Figure 1 plots these four quantities, S_t/D_t, $\sigma_{S,t}$, $\mu^A_{S,t}$, and θ^A_t, as a function of the fundamental D_t at $t = 0.1$. Unlike the constant-disagreement case, in general, dividend D_t and time of the day t are no longer sufficient to fully determine the state of the economy. Thus, we plot the average values of the variables conditional on t and D_t here.[19]

Price-Dividend Ratio. We first consider the circuit breaker's effect on the price-dividend ratio, shown in Panel A of Figure 1. Since agent A's belief about the dividend growth rate is constant over time, the price-dividend ratio under her belief is constant over different values of D_t, shown by the horizontal gray dashed line ($\widehat{S}^A_t/D_t = 1$). Due to the variation in δ_t, which is perfectly correlated with D_t, the price-dividend ratio under agent B's belief now increases with D_t, shown by the upward-sloping gray dashed line ($\widehat{S}^B_t/D_t$).

planner to discern relatively mild deviations from objective probabilities under normal market conditions Blume et al. (2018), it is more reasonable to assume that the planner can identify irrational panics.

[18] Our earlier result in Lemma 1 shows that this statement continues to hold in the general case with $\kappa > 0$.

[19] In the case considered here, δ_t follows a random walk. The additional state variable besides t and D_t is the Radon-Nikodym derivative η_t given in (6). For simplicity, we plot the variables of interest conditional on t and D_t, averaging over the conditional distribution for η_t.

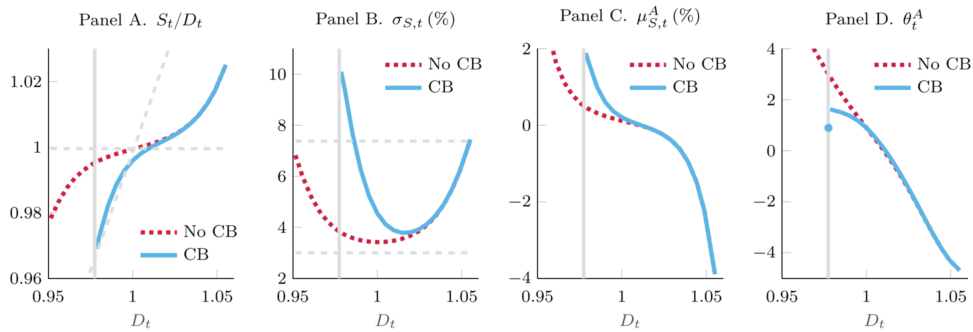

Figure 1. Price-dividend ratio, return volatility, instantaneous expected return, and agent A's portfolio. In the case of time-varying disagreement case, we plot all quantities as functions of dividend (D_t) at $t = 0.1$. In each panel, the gray vertical lines denote the circuit breaker threshold $\underline{D}_t$, and the gray dashed lines correspond to the two cases with only one of the agents present in the market. The variables of interest are conditional on D_t. The parameters are $T = 1$, $\mu = 0.04\%$, $\sigma = \nu = 3\%$, $\kappa = 0$, $\delta_0 = \bar{\delta} = 0$, $\omega = 0.9$, and $\alpha = 5\%$. (Color figure can be viewed at wileyonlinelibrary.com)

As discussed in Section II.A, when both agents are present, the price-dividend ratio without the circuit breaker is a wealth-weighted average of the prices from the two representative-agent economies populated by agents A and B, respectively. This is shown by the red dotted line in the figure, which does indeed lie between the two gray dashed lines. As D_t decreases, agent A becomes relatively more optimistic and holds more shares of stock. Consequently, the stock price reflects more of her belief. As D_t increases, the opposite is true. Agent B becomes relatively more optimistic, holds more shares of stock, and has a larger impact on the price.

In the presence of the circuit breaker, for any given level of dividend D_t above the circuit breaker threshold, the price-dividend ratio, shown by the solid blue line, is always lower than the value without the circuit breaker. Moreover, the gap between the two price-dividend ratios becomes negligible when D_t is sufficiently large and the market is far away from the circuit breaker. However, as D_t approaches the threshold $\underline{D}_t$ and the market moves closer to the circuit breaker, the difference becomes more pronounced.[20] When the circuit breaker is hit, the difference exceeds 2%.

We also note that in the presence of the circuit breaker, the stock price declines more rapidly as the dividend approaches the trigger threshold. The reason behind this behavior can be traced to how the stock price is determined upon market closure. As explained in Section II.B, at the instant when the circuit breaker is triggered, neither agent will be willing to take levered positions in the stock due to the inability to rebalance their portfolio thereafter. With bonds in zero net supply, the leverage constraint always binds for the relatively optimistic agent (agent A), and the market-clearing stock price has to be such

[20] In general, the threshold $\underline{D}(t, \delta_t)$ depends on both t and δ_t. Since our calibration of the δ process implies a one-to-one mapping between δ_t and D_t, the threshold becomes unique for any t.

that agent B is willing to hold all of his wealth in the stock, *regardless of his share of total wealth*. Indeed, we see the price-dividend ratio with the circuit breaker converging to $\widehat{S}_t^B/D_t$ when D_t approaches $\underline{D}_t$, instead of the wealth-weighted average of $\widehat{S}_t^A/D_t$ and $\widehat{S}_t^B/D_t$, given by the red dotted line. The lower stock price at the circuit breaker threshold also drives the stock price lower before market closure, with the effect becoming stronger as D_t moves closer to the threshold $\underline{D}_t$. This gives rise to the accelerated decline in the stock price as D_t drops, which also implies higher price sensitivity to dividend shocks.[21]

Conditional Return Volatility. The higher sensitivity of the price-dividend ratio to dividend shocks due to the circuit breaker manifests in elevated conditional return volatility, as shown in Panel B of Figure 1. Quantitatively, the impact of the circuit breaker on stock return volatility can be quite sizable. Without the circuit breaker, the conditional volatility of returns (red dotted line) peaks at about 7.3%. With the circuit breaker, the conditional volatility (blue solid line) becomes substantially higher as D_t approaches $\underline{D}_t$. In particular, the conditional volatility reaches 10% at the circuit breaker threshold.

Conditional Expected Return. Panel C of Figure 1 plots the conditional expected stock return under the objective probability measure, which is the same as agent A's belief. Even when there is no circuit breaker, the conditional expected return rises as the dividend falls. This is because the irrational agent B both gains wealth share and becomes more pessimistic as D_t falls, driving prices lower. Given that this price drop is not related to changes in expected *future* payoffs, the expected return rises. The presence of the circuit breaker accelerates the increase in the conditional expected return as D_t approaches the threshold $\underline{D}_t$.

Agents' Stock Holdings. We can also analyze the impact of the circuit breaker on the equilibrium stock price by connecting it to how the circuit breaker influences the equilibrium stock holdings of the two agents. Let us again start with the case without the circuit breaker, shown by the red dotted line in Panel D of Figure 1. The stock holding of agent A, $\widehat{\theta}_t^A$, continues to rise as D_t falls to $\underline{D}_t$ and beyond. This is the result of two effects: (i) with lower D_t the stock price is lower, implying a higher expected return under the belief of agent A, who is taking levered positions in the stock; and (ii) lower D_t also makes agent B, who is shorting the stock, wealthier and thus more capable of lending to agent A, who then takes on more levered positions.

[21] The circuit breaker rules out extremely low values for the price-dividend ratio during the trading session, which could occur at extremely low dividend values had trading continued. This could be one of the benefits of circuit breakers in the presence of market frictions. For example, when there are intraday mark-to-market requirements for some market participants, a narrower range for the price-dividend ratio can help reduce the chance of inefficient liquidations that could further destabilize the market. Formally modeling such frictions represents an interesting direction for future research.

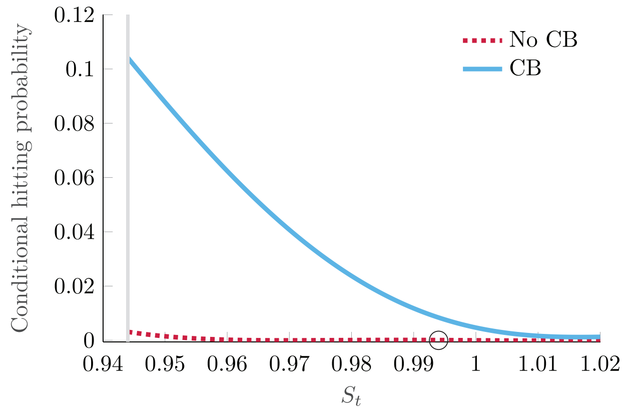

Figure 2. The "magnet effect." In the case of time-varying disagreement, we plot the conditional probabilities for the stock price to reach the circuit breaker threshold within the next h minutes from $t = 0.1$. Here, h is proportional to $(S_t - \underline{S})^2$: h is roughly 0.5 minutes when $S_t = 0.95$, 9.5 minutes when $S_t = 0.97$, and 62 minutes when $S_t = 1.01$. The black circle represents S_0. The gray vertical bar denotes the circuit breaker threshold $\underline{S}$. The parameters are $\mu = 0.04\%$, $\sigma = \nu = 3\%$, $\kappa = 0$, $\delta_0 = \bar{\delta} = 0$, $\omega = 0.9$, and $\alpha = 5\%$. (Color figure can be viewed at wileyonlinelibrary.com)

With the circuit breaker, while the stock holding θ_t^A takes similar values as $\widehat{\theta}_t^A$, its counterpart in the case without the circuit breaker, for large values of D_t it becomes visibly lower than $\widehat{\theta}_t^A$ as D_t approaches the circuit breaker threshold, as shown by the blue solid blue line. This is because agent A becomes increasingly concerned with the rising return volatility at lower D_t, which offsets the effect of a higher expected return. Finally, θ_t^A drops discretely when $D_t = \underline{D}_t$. With the leverage constraint binding, agent A will hold all of his wealth in the stock, which means θ_t^A will equal his wealth share ω_t^A. The stock price in equilibrium has to fall enough such that agent A has no incentive to sell more of his stock holding. This is indeed the case as shown in Panel A.

Magnet Effect: Accelerating Likelihood of Hitting the Circuit Breaker. Given our definition of the magnet effect, in Figure 2 we plot the two conditional hitting probabilities, $p_h(S_t)$ and $\widehat{p}_h(S_t)$. To avoid trivial limits of these conditional hitting probabilities, as in Proposition 7, we set h to be proportional to $(S_t - \underline{S})^2$, so it shrinks as S_t approaches the circuit breaker.

Comparing $p_h(S_t)$ and $\widehat{p}_h$, shown by the solid blue line and the red dashed line, respectively, we obtain the following results. First, the conditional hitting probability with the circuit breaker is always higher than that without the circuit breaker. This result is due largely to the fact that the circuit breaker tends to increase price volatility, which increases the hitting probability. Next, when S_t is sufficiently far from the circuit breaker threshold, the conditional hitting probabilities with and without the circuit breaker are both essentially zero. Third, the gap between the two quickly widens as the stock price moves closer to the threshold. The conditional hitting probability with the circuit breaker starts to increase substantially when S_t is still far away from the circuit breaker, at one, while the conditional hitting probability with the circuit breaker remains essentially zero and only picks up slightly when S_t is sufficiently close to the circuit breaker, around 0.95. This is the magnet effect as defined above. This effect arises from the fact that as the stock price approaches the circuit breaker threshold, its volatility increases significantly, as shown in Proposition 6 and Figure 1, which then leads to a higher hitting probability.[22]

The quantitative analysis above shows that under more general specifications of time-varying disagreement, the impact of the circuit breaker on market behavior is consistent with the analytical results obtained for the constant-disagreement case. In addition, the effects on price level, volatility, and expected returns can be large in magnitude.

C. Key Model Implications

As demonstrated in the theoretical analysis above, our model yields concrete predictions about how the circuit breaker affects the dynamics of the stock price, as captured by its conditional moments and trading behavior. To connect more directly with our empirical analysis below, we simulate the model with time-varying disagreement and examine the relationship between several *realized moments* of returns and the distance of the stock price from the circuit breaker threshold (i.e., DTCB). Returns are measured over one-minute intervals and the DTCB is defined as $S_t/S_0 - (1-\alpha)$. The results are plotted in Figure 3, shown by the blue solid lines. For comparison, we also show the results without the circuit breaker in red dotted lines. We have the following predictions:

(i) Realized return volatility increases dramatically as the DTCB falls, and the rate of increase accelerates substantially as the DTCB falls (until it is close to zero).[23] This differs substantially from the situation without the circuit breaker, in which case volatility increases only mildly.
(ii) Realized return skewness becomes negative and decreases further as the DTCB falls. This decline is reversed only when the DTCB is sufficiently small, at which point the fact that prices are bounded from below starts to have a significant effect on skewness.
(iii) The expected return rises as the DTCB becomes very small.

Comparing with the behavior of return moments without the circuit breaker, we see sharp differences. With no circuit breaker, volatility exhibits much

[22] From Panel C of Figure 1, we also see that as the price moves closer to the circuit breaker, its drift $\mu^A_{S,t}$ also becomes more positive, which tends to pull the price away from the trigger threshold. However, as we show in the proof of Proposition 7 in the Appendix, the volatility effect always dominates in the neighborhood of the threshold.

[23] Note that Panel A of Figure 3 is different from Panel B of Figure 1 due to a change in variable on the x-axis from D_t to DTCB.

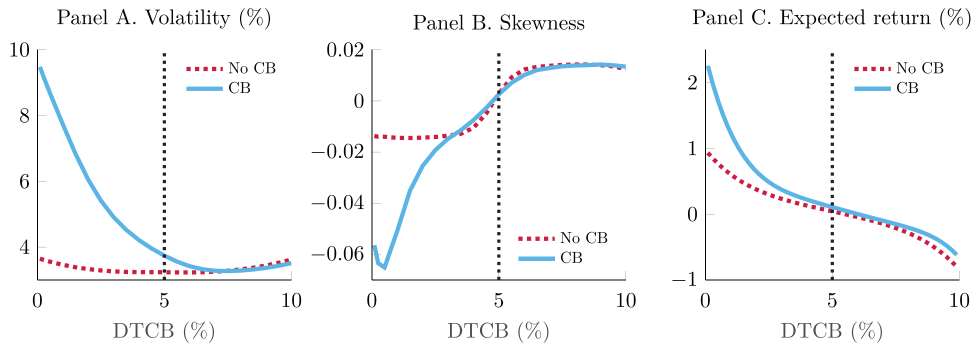

Figure 3. Model predicted one-minute conditional volatility, conditional skewness, and conditional expected return. In the case of time-varying disagreement, we plot all model predicted quantities as functions of the distance to the circuit breaker threshold (DTCB, defined as $S_t/S_0 - (1-\alpha)$). Both volatility and return are normalized to one day. The parameters are $\mu = 0.04\%$, $\sigma = \nu = 3\%$, $\kappa = 0$, $\delta_0 = \bar{\delta} = 0$, $\omega = 0.9$, and $\alpha = 5\%$. (Color figure can be viewed at wileyonlinelibrary.com)

smaller variation overall, increasing only mildly when the price level drops; skewness turns mildly negative as the price drops below its initial level and then flattens out. These patterns are in stark contrast to the fast and accelerating rise in volatility and negative skewness as well as its final reversal when the market moves closer to the circuit breaker when it is present.

Although our diffusion-based model does not yield a proper measure of trading volume, it does predict that the more optimistic investors will significantly reduce their stock positions when the circuit breaker is triggered. In the presence of transaction costs and execution delays, these investors will want to cut their stock positions earlier instead of waiting until the last moment. While we do not have account-level data to examine the directions of trades of different investors, we can examine the following prediction about total trading volume (from a generalized version of our model):

(iv) Trading volume increases as the DTCB falls.

In Section V, we empirically examine the predictions above on the behavior of price dynamics and trading activity.

IV. Impact of Circuit Breaker on Welfare

So far, we have focused on how the circuit breaker affects market behavior, such as the price level and dynamics. In this section, we examine how the circuit breaker impacts the welfare of market participants, which becomes possible in a general equilibrium setting like ours.

The welfare implication of a circuit breaker very much depends on agents' trading motives and the welfare measure used. If we interpret agent B's preference as a state-dependent utility function under the objective probability measure, as in (7), then there is no ambiguity about the welfare criteria. In this case, trading is motivated by risk sharing, and the introduction of a circuit breaker reduces welfare due to the potential loss of risk-sharing opportunities when the market is shut down.

If we instead interpret agent B's preference as a state-independent log utility function, as in (3), but under the subjective belief given in (5), then the welfare implication of the circuit breaker is less clear-cut. While the literature has proposed different welfare criteria for economies with heterogeneous beliefs,[24] we take the perspective of a paternalistic planner/regulator who considers welfare under the objective probability measure. A main rationale for policymakers to introduce a circuit breaker is to curb irrational trading, in particular "panic selling," under sudden market downturns. This is captured by the subjective belief deviation δ_t, which is on average zero but can become significant when the market moves substantially within a day. Under such an interpretation, a paternalistic planner/regulator will view the trading motive of agent B as speculation driven by irrational beliefs, and will see welfare gains from introducing a circuit breaker to curb such trading activities.

Welfare Measure. To formally examine these issues, we define an agent's welfare by the ex ante certainty equivalent wealth, CE. For agent A, we have

$$\ln(CE_0^A) = \mathbb{E}_0\left[u^A(W_T^A)\right] = \mathbb{E}_0\left[\ln(W_T^A)\right]. \tag{32}$$

For agent B, under the interpretation of state-dependent utility, we have

$$\ln(CE_0^B) \equiv \mathbb{E}_0\left[\tilde{u}^B(W_T^B, \eta_T)\right] = \mathbb{E}_0\left[\eta_T \ln(W_T^B)\right], \tag{33}$$

where the second equality follows from the definition of the state-dependent utility $\tilde{u}^B$ in (7). However, under the alternative interpretation of state-independent utility and irrational belief, agent B's certainty-equivalent wealth becomes

$$\ln(CE_0^B) \equiv \mathbb{E}_0\left[u^B(W_T^B)\right] = \mathbb{E}_0\left[\ln(W_T^B)\right]. \tag{34}$$

Here, the planner calculates agent B's expected utility under the objective probability measure $\mathbb{P}$ instead of his own belief $\mathbb{P}^B$.

The impact of the circuit breaker on welfare can then be defined as the change in agents' certainty equivalent wealth due to the introduction of the circuit breaker. In particular, we have

$$\Delta CE^i \equiv CE_0^i - \widehat{CE}_0^i, \quad i = A, B, \quad \mathit{and} \quad \Delta CE \equiv \omega\, \Delta CE^A + (1-\omega)\, \Delta CE^B, \tag{35}$$

where $\widehat{CE}^i$ denotes agent i's certainty equivalent wealth without the circuit breaker, ΔCE^i its change with the circuit breaker, and ΔCE the change in total

[24] See, for example, Brunnermeier, Simsek, and Xiong (2014) and Gilboa, Samuelson, and Schmeidler (2014).

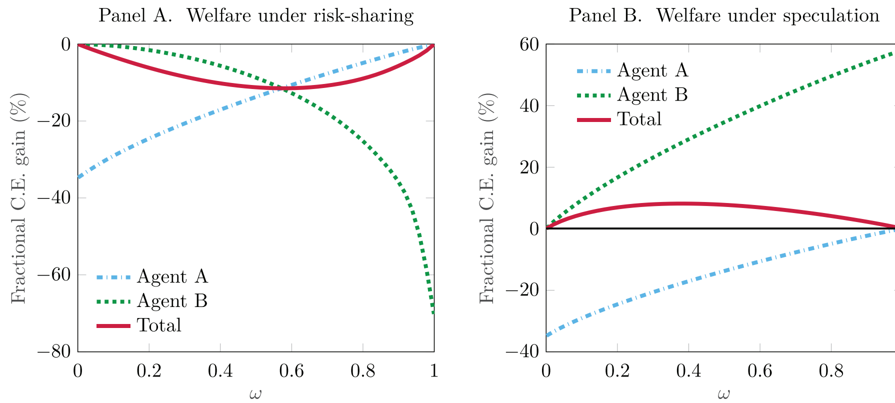

Figure 4. Welfare impact. Agents' gain in certainty equivalent wealth in a market with the circuit breaker versus a market without for different initial wealth shares ω of agent A (the case of time-varying disagreement). The blue dashed line corresponds to agent A and the green dotted line corresponds to agent B. The red solid line corresponds to the entire market, with proper weights for the agents. In Panel A, the certainty equivalent wealth is defined under agents' own beliefs. In Panel B, it is defined under the objective probability measure. The parameters are $\mu = 0.04\%$, $\sigma = 3\%$, $\nu = 6\%$, $\kappa = 0$, $\delta_0 = \bar{\delta} = 0$, $\omega = 0.9$, and $\alpha = 5\%$. (Color figure can be viewed at wileyonlinelibrary.com)

welfare of the market, given by the weighted average of changes in individual agents' welfare.

In Figure 4, we use the percentage change in certainty equivalent wealth to represent the effect of a circuit breaker on the agents' welfare, under the two alternative interpretations of agent B's trading motives. To better demonstrate the results, we set ν to twice the value of σ in (5), which increases the degree of heterogeneity between agents and potential gains from trading.

Welfare under Risk Sharing. Panel A of Figure 4 plots the circuit breaker's effect on agents' individual and total welfare. The blue dashed line shows the welfare change for agent A, the green dotted line shows the welfare change for agent B, and the red solid line plots the total welfare change.

For both agents, the introduction of the circuit breaker reduces welfare. This is a straightforward result of our model. In the absence of other market imperfections, a dynamically complete financial market allows efficient risk sharing. This is indeed the case in our setting without the circuit breaker. The presence of the circuit breaker, however, destroys market completeness, lowers the efficiency in risk sharing, and hence reduces welfare. This is what we refer to as the dark side of the circuit breaker, a topic we return to shortly.

In general, an agent's gain from risk sharing increases with the relative size of her/his counterparty, who demands a smaller premium. Thus, the welfare loss from the circuit breaker for agent A reaches its maximum when her wealth share ω approaches zero, it diminishes when ω increases, and it becomes zero when ω approaches one. For agent B, the reverse is true—his welfare loss increases with ω. It is also worth noting that for small values of ω (when it is close to zero), the welfare loss for agent B is almost zero. This is because with little wealth, agent A will demand a large premium from agent B to share his risk, who wants to trade large amounts. In equilibrium, the actual amount of trading is negligible even when the market is open. Thus, the welfare loss from a circuit breaker is limited. This property for risk-sharing trades is important when we consider both risk-sharing and speculation trades.

For the market as a whole, the welfare loss is maximized when ω is somewhere in the middle. This is not surprising given the welfare loss for the two agents and their relative weights in the market. The maximum welfare loss can exceed 10% of total wealth.

Welfare under Speculation. Panel B of Figure 4 plots the circuit breaker's welfare effect when agent B is viewed as having (time-varying) irrational beliefs. The blue dashed line shows the welfare loss of agent A, as measured by ΔCE^A. As in Panel A, it is always negative and decreases in magnitude with her initial wealth share ω. The green dotted line shows agent B's welfare change, now under the objective probability. In contrast to the welfare loss under his subjective measure, agent B's welfare now increases when a circuit breaker is in place. This is expected. With his incorrect belief, agent B generally incurs losses from trading with agent A. Therefore, the circuit breaker, when triggered, helps limit his trading and reduce his loss. Moreover, a larger wealth share of agent A, ω, allows her to take larger positions against B, leading to larger potential losses for agent B. Consequently, imposing the circuit breaker leads to larger welfare gains for agent B when ω is larger.

For the market as a whole, the net welfare gain is a weighted average of the two agents' welfare gains/losses. In general, it can be positive or negative. However, as shown in Panel B of Figure 4 by the red solid line, the net welfare gain from the circuit breaker is always positive. This is because, under the objective probability measure (and the same state-independent log utility function for both agents), the optimal allocation between the two agents is simply no trade. Any trading will then lead to less efficient allocations and lower total welfare. Since the circuit breaker reduces agents' trading opportunities, especially those for agent B based on irrational beliefs, it will increase welfare.

We thus conclude that in a complete market setting, if trading is driven only by agents' irrational beliefs, the circuit breaker helps curtail irrational trades and increases welfare for the market as a whole. Under this interpretation, the circuit breaker has a bright side.

Welfare under General Trading Motives. The two welfare effects considered above take extreme stands: agents trade either for risk-sharing motives due to heterogeneous preferences or for speculation motives due to irrational beliefs. As a result, the welfare implication of a circuit breaker is clear-cut: bad in the former case and good in the latter case.

In general, both trading motives are present in the market, which leads to richer welfare implications, depending on the equilibrium trade-off between

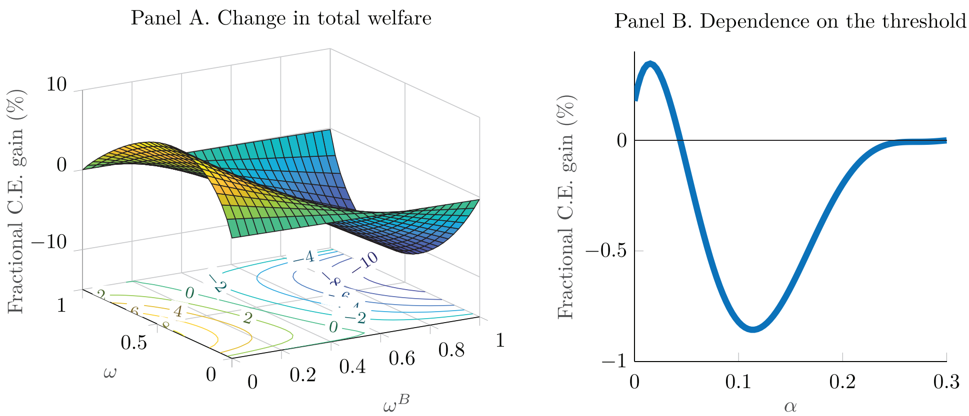

Figure 5. Welfare trade-offs. In the case of time-varying disagreement, the left panel shows the change in total welfare due to the circuit breaker in the presence of two types of trading motives: risk-sharing and irrational speculation. The right panel shows the dependence of the change in total welfare on the circuit breaker level. The parameters are $\mu = 0.04\%$, $\sigma = 3\%$, $\nu = 6\%$, $\kappa = 0$, and $\delta_0 = \bar{\delta} = 0$. In the left panel, $\alpha = 5\%$; in the right panel, $\omega = 0.50$ and $\omega^B = 0.41$. (Color figure can be viewed at wileyonlinelibrary.com)

the two effects. We thus extend our model to this more general case. Suppose there are three agents, A, B_1, and B_2. Agent A is the same as in the original model, agent B_1 has the state-dependent utility function as in (7) under the objective probability measure, and agent B_2 has the state-independent log utility function as in (3) under the subjective belief given in (5). Furthermore, the initial wealth share of agent A is still ω, while B_1 and B_2 have shares ω^B and $1-\omega^B$ of the remaining wealth. The change in total welfare due to a circuit breaker is then given by

$$\Delta CE = \omega\, \Delta CE^A + (1-\omega)\left[\omega^B \Delta CE^{B_1} + (1-\omega^B)\Delta CE^{B_2}\right], \tag{36}$$

where ΔCE^i is the change in certainty equivalent wealth for $i = A, B_1, B_2$, and CE^{B_1} and CE^{B_2} are given by (33) and (34), respectively.

The left panel of Figure 5 shows the change in total welfare as a function of the wealth share of agent A, ω, and the relative wealth share of agent B_1, ω^B. As expected, the welfare implication of the circuit breaker depends on both the wealth share of agent A and the relative wealth share of B_1 and B_2. Such a dependence exhibits rich patterns.

When ω^B is large (close to one), the rational trader with state-dependent utility is dominant and the circuit breaker reduces total welfare because the gain from risk sharing exceeds the loss from irrational speculation. This corresponds to the case shown in Figure 4, Panel A. The opposite is true when irrational traders are dominant, that is, when ω^B is small (close to zero), which corresponds to the case in Figure 4, Panel B.

For a given interior value of ω, the net welfare impact of the circuit breaker decreases with ω_B. It starts at a positive value when ω_B is small and more trading is driven by irrational speculation, and turns negative when ω_B becomes sufficiently large and more trading is driven by risk sharing. The turning point, however, depends on the value of ω, the wealth share of agent A. In particular, for a smaller value of ω, the critical value of ω_B for the circuit breaker's net welfare impact to turn negative is larger. This dependence is shown by the zero-value line on the $x-y$ plane. This result is driven by the fact that when ω is small, the welfare loss from a circuit breaker in reduced risk sharing is negligible, as discussed above, while the welfare gain in reduced speculation is significant and dominates. For larger values of ω, however, the welfare effect from risk sharing becomes significant and important in influencing the trade-off.

Optimal Circuit Breaker Design. So far, we have taken the circuit breaker, defined by α, as given and analyze its welfare impact under different distributions of agents with different trading motives. We can also take the distribution of agents as given and examine how different choices of α influence welfare. As an example, the right panel of Figure 5 plots the total welfare gain from a circuit breaker for different values of α, with $\omega = 0.50$ and $\omega^B = 0.41$. The initial wealth shares of the three agents, A, B_1, and B_2, are then 0.5, $(0.5)(0.41) = 0.205$, and $(0.5)(0.59) = 0.295$, respectively. As discussed above, a circuit breaker protects investors with irrational beliefs from speculation, but also hinders investors with state-dependent utility from full risk sharing. For small values of α the former effect dominates, yielding a welfare gain. For large values of α, the latter effect dominates, yielding a welfare loss.

The trade-off between these two effects is in general not monotonic and can exhibit rich patterns. In the current case, for example, as α increases from zero, corresponding to the case of no trade, the welfare gain from risk sharing exceeds the welfare loss from irrational speculation. Overall welfare is maximized at an interior α value of 1.2%. Beyond this point, welfare begins to decrease and turns negative. As α increases further, the welfare loss actually starts to decrease. This example shows that quantitatively the trade-off between the welfare gain and loss from a circuit breaker can be quite complex.[25]

To the extent that irrationality can also be interpreted as a form of market imperfection, our model can be a useful setup to illustrate the basic mechanisms behind the potential pros and cons of circuit breakers. Nonetheless, we refrain from making specific statements concerning the practical welfare and policy implications of circuit breakers. These statements should be based on a model that can quantitatively capture agents' trading motives and behavior as well as other important imperfections in the actual market.

[25] This example is for demonstrative purposes only. We did not try to search extensively in the parameter space for an "optimal" α with higher values, such as 5% or 7%. Given the parsimonious nature of the model, it is not intended for a quantitative calibration to the actual market. See also the discussion below.

V. Empirical Analysis

In Section III, we examine model predictions on how market dynamics change as the price approaches the circuit breaker, and we highlight four key predictions on the behavior of return volatility, skewness, expected return, and trading volume. In this section, we explore the data from the U.S. stock market and present a set of empirical results that are consistent with these predictions.

A. Data and Variables

The sample period we consider is May 1, 2013 to December 31, 2020. The choice of the start date is due to the fact that the current version of the MWCB for the United States was first implemented on April 8, 2013 (see Section I of the Internet Appendix for an overview of the evolution of the MWCB rule in the United States).

To study market dynamics, we employ transaction-level data for E-mini S&P 500 futures from the Chicago Mercantile Exchange (CME). One of the most actively traded financial products in the world, the E-mini S&P 500 futures closely tracks the S&P 500 index, the reference index for the current MWCB. Our analysis uses the lead month contracts, which are usually the most active in terms of trading volume.[26] Although the E-mini futures are traded almost around the clock, we restrict our attention to regular trading hours (9:30 a.m. to 4:00 p.m. ET) when the MWCB is active.

Since price dynamics can change rapidly as the distance to the circuit breaker changes, it is important to carry our analysis at a sufficiently high frequency, which we set to the minute level. We first compute second-level prices of the index futures using volume-weighted average transaction prices over each second. We then construct measures for (log) return, volatility, and skewness at the minute frequency. To reduce the impact of microstructure noise in the volatility measure, we apply the two-scale realized volatility (TSRV) measure of Zhang, Mykland, and Ait-Sahalia (2005).[27] Our skewness measure is the realized skewness introduced by Amaya et al. (2015).[28]

[26] All of our main empirical results continue to hold when we follow a deterministic rollover strategy by switching to the next contract one week before maturity.

[27] The TSRV realized volatility is constructed using the difference between two estimates of one-minute integrated realized volatility, one based on one-second returns and the other based a scaled average of one-minute integrated realized volatility computed using five-second returns. Taking the difference between the two estimates helps remove microstructure noise. In Section VI of the Internet Appendix, we show that our main results are robust to different measures of volatility, including TSRV with a different sampling frequency or simply the raw volatility measure.

[28] Amaya et al. (2015) define realized skewness as $\text{RDSkew}_t = (\frac{1}{N}\sum_{i=1}^{N} r_{t,i}^3)/(\frac{1}{N}\sum_{i=1}^{N} r_{t,i}^2)^{3/2}$, where N is the number of observations in a minute.

We also construct a measure of abnormal trading volume at the minute frequency, which is defined as

$$Abnormal\ volume_t \equiv \frac{volume\ in\ minute t}{average\ volume\ in\ the\ previous\ 6.5\ trading\ hours} - 1. \tag{37}$$

We exclude abnormal volume from the first and last day of each contract because trading volume tends to spike and fall, respectively, on these dates.

We construct a DTCB measure to determine how far the market is from the nearest circuit breaker threshold. It is worth noting that, in addition to the MWCB, the CME also imposes downside price limits for the E-mini S&P 500 future, with three price limit levels that exactly match the three circuit breaker threshold levels during regular trading hours,[29] and we need to account for a subtle difference between the two rules. Whereas the MWCB focuses on the distance between the S&P 500 index and its closing value on the previous day, the price limits on the E-mini focus on the distance between the E-mini price and its volume-weighted average price (VWAP) between 3:59:30 p.m. and 4:00 p.m. ET on the previous day. Although these two distances are nearly identical most of the time, it is possible for the E-mini price limit to be reached before the MWCB. To ensure consistency, we define $DTCB_t$ based on the E-mini price and omit observations for which the price limit was reached before the MWCB.[30] Specifically, at minute t,

$$DTCB_t \equiv \frac{P_t^{min} - (1-\alpha_t)\overline{P}_t^{close}}{\overline{P}_t^{close}}. \tag{38}$$

Here, P_t^{min} is the *minimum price level* for the E-mini during minute t, $\overline{P}_t^{close}$ denotes the volume-weighted average E-mini price for the last 30 seconds before 4:00 p.m. from the prior trading day, and α_t is the nearest active circuit breaker threshold. Specifically, α_t is equal to 7% at the beginning of a trading day; if the Level 1 circuit breaker is triggered before 3:25 p.m., α_t is raised to 13% after the market reopens, and α_t is raised to 20% after 3:25 p.m. or after the Level 2 circuit breaker is triggered. To reduce the noise caused by the discrete change in the circuit breaker threshold at 3:25 p.m., we drop observations after 3:25 p.m. from our analysis.

Finally, when examining the relationship between DTCB and conditional return moments or trading volume, we want to control for the well-known leverage effect on return volatility, which refers to the potential negative correlation between stock volatility and the price change (see Black (1976), Yu (2005), and Ait-Sahalia, Fan, Li (2013), among others). For this purpose, we construct

[29] See https://www.cmegroup.com/trading/equity-index/sp-500-price-limits-faq.html for more information on the price limits.

[30] This filter removes a total of eight observations from two dates, August 24, 2015 and March 9, 2020.

the following measure of how the average price level in the past 60 minutes compares to a moving-average (MA) price level from the past 21 trading days:

$$Lev_t = \frac{\textit{Average E-mini price over the past 60 minutes}}{\textit{Average E-mini price over the past 21 trading days}}. \tag{39}$$

To capture potential nonlinearity in the leverage effect, we also introduce a quadratic leverage factor, $QLev_t = (Lev_t - 1)^2$.

Summary statistics for the main variables and their correlation matrix are reported in Section VI of the Internet Appendix. Two points are worth noting. First, the first percentile of DTCB is 4.47%. This is because intraday movements in the S&P 500 index (SPX) (or E-mini) of 2.5% or more are rare. However, with a sample size of 686,697, we still have 248 minute-level observations with DTCB below 2%, which give us a window to examine market behavior near an MWCB.[31] Second, the correlation between DTCB and *Lev* (*QLev*) is 0.194 (−0.026). Despite the fact both the DTCB and leverage factor are driven by price movements, there are important differences between them. In *Lev*, we measure "current price levels" using average prices over the past 60 minutes rather than the past minute. More importantly, the benchmark for the current price level in *Lev* is the average price level over the past 21 days. In contrast, in DTCB we reset the benchmark daily to the previous closing price. It is therefore possible that, at some moments on two different trading days, the levels of the SPX are similar, resulting in similar values for *Lev*, yet the DTCB is drastically different due to different closing prices on the previous day.

B. Empirical Results

For a first look at the data, we run the following regressions of return moments (volatility, skewness, and mean return) and abnormal volume on a set of DTCB dummies while controlling for a time trend, intraday seasonality,[32] and leverage effects:

$$y_{d,m,t} = \sum_i \beta_i \times 1_{\{DTCB \in \mathrm{Bin}_i\}} + a_d + b_m + c\, Lev_t + d\, QLev_t + \varepsilon_{dmt}, \tag{40}$$

where $y_{d,m,t}$ is volatility, skewness, abnormal volume for day d, minute m, time t, all of which are contemporaneous with the right-hand-side variables, or the realized return in the next minute. The bins are defined as follows: $\mathrm{Bin}_1 = (0\%, 0.75\%]$, $\mathrm{Bin}_2 = (0.75\%, 1.25\%]$, $\mathrm{Bin}_3 = (1.25\%, 2\%]$, and

[31] For robustness, we also examine the empirical results when the four days in March 2020 are excluded. The results remain the same (see Section VI of the Internet Appendix).

[32] For example, both volatility and trading volume have well-documented U-shaped intraday patterns (see, e.g., Hong and Wang (2000)).

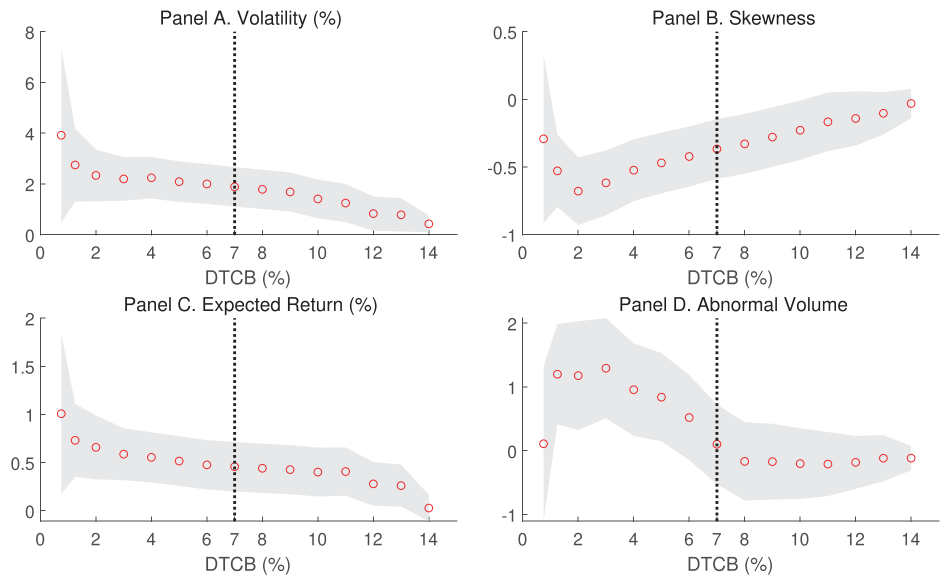

Figure 6. Binned scatterplot on regression coefficients of (40). This figure shows the β regression coefficients of (40) for volatility (two-scale realized volatility), skewness, expected return, and abnormal volume. $\text{Bin}_1 = (0\%, 0.75\%]$, $\text{Bin}_2 = (0.75\%, 1.25\%]$, $\text{Bin}_3 = (1.25\%, 2\%]$, and $\text{Bin}_i = ((i-1)\%, i\%]$ for $i = 4, \ldots, 14$. The red dots indicate β coefficients and the gray regions represent two standard deviations around the β coefficients. (Color figure can be viewed at wileyonlinelibrary.com)

$\text{Bin}_i = ((i-1)\%, i\%]$ for $i = 4, \ldots, 14$. The controls include daily and minute-of-day fixed effects (a_d, b_m), as well as the linear and quadratic leverage factor $(Lev_t, QLev_t)$. We then plot the coefficients β_i to examine how the variables of interest vary with the DTCB.

As Figure 6 shows, both realized volatility (Panel A) and expected returns Panel C) for the E-mini rise as DTCB decreases, and with these effects accelerating (the slopes steepen) as DTCB approaches zero. These patterns are consistent with the model predictions discussed in Section III (see Panels A and C of Figure 3). Panel B shows that realized skewness first decreases as DTCB declines and then starts to increase when DTCB falls below 2%. This pattern is qualitatively the same as the model prediction in Panel B of Figure 3, although the uptick in skewness when DTCB is near zero appears stronger in the data. Panel D of Figure 6 shows that abnormal volume rises steadily as DTCB declines, then drops when DTCB reaches the last bin before zero. The final dip in volume is likely due to the deterioration of market liquidity when the market approaches the circuit breaker threshold, a feature that our model abstracts from. Finally, notice that the confidence bands on the β estimates widen significantly when DTCB is near zero due to limited observations in this region.

We further examine the statistical significance of the empirical patterns demonstrated in Figure 6 using the regression specification

$$y_{d,m,t} = \beta \times DTCB_t + \gamma \times \min(DTCB_t - 2\%, 0) + a_d + b_m + c\, Lev_t + d\, QLev_t + Y_t'\delta + \varepsilon_{dmt}, \quad (41)$$

where $y_{d,m,t}$ is realized volatility, skewness, and abnormal volume for day d, minute m, and time t. When examining the prediction for expected returns, we replace $y_{d,m,t}$ with $r_{d,m,t+1}$, the realized E-mini log return in the next minute. For controls, similar to (40), we include daily and minute-of-day fixed effects (a_d, b_m), as well as linear and quadratic leverage factors (Lev_t, $QLev_t$). In addition, we include Y_t, a vector of lagged values of $y_{d,m,t}$ to control for serial correlations between y at different lags. To determine the number of lags to include in the regressions, we run an autoregressive (AR) model selection based on the Akaike information criterion (AIC).[33] The main coefficients of interest are β, which captures the baseline relationship between $y_{d,m,t}$ and $DTCB_t$, and γ, which captures the potential nonlinearity when DTCB is below 2%. Table II reports the regression results.

There is indeed a highly significant and sizable negative relationship between DTCB and realized volatility. On average, when the SPX E-mini price moves 1% closer to the circuit breaker, realized volatility of the E-mini futures rises by 17.5 bps, normalized to daily scale. Moreover, the slope coefficient on DTCB nearly doubles in magnitude as DTCB drops below 2%. The coefficient estimate is highly significant based on OLS standard errors, but insignificant based on Newey-West standard errors. This is because there are a total of 248 observations for which DTCB is below 2%, the majority of which are concentrated in a few windows from a few days in our sample, for which the correction for autocorrelation across observations makes a big difference.

Notice that when we exclude DTCB from the regression, the coefficient on the leverage factor is negative and statistically significant, suggesting that there is indeed a leverage effect in our sample. However, when DTCB is included, the coefficient on the leverage factor turns positive, likely because DTCB has subsumed the leverage effect from the leverage factor. However, there are a few reasons why DTCB is not just a leverage factor in disguise. First, the magnitude of the leverage effect is much smaller. When DTCB is not included in the regression, a one-standard-deviation decrease in the leverage factor corresponds to a 7 bps increase in realized volatility, about half the size of the effect from a one-standard-deviation decrease in DTCB. Second, the slope coefficient on DTCB becoming more negative as DTCB declines is a unique prediction of our model on volatility dynamics in the presence of a circuit breaker, which is present in the data even after controlling for nonlinearity in the leverage factor.

[33] For AR models with optimally selected lags, we verify that no remaining AR or MA structure is detected in the residuals.

Table II

Regression results

This table reports results of the regression $y_{d,m,t} = \beta \times DTCB_t + \gamma \times \min(DTCB_t - 2\%, 0) + a_d + b_m + c\,Lev_t + d\,QLev_t + Y_t'\delta + \varepsilon_{dmt}$, where $y_{d,m,t}$ is the realized volatility, skewness, and abnormal volume for day d, minute m, and time t or the return for time $t+1$, $DTCB_t$ is the distance to the circuit breaker for time t, Lev_t is the leverage variable measuring the ratio between a short-term and long-term average E-mini prices at time t, $QLev_t = (Lev_t - 1)^2$, and Y is a vector of lagged values of $y_{d,m,t}$. OLS and Newey-West standard errors (with 29 lags) are reported in parentheses and square brackets, respectively. Sample period: May 1, 2013 to December 31, 2020. The total number of observations is 686,697. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

	Volatility			Skewness			Return			Abnormal volume		
Lev	−2.843	8.555	8.441	−8.401	−16.761	−17.149	−2.648	1.116	1.039	0.166	24.004	24.687
	(0.170)***	(0.226)***	(0.229)***	(0.237)***	(0.328)***	(0.331)***	(0.076)***	(0.106)***	(0.107)***	(0.449)	(0.664)***	(0.669)***
	[0.304]***	[0.888]***	[0.744]***	[0.329]***	[0.672]***	[0.642]***	[0.144]***	[0.380]***	[0.316]***	[0.570]	[1.458]***	[1.392]***
QLev	−6.439	2.075	2.590	−17.945	−23.863	−22.293	−1.911	1.104	1.406	9.391	26.328	24.074
	(1.280)***	(1.280)	(1.287)	(1.804)***	(1.809)***	(1.819)***	(0.572)***	(0.574)*	(0.577)**	(3.442)***	(3.453)***	[3.463]***
	[6.675]	[6.563]	[6.594]	[2.300]***	[4.104]***	[3.993]***	[3.026]	[2.872]	[2.920]	[3.235]***	[8.665]***	[8.426]***
DTCB		−17.454	−17.263		12.085	12.701		−5.375	−5.254		−28.290	−29.277
(Distance to CB)		(0.230)***	(0.236)***		(0.328)***	(0.336)***		(0.107)***	(0.109)***		(0.582)***	(0.594)***
		[1.256]***	[1.050]***		[0.698]***	[0.628]***		[0.474]***	[0.381]***		[1.561]***	[1.454]***
$\min\{DTCB - 2\%, 0\}$			−12.684			−40.883			−7.656			69.168
			(3.443)***			(4.892)***			(1.543)***			(8.351)***
			[44.433]			[11.597]***			[15.880]			[17.508]***
r_t							−0.015	−0.012	−0.012			
							(0.001)***	(0.001)***	(0.001)***			
							[0.004]***	[0.004]***	[0.004]***			
Control for lags	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Date + Time FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Adjusted R^2	0.664	0.667	0.667	0.003	0.005	0.006	0.002	0.006	0.006	0.591	0.593	0.593

Next, the skewness of E-mini returns is positively related to DTCB. When the SPX moves 1% closer to the active circuit breaker, realized skewness drops by 0.12. Consistent with Figure 6, the coefficient γ is large and positive, enough to turn the overall relation between DTCB and skewness positive when DTCB is below 2%. This pattern is qualitatively consistent with what is shown in Panel B of Figure 3 and in Prediction (ii).

There is also a negative relationship between DTCB and the following one-minute return in the SPX E-mini futures, which is again consistent with the model prediction that the expected return rises when the market approaches the circuit breaker. Like volatility, the negative relationship between DTCB and expected returns strengthen when DTCB falls below 2%. The coefficient estimate more than doubles from -5.254 when DTCB is above 2% to -12.91 when DTCB is below 2%, but the difference is statistically insignificant based on the Newey-West standard error.

Finally, we find a negative and statistically significant relationship between DTCB and abnormal trading volume, although the relation appears to flatten when DTCB drops below 2%. Although we are not able to directly test the prediction about deleveraging by some investors, the findings on trading volume are consistent with elevated investor trading demand as the market approaches the circuit breaker.

VI. Model Robustness

To better illustrate how a circuit breaker impacts the market, our analysis has focused on a parsimonious version of our model. For example, the bond supply is assumed to be zero. The form of the circuit breaker is one-sided and determined only by price levels. Our setup, however, allows more general specifications. In this section, we examine the robustness of our results when the bond is in positive supply.[34]

Positive bond supply can potentially change equilibrium behavior because, upon triggering the circuit breaker, the two agents are no longer required to be fully invested in the stock for the market to clear. For example, if the optimistic agent is sufficiently wealthy, she could hold the entire stock market without taking on leverage at the market closure. Consequently, the pessimistic investor need not be the only marginal investor at that instant.

Instead, depending on the wealth distribution and the amount of bond supply, there are four possible scenarios at the time of market closure τ: (i) the optimistic agent faces a binding leverage constraint, while the pessimistic agent is unconstrained; (ii) the optimistic agent is unconstrained, while the pessimistic

[34] We have also considered other extensions of the model, including differences between continuous-time and discrete-time trading and different types of circuit breakers such as two-sided circuit breakers, multitiered circuit breakers, and circuit breakers based on triggers beyond price levels. In contrast to trading halts triggered by price-level thresholds, such as circuit breakers, markets typically have prescheduled trading halts such as overnight closures. In general, their impact on market behavior is distinct. Such a comparison is provided in Section V of the Internet Appendix.

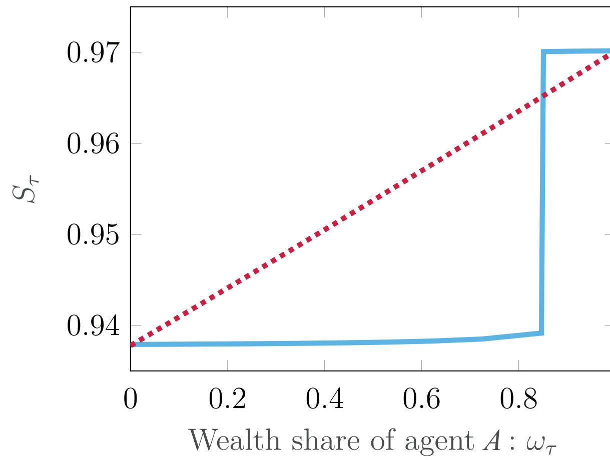

Figure 7. Stock price at time τ with positive bond supply. Blue solid lines correspond to the case with the circuit breaker. Red dotted lines correspond to the case without the circuit breaker. The parameters are $\tau = 0.25$, $D_\tau = 0.97$, and $\Delta = 0.17$. (Color figure can be viewed at wileyonlinelibrary.com)

agent faces a binding short-sale constraint; (iii) the optimistic agent faces a binding leverage constraint, while the pessimistic agent faces a binding short-sale constraint; and (iv) neither agent is constrained. In particular, scenario (ii) is the opposite of scenario (i), in which case the equilibrium price level can become higher and volatility lower in the economy with a circuit breaker. We now examine the conditions (wealth distribution and size of bond supply) that determine which of the four scenarios occur in equilibrium, and how they affect our model's main prediction that the presence of circuit breakers amplifies return volatility.

As in II.B, we first consider the problem at the instant before market closure, which provides us with much of the intuition about the effect of positive bond supply. Suppose the total supply of the bond is Δ. At time τ, the two agents have one last chance to trade and then have to hold their positions until time T. Similar equilibrium conditions as those given in Section II.B apply here, with the exception that $\phi_\tau^A + \phi_\tau^B = \Delta$.

To illustrate, in Figure 7 we plot the equilibrium stock price as a function of the wealth share of agent A for $\tau = 0.25$ and $D_\tau = 0.97$ (blue solid line), and compare it to the stock price without the circuit breaker (red dotted line). The bond supply is assumed to be $\Delta = 0.17$. Notice that because of the one-to-one mapping between D_t and δ_t, we know that agent A is relatively more optimistic for this value of D_τ.

Again, the stock price in the market without the circuit breaker is a weighted average of the valuations of the optimist and pessimist, with the weight depending on their respective wealth shares. Since agent A is more optimistic, the stock price without the circuit breaker naturally increases in her wealth share. When $\Delta = 0$, the stock price with the circuit breaker will always be equal to the valuation of agent B (the pessimist) at time τ. However, this is no longer the case with $\Delta > 0$. As Figure 7 shows, when agent A's wealth share is not too high, the stock price with circuit breaker will be at or slightly above agent B's valuation. However, when her wealth share is sufficiently high, the stock price will rise sharply and reach the valuation of agent A (the optimist).

The intuition is as follows. When agent A's wealth share ω_τ^A is not too high, she will be fully invested in the stock at time τ but is not able to clear the stock market by herself. In this case, agent A's leverage constraint will be binding, agent B will hold all of the riskless bond and the remaining stock, and the market-clearing price has to agree with agent B's (the pessimist) valuation. This is scenario (i) above. The reason that the pessimist's valuation increases slightly with ω_τ^A is that, as agent A becomes wealthier, she will hold more of the stock, making agent B's portfolio at time τ less risky.

When agent A's wealth share becomes sufficiently high, she will be able to hold the entire stock market without borrowing. In such cases, agent A will hold all of the stock and potentially some bonds, while agent B will invest all of his wealth in the bonds. As long as the stock price is above agent B's private valuation, he will want to short the stock, but the short-sale constraint would be binding (an arbitrarily small short position can lead to negative wealth). Consequently, agent A (the optimist) becomes the marginal investor at τ and the market-clearing price has to agree with her valuation.[35] This equilibrium is qualitatively different. The switch of marginal investor from the pessimist to the optimist means that the price-dividend ratio will be higher and conditional return volatility lower with the circuit breaker.

The analysis above highlights the key differences between the economies with positive and zero bond supply. When $\Delta > 0$, the circuit breaker equilibrium is similar to the case with $\Delta = 0$ as long as agent A's wealth share is not dominant. However, if agent A's wealth share becomes sufficiently high, the property of the equilibrium changes qualitatively, with the price level rising and volatility declining in the presence of a circuit breaker.

To comprehensively examine the different cases, in Figure 8 we present a heat map for average daily return volatility in the economies with and without circuit breakers. The heat map covers a wide range of values for bond supply (Δ) and initial wealth share for agent A (ω). The red region indicates the scenarios under which the introduction of a circuit breaker amplifies market volatility, whereas the blue region indicates the opposite. Quantitatively, the volatility amplification effect occurs for most parameter areas and is stronger when the net bond supply is small relative to the net supply of the stock, and when agent A's initial wealth share is not too low or too high.

For the actual market, the net supply of riskless bonds relative to the stock market is likely small. For example, the market for equity is about \$41 trillion in 2020, while the total size of the U.S. corporate bond market is about \$10

[35] There is also a knife-edge case in which agent A does not hold any bonds, and the stock price is below her valuation, but she faces a binding leverage constraint. In this case, the stock price and the wealth distribution have to satisfy the condition $\omega_\tau^A = \frac{S_\tau}{S_\tau + \Delta}$.

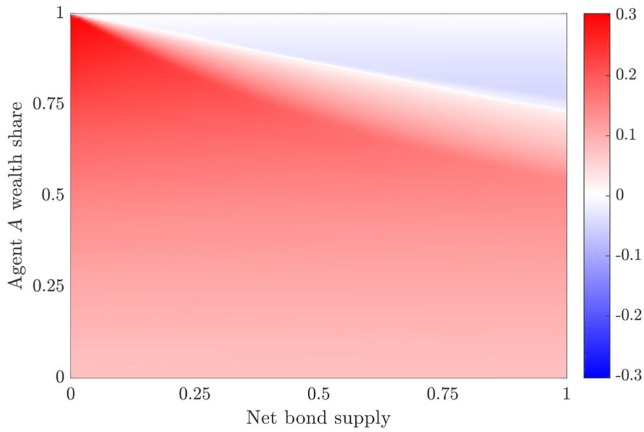

Figure 8. Impact of net bond supply on volatility. Heat map for the ratio of average daily volatilities in the economies with and without circuit breakers. For normalization, we subtract one from the ratios.

trillion, of which about \$3.5 trillion is rated at A or above. If we treat these highly rated bonds as essentially riskless, the relative size of the market for riskless bonds would be 9%. At such a level of riskless bond supply, we still get volatility amplification for most values of ω.

VII. Conclusion

In this paper, we build a dynamic model to examine the mechanism through which MWCBs affect trading and price dynamics in the stock market. As we show, a downside circuit breaker tends to lower the price-dividend ratio and reduce daily price ranges, but increase conditional and realized volatility. It also raises the probability of the stock price reaching the circuit breaker limit as the price approaches the threshold (the so-called magnet effect). The effects of circuit breakers can be further amplified when some agents' willingness to hold the stock is sensitive to recent shocks to fundamentals, which can be due to behavioral biases, institutional constraints, and the like.

Our results demonstrate some of the negative impacts of circuit breakers even without any other market frictions, and they highlight the source of these effects, namely, the tightening of leverage constraints when levered investors cannot rebalance their portfolios during trading halts. These results also shed light on the design of circuit breaker rules. Using historical price data from a period when circuit breakers have not been implemented can lead one to significantly underestimate the likelihood of a circuit breaker being triggered, especially when the threshold is relatively tight.

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Appendix

In this appendix, we present the key components of the proofs for the analytical results in Sections II and III, while deferring some standard derivations and proofs of more technical claims to the Internet Appendix.

A Proof of Proposition 1

From the solution to the social planner's problem (10), deriving the results simply utilizes the (exponential) affine structure of (D, δ), which is given in the Internet Appendix Section II.A.

B Proof of Proposition 2

We first prove that $S_\tau \leq \min\{\widehat{S}^A_\tau, \widehat{S}^B_\tau\}$; we then show that S_τ cannot be strictly less than the minimum value. Suppose the market closes at time $\tau < T$. The two agents' optimization problems at time τ are specified in (19), together with the portfolio constraints $\theta^i_\tau \geq 0$ and $\phi^i_\tau \geq 0$ as implied by the Inada condition. The Lagrangian for agent i is

$$L = \mathbb{E}^i_\tau\left[\ln\left(\theta^i_\tau D_T + \phi^i_\tau\right)\right] + \zeta^i\left(W^i_\tau - \theta^i_\tau S_\tau - \phi^i_\tau\right) + \xi^i_\theta\,\theta^i_\tau + \xi^i_\phi\,\phi^i_\tau, \quad i \in A, B,$$

and the first-order conditions (FOC) with respect to θ^i_τ and ϕ^i_τ are

$$0 = \mathbb{E}^i_\tau\left[\frac{D_T}{\theta^i_\tau D_T + \phi^i_\tau}\right] - \zeta^i S_\tau + \xi^i_\theta, \quad 0 = \mathbb{E}^i_\tau\left[\frac{1}{\theta^i_\tau D_T + \phi^i_\tau}\right] - \zeta^i + \xi^i_\phi. \tag{A1}$$

Furthermore, the market-clearing conditions at time τ are given in (20).

First, consider the events when agent A's valuation of the stock in a single-agent economy is higher than that of agent B, that is, $\widehat{S}^A_\tau(\omega) \geq \widehat{S}^B_\tau(\omega)$, for some ω. This implies that the condition $\delta_\tau \leq \underline{\delta}(\tau) = -\frac{a(\tau,T)}{b(\tau,T)}$ holds for these ω. The other case, where $\widehat{S}^A_\tau(\omega) < \widehat{S}^B_\tau(\omega)$ for some ω, can be proved similarly. We examine the following three scenarios:

(i) Agent B (pessimist) is unconstrained and finds it optimal to hold all of his wealth in the stock, while agent A (optimist) would like to put more than 100% of her wealth in the stock but faces a binding leverage constraint. In this case,

$$\theta^{i^*}_\tau = W_{i,\tau}/S_\tau, \;\; \phi^{A^*}_\tau = \phi^{B^*}_\tau = 0; \quad \xi^A_\theta = \xi^B_\theta = 0, \;\; \xi^A_\phi > 0, \;\; \xi^B_\phi = 0,$$

where $\cdot^*$ denotes optimal holdings. Then from the FOC (A1) of the unconstrained agent B, we get

$$S_\tau = \frac{\mathbb{E}^B_\tau[D_T/(\theta^{B*}_\tau D_T + \phi^{B*}_\tau)]}{\mathbb{E}^B_\tau\left[1/(\theta^{B*}_\tau D_T + \phi^{B*}_\tau)\right]} = \frac{1}{\mathbb{E}^B_\tau[1/D_T]} = \widehat{S}^B_\tau.$$

(ii) The price is so low that both agents would prefer to take levered positions in the stock. But the circuit breaker constrains both agents from borrowing. In this case, both agents submit demands proportional to their wealth:

$$\theta_\tau^{i^*} = W_{i,\tau}/S_\tau, \;\; \phi_\tau^{A^*} = \phi_\tau^{B^*} = 0; \quad \xi_a^A = \xi_a^B = 0, \;\; \xi_b^A > 0, \;\; \xi_b^B > 0,$$

and the market for the stock clears. Hence,

$$S_\tau = \frac{\mathbb{E}_\tau^B[D_T/(\theta_\tau^{B*}D_T + \phi_\tau^{B*})]}{\mathbb{E}_\tau^B\big[1/(\theta_\tau^{B*}D_T + \phi_\tau^{B*})\big] + \xi_b^B} = \frac{1}{\mathbb{E}_\tau^B[1/D_T] + \xi_b^B\theta_\tau^{B*}} < \frac{1}{\mathbb{E}_\tau^B[1/D_T]} = \widehat{S}_\tau^B.$$

(iii) For any $S_\tau > \widehat{S}_\tau^B$, agent B will prefer to hold less than 100% of his wealth in the stock. Agent A will need to take a levered position to clear the stock market but cannot do so because of the circuit breaker. Thus, this cannot be an equilibrium.

Next we prove that the market-clearing price is given by (25) before τ. To this end, agent's indirect utility functions are

$$V^i(W_{\tau\wedge T}^i, \tau\wedge T) = \begin{cases} \ln(W_T^i), & \text{if} \quad \tau \geq T, \\ V^i(W_\tau^i, \tau), & \text{if} \quad \tau < T. \end{cases} \tag{A2}$$

Substituting them into the planner's problem (21) and taking the FOC, we get the wealth of agent A at time $\tau\wedge T$:

$$W_{\tau\wedge T}^A = \frac{\omega S_{\tau\wedge T}}{\omega + (1-\omega)\eta_{\tau\wedge T}}. \tag{A3}$$

Taking the equilibrium allocation $W_{\tau\wedge T}^A$, the state-price density for agent A at time $\tau\wedge T$ can be expressed as her marginal utility of wealth times a constant ξ: $\pi_{\tau\wedge T}^A = \xi\, \frac{\partial V^A(W,\tau\wedge T)}{\partial W}|_{W=W_{\tau\wedge T}^A}$. The price of the stock at any time $t \leq \tau\wedge T$ is then given by

$$S_t = \mathbb{E}_t\left[\frac{\pi_{\tau\wedge T}^A}{\pi_t^A} S_{\tau\wedge T}\right], \tag{A4}$$

where $\pi_t^A = \mathbb{E}_t[\pi_{\tau\wedge T}^A]$. Combining the previous two equations, we confirm (25).

Finally, we prove $S_\tau = \min\{\widehat{S}_\tau^A, \widehat{S}_\tau^B\}$ when $\tau < T$. Given the Brownian filtration, it follows from the martingale representation theorem (see, e.g., Karatzas and Shreve (1991, Chap. 3, Th. 4.15)) that both $\mathbb{E}_t[\pi_{\tau\wedge T}^A S_{\tau\wedge T}]$ and $\pi_t^A = \mathbb{E}_t[\pi_{\tau\wedge T}^A]$ in (A4) can be represented as stochastic integrals with respect to the Brownian motion Z_t; in particular, they have continuous paths for $t \in [0, \tau\wedge T]$. Therefore, S in (A4) is also continuous on the same time interval. Suppose that $S_\tau < \min\{\widehat{S}_\tau^A, \widehat{S}_\tau^B\}$ with positive probability under $\mathbb{P}$. Due to the path continuity of S, there exists $t < \tau\wedge T$, that is, before the circuit breaker is triggered, such that $S_t < \min\{\widehat{S}_t^A, \widehat{S}_t^B\}$ with positive probability under $\mathbb{P}$. However, since neither agent is constrained before market closure, both would want to take on leverage to invest in the undervalued stock when the stock price is less than the pessimistic agent's valuation. This cannot be an equilibrium. Therefore, $S_\tau = \min\{\widehat{S}_\tau^A, \widehat{S}_\tau^B\}$.

C Proof of Proposition 3

Recall from (15) that the definition of τ' in (28) is equivalent to

$$\tau' = \inf\{t \geq 0 \,:\, \widehat{S}_t^{min} \leq \underline{S}\}. \tag{A5}$$

We first show that the map F admits a fixed point. To this end, when $S_0 = 0$, $\underline{S} = (1-\alpha)S_0 = 0$, we have $\underline{D} \equiv 0$, and hence $\tau' = \infty$ and $F(0) = [\omega\mathbb{E}[D_T^{-1}] + (1-\omega)\mathbb{E}^B[D_T^{-1}]]^{-1} > 0$. When S_0 is sufficiently large such that $\underline{D}(0, \delta_0) > D_0$, then $\tau' = 0$ almost surely. As a result,

$$F(S_0) = \widehat{S}_0^{min} < \underline{D}(0, \delta_0) \cdot \begin{cases} e^{(\mu-\sigma^2)T}, & \text{if} \quad \delta_0 \geq \underline{\delta}(0), \\ e^{(\mu-\sigma^2)T - a(0,T) - b(0,T)\delta_0}, & \text{if} \quad \delta_0 \leq \underline{\delta}(0), \end{cases} = \underline{S} \;\leq\; S_0.$$

We have therefore shown that $F(0) > 0$ and $F(S_0) < S_0$ for sufficiently large S_0. If F is also continuous, then there exists at least one fixed point S_0 such that $F(S_0) = S_0$. The continuity of F is proved in the Internet Appendix Section II.B.

Finally, for a fixed point S_0, introduce S' as in (29); we prove that S' reaches the threshold $\underline{S}$ at τ' for the first time. Define

$$\sigma = \inf\{t \geq 0 \,:\, S_t' \leq \underline{S}\}.$$

Given that $S'_{\tau'} = \underline{S}$ when $\tau' < T$, we have $\sigma \leq \tau'$. We only need to show that σ cannot be smaller than τ' with positive probability. Assuming otherwise, that is, $\sigma < \tau' < T$ happens with positive probability. Then, $S'_\sigma = \underline{S}$ when $\sigma < \tau' < T$. In this case, (29) implies that

$$\text{either} \quad \mathbb{E}_\sigma[(\widehat{S}^{min}_{\tau'\wedge T})^{-1}] \geq \underline{S}^{-1} \quad \text{or} \quad \mathbb{E}^B_\sigma[(\widehat{S}^{min}_{\tau'\wedge T})^{-1}] \geq \underline{S}^{-1}. \tag{A6}$$

However, $\widehat{S}^{min}_{\tau'\wedge T} \geq \underline{S}$ and this inequality is strict with positive probability (this situation happens when $\tau' = T$). Hence, $\mathbb{E}_\sigma[(\widehat{S}^{min}_{\tau'\wedge T})^{-1}] < \underline{S}^{-1}$ and $\mathbb{E}^B_\sigma[(\widehat{S}^{min}_{\tau'\wedge T})^{-1}] < \underline{S}^{-1}$, contradicting (A6). Therefore, σ and τ' must be the same, and τ' satisfies (8).

D Proof of Proposition 4

Lemma 1 is proved in the Internet Appendix Section II.C. In the proof, we first show that $\underline{\delta}(t) > 0$ for any $t < T$. Therefore, agent B must be the marginal agent if $\tau = 0$, because $\delta_0 = 0 < \underline{\delta}(t)$. When $0 < \tau < T$, using the explicit solution of (5) with $\delta_0 = \bar{\delta} = 0$, we prove that $\delta_\tau < \underline{\delta}(\tau)$. Hence, agent B is the marginal agent in this case as well.

We now prove Proposition 4 first for constant δ then stochastic δ.

Constant δ. Let us consider the case in which δ is a negative constant, that is, agent B is more pessimistic. We prove in the Internet Appendix Section II.D that the map F is decreasing, and hence there can be only one fixed point for F.

Stochastic δ. We first claim the following result, proved in the Internet Appendix Section II.E.

LEMMA A1: *Suppose that $\bar{\delta} = \delta_0 = 0$ and $\omega < 1 - \alpha$. Then, $F'(S_0) < 1$ for* any *fixed point S_0 of F.*

With the previous result, now suppose that F admits more than one fixed point, say, S_0 and $\widetilde{S}_0$. Because $F'(S_0)$ and $F'(\widetilde{S}_0) < 1$, the continuous function $S \mapsto F(S)$ must cross the 45° line $S \mapsto S$ at a point S_0^{mid} between S_0 and $\widetilde{S}_0$. The point S_0^{mid} is another fixed point for F, but $F'(S_0^{mid})$ must be at least one to cross the 45° line. This contradicts $F'(S_0^{mid}) < 1$. Therefore, there can be only one fixed point for F.

E Proofs for Section III

Proof of Proposition 5: Comparing (16) and (30), the statement is equivalent to $\mathbb{E}_t[\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}]] > \mathbb{E}_t[D_T^{-1}]$. Due to the tower property of conditional expectation, it suffices to prove

$$\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}] \geq \mathbb{E}_{\tau\wedge T}[D_T^{-1}], \tag{A7}$$

where the inequality is strict with positive probability. We show later in the proof of Proposition 6 that stock volatility is strictly positive, and hence the circuit breaker is triggered before T with positive probability under both $\mathbb{P}$ and $\mathbb{P}^B$. Due to (15), (A7) is equivalent to $\widehat{S}^B_{\tau\wedge T} \leq \widehat{S}^A_{\tau\wedge T}$, which holds for negative constant δ or stochastic δ due to Lemma 1. Therefore, (A7) is confirmed.

Proof of Proposition 6: Other than the results stated in the proposition, we also have

$$\lim_{D_t\uparrow\infty} \widehat{S}_t - S_t = 0 \quad \text{and} \quad \lim_{D_t\uparrow\infty} \sigma_t^S = \lim_{D_t\uparrow\infty} \sigma_t^{\widehat{S}} = \sigma, \quad \text{for any } t < T.$$

We prove these additional results in what follows as well.

Statements in (i). The statements and $\lim_{D_t\uparrow\infty} \widehat{S}_t - S_t = 0$ are proved in the Internet Appendix Section II.F by examining the limiting behavior of $D_t \uparrow \infty$ and $D_t \downarrow \underline{D}_t$.

To prove the statements in (ii) and (iii), we introduce

$$y_t \equiv \left(\frac{\sigma}{2} - \frac{\delta}{\sigma}\right)t + Z_t \quad \text{and} \quad d \equiv \frac{1}{\sigma}\log\left[(1-\alpha)S_0 e^{-(\mu-\sigma^2+\delta)T}\right].$$

We then have $D_t = e^{(\mu-\sigma^2+\delta)t+\sigma y_t}$, $\underline{D}_t = e^{(\mu-\sigma^2+\delta)t+\sigma d}$, and $\tau = \inf\{s \geq t : y_s \leq d\}$ is the circuit breaker triggering time. Denote

$$f^A(y_t,t) \equiv \mathbb{E}_t\left[\mathbb{E}^B_{\tau\wedge T}[D_T^{-1}]\right], \quad \widehat{f}^A(y_t,t) \equiv \mathbb{E}_t[D_T^{-1}], \quad \text{and} \quad f^B(y_t,t) \equiv \mathbb{E}^B_t[D_T^{-1}]. \tag{A8}$$

It then follows from (16) and (30) that

$$\widehat{S}_t = \left[\omega^A \widehat{f}^A(y_t,t) + \omega^B f^B(y_t,t)\right]^{-1}, \quad S_t = \left[\omega^A f^A(y_t,t) + \omega^B f^B(y_t,t)\right]^{-1}. \tag{A9}$$

The functions $\widehat{f}^A$ and f^B have explicit expressions

$$\widehat{f}^A(y_t,t) = \mathbb{E}_t[D_T^{-1}] = e^{-(\mu-\sigma^2+\delta)T}\mathbb{E}_t\left[e^{-\sigma y_T}\right] = e^{-(\mu-\sigma^2+\delta)T-\sigma y_t+\delta(T-t)}, \tag{A10}$$

$$f^B(y_t,t) = e^{-(\mu-\sigma^2+\delta)T-\sigma y_t}. \tag{A11}$$

The function f^A also admits a closed-form expression presented in Proposition IA1.

Statements in (ii). Because the volatility of y_t is one, the volatility of $\widehat{S}$ is $\partial_y \ln(\widehat{S}_t)$. It follows from (A9) that

$$\begin{aligned}
\sigma_t^{\widehat{S}} &= \frac{\partial_y \widehat{S}_t}{\widehat{S}_t} = -\frac{\partial_y\left(\omega_t^A \widehat{f}^A(y_t,t) + (1-\omega_t^A)f^B(y_t,t)\right)}{\omega_t^A \widehat{f}^A(y_t,t) + (1-\omega_t^A)f^B(y_t,t)} \\
&= -\frac{\partial_y \omega_t^A(\widehat{f}^A - f^B)}{\omega_t^A \widehat{f}^A + (1-\omega_t^A)f^B} - \frac{\omega_t^A \widehat{f}^A}{\omega_t^A \widehat{f}^A + (1-\omega_t^A)f^B}\frac{\partial_y \widehat{f}^A}{\widehat{f}^A} - \frac{(1-\omega_t^A)f^B}{\omega_t^A \widehat{f}^A + (1-\omega_t^A)f^B}\frac{\partial_y f^B}{f^B} \\
&= \frac{(1-\omega_t^A)\omega_t^A f^B}{\omega_t^A \widehat{f}^A + (1-\omega_t^A)f^B}\frac{\delta}{\sigma}\left(e^{\delta(T-t)} - 1\right) + \sigma,
\end{aligned} \tag{A12}$$

where the third identity follows from $-\partial_y \omega_t^A = \omega_t^A(1-\omega_t^A)\frac{\delta}{\sigma}$, $-\frac{\partial_y \widehat{f}^A}{\widehat{f}^A} = -\frac{\partial_y f^B}{f^B} = \sigma$, and $\frac{\widehat{f}^A}{f^B} = e^{\delta(T-t)}$. Note that $\delta(e^{\delta(T-t)} - 1) > 0$ when $t < T$. We then have from (A12) that $\lim_{y\downarrow d}\sigma_t^{\widehat{S}} > \sigma$, for $t < T$. When $y \uparrow \infty$, $f^B \downarrow 0$. Therefore $\sigma^{\widehat{S}} \to \sigma$.

To prove the statements about σ^S in (ii), we collect several properties of functions f^A, $\widehat{f}^A$, and f^B in the following result, whose proofs are given in the Internet Appendix Section II.H.

LEMMA A2: *When $\delta < 0$,*

(i) *$f^A(d,t) = f^B(d,t)$ for any $t \leq T$.*

(ii) $f^B(y,t) > f^A(y,t)$ for any $y > d$ and $t < T$, $\lim_{y\uparrow\infty} f^A(y,t) = \lim_{y\uparrow\infty} \widehat{f}^A(y,t)$.

(iii) $\frac{\partial_y f^B(y,t)}{f^B(y,t)} \equiv -\sigma$ and $\lim_{y\uparrow\infty} \frac{\partial_y f^A(y,t)}{f^A(y,t)} = -\sigma$.

(iv) If $\delta \leq -\frac{\sigma^2}{2}$, then $\partial_y f^B(y,t)|_{y=d} > \partial_y f^A(y,t)|_{y=d} > -(\sigma - \frac{2\delta}{\sigma}) f^A(y,t)|_{y=d}$ for any $t < T$;

(v) If $\delta \leq -\frac{\sigma^2}{2}$, then $\partial_y (\frac{\partial_y f^A(y,t)}{f^A(y,t)})|_{y=d} > 0$ for any $t < T$.

Limit of σ^S. Using (30) and following similar derivations in (A12), we obtain

$$\sigma_t^S = \frac{\partial_y S_t}{S_t} = \sigma - \frac{\partial_y \omega_t^A (f^A - f^B)}{\omega_t^A f^A + (1-\omega_t^A) f^B} + \frac{\omega_t^A f^A}{\omega_t^A f^A + (1-\omega_t^A) f^B}\left(-\frac{\partial_y f^A}{f^A} - \sigma\right). \tag{A13}$$

Sending $y_t \downarrow d$ (equivalently $D_t \downarrow \underline{D}_t$), Lemma A2(i) implies

$$\lim_{D_t \downarrow \underline{D}_t} \sigma_t^S = \sigma + \lim_{y_t \downarrow d} \frac{\omega_t^A f^A}{\omega_t^A f + (1-\omega_t^A) f^B}\left(-\frac{\partial_y f^A}{f^A} - \sigma\right). \tag{A14}$$

Comparing the previous expression with (A12), the statement $\lim_{D_t \downarrow \underline{D}_t} \sigma_t^S > \lim_{D_t \downarrow \underline{D}_t} \sigma_t^{\widehat{S}}$ is equivalent to

$$\lim_{y \downarrow d} \frac{\omega_t^A f^A}{\omega_t^A f^A + (1-\omega_t^A) f^B}\left(-\frac{\partial_y f^A}{f^A} - \sigma\right) > \lim_{y \downarrow d} \frac{(1-\omega_t^A)\omega_t^A f^B}{\omega_t^A \widehat{f}^A + (1-\omega_t^A) f^B} \frac{\delta}{\sigma}\left(e^{\delta(T-t)} - 1\right). \tag{A15}$$

This is satisfied when $\delta \leq -\sigma^2$, as proved in the Internet Appendix Section III.

When $y_t \uparrow \infty$ (equivalently $D_t \uparrow \infty$), $\eta_t \to 1$ and $\omega_t^A \to 1$, $\partial_y \omega_t^A \to 0$. Therefore, $\lim_{y\uparrow\infty} \frac{\partial_y f^A}{f^A} = -\sigma$ in Lemma A2(iii) and (A13) together imply $\lim_{D_t \uparrow \infty} \sigma_t^S = \sigma$.

Statements in (iii). We first prove the statements in the limit $\omega \to 1$. We then apply a continuity argument to obtain the statements when ω is sufficiently close to one.

As $\omega \to 1$, we obtain from (IA20) in the Internet Appendix that $\widehat{S}_t / S_t \to e^{-\delta(T-t)} > 1$ due to $\delta < 0$.

As $\omega \to 1$, $\widehat{S}_t = e^{(\mu - \sigma^2)(T-t)} D_t$, and hence $\sigma_t^{\widehat{S}} \equiv \sigma$. We claim that σ_t^S increasingly converges to a limit higher than σ as $D_t \downarrow \underline{D}_t$. To prove this claim, note that when $\omega \to 1$, we have $\omega^A \to 1$, then equation (A9) yields $S_t = (f^A(y_t, t))^{-1}$. Therefore,

$$\lim_{\omega \to 1} \sigma_t^S = -\frac{\partial_y f^A(y_t, t)}{f^A(y_t, t)}.$$

Thanks to Lemma A2(v), $\partial_y(\partial_y f^A/f^A)|_{y=d} > 0$. Therefore, in the limit $\omega \to 1$, σ_t^S increases to its limit as $y_t \downarrow d$. Lemma A2(i), (iii), and (iv) imply that

$$\sigma = -\frac{\partial_y f^B(y,t)|_{y=d}}{f^B(d,t)} < -\frac{\partial_y f^A(y,t)|_{y=d}}{f^A(d,t)} = \lim_{\omega\to 1}\sigma_t^S.$$

Finally, we prove the statements when ω is close to one using a continuity argument in the Internet Appendix Section II.J.

Proof of Proposition 7: Denote $\Delta S_t = S_t - \underline{S}$. Then $h = (\Delta S_t)^2$ from the choice of h. We present the following results, whose proofs are given in the Internet Appendix Section II.K.

LEMMA A3: *When $\delta \le 0$ and $S_t > \underline{S}$, we have the following statements for any $t < T$:*

$$\lim_{S_t \downarrow \underline{S}} p_{(\Delta S_t)^2}(S_t) = 2N\left(-\frac{1}{\underline{S}\sigma_t^S|_{S_t=\underline{S}}}\right), \tag{A16}$$

where $N[\cdot]$ is the distribution function of a standard normal. For the complete market, the same statement holds with volatility replaced by $\sigma_t^{\widehat{S}}|_{\widehat{S}_t=\underline{S}}$ on the right-hand side. Moreover,

$$\partial_{\Delta S_t} p_{(\Delta S_t)^2}(S_t)\big|_{S_t=\underline{S}} = -\frac{\sigma - \frac{2\delta}{\sigma}}{\underline{S}\sigma_t^S|_{S_t=\underline{S}}} N\left(-\frac{1}{\underline{S}\sigma_t^S|_{S_t=\underline{S}}}\right). \tag{A17}$$

In the limit $\omega \to 1$, $\partial_{\Delta S_t}\hat{p}_{(\Delta S_t)^2}(S_t)|_{S_t=\underline{S}} = -\frac{1}{\underline{S}}N(-\frac{1}{\sigma\underline{S}})$.

Because $\delta \le 0$, (A17) shows that $\partial_{\Delta S_t} p_{(\Delta S_t)^2}(S_t)|_{S_t=\underline{S}} < 0$, and hence the statement in (i) immediately follows. For the statement in (ii), equation (A16) shows that the limit of the hitting probability is increasing in volatility at the circuit breaker $\underline{S}$. We have seen from Proposition 6(ii) that volatility at the threshold is higher with the circuit breaker than without. Therefore, $\lim_{S_t\downarrow\underline{S}} p_{(\Delta S_t)^2}(S_t) > \lim_{S_t\downarrow\underline{S}} \hat{p}_{(\Delta S_t)^2}(S_t)$. Then the continuity of the hitting probability in S_t implies that the previous inequality still holds in the prelimit in a neighborhood of $\underline{S}$. When ω is sufficiently close to one, the statement is proved in the Internet Appendix Section II.L.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.