Frankenmandering: Repeated Social Graph Gerrymandering

Paper #1108

Abstract. Opinion influence by means of manipulating a social graph structure, as well as election manipulation by means of artificial grouping (gerrymandering), are well known fields of application and investigation in computational social choice, and multi-agent learning and systems. However, while they are studied separately, in real life, both of these manipulations (social graph alteration and gerrymandering) occur simultaneously. In this paper, we offer the first model of such a simultaneous process, which takes the form of repeated gerrymandering with an underlying social graph for opinion diffusion. We term this process model "Frankenmandering", and provide the first steps in its analysis: examples of principal feasibility and the impact of the underlying social graph.

1 Introduction

Influencing the spread of opinion in a social network has been studied from two main perspectives: forcing changes in network topology [41, 25, 5], or placing "influencers" in the network [6, 30, 12]. Former can be deployed by social network management, while the latter is an open market. Fortunately, neither is easy to implement effectively[10], though possible [47, 7, 30, 42, 12].

However, in district-based election schemes, political parties have found and favour another dubious tactic: Gerrymandering – altering the grouping of voters to skew the representative's election process. This has been studied both in practical political terms [3, 43, 28] and more theoretical terms [15, 9, 27]. Strangely enough, Gerrymandering remains (as a task) a single-shot process, a way to exploit the existing set of opinions, rather than means to both exploit and *form* them. Strangely, this is in spite of the fact that some studies of the long-term effects of Gerrymandering do exist in such areas as political science (e.g., [31]). Doubly strange, if we notice that the question of grouping people for forced interaction, targetting their opinions in an elections context, has been raised (e.g., [32]).

In this paper, we bridge the gap and show that gerrymandering can be effectively used as an opinion *formation* tool with *long-term* electoral effects. More specifically, we recognise that gerrymandering implies formation (or modification) of social opinion networks, and, therefore, can be used as an opinion control method in *long term*. We term this new model of repeated gerrymandering process over social networks **"Frankenmandering"**.

2 Model and Technical Background

We will begin by formally describing the formal model of interaction that we target (The Model), and then provide some technical details on how different components of The Model can be instantiated and initialised.

However, first, we would like to depict an intuitive picture of what kind of interaction we are seeking to capture. Imagine, then, a vast, sprawling, multicultural city, with its many neighbourhoods, cul-de-sacs, mews, condominium high-rises, bridges, highways, and inhabitants of every age and persuasion. The city council, which presides over the city's bylaws, is naturally composed of the representatives of various groups of the city's inhabitants. Mostly, these groups are based on where people live. In other words, the city is broken down into *districts*, and each district has an elected *representative* sitting on the city council. Some people like the representative of their district and even tend to adopt some of the representative's views, seeing them as beneficial. Others will be politically misaligned, think that the representative can do no good, brood on it and deepen in their disagreement. None will stay silent and will continue to discuss the matter with their friends and neighbours, especially those from a different district, whether to reconcile afterwards or make new frenemies. These changes in people's opinions lead to the change in the elected representatives, and the grumbling cycle repeats.

Sometimes, however, the boundaries between different districts change, and people may suddenly find themselves under the influence of a very different kind of a representative. Not because they moved or their opinion changed, but because the balance of opinions within the district has changed. Some will be happy about it, they will say "this is my kind of a neighbourhood now". Others will be less enthused, saying "there's a reason we were in different districts, there's literally a river separating us". This too will influence the opinions of the city's inhabitants.

But why were the district boundaries changed? One answer is: Gerrymandering – the Mayor wanted to influence the composition of the City Council, so she redrew the boundaries to change the balance of opinions in different districts and, thus, change what opinions are brought in by the representatives. But a smarter Mayor has a different agenda. She's not after the representatives. She's there to change the mind of the city's inhabitants by "forcing" on them the influence of a specific kind of representatives. Because her elections depend on the opinion of *all* people, and if the majority of all inhabitants will see things her way, then, over time, so will their elected representatives. And then the Mayor can truly rule the City. It is this long-term, repeated redistricting that we are formalising in our model.

2.1 *The Model*

Given a set of n voters V, each characterised by a position in some "physical" space $p_v \in \mathbb{R}^d$ and an opinion $c_v \in \mathbb{R}^m$. We will use bold-face $\mathbf{c} \in (\mathbb{R}^m)^n$ to denote the vector of all opinions of all voters, and use functional form $c_v(l)$ to refer to the l'th coordinate of the preference vector c_v of voter $v \in V$. In addition, for a subset

$D \subset V$, we will denote the sub-vector of opinions within the subset by $\mathbf{c}|_D = \{c_v\}_{v \in D} \in (\mathbb{R}^m)^{|D|}$. Let there also be a directed (social) graph $G = (V, E, w)$ with $w : E \to \mathbb{R}$ being an edge weight function. In addition, let $\mathbb{L}[G] : (\mathbb{R}^m)^n \to (\mathbb{R}^m)^n$ be a graph-parameterised opinion dynamics function, and $d : (\mathbb{R}^m)^n \to \mathbb{R}$ a social opinion evaluation function. Finally, let there be a representative selection function $\mathbb{F}$, so that for any $\widetilde{V} \subseteq V$, $\mathbb{F}\left(\mathbf{c}|_{\widetilde{V}}\right) \in \widetilde{V}$.

Set $\mathbf{c}^0$ to be the initial opinion of all voters, and consider the following process iterated for every time period t:

1. Districts, $\mathbf{D}^t = \{D_j^t\}_{j=1}^K$, are being selected, so that $\forall i, D_i^t \subseteq V$, $\bigcup_{j=1}^K D_i^t = V$, and for any i, j holds $D_i^t \cap D_j^t = \emptyset$;
2. Local elections are run, producing district representatives $r_j^t = \mathbb{F}\left(\mathbf{c^t}|_{D_j^t}\right) \in D_j^t$
3. Representatives becomes local "influencers". I.e., Graphs, $\{H_j^t = (D_j^t, E_j^t, w_j^t)\}_{j=1}^K$, are built so that $(r_j^t, v) \in E_j^t$ for all $v \in D_j^t$, and $w_j^t : E_j^t \to \mathbb{R}$ reflect the influence of the chosen representative on the "constituents" of its district;
4. Social influence is then exercised on all opinions via $\mathbb{L}^t = \mathbb{L}[G \cup \bigcup_{j=1}^K H_j^t]$, so that : $\mathbf{c}^t = \mathbb{L}^t(\mathbf{c}^{t-1})$

2.2 *District Geometry and Initial Opinion Distribution*

In our initial studies, we will assume that voters are placed on a regular (2D) grid, though they do not necessarily *form* that grid. This initial simplification pursues several goals.

First, such a positioning will simulate well the real-life geographical maps used in applied gerrymandering. Such maps are commonly reduced to planar graphs, and we will have the benefit of prior work on algorithmic gerrymandering as experimentation baselines (e.g., [15, 14]). Second, grids and planar graphs are a convenient medium for the application of spatial distribution models, such as Gaussian and Dirichlet Processes [16, 20, 38], to capture and track the distribution of opinions in geographical space and over the social network.

Among other things, these mathematical models allow us to:

- Capture spatial autocorrelation between opinions of different voters without artificial boundaries
- Model neighbourhood-wide "popularity" effects
- Generate realistic opinion distribution patterns with local clustering and regional variation
- Create natural frontiers between opinion clusters
- Provide a tunable mathematical framework for extensive experimentation

In particular, for bi-polar opinions we use a Hierarchical Beta-Occupancy Model, instantiated by Algorithm 1 to generate the initial opinion distribution for experimentation. The graph of interest in Algorithm 1 can be either the lattice structure associated with the "geographic" positions p_v of voters or the social graph of The Model.

Speaking of the "geography" and voter locations $p_v \in \mathbb{R}^d$, we must note that it is common practice to restrict district formation. Specifically, it is required that there are contiguous non-overlapping regions of the location space $A_j \subset \mathbb{R}^d$, $j \in \{1, ..., K\}$, so that $D_j \subset A_j$. Further restrictions come in the form of an *exclusion* area $A^\emptyset \subset \mathbb{R}^d$, and the requirement that no district-forming area A_j intersects with $A^\emptyset$. The exclusion area dictates geographic limitations, such as a lake or a river, that a district cannot cross/bridge. In practice, following a classical robotics trick, geographic limitations tend to be resolved by "tessellating" the *permissible* space of locations, and then forming districts by collecting connected "shards". This can further be reduced to a complementary graph of the tessellation, where each shard is represented by the graph's node and each edge represents a side shared by two shards. Which naturally leads to redistricting being reduced to graph separation, as in, e.g., [14, 15]. In our feasibility study in Section 4 we adopt this view of redistricting as well, making sure that the "geography" graph is planar.

Algorithm 1 Hierarchical Beta Occupancy Model

Require: Graph $G = (V, E)$
Require: Baseline distribution parameters $\widehat{\alpha}, \widehat{\beta}$
Require: Neighbourhood sensitivity parameter $f_{\text{influence}}$
Ensure: $|V| > 0$, $0 < f_{\text{influence}} < 1$
  $Z = \emptyset$ ▷ Set of opinionated voters
  $U = V$ ▷ Set of voters without an opinion
  while $U \neq \emptyset$ **do**
    Randomly select a voter $v \in U$
    Let $N^o = N(v) \cap Z$ ▷ All opinionated neighbours
    if $N^o = \emptyset$ **then**
      Assign $c_v \sim \text{Beta}(\widehat{\alpha}, \widehat{\beta})$
    else
      $\widehat{c} = \frac{1}{|N^o|} \sum_{u \in N^o} c_u$ ▷ Mean neighbourhood opinion
      $\delta = sign(\widehat{c} - 0.5)$
      $\alpha_{\text{new}} = \widehat{\alpha}(1 + \delta * f_{\text{influence}})$
      $\beta_{\text{new}} = \widehat{\beta}(1 - \delta * f_{\text{influence}})$
      Draw $X \sim \text{Beta}(\alpha_{\text{new}}, \beta_{\text{new}})$
      Set $c_v = X(1 - f_{\text{influence}}) + \widehat{c} \cdot f_{\text{influence}}$
    end if
    $Z \leftarrow Z \cup \{v\}$ ▷ v is now opinionated
    $U \leftarrow U \setminus \{v\}$
  end while
  return $G = (V, E)$ with opinions c_v for all $v \in V$ assigned

2.3 *Social Graph*

The same planar positioning also simplifies the social network generation. Such popular small-world network generators as Newman-Watt-Strogatz(NWS) [45, 36, 35] easily originate from an initial grid. Of course, it has been argued that Watts-Strogatz networks do not possess the scale-free property of the naturally occurring social networks, so that a Barabási-Albert (BA) [2] model would be preferred. Indeed, the fact that we place the voters on a lattice does not prevent us from using a BA generator for the underlying social network.

However, we can also reconcile the issue by using some of the more modern approaches, which explicitly create planar graphs with small-world and scale-free properties (e.g., [34]), and can be modified to run over a grid structure. Though it is less clear if thus obtained social graph will have the same properties as the more classical generators [11].

Of course, the obvious benefit of using these classical generators is that the outcomes of the opinion dynamics on them are relatively well studied [39, 37, 44], which allows us to form some baseline expectations and distinguish between natural and manipulated outcomes of opinion dynamics.

2.4 Local Representative Election

The grand formalism of The Model does not limit the manner in which a district representative is chosen, but merely suggests that it exists and it is uniform for all districts. However, to begin our analysis of The Model we must choose a rule. Now, by its structure, The Model favours spatial social choice – a long-standing theoretical and practical modelling technique of political elections [17, 18, 33, 40]. More recent research even provides some bounds and assurances as to the quality of the outcome (e.g., [4, 1], with the former explicitly targetting district-based voting) and computational feasibility under uncertainty (e.g., [21]). This real-life connection and relative computational ease makes the study of The Model under spacial/metric voting particularly attractive.

Thus, we will follow suit and begin by choosing a "median" representative:

$$r_j = \arg\min_{l^* \in D_j} \sum_{\lambda \in D_j} \|c_l - c_\lambda\|$$

2.5 Opinion Dynamics

There are multiple opinion dynamics we can target, starting from the (weighted) Ising model and to aggregate dynamics with backlash/backfire [23, 26]. The latter is of particular interest, and we can have a look at two variants of it: general and multi-issue.

During an interaction of two voters $v, v \in V$, connected by an edge $e = (uv) \in E$, the opinion c_v will undergo a change depending on the relative proximity of c_u:

$$c_v^{t+1} = \begin{cases} c_v^t + \mu^+(c_u^t - c_v^t) & \|c_v - c_u\| < \epsilon^+ \\ c_v^t - \mu^-(c_u^t - c_v^t) & \|c_v - c_u\| \geq \epsilon^- \\ c_v^t & otherwise \end{cases}$$

where $\epsilon^- >> \epsilon^+$ are, respectively, the "backfire" and "confirmation" thresholds, and μ^+, μ^- are the corresponding degrees of sensitivity. Sensitivity arguments can be a function of the general edge weight $w(e)$ as described by the social network graph $G = (V, E, w)$. These effects are aggregated (and the influence coefficients are possibly normalised) across all edges.

In the multi-issue variant the above dynamic can be applied per opinion coordinate. In addition, sensitivity coefficients can be correlated with the overall (normalised) distance of opinions, so that overall close opinions will depress the backfire sensitivity, while the overall distant opinions will depress the confirmation coefficients. In fact, borrowing some inspiration from [13], the dynamic may be written as:

$$c_v^{t+1} = c_v^t + \mu_{uv}^t(c_u^t - c_v^t),$$

where $\mu^t \propto w_{uv}\langle c_u^t, c_v^t \rangle$, so that the update coefficient is proportional to: a) the strength/magnitude of the individual opinions; b) their alignment (cos of the angle between the opinion vectors, or another distance); and c) the strength of the influence, w_{uv} of voter u on voter v. However, in this paper, we will be using an even more generic form of neighbourhood influence aggregation, formally described as follows.

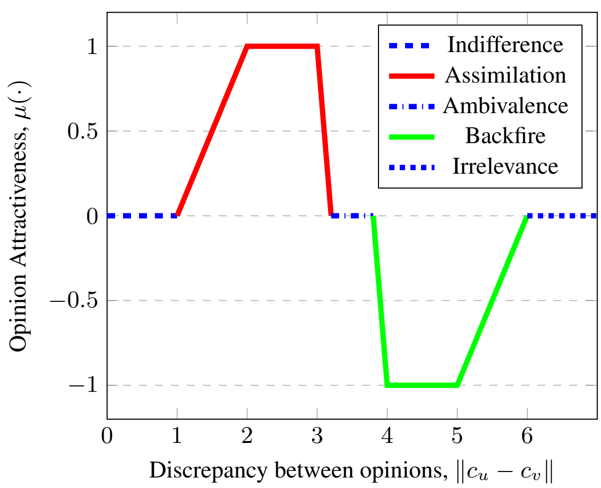
Let us define for a $c \in \mathbb{R}^m$ a unit vector with the same direction $\vec{\mathbb{1}}(c) = \frac{1}{\|c\|}c$, extending this function so that $\vec{\mathbb{1}}(\vec{0}) = \vec{0}$. Let $\mu : \mathbb{R} \to \mathbb{R}$ be the *discrepancy response function (DRF)*, which we presume to be bounded so that there are $\mu^- < \mu^+ \in \mathbb{R}$ so that $\mu^- \leq \mu(x) \leq \mu^+$. Now, given a (directed) social graph $G = (V, E, w)$, let us denote the lem influencing and *following* neighbours of a voter $v \in V$ by $N^\downarrow(v) = \{u \in V | (uv) \in E\}$ and, respectively $N^\uparrow(v) = \{u \in V | (vu) \in E\}$ Then, opinion dynamics over a (directed) social graph $G = (V, E, w)$ are described by the following equation:

$$c_v^{t+1} = c_v^t + \frac{1}{Z} \sum_{u \in N^\downarrow(v)} \mu\left(\|(c_u^t - c_v^t)\|\right) w_{uv} \vec{\mathbb{1}}(c_u^t - c_v^t),$$

where Z is a normalisation factor. The normalisation factor, among other things, dictates an opinion's sensitivity to the cardinality of a voter's neighbourhood. So, for example, by setting $Z \propto \sum_{u \in N^\downarrow(v)} w_{uv}$ we normalise the relative importance of neighbours without regard to their number, while by setting $Z \propto \sum_{u \in N^\downarrow(v)} w_{uv}\mu\left(\|(c_u^t - c_v^t)\|\right)$ we also normalise the response to the discrepancy in the neighbours opinion.

Now the *discrepancy response function (DRF)*, μ, is what dictates whether a voter is attracted or repulsed by the opinion of its neighbour(s). We will be using DRF with two distinct phases of *similarity bias* (or "assimilation"), wherein somewhat different but close opinions are attractive, and *backfire*, wherein opinions that are too distinct to a voter will be repulsive. Furthermore, we will allow for three neutral ranges of discrepancy in opinions: "indifference" (wherein voters cannot tell the difference between each other's opinion), "ambivalence" (wherein voters cannot clearly define their reaction to the discrepancy) and "irrelevance" (where the opinion of the other voter is so distinct that it is disregarded). Figure 1 depicts the overall shape of the DRFs that we will employ.

Figure 1. Discrepancy Response Function used in Section 4.2



3 Creating Frankenmandering

The Model only describes the interaction between a social graph opinion dynamics and a redistricting process. But on it own, The Model lacks purpose. To develop one we expand the concept of Gerrymandering [22, 19, 31] – a *single instance* redistricting aimed at ensuring that representatives hold a specific type of opinion.

3.1 Gerrymandering

To begin, let us recast the classical Gerrymandering problem in terms and formalism of our Model. In this case, the opinions represent a preference order over m candidates, and the representative is there to support the aggregate preference order of its district. The winner of an election is determined by aggregating the preferences of the representatives, and Gerrymandering targets this final (single-shot) outcome.

E.g., the following procedure may be followed. Let a (plurality) ballot projection $\beta : \mathbb{R}^m \to \mathbb{R}^m$ such that if $b_v = \beta(c_v)$ for some $v \in V$ then b_v is a one-hot vector and $b_v(w) = 1$ implies $w \in \arg\max_{i\in[1:m]} c_v(i)$.

Then the representative for district D_j is defined by

$$r_j = \arg\max_{r\in D_j} \left\langle \beta(c_r), \sum_{v\in D_j} \beta(c_v) \right\rangle$$

That is the ballot of the representative corresponds to the choosing the candidate with the highest number of plurality votes in the district.

A constructive Gerrymandering algorithm then accepts a desired candidate $w \in [1 : m]$, and seeks re-districting, $\{D_j\}_{j=1}^{K}$, that makes w a winner. I.e., $w = \arg\max_{i\in[1:m]} b(i)$, where $b = \sum_{j=1}^{K} b_r$.

A destructive Gerrymandering algorithm seeks a re-districting so that $l^* < \max_{i\in[1:m]} b(i)$, preventing l^* from winning the election.

The softer versions of the above also exist, where redistricting seeks to maximise (or minimise) the number of votes received by the idealised candidate.

3.2 *Gerrymandering a Dynamic Voter Base*

Noticeably, Gerrymandering aims at a static voter base, i.e., it is a single shot operation whose after-effects on the voter's opinions are not taken into account. In fact, we did not find any work on *repeatedly* gerrymandering a population of voters, though some works (e.g., [31]) in political science do discuss whether the fact of gerrymandering influences such population metrics as polarisation.

In this work, we'd like to suggest that (repeated) re-districting can be used as a control mechanism of opinion dynamics, thus having long-term and, potentially, permanent effect on future elections. We term such an application of re-districting: **Frankenmandering**.

Under The Model, we set the number of its iterations to T, provide a "reference opinion" c^*, and define **Strong Frankenmandering** as minimising the total distance from actual voter opinions to the ideal one. Formally, it is captured by the following optimisation problem:

$$\begin{array}{rl}
\min\limits_{\mathbf{D}^1,\ldots,\mathbf{D}^T} & \sum\limits_{v\in V} \|c^* - c_v^T\| \\
 & s.t. \\
\forall j \in [1:K]\ \forall t \in [1:T] & r_j^t = \mathbb{F}\left(\mathbf{c^t}|_{D_j^t}\right) \\
\forall j \in [1:K]\ \forall t \in [1:T] & H_j^t = (D_j^t, E_j^t, w_j^t) \\
\forall t \in [1:T] & \mathbb{L}^t = \mathbb{L}[G \cup \bigcup\limits_{j=1}^{K} H_j^t] \\
\forall t \in [1:T] & \mathbf{c}^t = \mathbb{L}^t(\mathbf{c}^{t-1})
\end{array}$$

Notice that Frankenmandering is a control problem. Specifically, optimising opinion diffusion over a social network via (restricted) graph connectivity control. In its naive form above it poses a much stronger requirement on the final outcome than Gerrymandering at time $t = T$ would do. However, it is readily changeable to a more recognisable variant of Gerrymandering if the objective function is swapped with one of the standard Gerrymandering outcomes at time $t = T$. We call this form: **Senat Frankenmandering**.

$$\begin{array}{rl}
\min\limits_{\mathbf{D}^1,\ldots,\mathbf{D}^T} & \sum\limits_{j=1}^{K} \|c^* - \mathbf{c}^T(r_j^T)\| \\
 & s.t. \\
\forall j \in [1:K]\ \forall t \in [1:T] & r_j^t = \mathbb{F}\left(\mathbf{c^t}|_{D_j^t}\right) \\
\forall j \in [1:K]\ \forall t \in [1:T] & H_j^t = (D_j^t, E_j^t, w_j^t) \\
\forall t \in [1:T] & \mathbb{L}^t = \mathbb{L}[G \cup \bigcup\limits_{j=1}^{K} H_j^t] \\
\forall t \in [1:T] & \mathbf{c}^t = \mathbb{L}^t(\mathbf{c}^{t-1})
\end{array}$$

4 Frankenmandering Feasibility

We would like to present two hand-crafted examples. The first example is to show that Frankenmandering, as a repeated extension of Gerrymandering, is possible. The second is to show that the presence of a social network can amplify even a single-step Gerrymandering solution into a Frankenmandering monster.

4.1 *Frankenmandering Inchworm*

In this first example, we demonstrate that by carefully selecting the districts $\mathbf{D}^t$ at each step, we can cause all opinions of a population to indefinitely shift in the positive direction. We assume that both the underlying physical network is a complete graph (i.e. any districting is possible) and that social network G is the empty graph (we ignore opinion dynamics within the voter population).

We begin with the following profile of opinions for $n = 10$ voters: $\{0, 0, 0, 1, 2, 3, 4, 5, 5, 5\}$. In each iteration, we create only a single district d^t of size 3 where the median voter is selected as the representative. The representative then exerts influence on the two other voters in d^t according to the following update rule, which depicts the update rule for voter v's opinion c_v:

$$c_v^{t+1} = \begin{cases} c_v^t + \text{sign}(c_u^t - c_v^t) & \|c_v - c_u\| < 3 \\ c_v^t - \text{sign}(c_u^t - c_v^t) & \|c_v - c_u\| \geq 3 \end{cases}$$

The Frankenmandering Problem, then, is to select a sequence of size-3 districts d^t such that we shift the entire profile of voter opinions by exactly $+1$; i.e. we end up with the target opinion profile $\{1, 1, 1, 2, 3, 4, 5, 6, 6, 6\}$. The intuition behind the solution is that district sequence proceeds in two phases: The first phase uses the backfire effect to "push" the most positive voters away from the main body while keeping the median voter close to the least positive voters to attract them toward a central value. The second phase selects increasingly higher opinion median voters to "pull" voters toward increasingly positive values. While this shift occurs, it is important to keep a "ladder" of voters with intermediate opinion values that can be used as median voters. The movement of voters is reminiscent of the locomotion of the inchworm.

Figure 2 illustrates a sequence of arrow diagrams that depicts the selection of districts that causes all opinions to shift exactly $+1$. In each arrow diagram, each voter is positioned (vertically) according to her opinion. The sequence of arrow diagrams proceeds from left to right. In each iteration t, the 3 voters shaded in orange are selected

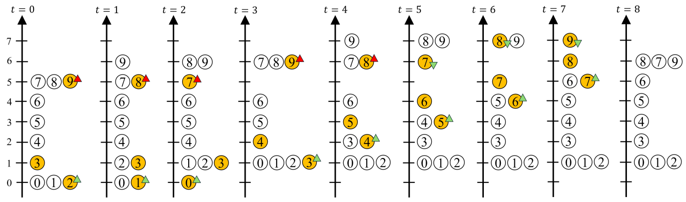


Figure 2. A Sequence of Gerrymandered Districts to Shift All Voters

to form the district d^t. The median voter becomes the representative, whose opinion remains uninfluenced. The voters' opinions may shift, and these shifts depicted as a small arrow: green arrows denote an attractive effect toward the representative, and red arrows denote the backfire effect pushing the voter away from the representative. These shifts are then reflected at the next arrow diagram at $t + 1$. The final ($t = 8$) diagram reproduces the initial ($t = 0$) opinion profile, with all opinions shifted $+1$. The same pattern can then be repeated to shift the entire population's opinions indefinitely.

It should be noted that this example does not require that the physical network be fully connected. The graph in Figure 3 show all the adjacencies required between voters to form contiguous districts. Note the required adjacencies form a planar graph.

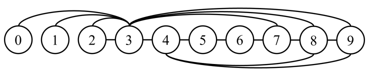


Figure 3. Physical Voter Adjacencies Required by the Example of Fig 2

4.2 Social Network Effect Example

In this second example, we show that we can achieve the same effect of shifting the opinions of the entire population of voters by defining only one district amongst the voters that remains unchanged. We do this by leveraging social influence between the voters. Let us define an initial opinion profile on $n = 6$ voters of $\{0, 1, 2, 3, 4, 6\}$, and a target opinion profile of $\{1, 2, 3, 4, 5, 7\}$. We will fix a district d^* to include exactly the first two voters and the last voter. The social network will be a line graph, with each voter connected to (up to) two most similar peers. Finally, we define the following update rule for opinion dynamics interactions:

$$c_v^{t+1} = \begin{cases} c_v^t & \|c_v - c_u\| < 2 \\ c_v^t + \text{sign}(c_u^t - c_v^t) & \|c_v - c_u\| < 4 \\ c_v^t - \text{sign}(c_u^t - c_v^t) & \|c_v - c_u\| < 6 \\ c_v^t & \|c_v - c_u\| \geq 6 \end{cases}$$

As before, the opinion dynamics captures the attractive effect of interacting with like-minded individuals and the backfire effect of interacting with those who are significantly different. New here are two plateaus where there is no net effect: either when individuals are highly similar or radically different[2].

The sequence of arrow diagrams in Figure 4 shows how this is accomplished. As in Figure 2, the vertical positions in each arrow diagram denotes the opinions of the voters. The social network among

[2] Interactions with highly similar individuals may focus on common grounds, while radically different individual may be simply ignored. More pragmatically, these changes are necessary to prevent a "bunching up" of lower range voters as they are brought closer to the median, and to avoid a "runaway effect" of upper range voters as they are pushed further away.

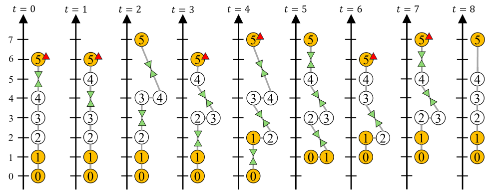


Figure 4. A Fixed Gerrymandered District to Shift All Voters

the voters is depicted as a gray line graph. Only voters 0, 1, and 5 form the fixed district d. Influence from the representative is depicted by small arrows as before (in this case, these are exactly the red arrows). Social influence between voters through the social network are drawn directly on the network (in this case, these are exactly the green arrows). By $t = 8$, the same opinion profile is regenerated, with all opinions shifted $+1$ from $t = 0$. With no further intervention, this system will shift opinions of all voters indefinitely in the positive direction.

5 Discussion and Future Work

In this paper, we present a novel interaction model of repeated redistricting with an underlying social network. The redistricting forces some nodes to become temporary opinion influencers in the social network, wherein the opinion undergo a non-trivial dynamic with "assimilation" and "backfire" properties. We term the strategic use of such redistricting **Frankenmandering**, and formulate an opinion control problem that corresponds to it. We show the feasibility of Frankenmanding by providing two key examples, where the opinions essentially "inchworm" in the desired direction.

Now, currently we seek two algorithmic solutions to Frankenmandering. First, seeing as Frankenmandering is a time-extended decision policy in a dynamic system, we are developing a reinforcement learning algorithm to solve Frankenmandering. In spite of its ostensibly high dimensionality, deep learning solution of influencer placing in an opinion network do exist, e.g. [12, 8, 46]. Our current designs follow these and other approaches capable of some reasoning over graph embeddings [24, 29]. Second, we are considering an ad-hoc algorithm based on reproducing the "inchworm" solution on a more general graph by seeking to reorder the "geography" graph and social graph to align with the order implied by the geodesic distance of voter opinions from the ideal opinion that we seek to achieve.

We submit our model and preliminary finding to the community, seeking comments, critique and cooperation to facilitate our progress.

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