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Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes

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For complex systems that experience Multiple Dependent Competing Failure Processes (MDCFP), the dependency among the failure processes presents challenging issues in reliability modeling. This article, develops reliability models and preventive maintenance policies for systems subject to MDCFP. Specifically, two dependent/correlated failure processes are considered: *soft failures* caused jointly by continuous smooth degradation and additional abrupt degradation damage due to a shock process and *catastrophic failures* caused by an abrupt and sudden stress from the same shock process. A general reliability model is developed based on degradation and random shock modeling (i.e., extreme and cumulative shock models), which is then extended to a specific model for a linear degradation path and normally distributed shock load sizes and damage sizes. A preventive maintenance policy using periodic inspection is also developed by minimizing the average long-run maintenance cost rate. The developed reliability and maintenance models are demonstrated for a micro-electro-mechanical systems application example. These models can also be applied directly or customized for other complex systems that experience multiple dependent competing failure processes.

Keywords: Multiple dependent competing failure processes, degradation, random shocks, preventive maintenance, micro-electro-mechanical systems reliability

1. Introduction

The failure of complex systems is often the result of forces and stresses generated either during the intended operation of the systems themselves or from external sources. Common failure mechanisms and causes include wear degradation, corrosion, fracture, shock loads, fatigue, etc. For systems that suffer multiple failure mechanisms, competing failure processes may occur and any of them can cause the system to fail. These multiple competing failure processes may be independent or dependent. When they are *dependent*, it creates a unique and challenging problem to analyze and predict the system reliability performance. In this article, we develop models to analyze the reliability of complex systems that experience Multiple Dependent Competing Failure Processes (MDCFP), particularly due to degradation and/or shock loads.

Reliability analysis for systems that experience only degradation or only random shocks has been extensively explored in the literature. As an effective alternative to compensate for insufficient failure data, degradation modeling can provide a greater understanding of the physics of failure and offers an indirect method to predict reliability. Therefore, degradation modeling and analysis has attracted considerable attention from researchers in statistics and reliability since the early 1990s (see, for example, Lu and Meeker (1993), Singpurwalla (1995), and Kharoufeh and Cox (2005)). Random shock modeling has been extensively studied for cases in which devices are exposed to external shock environments, such as sudden and unexpected usage loads and accidental dropping onto hard surfaces. In the literature, there are four categories of random shock models; (i) *extreme shock model*: failure occurs when the magnitude of any shock exceeds a specified threshold; (ii) *cumulative shock model*: failure occurs when the cumulative damage from shocks exceeds a critical value; (iii) *run shock model*: failure occurs when there is a run of k shocks exceeding a critical magnitude; and (iv) δ-*shock model*: failure occurs when the time lag between two successive shocks is shorter than a threshold δ (Nakagawa, 2007; Liu *et al.*, 2008). Optimal maintenance models have been established for different random shock models (Wang and Zhang, 2005; Chien *et al.*, 2006).

For a failure process involving both degradation and shocks, Klutke and Yang (2002) derived an availability model for an inspected system subject to continuous smooth degradation and shocks that also cause additional

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degradation damage. Kharoufeh *et al.* (2006) derived the system lifetime distribution and the limiting average availability for a similar failure process. Recently, reliability modeling for systems with multiple *independent* competing failure processes has been investigated by several researchers. This work includes on independent multiple catastrophic and degradation failure processes (Huang and Askin, 2003) and two random shock processes using the extreme shock model and the δ-shock model (Wang and Zhang, 2005). For three independent failure processes including two degradation processes and one random shock process, Li and Pham (2005a, 2005b) analyzed the reliability for a multi-state degraded system and developed an inspection-maintenance model.

However, relatively little research has been devoted to the reliability analysis of systems with MDCFP. The dependency among the failure processes presents challenging issues in reliability modeling. In this article, we develop reliability models and preventive maintenance policies for complex systems that experience MDCFP. Specifically, we consider two dependent/correlated failure processes: the *soft failure* process caused by continuous smooth degradation and additional abrupt degradation damages due to a shock process and *catastrophic failures* caused by the stress from the same shock process. These two failure processes are *competing*, which means that either failure processes can cause the component or system to fail. The failure is caused by whichever failure process reaches the critical threshold first. In addition, these two failure processes are *dependent* or *correlated* because the effects from the same shock process contribute to both failure processes. For this article, dependence is considered in a probabilistic or statistical sense and the failure times of the two failure mechanisms are taken to be *dependent*. This does not imply that one failure causes the other or is physically dependent on the other. They could also be called correlated failure processes.

Different maintenance strategies for degrading systems have been extensively examined in the literature; see, for example, Li and Pham (2005a), Liao *et al.* (2006), Tang and Lam (2006), Zequeira and Bérenguer (2006), and Zhu *et al.* (2010), among others. The maintenance model described in this article differs from other studies in that we consider an inspection-based maintenance policy for *two competing dependent failure processes* including a degrading process and a sudden failure process.

We demonstrate the developed reliability model and maintenance optimization for MDCFP on a realistic example considering a Micro-Electro-Mechanical Systems (MEMS) device. MEMS reliability studies are becoming an increasingly important factor in creating the conditions to achieve widespread acceptance of MEMS devices, both for large-volume commercialization and for critical applications. Experiments have been performed to investigate failure modes and mechanisms for MEMS devices, particularly in extreme environments (e.g., shock, vibration; Miller *et al.*, 1998). The issue of multiple failure processes is of particular interest to MEMS researchers, as it is a critical problem that MEMS have experienced in the field. According to data collected during reliability tests on micro-actuator systems performed by the Sandia National Laboratory (Tanner and Dugger, 2003), both degradation failures caused by wear as well as random shocks and catastrophic failures caused by random shocks have a significant impact on the reliability of MEMS. These two competing failure processes are *dependent* in that the same random shock process contributes to both cumulative wear damage and catastrophic failure. Few studies have been performed that probabilistically analyze degradation and shock events to alow the design of dynamically reliable MEMS devices. Preliminary studies have been reported in Peng *et al.* (2009a) and Feng and Coit (2010), in which a model was developed specifically for the MEMS reliability problem. In Feng and Coit (2010), some key theoretical arguments were presented and formulas for the reliability were presented; however, no numerical examples, mathematical developments, or maintenance models were constructed. In this article, a general reliability model is developed based on general degradation path and random shock models, which can be readily applied to many systems that experience multiple *dependent* failure processes.

The remainder of this article is organized as follows. Section 2 lists the assumptions and notations used in the reliability and maintenance modeling studies. In Section 3, we develop the reliability models for systems experiencing MDCFP due to degradation and random shocks. Section 4 presents the periodic inspection-maintenance strategy with an optimal inspection interval that is obtained by minimizing the average long-run maintenance cost per unit time. In Section 5, the developed reliability model and maintenance strategy are implemented for an example MEMS application. Section 6 summarizes the article and concluding remarks are made.

2. System description

As shown in Fig. 1, a system may fail due to two competing yet dependent failure modes that involve the same shock process: (i) soft failures caused jointly by continuous wear degradation with additional abrupt degradation damages from a random shock process; and (ii) catastrophic/hard failures caused by stress from the same random shock process. The continuous wear degradation is an aging process during field operation. Shock loads can cause additional abrupt damage that contributes to the degradation process; e.g., wear debris of a relatively large size due to a shock load. Soft failure occurs when a critical threshold H is exceeded by the total wear volume (e.g., stiction, immobilization due to wear debris). In addition, the same random shock process can cause catastrophic/hard failures when the load magnitude from a single shock exceeds a critical

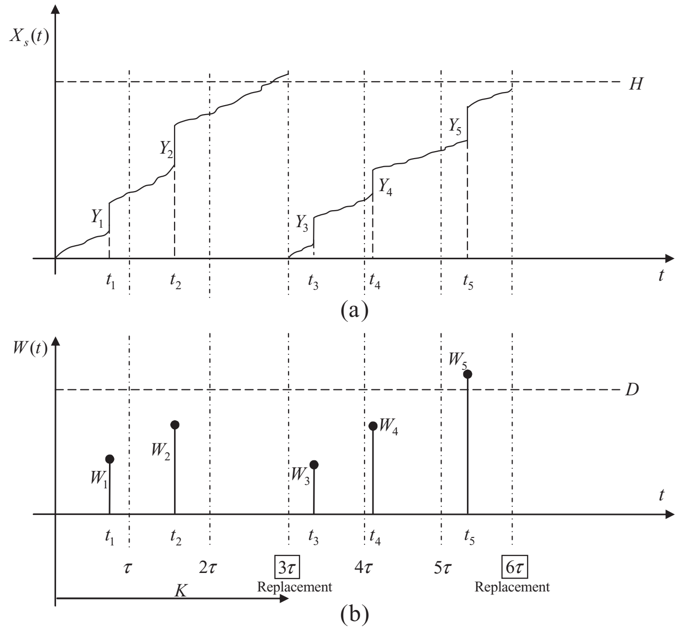

Fig. 1. Two dependent competing failure processes: (a) soft failure process and (b) hard failure process.

strength level D (e.g., fracture, delamination). The system fails when either of the two competing failure modes occurs.

For the maintenance policy, we consider a non-repairable system. A periodic inspection-maintenance policy can be implemented to minimize the impact of unexpected failures and related costs. The system is inspected at periodic intervals, τ, and the inspections are assumed to be instantaneous, perfect and non-destructive. No continuous monitoring is performed on the system due to cost concerns and impracticality. If the system fails, it remains idle and no maintenance actions are taken until next scheduled inspection. For example, for systems that are operating in remote areas, it is very expensive to dispatch maintenance crews to repair a failed system until a scheduled periodic maintenance/inspection time. The inspection determines whether the system has failed due to one of the two competing failure modes; i.e., soft failures and catastrophic failures. If one of the failure modes has occurred, the system is replaced instantly with a new one. If neither of the failure modes occurs, the system remains undisturbed until the next inspection period.

2.1. *Notation*

The notation used in formulating the reliability and maintenance models in Sections 3 and 4 is now listed.

D = maximal fracture strength or threshold for catastrophic failures;
$N(t)$ = number of shock loads that have arrived by time t;
λ = arrival rate of random shocks;
W_i = size/magnitude of the ith shock load;
$F_W(w)$ = cumulative distribution function (cdf) of W_i;
H = critical wear degradation failure threshold;
$X(t)$ = wear volume due to continuous degradation at t;
$X_S(t)$ = total wear volume at t due to both continual wear and instantaneous damage;
Y_i = damage size caused by the ith shock load;
$S(t)$ = cumulative shock damage size at t;
$G(x, t)$ = cdf of $X(t)$ at t;
$F_X(x, t)$ = cdf of $X_S(t)$ at t;
$f_Y(y)$ = probability density function (pdf) of Y_i;
$f_Y^{\langle k \rangle}(y)$ = pdf of the sum of k independent and identically distributed (i.i.d.) Y_i variables
$f_T(t)$ = pdf of the failure time, T;
$F_T(t)$ = cdf of the failure time, T;
τ = periodic inspection interval;
$C(t)$ = cumulative maintenance cost by time t;
$CR(\tau)$ = average long-run maintenance cost rate;
$E[K]$ = expected value of the first renewal cycle length, K;
$E[TC]$ = expected value of the total maintenance cost of the first renewal cycle, TC;
N_I = number of inspections during the first renewal cycle;
ρ = system downtime;
C_I = inspection cost per unit;
C_R = replacement cost per unit;
C_F = penalty cost rate during downtime;

2.2. *Assumptions*

The specific assumptions used for the reliability and maintenance modeling in this article are summarized as follows. The details of each assumption are explained in the corresponding sections.

1. Soft failure occurs when the overall degradation is beyond a threshold value H. The total degradation is accumulated by both continuous degradation over time and abrupt damage due to random shocks (according to a *cumulative shock* model).
2. Hard/catastrophic failure occurs when the shock load itself exceeds the maximum strength of the materials D (according to an *extreme shock* model).
3. Random shocks arrive according to a Poisson process. W_i are used to denote the sizes/magnitudes of shock loads themselves, which are i.i.d. random variables. Y_i are used to denote the damage sizes caused by the ith shock load, which are also i.i.d. random variables.
4. The cumulative distribution function for the wear volume due to continuous degradation, $X(t)$, is denoted as $G(x, t)$ in general. In the more specific model, the continuous degradation follows a linear path $X(t) = \varphi + \beta t$, where the initial value φ is a constant and the

degradation rate β follows a normal distribution, $\beta \sim N(\mu_\beta, \sigma_\beta^2)$.

5. The system is inspected at periodic intervals. Inspections are assumed to be instantaneous, perfect, and non-destructive. For this model formulation, no continuous monitoring is performed on the system due to cost concerns and practicality issues.
6. If the system fails, it will remain idle and no maintenance actions will be taken until the next scheduled inspection. For example, for a system operating in remote areas it is very expensive and impractical to repair a failed system until a scheduled inspection time.
7. The system is non-repairable. If it is detected to have failed, it will be replaced instantly with a new one.

3. Reliability analysis for MDCFP with degradation and shocks

3.1. *Modeling for catastrophic failures due to shocks*

Catastrophic/hard failures can occur according to one of the four random shock models: (i) extreme shock model; (ii) cumulative shock model; (iii) run shock model; or (iv) δ-shock model. Regardless of the physical origin, shock loads can be characterized by their shapes, magnitudes, durations, and locations. The commonalities associated with different shock models imply that the same mathematical framework can be used to analyze the reliability of systems exposed to different types of shocks. Mathematically, shocks arriving at random time intervals can be modeled as a renewal process with exponential, Weibull, or gamma-distributed interarrival times. The sizes of shocks can be modeled by a continuous random variable such as a normal or exponential distributed variable. The magnitudes of shock damage that influence wear degradation can also be modeled as i.i.d. random variables.

Figure 1(b) shows an extreme shock model; that is, a system fails due to fracture when the shock load/stress exceeds the maximal fracture strength D. In this model, shocks arrive according to a Poisson process $\{N(t), t \geq 0\}$ with rate λ. The size of the ith shock load arriving at t_i is denoted as W_i for $i = 1, 2, \ldots, \infty$. If the shock loads are i.i.d with $F_W(w)$ as the cdf of W_i, then according to the stress–strength model (for more see Kotz *et al.* (2003)), the probability that the device survives the applied stress from the ith shock is

$$P(W_i < D) = F_W(D) \quad \text{for} \quad i = 1, 2, \ldots, \infty. \tag{1}$$

If the W_i are assumed to be i.i.d. random variables distributed as a normal distribution, $W_i \sim N(\mu_W, \sigma_W^2)$, then the probability of survival becomes (Feng and Coit, 2010):

$$P_\mathrm{L} = F_W(D) = \Phi\left(\frac{D - \mu_W}{\sigma_W}\right) \quad \text{for} \quad i = 1, 2, \ldots, \infty, \tag{2}$$

where $\Phi(\cdot)$ is the cdf of a standard normally distributed variable.

3.2. *Modeling for soft failures due to degradation and shocks*

Soft failures can occur when the overall degradation is beyond a threshold level H, which can be a constant or a random variable. As shown in Fig. 1(a), the total degradation accrued by the system, $X_\mathrm{s}(t)$, is the sum of the degradation due to continual *wear* and the instantaneous damages due to shocks. The degradation due to wear over time, $X(t)$, may follow one of the various degradation path models, such as a linear degradation path with random coefficients (Christer and Wang, 1992; Lu and Meeker, 1993) or a randomized logistic degradation path (Li and Pham, 2005b). For illustration, a linear degradation path is shown in Fig. 1(a), $X(t) = \varphi + \beta t$, where the initial value φ and the degradation rate β can be constants or random variables.

In addition, degradation changes or shifts can accumulate instantaneously (e.g., in the form of debris) when a shock arrives. The instantaneous increases in the total degradation are measured by the shock damage sizes, which are assumed to be i.i.d random variables, denoted as Y_i for $i = 1, 2, \ldots, \infty$. The cumulative damage size due to random shocks until time t, $S(t)$, is given as

$$S(t) = \begin{cases} \sum_{i=1}^{N(t)} Y_i, & \text{if} \quad N(t) > 0, \\ 0, & \text{if} \quad N(t) = 0, \end{cases} \tag{3}$$

where $N(t)$ is the total number of shocks that have arrived by time t. This implies a *cumulative shock* model. The overall degradation of the system, considering both wear degradation and random shock damages, is expressed as $X_\mathrm{s}(t) = X(t) + S(t)$. Then the probability that the total degradation at time t is less than x, $F_X(x, t)$, can be derived as

$$F_X(x, t) = P(X_\mathrm{s}(t) < x) = \sum_{i=0}^{\infty} P(X(t) + S(t) < x \mid N(t) = i) P(N(t) = i). \tag{4}$$

Furthermore, if we consider $G(x, t)$ to be the cdf of $X(t)$ at t, $f_Y(y)$ to be the pdf of Y_i, and $f_Y^{<k>}(y)$ to be the pdf of the sum of k i.i.d. Y_i variables, then the cdf of $X_\mathrm{s}(t)$ in Equation (4) can be derived using a convolution integral:

$$F_X(x, t) = G(x, t)\exp(-\lambda t) + \sum_{i=1}^{\infty}\left(\int_0^x G(x-u, t) f_Y^{\langle i \rangle}(u)\mathrm{d}u\right)\frac{\exp(-\lambda t)(\lambda t)^i}{i!}. \tag{5}$$

If the shock damage sizes are i.i.d. normal random variables, $Y_i \sim N(\mu_Y, \sigma_Y^2)$, and the degradation path is linear with a constant initial value φ and a normal-distributed degradation rate β, $\beta \sim N(\mu_\beta, \sigma_\beta^2)$, then a more specific model can be determined based on Equation (4) (Peng *et al.*, 2009a):

$$F_X(x,t)=\sum_{i=0}^{\infty}\Phi\left(\frac{x-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta^2 t^2+i\sigma_Y^2}}\right)\frac{\exp(-\lambda t)(\lambda t)^i}{i!}. \tag{6}$$

The probability that no soft failure occurs before time t is expressed as

$$P(X_s(t)<H)=F_X(H,t). \tag{7}$$

3.3. *System reliability analysis*

For systems experiencing the two dependent competing failure processes; i.e., catastrophic failures and soft failures, the occurrence of either failure process can cause the system to fail. Since the shocks contribute to both types of failures, their occurrences are dependent. They are dependent in a probabilistic or statistical sense, but this does not imply that one failure causes or is physically dependent on the other. Therefore, the system reliability at time t is the probability that the system survives each of the $N(t)$ shock loads ($W_i<D$ for $i=1,2,\ldots,\infty$) and the total degradation is less than the threshold level ($X_s(t)<H$):

$$\begin{aligned}R(t)=&\;P(X(t)<H,N(t)=0)\\&+\sum_{i=1}^{\infty}P\Bigg(W_1<D,\ldots,W_{N(t)}<D,X(t)\\&+\sum_{j=1}^{N(t)}Y_j<H,N(t)=i\Bigg)\\=&\;P(X(t)<H,N(t)=0)\\&+\sum_{i=1}^{\infty}F_W(D)^iP\Bigg(X(t)+\sum_{j=1}^{N(t)}Y_j<H|N(t)=i\Bigg)\\&\times P(N(t)=i).\end{aligned} \tag{8}$$

By using Equations (5) and (8), the system reliability function can be derived for the general case as

$$\begin{aligned}R(t)=&\;G(H,t)\exp(-\lambda t)+\sum_{i=1}^{\infty}F_W(D)^i\\&\times\left(\int_0^H G(H-u,t)f_Y^{(i)}(u)\mathrm{d}u\right)\frac{\exp(-\lambda t)(\lambda t)^i}{i!}.\end{aligned} \tag{9}$$

Based on the general case in Equation (9), the reliability function for the more specific case with normally distributed W_i, Y_i, and β can be expressed as

$$\begin{aligned}R(t)=&\;\Phi\left(\frac{H-\mu_\beta t-\varphi}{\sigma_\beta t}\right)\exp(-\lambda t)+\sum_{i=1}^{\infty}P_L^i\\&\times\Phi\left(\frac{H-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta^2t^2+i\sigma_Y^2}}\right)\frac{\exp(-\lambda t)(\lambda t)^i}{i!},\end{aligned} \tag{10}$$

where P_L is given by Equation (2). Then the pdf of the failure time, $f_T(t)$, for the specific case is derived as

$$\begin{aligned}f_T(t)=&-\frac{\mathrm{d}R(t)}{\mathrm{d}t}=-\sum_{i=1}^{\infty}P_L^i\phi\left(\frac{H-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta^2t^2+i\sigma_Y^2}}\right)\\&\times\left(\frac{-\mu_\beta\left(\sigma_\beta^2t^2+i\sigma_Y^2\right)-\sigma_\beta^2t\left(H-(\mu_\beta t+\varphi+i\mu_Y)\right)}{\left(\sigma_\beta^2t^2+i\sigma_Y^2\right)^{\frac{3}{2}}}\right)\\&\times\frac{\exp(-\lambda t)(\lambda t)^i}{i!}-\sum_{i=1}^{\infty}P_L^i\Phi\left(\frac{H-(\mu_\beta t+\varphi+i\mu_Y)}{\sqrt{\sigma_\beta^2t^2+i\sigma_Y^2}}\right)\\&\times\frac{\lambda\exp(-\lambda t)(\lambda t)^{i-1}(-\lambda t+i)}{i!}-\phi\left(\frac{H-\mu_\beta t-\varphi}{\sigma_\beta^t}\right)\\&\times\left(\frac{-H+\varphi}{\sigma_\beta t^2}\right)\exp(-\lambda t)+\lambda\Phi\left(\frac{H-\mu_\beta t-\varphi}{\sigma_\beta^t}\right)\exp(-\lambda t),\end{aligned} \tag{11}$$

where $\phi(\cdot)$ is the pdf of a standard normally distributed variable.

4. Maintenance modeling and optimization

To evaluate the performance of the maintenance policy, we use an average long-run maintenance cost rate model, in which the periodic inspection interval τ is the decision variable. A cycle is defined as either a time interval between the installation of a system and the first replacement or a time interval between two consecutive replacements. The successive cycles together with the costs incurred in each cycle constitute a renewal process. Let $C(t)$ denote the cumulative maintenance cost until time t. From basic renewal theory, the average long-run total maintenance cost per unit time, $\lim_{t\to\infty}(C(t)/t)$, can be evaluated by (Ross, 1996; Li and Pham, 2005a):

$$\begin{aligned}&\lim_{t\to\infty}(C(t)/t)\\&=\frac{\text{Expected maintenance cost incurred in a cycle}}{\text{Expected length of a cycle}}\\&=\frac{E[TC]}{E[K]},\end{aligned}$$

where TC is the total maintenance cost of a renewal cycle, and K is the length of a cycle as shown in Fig. 1.

The total maintenance cost includes the inspection cost, replacement cost, and penalty cost during system downtime. The penalty cost is based on the revenue or profit that is unattainable because the system is not functioning. Then the expected total maintenance cost of a renewal cycle is given as

$$E[TC] = C_{\mathrm{I}} E[N_{\mathrm{I}}] + C_{\mathrm{F}} E[\rho] + C_{\mathrm{R}}, \tag{12}$$

where C_{I} is the cost associated with each inspection, C_{F} is the penalty cost rate during downtime, C_{R} is the replacement cost, N_{I} is the number of inspections, and ρ is the system downtime or the time from the system failure until the next inspection at which the failure will be detected.

The number of inspections in a renewal cycle, N_{I}, is related to the failure time of the system, T, in a way that $(N_{\mathrm{I}} - 1)\,\tau < T < N_{\mathrm{I}}\,\tau$. Therefore, the expected number of inspections in a renewal cycle $E[N_{\mathrm{I}}]$ is derived as

$$\begin{aligned} E[N_{\mathrm{I}}] &= \sum_{i=1}^{\infty} i\,P(N_{\mathrm{I}} = i) = \sum_{i=1}^{\infty} i\,P((i-1)\tau < T \le i\tau) \\ &= \sum_{i=1}^{\infty} i(F_T(i\tau) - F_T((i-1)\tau)) \end{aligned} \tag{13}$$

where $F_T(t)$ is the cdf of T, which can be calculated from the reliability function $R(t)$ in Equation (9) or Equation (10).

The system downtime is the time from a system failure until the end of the renewal cycle, or $\rho = N_I\tau - T$. Then the expected value of system downtime in a renewal cycle $E[\rho]$ is given as

$$\begin{aligned} E[\rho] &= \sum_{i=1}^{\infty} E[\rho | N_{\mathrm{I}} = i] P(N_{\mathrm{I}} = i) \\ &= \sum_{i=1}^{\infty} \left(\int_{(i-1)\tau}^{i\tau} (i\tau - t) \mathrm{d}F_T(t) \right) (F_T(i\tau) - F_T((i-1)\tau)). \end{aligned} \tag{14}$$

Similarly, the length of a renewal cycle is related to the number of inspections in a cycle, and it is determined as $N_{\mathrm{I}}\tau$. The expected length of a renewal cycle $E[K]$ is derived as

$$\begin{aligned} E[K] &= \sum_{i=1}^{\infty} E[K | N_{\mathrm{I}} = i] P(N_{\mathrm{I}} = i) \\ &= \sum_{i=1}^{\infty} i\tau (F_T(i\tau) - F_T((i-1)\tau)). \end{aligned} \tag{15}$$

Based on Equations (12) to (15), the average long-run maintenance cost rate as a function of τ, $CR(\tau)$, is given as

$$CR(\tau) = \frac{C_{\mathrm{I}} \left(\sum_{i=1}^{\infty} i(F_T(i\tau) - F_T((i-1)\tau)) \right) + C_{\mathrm{F}} \left(\sum_{i=1}^{\infty} \left(\int_{(i-1)\tau}^{i\tau} (i\tau - t) \mathrm{d}F_T(t) \right) (F_T(i\tau) - F_T((i-1)\tau)) \right) + C_{\mathrm{R}}}{\sum_{i=1}^{\infty} i\tau (F_T(i\tau) - F_T((i-1)\tau))}. \tag{16}$$

To obtain an analytical result of the optimal solution, we calculate the first derivative of the objective function in Equation (16), as given below:

$$CR'(\tau) = \frac{\left(\sum_{i=1}^{\infty} i\tau (F_T(i\tau) - F_T((i-1)\tau)) \right) (\mathrm{d}u/\mathrm{d}t) - u \left(\sum_{i=1}^{\infty} \left(i^2 \tau f_T(i\tau) - i(i-1)\tau f_T((i-1)\tau) + i F_T(i\tau) - i F_T((i-1)\tau) \right) \right)}{\left(\sum_{i=1}^{\infty} i\tau (F_T(i\tau) - F_T((i-1)\tau)) \right)^2},$$

where

$$\begin{aligned} u &= C_{\mathrm{I}} \left(\sum_{i=1}^{\infty} i (F_T(i\tau) - F_T((i-1)\tau)) \right) \\ &\quad + C_{\mathrm{F}} \left(\sum_{i=1}^{\infty} \left(\int_{(i-1)\tau}^{i\tau} (i\tau - t) \mathrm{d}F_T(t) \right) \times (F_T(i\tau) - F_T((i-1)\tau)) \right) + C_{\mathrm{R}}, \\ \frac{\mathrm{d}u}{\mathrm{d}\tau} &= C_{\mathrm{I}} \left(\sum_{i=1}^{\infty} i^2 f_T(i\tau) - i(i-1) f_T((i-1)\tau) \right) \\ &\quad + C_{\mathrm{F}} \left(\sum_{i=1}^{\infty} \left(\begin{array}{l} \left(\int_{(i-1)\tau}^{i\tau} (i\tau - t) \mathrm{d}F_T(t) \right) (i f_T(i\tau) \\ \quad - (i-1) f_T((i-1)\tau)) \\ \quad + (F_T(i\tau) - F_T((i-1)\tau)) \\ \quad \times \left(i \int_{(i-1)\tau}^{i\tau} f_T(t) \mathrm{d}t + (1-i) \right. \\ \quad \left. \times \tau f_T((i-1)\tau) \right) \end{array} \right) \right). \end{aligned} \tag{17}$$

The optimal solution can be found by setting $CR'(\tau)$ equal to zero. By minimizing $CR(\tau)$ using the analytical or numerical methods, we can obtain the optimal time interval of periodic inspection for the preventive maintenance policy.

5. Numerical example: an MEMS application

A microengine consists of orthogonal linear comb drive actuators that are mechanically connected to a rotating gear (Tanner and Dugger, 2003). The linear displacement of the comb drives is transformed to the gear via a pin joint. The dominant failure mechanism is identified as the visible wear on rubbing surfaces between the gear and the pin joint, which often results in a seized microengine or a broken pin joint. The wear volume is primarily caused by the aging degradation process. In addition, shock tests on microengines reveal that shock loads may cause substantial wear debris between the gear and the pin joint, as well as the fracture of springs. Therefore, microengines experience these two competing failure processes: soft failures due to aging degradation and debris from shock loads

Table 1. Parameter values for microengine reliability analysis

Parameters	*Values*	*Sources*
H	0.00125 μm^3	Tanner and Dugger (2003)
D	1.5 GPa (for polysilicon material used for springs)	Tanner and Dugger (2003)
φ	0	Tanner and Dugger (2003)
β	$\sim N(\mu_\beta, \sigma_\beta^2)$ $\mu_\beta = 8.4823 \times 10^{-9}$ μm^3 and $\sigma_\beta = 6.0016 \times 10^{-10}$ μm^3	Tanner and Dugger (2003) Peng *et al.* (2009a)
λ	2.5×10^{-5}	Assumption
Y_i	$\sim N(\mu_Y, \sigma_Y^2)$ for $i = 1, 2, \ldots, \infty$ $\mu_Y = 1 \times 10^{-4}$ μm^3 and $\sigma_Y = 2 \times 10^{-5}$ μm^3	Assumption
W_i	$\sim N(\mu_W, \sigma_W^2)$ for $i = 1, 2, \ldots, \infty$ $\mu_W = 1.2$ GPa and $\sigma_W = 0.2$ GPa	Assumption

and catastrophic failures due to spring fracture. Consider microengines that are used in a load-sharing redundant system where the failure of one engine is not self-announcing. The inspection-based maintenance policy is implemented to enhance system performance. We use the models proposed in this article to study the microengine reliability and to determine its periodic inspection interval that can be used in practice.

5.1. *Reliability analysis*

The parameters in Equations (10) and (11) for reliability analysis of the microengine are provided in Table 1. From Sandia's experimental results, we obtained the critical failure threshold for wear degradation, H, the maximum fracture strength, D, and the parameters in the linear wear degradation path, $X(t) = \varphi + \beta t$. It was assumed that β is a normally distributed random variable; that is, the wear volume at any time t varies from unit to unit following a normal distribution (Peng *et al.*, 2009b). The parameters for random shocks were assumed for the purpose of illustration, including the arrival rate λ, shock damage sizes, Y_i, and shock load sizes, W_i, for i =1, 2, ..., ∞. We assumed that the sizes of the shock loads follow a normal distribution and, consequently, the resulting damage sizes on the wear volume also follow a normal distribution.

The probability that a device survives an applied stress from a shock, P_L, was calculated to be 93.32% using Equation (2). Based on Equations (10) and (11), the reliability function $R(t)$ and the pdf of failure time $f_T(t)$ are plotted in Fig. 2. A sensitivity analysis was performed to assess the effects of the model parameters on $R(t)$ and $f_T(t)$. The model parameters that we are interested in include the wear degradation failure threshold value, H, and the arrival rate of random shocks, λ. The results are shown in Figs. 3 and 4, respectively.

Figure 3 indicates that the degradation failure threshold value H has a significant effect on both the reliability function and the failure time distribution. When H increases from 0.001×10 μm^3 to 0.001×30 μm^3, both $R(t)$ and $f_T(t)$ are not sensitive to H before t reaches 0.5×10^5 revolutions approximately. For the region of t larger than 0.5×10^5 revolutions, both $R(t)$ and $f_T(t)$ shift to the right when H increases, which implies a better reliability performance for a larger value of H.

In Fig. 4, we can observe that both the reliability function and the failure time distribution are sensitive to the arrival rate of random shocks, λ. When λ increases from 2×10^{-5}

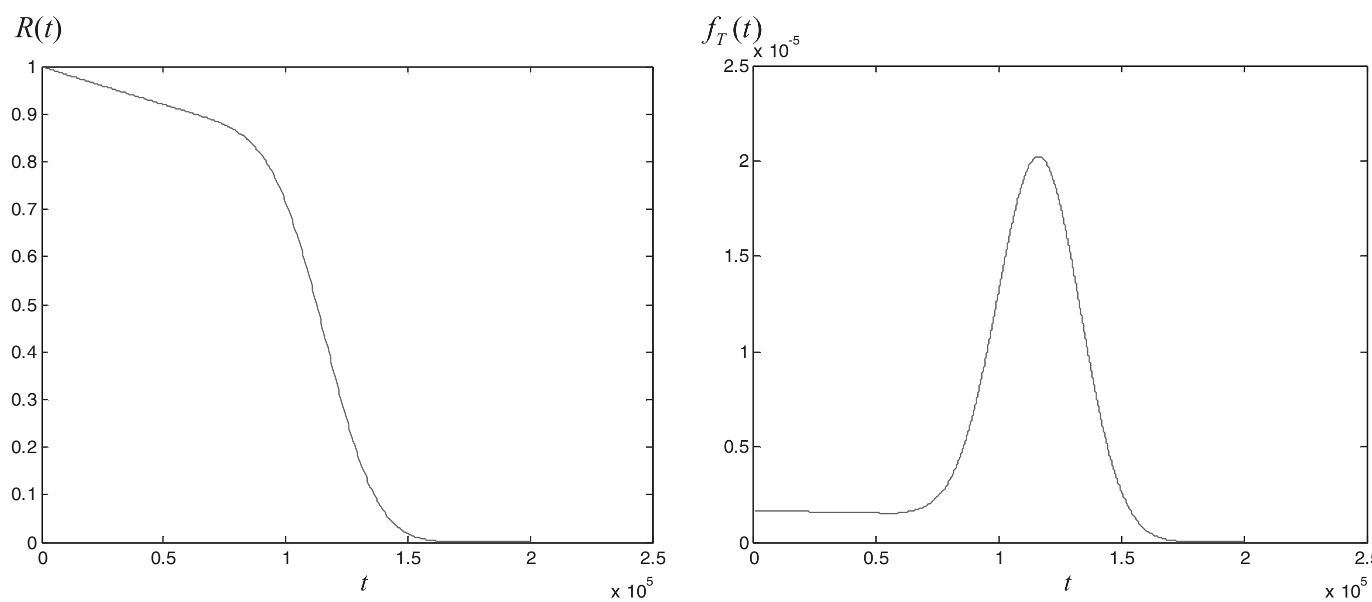

Fig. 2. Plots of reliability function $R(t)$ and failure time distribution $f_T(t)$.

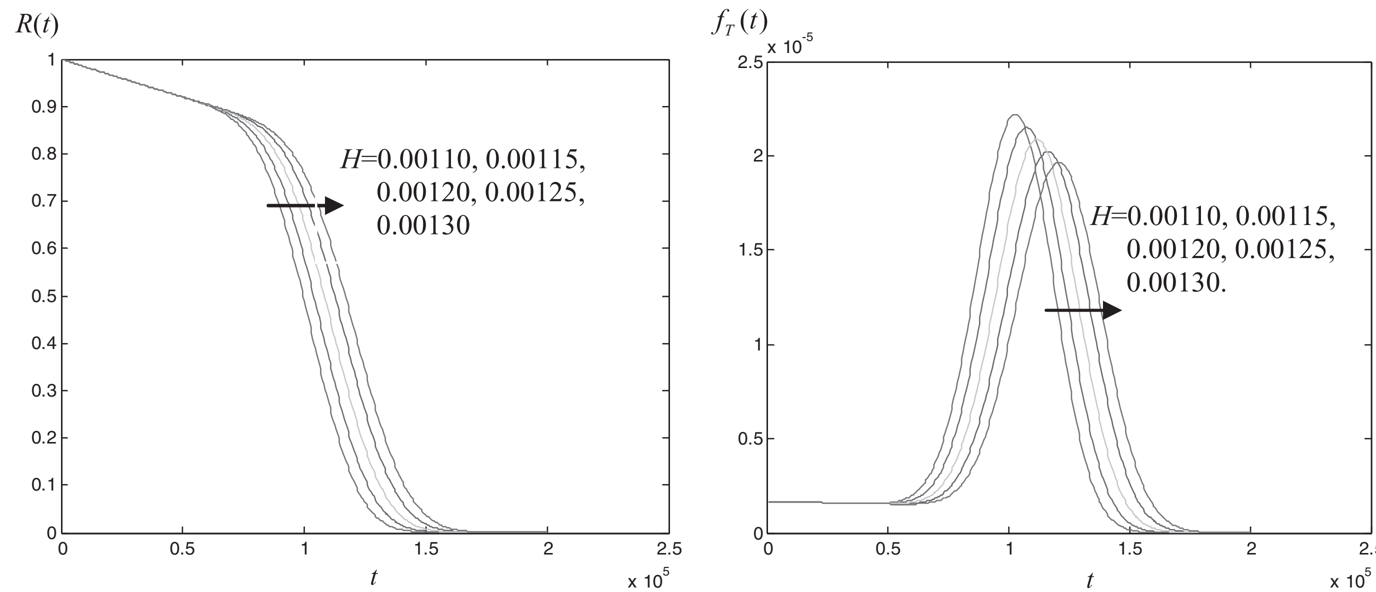

Fig. 3. Sensitivity analysis of $R(t)$ and $f_T(t)$ on H.

to 4×10^{-5}, both $R(t)$ and $f_T(t)$ shift to the left. This suggests that the reliability performance deteriorates when microengines are operating in an environment with higher arrival rates of random shocks.

5.2. *Optimal maintenance policy*

The cost parameters in Equation (17) are typical values used in order to illustrate the optimization of the average long-run maintenance cost rate. For an example of $C_{\mathrm{I}} = \$1$, $C_{\mathrm{F}} = \$50$, and $C_{\mathrm{R}} = \$10$, we can find the minimum average long-run maintenance cost rate of \$6.8309/cycle, which is obtained at $\tau^* = 1.2657 \times 10^5$, the optimal number of revolutions for periodical inspection. The optimal solution is verified through numerical calculation. Figure 5 illustrates $CR(\tau)$ as a function of τ.

A sensitivity analysis was performed to analyze the effects of the model parameters on the optimal solutions. The model parameters of interest include the arrival rate of random shocks, λ, the soft failure threshold value, H, and the probability that the device survives the applied stress, P_{L}

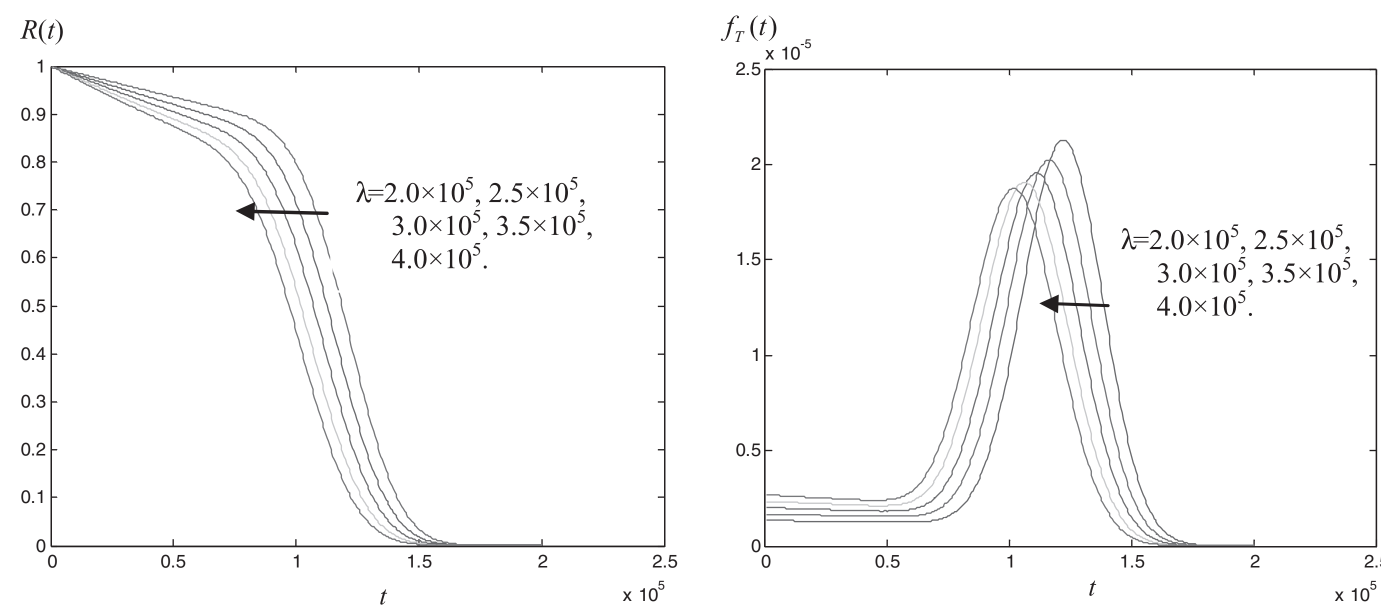

Fig. 4. Sensitivity analysis of $R(t)$ and $f_T(t)$ on λ.

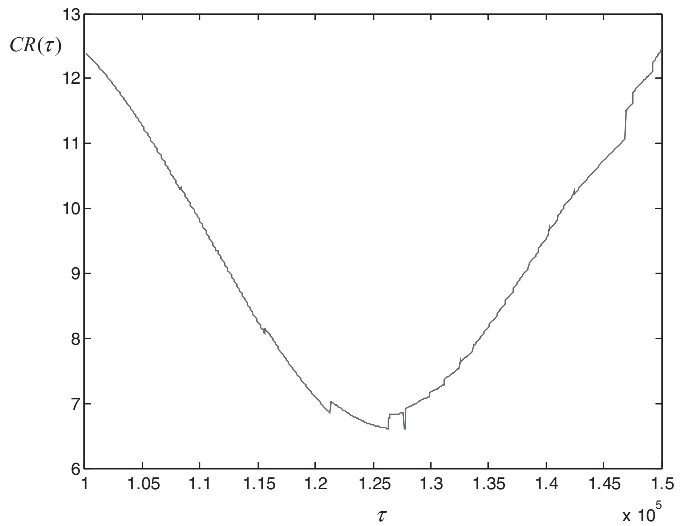

Fig. 5. Average long-run maintenance cost rate versus inspection interval.

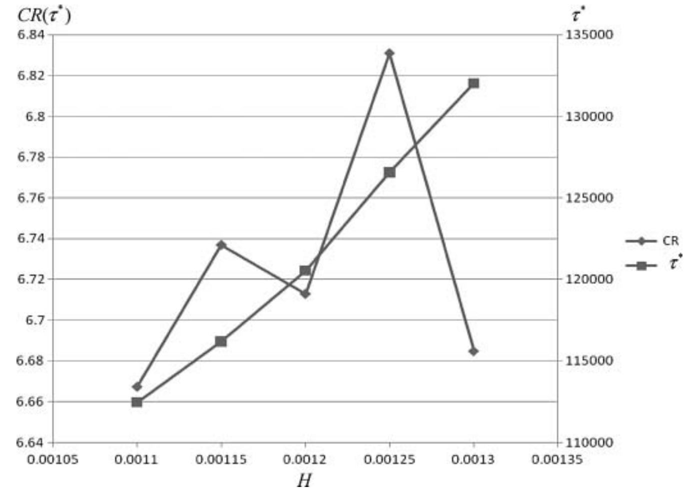

Fig. 7. Sensitivity analysis of $CR(\tau^*)$ and τ^* on H.

(related to the maximum fracture strength, D). The results are shown in Figs. 6 to 8, respectively.

When λ increases from 2×10^{-5} to 4×10^{-5} as shown in Fig. 6, the minimum average long-run maintenance cost rate, $CR(\tau^*)$, increases from \$6.5979 to \$7.218, and the optimal inspection interval decreases from 1.3206×10^5 to 1.1163×10^5 revolutions. This implies that a higher arrival rate of random shocks leads to a higher potential of hard failures. As a result, a microengine should be inspected more frequently when operating in an environment with a higher shock arrival rate.

As shown in Fig. 7, when the soft failure threshold value H increases from $0.001 \times 10\ \mu\text{m}^3$ to $0.001 \times 30\ \mu\text{m}^3$, the minimum average long-run maintenance cost rate fluctuates around \$6.70, and the optimal inspection interval increases from 1.1247×10^5 to 1.3203×10^5 revolutions. This indicates that a larger threshold value H results in a longer inspection interval, whereas the average cost rate is insensitive to the variation of H.

For the same random shock process (with the same μ_W and σ_W), the survival probability P_L from shock loads is related to the maximum fracture strength, in a way that P_L increases as D increases (Equation (2)). When P_L rises from 0.85 to 0.95 as shown in Fig. 8, the minimum average long-run maintenance cost rate reduces from \$8.9372 to \$6.0236, and the optimal inspection interval increases from 1.2129×10^5 to 1.2652×10^5 revolutions. This implies that the enhanced material/structure strength can significantly reduce the average long-run cost rate and the periodic inspection frequency.

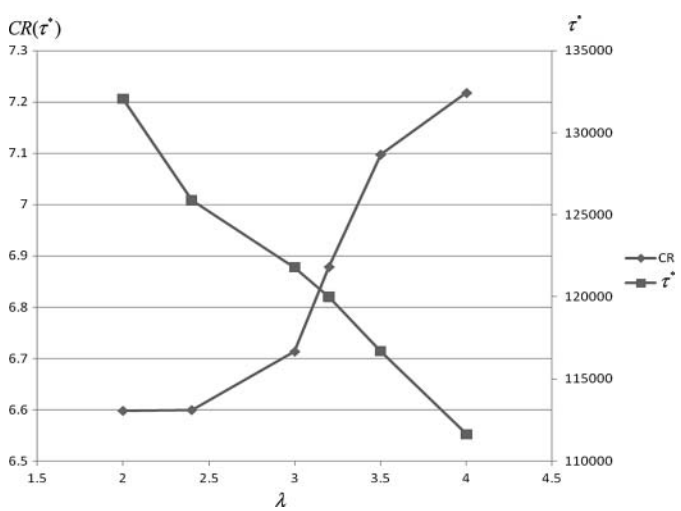

Fig. 6. Sensitivity analysis of $CR(\tau^*)$ and τ^* on λ.

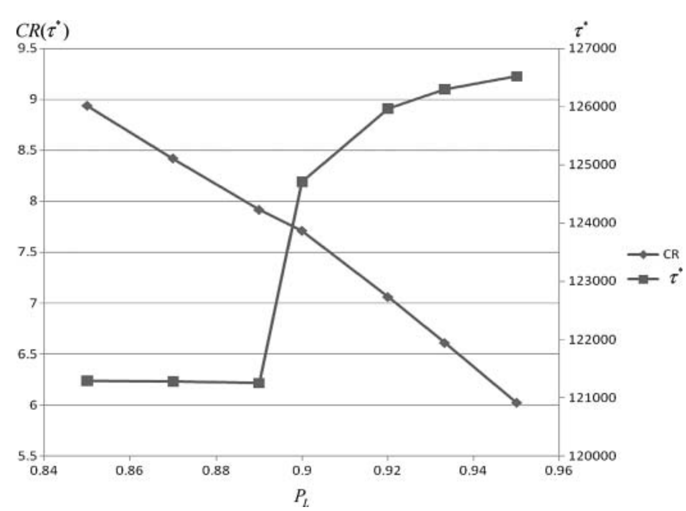

Fig. 8. Sensitivity analysis of $CR(\tau^*)$ and τ^* on P_L.

6. Conclusions and future directions

In this article, we develop reliability and preventive maintenance models for complex systems that experience MDCFP. Specifically, we consider two dependent/correlated failure processes: *soft failures* caused by continuous smooth degradation and additional abrupt degradation damages due to a shock process and *catastrophic failures* caused by the stress from the same shock process. These two failure processes are *competing yet dependent* because the effects from the same shock process contribute to both failure processes.

A general reliability model is developed based on general degradation path and random shock models, where *extreme* and *cumulative* shock models are used in the modeling of the catastrophic and soft failure processes, respectively. The general reliability model is then extended to a specific model for a linear degradation path with a normal-distributed degradation rate and a random shock process with normal-distributed shock load sizes and shock damage sizes. Based on the reliability analysis, the average long-run maintenance cost rate is evaluated and optimized for the preventive maintenance policy where the periodic inspection interval is the decision variable. Both the reliability model and the maintenance optimization model for MDCFP are demonstrated on a realistic example considering a microengine. To the best of our knowledge, this is among the first studies for the design of dynamically reliable MEMS by probabilistically analyzing degradation and shocks.

For future research directions, the general reliability model developed in this article can be extended to other systems that experience multiple dependent failure processes. For example, a specific model can be constructed for a nonlinear degradation path model (e.g., fatigue crack growth that follows the power law), random degradation failure threshold (e.g., H is a normal random variable), or different distributions of shock damage sizes (e.g., exponential distribution). In addition, reliability models can be developed for two or more dependent competing failure processes that involve different degradation and/or shock processes (e.g., two degradation processes and one shock process that are dependent).

In this article, we develop a maintenance model for a non-repairable system and the decision variable is the inspection interval. The maintenance model for a system that experiences multiple dependent failure processes should be developed for specific applications. For example, maintenance models can be constructed for repairable systems, for which a preventive maintenance level needs to be optimized in order to minimize the maintenance cost or maximize the system availability.

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