نام و نام خانوادگی: کیمیا مهدی نژاد
شماره دانشجویی: ۹۸۱۲۷۶۲۳۴۹

.۱

$$G_\partial = \int_{-\infty}^{\infty} g_\partial \, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} e^{\frac{-(t-\mu)^2}{2\delta^2}}\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} e^{\frac{-t^2-\mu^2+2t\mu-2\delta^2 j\omega t}{2\delta^2}}\, dt$$

$$= \int_{-\infty}^{\infty} e^{\frac{-t^2+2t(\mu-2\delta^2 j\omega)-\mu^2}{2\delta^2}}\, dt = \int_{-\infty}^{\infty} e^{\frac{-t^2+2t(\mu-2\delta^2 j\omega)-\mu^2}{2\delta^2}}\, dt$$

$$= \int_{-\infty}^{\infty} e^{\frac{-(t^2-2t(\mu-2\delta^2 j\omega)+(\mu-2\delta^2 j\omega)^2)+((\mu-2\delta^2 j\omega)^2-\mu^2)}{2\delta^2}}\, dt$$

$$= \int_{-\infty}^{\infty} e^{\frac{-\left(t-(\mu-2\delta^2 j\omega)\right)^2}{2\delta^2}} e^{\frac{((\mu-2\delta^2 j\omega)^2-\mu^2)}{2\delta^2}}\, dt$$

$$= e^{\frac{((\mu-2\delta^2 j\omega)^2-\mu^2)}{2\delta^2}} \int_{-\infty}^{\infty} e^{\frac{-\left(t-(\mu-2\delta^2 j\omega)\right)^2}{2\delta^2}}\, dt \xRightarrow{guassian\ signal\ integral\ is\ 1}$$

$$e^{\frac{((\mu-2\delta^2 j\omega)^2-\mu^2)}{2\delta^2}} = e^{\frac{-\mu^2}{2\delta^2}} e^{\frac{(\mu-2\delta^2 j\omega)^2}{2\delta^2}}$$

قسمت $e^{\frac{-\mu^2}{2\delta^2}}$ ضریب و قسمت $e^{\frac{(\mu-2\delta^2 j\omega)^2}{2\delta^2}}$ سیگنال گوسی می باشد. (سیگنال گوسی به فرمت $f(x) = ae^{\frac{-(x-b)^2}{2c^2}}$ می باشد.)

.۲

$$x(t) \overset{F}{\rightarrow} X(\omega) \implies X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \cdots + \int_{-5\mathrm{T}_1}^{-3\mathrm{T}_1} 1 \times e^{-j\omega t}\, dt + \int_{-\mathrm{T}_1}^{\mathrm{T}_1} 1 \times e^{-j\omega t}\, dt + \int_{3\mathrm{T}_1}^{5\mathrm{T}_1} 1 \times e^{-j\omega t}\, dt + \cdots$$

$$= \cdots + \frac{-1}{j\omega} e^{-j\omega t}\Big|_{-4\mathrm{T}_1}^{-2\mathrm{T}_1} + \frac{-1}{j\omega} e^{-j\omega t}\Big|_{-\mathrm{T}_1}^{\mathrm{T}_1} + \frac{-1}{j\omega} e^{-j\omega t}\Big|_{2\mathrm{T}_1}^{4\mathrm{T}_1} + \cdots$$

$$\xRightarrow{\frac{-1}{j\omega}e^{-j\omega t}\Big|_{-4\mathrm{T}_1}^{-2\mathrm{T}_1}=\frac{-\mathrm{T}_1}{2\pi j}\left(e^{-j\times\frac{2\pi}{\mathrm{T}_1}\times-2\mathrm{T}_1}-e^{-j\times\frac{2\pi}{\mathrm{T}_1}\times-4\mathrm{T}_1}\right)=\frac{-\mathrm{T}_1}{2\pi j}\left(e^{4\pi j}-e^{8\pi j}\right)=\frac{-\mathrm{T}_1}{2\pi j}e^{\frac{2}{4}}}$$

$$= \cdots + \frac{-\mathrm{T}_1}{2\pi j}e^{\frac{2}{4}} + \frac{-\mathrm{T}_1}{2\pi j}e^{\frac{1}{-1}} + \frac{-\mathrm{T}_1}{2\pi j}e^{\frac{4}{2}} + \cdots = \frac{-\mathrm{T}_1}{2\pi j}\left(\cdots + e^{\frac{2}{4}} + e^{\frac{1}{-1}} + e^{\frac{4}{2}} + \cdots\right)$$

.۴

$$M^n\big(x(t)\big) = x(t) * h(nt) \xRightarrow{x(t)*y(t)\overset{F}{\rightarrow}X(\omega)\,Y(\omega)} F\left(M^n\big(x(t)\big)\right) = X(\omega)\,H(n\omega)$$

$$\xRightarrow{h(nt)=\ x(t)*h\left((n-1)t\right)\overset{F}{\rightarrow}H(n\omega)=X(\omega)\ H\left((n-1)\omega\right)} F\left(M^n\big(x(t)\big)\right) = X(\omega)^n\,H(\omega)$$

.۵

الف)

$$\int_{-\infty}^{\infty} f(t)\,g^*(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,G^*(\omega)\,d\omega$$

$$\xRightarrow{f(t)=\frac{1}{2\pi}\int_{\omega}F(\omega)\,e^{-j\omega t}\,d\omega\ ,g(t)=\frac{1}{2\pi}\int_{\omega}G(\omega)\,e^{-j\omega t}\,d\omega \Rightarrow g^*(t)=\frac{1}{2\pi}\int_{\omega}G^*(\omega)\,e^{-j\omega t}\,d\omega}$$

$$\Rightarrow \int_{-\infty}^{\infty} f(t)\,g^*(t)\,dt = \int_t f(t)\left(\frac{1}{2\pi}\int_{\omega}G(\omega)\,e^{-j\omega t}\,d\omega\right)^* dt$$

$$= \int_t f(t)\left(\frac{1}{2\pi}\int_{\omega}G^*(\omega)\,e^{-j\omega t}\,d\omega\right)dt = \frac{1}{2\pi}\int_t f(t)\int_{\omega}G^*(\omega)\,e^{-j\omega t}\,d\omega\ dt$$

$$= \frac{1}{2\pi}\int_t\int_{\omega} f(t)\,G^*(\omega)\,e^{-j\omega t}\,d\omega\ dt = \frac{1}{2\pi}\int_{\omega}\int_t f(t)\,G^*(\omega)\,e^{-j\omega t}\,dt\ d\omega$$

$$= \frac{1}{2\pi}\int_{\omega}\int_t G^*(\omega)\,f(t)\,e^{-j\omega t}\,dt\ d\omega = \frac{1}{2\pi}\int_{\omega}G^*(\omega)\,F(\omega)\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, G^*(\omega)\, d\omega$$

ب)

$$\xRightarrow{f(t)=\frac{\sin t}{t},\, g^*(t)=e^{-2t}\,u(t)} \int_{-\infty}^{\infty} \frac{\sin t}{t}\; e^{-2t}\,u(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F\left(\frac{\sin t}{t}\right) F\big(e^{-2t}\,u(t)\big)\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\, rect\left(\frac{\omega}{2}\right) \frac{1}{2+j\omega}\, d\omega = \pi\int_{-1}^{1} rect\left(\frac{\omega}{2}\right)\frac{1}{2+j\omega}\, d\omega = \pi\int_{-1}^{1}\frac{1}{2+j\omega}\, d\omega$$

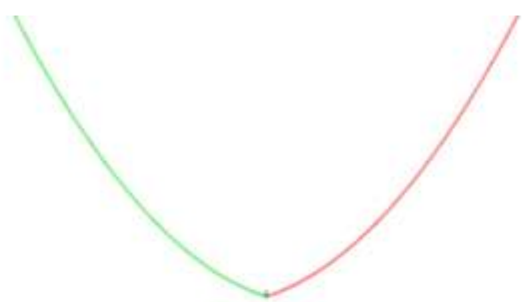
$$= \pi\, \frac{\log(2+j\omega)}{j}\Bigg|_{-1}^{1} = \pi\left(\frac{\log(2+j)}{j} - \frac{\log(2-j)}{j}\right) = \frac{\pi}{j}\log\left(\frac{2+j}{2-j}\right)$$

۶.

الف)

$$X(\omega) = \begin{cases} 1+\dfrac{t}{2\pi}\ ; & -2\pi \le t \le 0 \\ 1-\dfrac{t}{2\pi}; & 0 \le t \le 2\pi \end{cases} \xRightarrow{y(t)=x(t)*x(t) \xrightarrow{F} Y(\omega)=X(\omega)\,X(\omega)=\left(X(\omega)\right)^2}$$

$$\left(X(\omega)\right)^2 = \begin{cases} 1+\dfrac{t}{\pi}+\dfrac{t^2}{4\pi^2}\ ; & -2\pi \le t \le 0 \\ 1-\dfrac{t}{\pi}+\dfrac{t^2}{4\pi^2}; & 0 \le t \le 2\pi \end{cases}$$



ب)

$$z(t) = y(t)\ \cos 100t \xRightarrow{c(t)=\cos 100t} Z(\omega) = \frac{1}{2\pi}\ Y(\omega)\ * C(\omega)$$

$$\cos^2 100t = \frac{1+\cos 200t}{2} = \frac{1}{2} + \frac{\cos 200t}{2}$$

$$\Rightarrow C(\omega) = \int_{-\infty}^{\infty} \frac{1}{2} \times\ e^{-j\omega t}\, dt + F\left(\frac{\cos 200t}{2}\right)$$

$$\xRightarrow{Cos\ Domain\ is\ -2\pi\ to\ 2\pi} \frac{-1}{2j\omega} e^{-j\omega t}\Big|_{-2\pi}^{2\pi} + \frac{\pi}{2}\big(\delta(\omega-200)+\delta(\omega+200)\big)$$

$$= \frac{-1}{2j\omega}\ \left(e^{-2\pi j\omega} - e^{2\pi j\omega}\right) + \frac{\pi}{2}\big(\delta(\omega-200)+\delta(\omega+200)\big)$$

$$= \frac{-1}{2j\omega} e^{-1} + \frac{\pi}{2}\big(\delta(\omega-200)+\delta(\omega+200)\big)$$

$$\Rightarrow Z(\omega) = \frac{1}{2\pi}\ \big(X(\omega)\big)^2\ * \left(\frac{-1}{2j\omega} e^{-1} + \frac{\pi}{2}\big(\delta(\omega-200)+\delta(\omega+200)\big)\right)$$

$$= \frac{1}{2\pi}\ \big(X(\omega)\big)^2\ * \frac{-1}{2j\omega} e^{-1} + \frac{1}{2\pi}\frac{\pi}{2}\big(X^2(\omega-200)+X^2(\omega+200)\big)$$

$$\frac{1}{2\pi}\ \big(X(\omega)\big)^2\ * \frac{-1}{2j\omega} e^{-1} = \frac{1}{2\pi}\int \big(X(\omega)\big)^2 \frac{-1}{2j\omega} e^{-1-\tau} d\tau$$

$$= \frac{-1}{4\pi j\omega}\int \big(X(\tau)\big)^2 e^{-1-\tau} d\tau = \frac{-1}{4\pi j\omega}\left(\int_{-2\pi}^{0}\left(1+\frac{\tau}{2\pi}\right)^2 e^{-1-\tau} d\tau + \int_{0}^{2\pi}\left(1-\frac{\tau}{2\pi}\right)^2 e^{-1-\tau} d\tau\right)$$