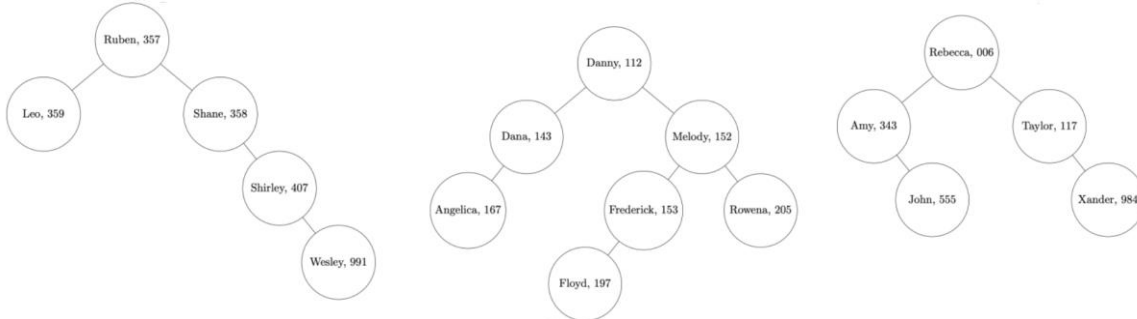


# Question 1: Quasi-balanced Search Trees

## Quasi-balanced Search Trees

In the following diagram, which we call a *quasi-balanced search tree (QUBSET)*, every node represents a student. Each student has a name and each student has a student number, both of which are shown in each node. Every diagram shown is structured by the same set of rules. *You can assume for the whole question that all names/IDs are unique.*



## Part A

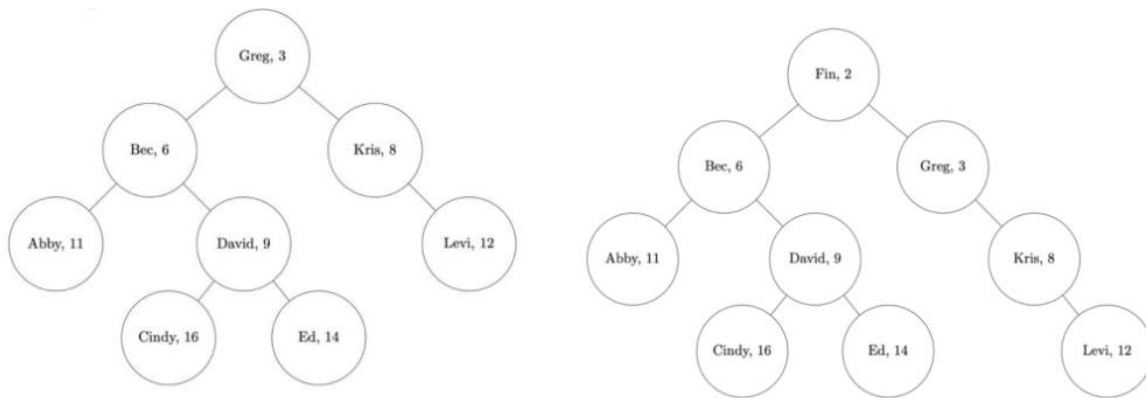
i. If you remove the student IDs and only consider the names, what do you notice about the order/structure? Explain your answer. **0.5 marks**

ii. If you remove the names and only consider the student IDs, what do you notice about the order/structure? Explain your answer. **0.5 marks**

*At the bottom of this page, we provide two examples that show what happens when we add a new student (node) to our diagram.*

## Part B

If we add the student Fin who has a student ID of 2, the *QUBSET* changes from left to right.



Write down all the missing steps in this process. You should provide just as much detail as the examples shown at the bottom. **2 marks**

## Part C

Given an arbitrary *QUBSET*,  $T$ , and a new student  $S$ , write a new function `add_student( $T$ ,  $S$ )` that adds the new student to the diagram. Assume the operation is done in place (there should be no return value). You can assume  $T_{\text{name}}$  and  $T_{\text{id}}$  give the student name/ID respectively.  $T$  and  $S$  are of the same type and you can assume  $S$  has no children. Your pseudocode should look like the pseudocode that is given in Lecture 9. **3 marks**

## Part D

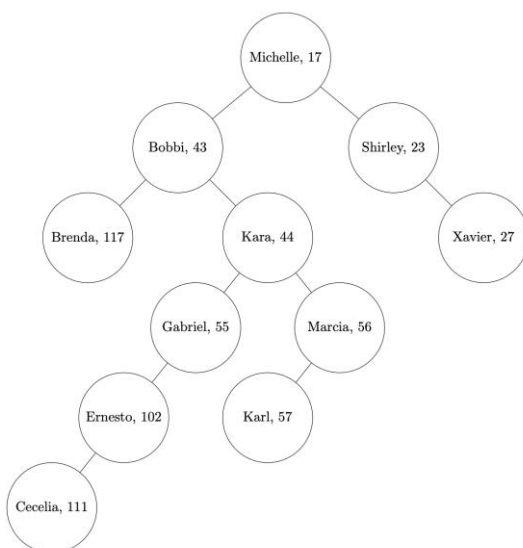
We want the height of *QUBSET* to be as small as possible. Give an example of the worst case height when we add 5 students to an empty *QUBSET*. **1 mark**

## Part E

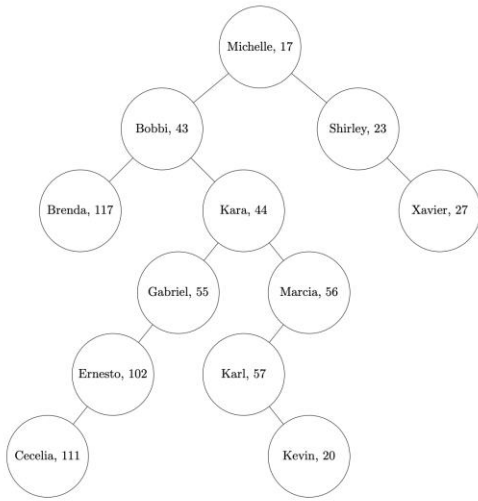
Suppose we have a group of  $n$  students that are listed in alphabetical order. If we add them to a binary search tree (sorted just by name and ignoring their student IDs), it can be shown that it degenerates to a tree of height  $n$ . Explain why the *QUBSET* we have used in this question is likely to have a height much smaller than  $n$ . **1 mark**

### Example 1

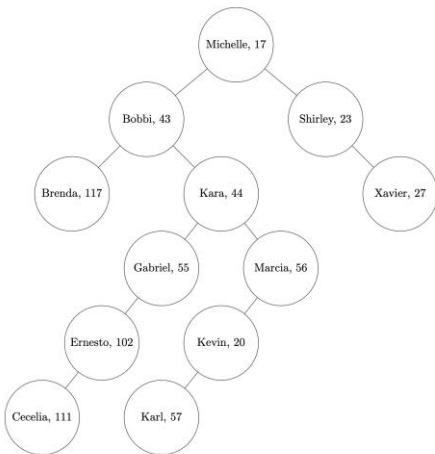
We start with the following *QUBSET*



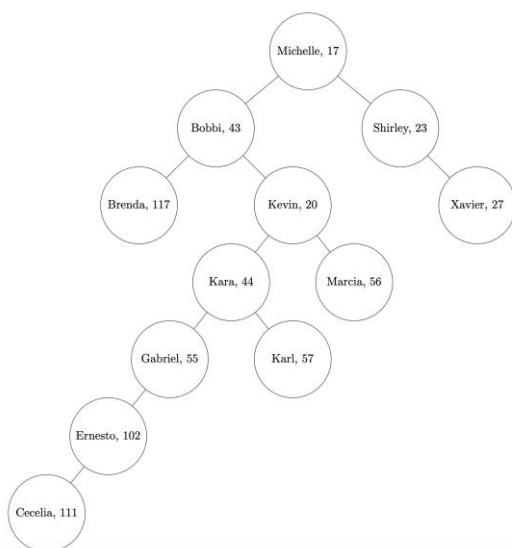
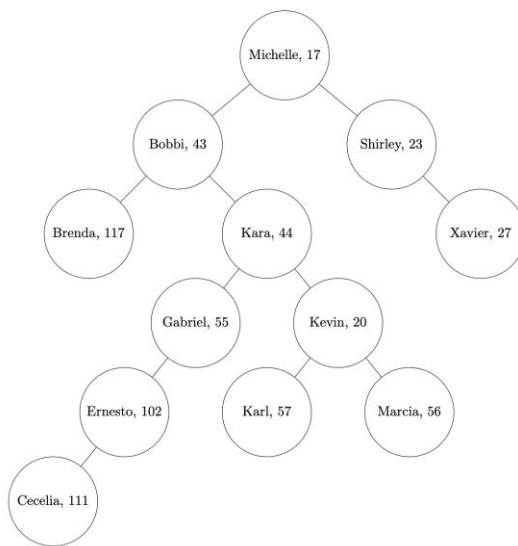
Now suppose we add Kevin who has a student ID of 20 to the *QUBSET* following the rule in part a) i.



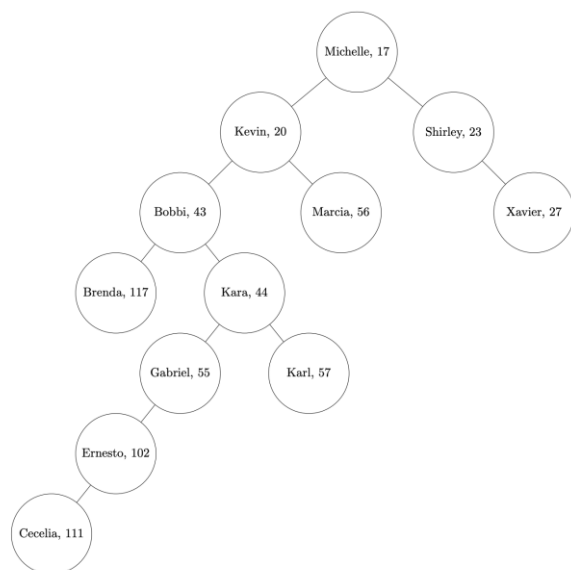
After this, the tree does not follow the rule in part a) ii. We continue to update the diagram until it follows the *QUBSET* rule.



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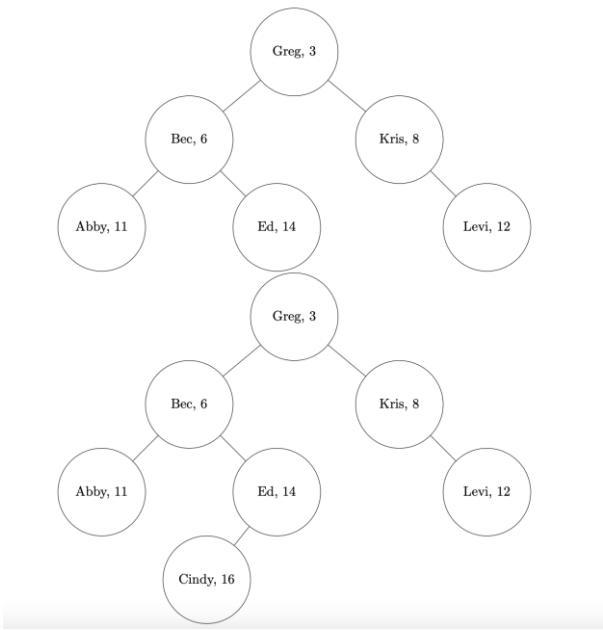
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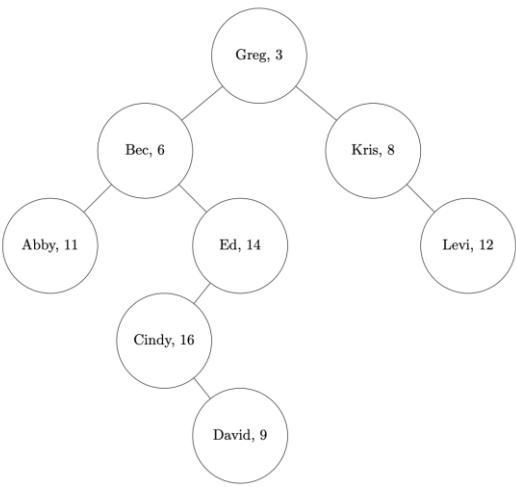
Now we satisfy the *QUBSET* rule in a) ii. and stop.

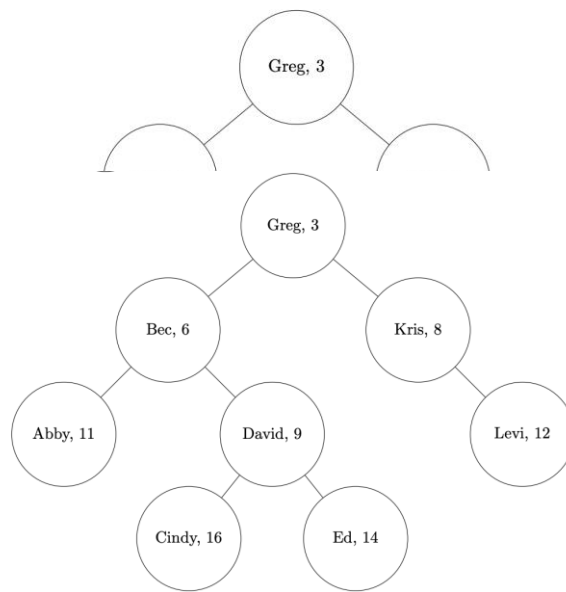
Example 2

If we start with the following *QUBSET* and then add Cindy who has a student ID of 16, we do not need to change anything afterwards.



Now suppose we add David who has a student ID of 9 into our *QUBSET*. The following steps occur





And after this we are done.