Chapter 7

Prospect Theory: Much Ado About Nothing?

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Prospect theory is a paradigm challenging the expected utility paradigm. One of the fundamental components of prospect theory is the S-shaped value function. The value function is mainly justified by experimental investigation of the certainty equivalents of prospects confined either to the negative or to the positive domain, but not of mixed prospects, which characterize most actual investments. We conduct an experimental study with *mixed* prospects, using, for the first time, recently developed investment criteria called Prospect Stochastic Dominance (PSD) and Markowitz Stochastic Dominance (MSD). We reject the S-shaped value function, showing that *at least* 62%–76% of the subjects cannot be characterized by such preferences. We find support for the Markowitz utility function, which is a reversed S-shaped function—exactly the opposite of the prospect theory value function. It is possible that the previous results supporting the S-shaped value function are distorted because the prospects had only positive or only negative outcomes, presenting hypothetical situations which individuals do not usually face, and which are certainly not common in financial markets.
(*Prospect Theory; Prospect Stochastic Dominance; Markowitz Stochastic Dominance*)

Introduction

Most models in economics and finance that deal with decision making under uncertainty and asset pricing rely on the von Neuman and Morgenstern (1944) expected utility theory. Even though some experimental studies reveal a few contradictions to the expected utility paradigm (see, for example Allais 1953 and Markowitz 1952b), it is still one of the pillars of economics. A challenging paradigm to the expected utility paradigm is prospect theory, developed by Kahneman and Tversky (1979). The main features of prospect theory are:[1]

(a) Investors make decisions based on *change* of wealth rather than on *total* wealth, in contrast to what is advocated by expected utility theory. It is interesting to note that using the change in wealth was postulated in the literature as early as 1952 (see Markowitz 1952b, p. 155).

(b) Investors maximize the expectation of a value function, $V(x)$, where x stands for the *change* in wealth (rather than total wealth). $V(x)$ is S-shaped: $V'(x) > 0$ for all $x \neq 0$,[2] $V''(x) > 0$ for $x < 0$, and $V''(x) < 0$ for $x > 0$. The parameters of the value function may change with wealth (hence, sometimes the notation $V_w(x)$ is used where w denotes wealth), but the S-shaped property of the value function is general to all initial wealth levels.

(c) Investors subjectively distort probabilities. They make decisions based on the subjective cumulative distribution F^*, which is given by $F^* = T(F)$, where F

[1] Prospect theory was extended by Tversky and Kahneman (1992). The extended version is known as cumulative prospect theory. In what follows, we do not make a distinction between prospect theory and cumulative prospect theory since we are dealing with the S-shaped value function which is a key feature in both frameworks.

[2] In general, $V'(x)$ may be nonexistent at $x = 0$. This is the case for the two-part power value function suggested by Tversky and Kahneman (1992), and for the value function with a "kink" at the origin, as suggested by Benartzi and Thaler (1995).

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denotes the objective cumulative distribution and T is some subjective transformation such that $T'(\cdot) > 0$, $T(0) = 0$, and $T(1) = 1$ (this is the main modification of cumulative prospect theory in comparison to prospect theory; see Footnote 1).

(d) The "framing" of alternative outcomes may strongly affect subjects' choices (see Tversky and Kahneman 1981).

These four properties found in experimental studies are the backbone of prospect theory. Prospect theory recently gained much popularity among economists, and there is a stream of papers that build economic models based on this theory.[3] There have been many empirical and experimental attempts to test prospect theory, most of which support the theory.[4] The purpose of this study is to report the results of an investment decision-making experiment carried out with three groups of subjects with distinctively varied characteristics: business school students; faculty of social science (mainly of business schools and economic departments), and practitioners who are financial analysts, mutual fund managers and portfolio managers. In this study we employ a recently developed investment criterion called Prospect Stochastic Dominance (PSD). We also employ for the first time a new criterion called Markowitz Stochastic Dominance (MSD), which is developed in this study. PSD is a criterion that determines the dominance of one investment alternative over another *for all prospect theory S-shaped value functions.* MSD is a criterion that determines the dominance of one investment alternative over another *for all reverse S-shaped functions,* as suggested by Markowitz (1952b). The PSD and MSD criteria allow us to test the prospect theory S-shaped value function hypothesis and the Markowitz reverse S-shaped hypothesis in a framework which avoids the serious problems of the more traditional certainty equivalent approach.

The structure of this paper is as follows: In §1 we briefly discuss the main classes of preferences suggested in the literature. In §2 we review the previous empirical evidence supporting the S-shaped value function, and we show why this evidence is very problematic and may be severely biased by having nonmixed payoffs and by the "certainty effect." This section also presents the PSD and MSD criteria. In §3 we describe the experiments and discuss the results. Concluding remarks are given in §4.

1. Characterization of Preferences

There is an ongoing debate in the literature regarding the shape of the utility (or value) function. Figure 1 provides the main utility functions advocated in the literature. Figure 1a depicts the classical utility function which is concave everywhere, in accordance with the notion of decreasing marginal utility. Such a function implies risk aversion, meaning that individuals would never accept any fair bet (let alone unfair bets). Friedman and Savage (1948) claim that the fact that investors buy insurance, buy lottery tickets, and buy both insurance and lottery tickets simultaneously, plus the fact that most lotteries have more than one big prize, imply that the utility function must have two concave regions with a convex region in between, as represented in Figure 1b.

Markowitz (1952b) points out several severe problems with the Friedman and Savage utility function.[5] However, he shows that the problems are solved if the first inflection point of the Friedman and Savage utility function is exactly at the individual's current wealth. Thus, Markowitz introduces the idea that decisions are based on *change* in wealth. Hence, the Markowitz utility function can be also thought of as a value function.

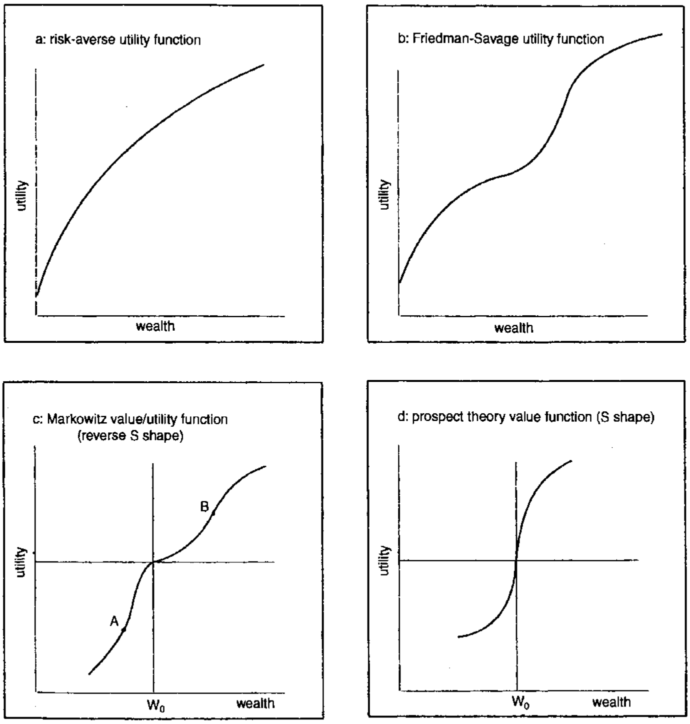
By analyzing several hypothetical gambles, Markowitz suggests that individuals are risk averse for losses and risk seeking for gains, as long as the possible outcomes are not very extreme. For extreme outcomes, Markowitz argues that individuals become

[3] For example, Thaler (1985) uses elements of prospect theory to model consumer behavior. Benartzi and Thaler (1995) employ prospect theory to explain the equity premium puzzle. Several recent studies investigate the implication of prospect theory to asset pricing (Levy et al. 2000, Barberis et al. 2001) and to asset allocation (Shefrin and Statman 1993, Levy 2000).

[4] For a comprehensive survey see Edwards (1996).

[5] For example, Markowitz argues that individuals with the Friedman and Savage utility function and wealth in the convex region would wish to take large symmetric bets, which is in contradiction to empirical observation.

Figure 1 Alternative Shapes of the Utility/Value Function



risk averse for gains, but risk seeking for losses. Thus, Markowitz suggests a utility function which is characterized by three inflection points, as shown in Figure 1c. Notice that the central part of this function (the range between Points A and B in Figure 1c) has a reversed S-shape. In the present study we confine our analysis to segment AB of the utility function, i.e., the reversed S-shape range.

The most well-known and most investigated class of value function is the prospect theory S-shaped function suggested by Kahneman and Tversky. Based on their experimental results with bets which are either negative or positive, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) claim that the value function is concave for gains and convex for losses, yielding an S-shaped function, as shown in Figure 1d.

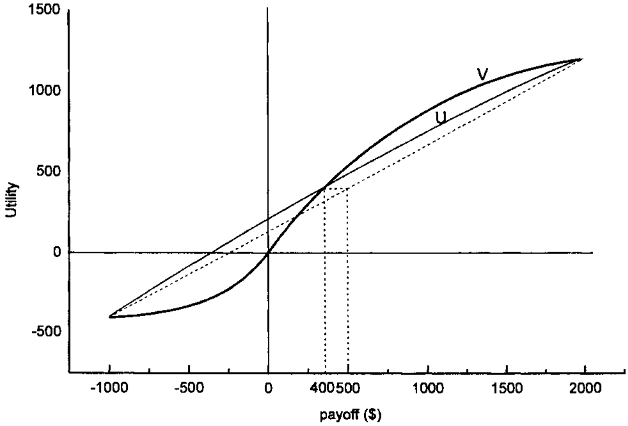
2. Methodology

2.1. The Certainty Equivalent Approach

When researchers investigate the value function they typically wish to characterize the shape of the function (concave or convex) for both the positive domain (gains) and the negative domain (losses). When the certainty equivalent method is employed, this implies finding the certainty equivalent of two positive outcomes and then *separately* finding the certainty equivalent of two negative outcomes (as done by Swalm

1966 and Kahneman and Tversky 1979). In this technique, the shape of the value function is separately investigated in the positive and the negative ranges. Researchers confined themselves to positive or to negative outcomes presumably because for a bet with both positive outcomes and negative outcomes (a mixed bet), the certainty equivalent approach cannot indicate in a simple way whether the value function is convex or concave. To see this, suppose that the certainty equivalent of $-\$1{,}000$ with probability $\frac{1}{2}$ and $+\$2{,}000$ with a probability $\frac{1}{2}$ is found to be \$400. This certainty equivalent can be explained by a standard utility function which is concave everywhere, but also by an S-shaped function (or even by a reverse S-shaped function). Figure 2 demonstrates this point with the bet mentioned above. The mean outcome is \$500, and a risk-averse utility function, U, or an S-shaped value function, V, can both justify the certainty equivalent of \$400 assigned to this bet. Hence, in the certainty equivalent framework it is practically impossible to draw any conclusions regarding the shape of preference with mixed bets. Such an ambiguous result regarding the value function can be avoided by restricting outcomes either to the positive domain or to the negative domain (as in Table 1 of Kahneman and Tversky 1979, p. 268, and also in Tasks 1 and 2 of Experiment 3 of this study, given in Table 3). Indeed, to circumvent this difficulty, researchers typically study separately the negative and the positive segments (see Swalm 1966, Kahneman and Tversky 1979, and Tversky and Kahneman 1992). While technically one can easily pinpoint the shape of the utility function with the certainty equivalent approach, this approach presents subjects with an unrealistic situation, because in real life virtually all investments—e.g., stocks, bonds, options, and real estate—yield an uncertain distribution of outcomes covering the positive as well as the negative domain. As the way an experiment is "framed" is crucial (see Kahneman and Tversky 1981), it is possible that framing the experiment in an unrealistic way with hypothetical distributions that investors never, or almost never, face in reality, distorts the results. Moreover, by the certainty equivalent approach some of the outcomes are certain. It is well known that in such cases the "certainty effect" may strongly affect choices (see Allais 1953, and Tversky and Kahneman 1981). Thus, in the certainty equivalent approach we do not know whether the subjects' choices indicate an S-shaped function, the distortion of probability, or both. In the methodology employed in this study there are no certain outcomes, hence there is no "certainty effect."

Figure 2 The Certainty Equivalent of a Mixed Prospect



The recently developed Prospect Stochastic Dominance (PSD) analysis and the Markowitz Stochastic Dominance (MSD) analysis developed in this paper circumvent the above difficulties. PSD is a criterion which can be applied to any prospect: negative, positive, and, most importantly, mixed, which is the case closest to the characteristics of actual financial investments. PSD analysis is insensitive to the details of any specific value function as long as it is S-shaped: If Prospect F dominates Prospect G by PSD, then F is preferred over G by *any* prospect theory S-shaped value function. Thus, by employing PSD we do not try to figure out the precise shape of the value function, but we can test whether it belongs to a given family of functions, i.e., the S-shape family. The same is true of the MSD criterion and the reversed S-shape family. Similarly to hypothesis testing in statistics, with PSD and MSD we can either reject a given hypothesis, or support it by not being able to reject it. Using PSD and MSD analyses with mixed prospects and a large and diversified subject population, we reject the hypothesis of an S-shaped prospect theory value function and support the Markowitz reverse S-shaped function. The stochastic dominance methodology and the competing PSD and MSD decision rules are discussed next.

2.2. Stochastic Dominance, Prospect Stochastic Dominance, and Markowitz Stochastic Dominance

We present below several stochastic dominance rules, focusing on a recently developed rule called prospect stochastic dominance (PSD) and a new rule, which is developed here, called Markowitz stochastic dominance (MSD). The advantage of the stochastic dominance approach is that we have a decision rule which holds for all utility or value functions of a certain class. Specifically, PSD is a criterion which is valid for *all* S-shaped value functions. Moreover, stochastic dominance rules can be employed with mixed prospects, with no certainty effect, and with change of wealth (rather than total wealth). Let us elaborate.

In prospect theory decisions are based on change in wealth rather than on total wealth. In the expected utility framework decisions are based on total wealth. Recall, however, that if one prospect dominates another by first-degree stochastic dominance (FSD) or second-degree stochastic dominance (SSD) when the outcomes are given in terms of change in wealth, then the dominance relation also holds in terms of total wealth, for any initial wealth level.[6] For example, suppose that an investor faces the following two prospects, where the outcomes are in terms of change in wealth: Prospect F is $\{-\$1, \frac{1}{2}; \$4, \frac{1}{2}\}$ (i.e., $-\$1$ or $\$4$ are received with an equal probability of $\frac{1}{2}$), and Prospect G is $\{-\$2, \frac{1}{2}; \$1, \frac{1}{2}\}$ (i.e., $-\$2$ or $\$1$ are received with an equal probability of $\frac{1}{2}$). F dominates G by FSD, and therefore any investor with a nondecreasing utility (or value) function will prefer F over G, independent of his initial wealth. The same argument also holds for PSD and MSD, hence one can safely ignore the initial wealth. Therefore, without loss of generality we formulate below stochastic dominance rules in terms of changes in wealth.

The following classes of preferences are defined:

$\mathbf{U}_1$—The class of all monotonic nondecreasing utility functions U, i.e., $U \in \mathbf{U}_1$ if $U' \geq 0$.

$\mathbf{U}_2$—the class of all nondecreasing concave utility functions, i.e., $U \in \mathbf{U}_2$ if $U' \geq 0$ and $U'' \leq 0$. $\mathbf{U}_2$ is a subset of $\mathbf{U}_1$.

$\mathbf{V}_{KT}$—the class of all prospect theory value functions (where the subscript KT denotes Kahneman and Tversky): functions which are S-shaped with an inflection point at $x = 0$. Thus, $V \in \mathbf{V}_{KT}$ if $V' \geq 0$ for

[6] FSD and SSD can be stated either in terms of total wealth or in terms of change of wealth without contradicting expected utility because $F(x) \leq G(x)$ for all $x \Leftrightarrow F(w+x) \leq G(w+x)$ for all $x \Leftrightarrow E_F U_F(w+x) \geq E_G U(w+x)$ for all non-decreasing $U(w+x)$, (FSD, see Equation (1) below). Similarly,

$$\int_{-\infty}^{x} (G(t) - F(t))\,dt \geq 0 \quad \text{for all } x$$
$$\Leftrightarrow \quad \int_{-\infty}^{x} [(G(w+t) - F(w+t)]\,dt \geq 0 \quad \text{for all } x$$
$$\Leftrightarrow \quad E_F U(w+x) \geq E_G U(w+x)$$

for all nondecreasing concave U(w+x), (SSD, see Equation (2) below). (The two distributions F and G are shifted by w without affecting the relationship between them). Thus, we can conclude that without loss of generality, in testing whether FSD or SSD prevail, we can focus on change of wealth and ignore the initial wealth. By both rules, dominance prevails with change of wealth if and only if it prevails with total wealth. For a review article of stochastic dominance rules, see Levy (1992).

all $x \neq 0$, $V'' \geq 0$ for $x > 0$, and $V'' \geq 0$ for $x < 0$. Notice that since the value function is nondecreasing ($V' \geq 0$), $\mathbf{V}_{\mathrm{KT}}$ is a subset of $\mathbf{U}_1$. We call V a value function, rather than a utility function, to be consistent with the terminology of Kahneman and Tversky.

In the discussion of Markowitz utility functions we confine ourselves to the range between the two extreme inflection points of the Markowitz utility function (Points A and B in Figure 1c), which are expected to be at extreme wealth levels (see Markowitz 1952b). Thus we define:

$\mathbf{V}_{\mathrm{M}}$—the class of all Markowitz utility functions (where the subscript M denotes Markowitz): functions which are reverse S-shaped with an inflection point at $x = 0$. Thus, $V \in \mathbf{V}_{\mathrm{M}}$ if $V' \geq 0$ for all $x \neq 0$,[7] $V'' \geq 0$ for $x > 0$, and $V'' \leq 0$ for $x < 0$. Notice that since the value function is nondecreasing ($V' \geq 0$), $\mathbf{V}_{\mathrm{M}}$ is also a subset of $\mathbf{U}_1$. As Markowitz's function, like the prospect theory value function, depends on change of wealth, we denote it by $\mathbf{V}_{\mathrm{M}}$, rather than $\mathbf{U}_{\mathrm{M}}$.

We turn now to a few decision rules which are known as stochastic dominance rules. As explained above, because all of these rules (not only FSD as in the example above) are invariant to the initial wealth, they can be written in terms of the change of wealth x. We briefly describe the well-known FSD and SSD rules. We then present the prospect stochastic dominance rule (PSD), which corresponds to all S-shaped value functions, and the Markowitz stochastic dominance rule (MSD), which corresponds to all reverse S-shaped value functions. All these rules will be examined in the experiment. We focus on PSD and MSD, which may indicate whether the prospect theory value function is valid in the general case, where the subjects have to make choices between mixed prospects with negative as well as positive outcomes, and in which there is no certainty effect.

FSD (First-Degree Stochastic Dominance). Let F and G be two distinct prospects with cumulative distributions F and G, respectively. Then,

$$F(x) \leq G(x) \text{ for all } x \quad \Leftrightarrow \quad E_F U(x) \geq E_G U(x) \text{ for all } U \in \mathbf{U}_1, \tag{1}$$

where there is a strict inequality for some $x = x_0$, and a strict inequality for some $U_0 \in \mathbf{U}_1$.

SSD (Second-Degree Stochastic Dominance). Define F and G as above. Then,

$$\int_{-\infty}^{x} [G(t) - F(t)]\, dt \geq 0 \text{ for all } x \quad \Leftrightarrow \quad E_F U(x) \geq E_G U(x) \text{ for all } U \in \mathbf{U}_2, \tag{2}$$

where there is a strict inequality for some $x = x_0$, and a strict inequality for some $U_0 \in \mathbf{U}_2$.

For proof of the FSD and SSD rules, see Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970). For a survey on stochastic dominance rules, see Levy (1992).

PSD (Prospect Stochastic Dominance). Define F and G as above. Then F dominates G for all S-shape utility/value functions, $V \in \mathbf{V}_{\mathrm{KT}}$, if and only if

$$\int_{y}^{0} [G(t) - F(t)]\, dt \geq 0 \text{ for all } y \leq 0 \quad \text{and} \quad \int_{0}^{x} [G(t) - F(t)]\, dt \geq 0 \text{ for all } x \geq 0. \tag{3}$$

(Once again, we require a strict inequality for some pair (y_0, x_0) and for some $V_0 \in \mathbf{V}_{\mathrm{KT}}$.) A proof of PSD and more detail can be found in Levy (1998), and Levy and Wiener (1998).

MSD (Markowitz Stochastic Dominance). Define F and G as above. Then F dominates G for all reverse S-shaped value functions, $V \in \mathbf{V}_{\mathrm{M}}$, if and only if

$$\int_{-\infty}^{y} [G(t) - F(t)]\, dt \geq 0 \quad \text{for all } y \leq 0 \quad \text{and} \quad \int_{x}^{\infty} [G(t) - F(t)]\, dt \geq 0 \text{ for all } x \geq 0, \tag{4}$$

(with at least one strict inequality). We call this dominance relation MSD—Markowitz Stochastic Dominance.

Proof. See Appendix A.

Although Conditions (3) and (4) look similar, we will show below that it is possible that F dominates G by PSD, but G dominates F by MSD.

While the intuitions of FSD and SSD are relatively straightforward, the intuition for PSD and MSD is less transparent. The intuition for SSD and the risk-seeking dominance (RSD) rule (presented below) are important in particular because both PSD and MSD

[7] As in the case of S-shaped functions, in the general case we allow for $V'(0)$ to be nonexistent.

preferences contain both risk-averse and risk-seeking segments. Therefore, once we understand the intuition for SSD and RSD, the intuition for PSD and MSD becomes straightforward.

FSD. $F(x) \leq G(x)$ for all x implies that $1 - F(x) \geq 1 - G(x)$ for all x, or $\Pr_F(X \geq x) \geq \Pr_G(X \geq x)$ for all x; Thus, the probability of getting x or more is higher under F than under G for all x; consequently, any individual who prefers more money over less money will prefer F.

SSD. For any utility function the difference in expected utility between prospects F and G is given by:

$$\Delta \equiv E_F U(x) - E_G U(x) = \int_{-\infty}^{\infty} [G(t) - F(t)] U'(t)\, dt. \quad (5)$$

Equation (5) is obtained by integration by parts of the difference in expected utility of the two prospects, see Hadar and Russell (1969), Hanoch and Levy (1969), Rothschild and Stiglitz (1970), and Levy (1998). The condition for SSD dominance of F over G is that $\int_{-\infty}^{x} [G(t) - F(t)]\, dt \geq 0$ for all x. Graphically, this means that the area enclosed between G and F from $-\infty$ to any value x is positive. Thus, the SSD condition states that for any negative area ($F > G$) there is a larger *preceding* positive area ($G > F$). In calculating Δ these areas are multiplied by U' (see Equation (5)), and as U' is a declining function for risk averters ($U'' \leq 0$) the SSD condition ensures that the positive contribution to Δ of the positive areas is greater than the negative contribution to Δ of the negative areas, and therefore $\Delta \geq 0$.

RSD. The dominance condition of F over G for risk-seeking investors ($U' \geq 0$, $U'' \geq 0$ in the whole domain of x) is that $\int_{x}^{\infty} [G(t) - F(t)]\, dt \geq 0$ for all x (see Levy 1998, pp. 330–332). Graphically this means that the *last* area (largest x) enclosed between G and F is positive (which is not required by SSD). There may be a negative area before the last positive area, but it must be smaller than the last positive area, and so forth—for each negative area there is a larger positive area which follows (at larger x). The explanation for the RSD criterion is that we have, as before, $\Delta \equiv E_F U(x) - E_G U(x) = \int_{-\infty}^{\infty} [G(t) - F(t)] U'(t)\, dt$ (see Equation (5)), but this time U' is an increasing function ($U'' \geq 0$). If the RSD criterion holds, the contribution of the positive areas to Δ is larger than the negative contribution of the negative areas, and we have $\Delta \geq 0$ or $E_F U(x) \geq E_G U(x)$ for all risk-seeking investors. Having the intuitive explanation for risk-averse and risk-seeking dominance, the intuition of PSD and MSD becomes straightforward.

PSD. In this case we have $U'' \geq 0$ for $x < 0$, and $U'' \leq 0$ for $x > 0$, and the condition for the dominance of F over G by PSD is: $\int_{0}^{x} [G(t) - F(t)]\, dt \geq 0$ for all $x > 0$ and $\int_{y}^{0} [G(t) - F(t)]\, dt \geq 0$ for all $y < 0$ (see Equation (3)). The intuition for this condition relates directly to the preceding explanations about SSD and RSD. We want F to be preferred over G by any S-shaped utility function. Consider an S-shaped utility function which is almost linear and with a very small slope for $y < 0$ (i.e., $U' \approx 0$ for $y < 0$). For such a function the contribution of the negative domain to Δ of Equation (5) is negligible, and we should only consider the risk-averse condition for the positive range, which is $\int_{0}^{x} [G(t) - F(t)]\, dt \geq 0$ for all $x > 0$. Similarly, considering an S-shaped function which is almost linear and flat in the positive range yields the risk-seeking condition for the negative range, which is $\int_{y}^{0} [G(t) - F(t)]\, dt \geq 0$ for all $y < 0$. Therefore, for F to dominate G by PSD for all S-shaped functions, both of these conditions must hold.

MSD. Similar to the logic of the PSD intuition, consider a reverse S-shaped function which is almost linear and flat in the negative domain. For such a function the contribution of the negative domain to Δ is negligible, and we should only consider the risk-seeking condition for the positive range, which is $\int_{x}^{\infty} [G(t) - F(t)]\, dt \geq 0$ for all $x > 0$. A reverse S-shaped function which is almost linear and flat in the positive domain dictates the risk-averse condition for the negative range, which is $\int_{-\infty}^{y} [G(t) - F(t)]\, dt \geq 0$ for all $y \leq 0$. As we encompass all reverse S-shaped functions by MSD, the MSD rule is a combination of these two conditions (for formal proof of the MSD rule, see Appendix A).

MSD is generally not "the opposite" of PSD. In other words, if F dominates G by PSD, this does not necessarily mean that G dominates F by MSD. This is easy to see, because having a higher mean is a necessary condition for dominance by both rules.[8] Therefore, if F dominates G by PSD, and F has a higher

[8] To see this, choose $y = -\infty$ and $x = +\infty$ in Equation (3), and $y = x = 0$ in Equation (4), and employ Equation (5) with $U(x) = x$.

mean than G, G cannot possibly dominate F by MSD. However, as Corollary 1 below states, PSD and MSD *are* opposite if the two distributions have the same mean.

Corollary 1. *Let F and G have the* same mean. *Then F dominates G by PSD if and only if G dominates F by MSD.*

Proof. From the relation $E_F(x) - E_G(x) = \int_{-\infty}^{\infty}[G(t) - F(t)]\,dt$ (Equation (5) with $U(x) = x$) we know that if F and G have the same mean, then $\int_{-\infty}^{\infty}[G(t) - F(t)]\,dt = 0$, or $\int_{-\infty}^{0}[G(t) - F(t)]\,dt + \int_{0}^{\infty}[G(t) - F(t)]\,dt = 0$. If F dominates G by PSD, then each of these two terms must equal zero. To see this claim, recall that by the equal-mean condition, both terms cannot be negative. If one is positive, by the equal mean condition the other must be negative, and Equation (3) does not hold, so F cannot dominate G by PSD. By the same argument, if G dominates F by MSD we must also have $\int_{-\infty}^{0}[G(t) - F(t)]\,dt = \int_{0}^{\infty}[G(t) - F(t)]\,dt = 0$. Thus, if one distribution dominates the other by either PSD or MSD, this implies $\int_{-\infty}^{0}[G(t) - F(t)]\,dt = 0$ and $\int_{0}^{\infty}[G(t) - F(t)]\,dt = 0$. Now, let us prove the claim of Corollary 1. F dominates by PSD only if for every $y < 0$, $\int_{y}^{0}[G(t) - F(t)]\,dt \geq 0$. But because in the case of equal means we have $\int_{-\infty}^{0}[G(t) - F(t)]\,dt = 0$, this implies that $\int_{-\infty}^{y}[G(t) - F(t)]\,dt \leq 0$ for every $y < 0$, or $\int_{-\infty}^{y}[F(t) - G(t)]\,dt \geq 0$ for every $y < 0$, which is the condition for the MSD dominance of G over F in this range (see Equation (4)). Similarly, $\int_{0}^{x}[G(t) - F(t)]\,dt \geq 0$ for all $x > 0$ implies $\int_{x}^{\infty}[G(t) - F(t)]\,dt \leq 0$ for all $x > 0$, which in turn leads to $\int_{x}^{\infty}[F(t) - G(t)]\,dt \geq 0$ for all $x > 0$. Thus, for F and G with equal mean, F dominates G by PSD if and only if G dominates F by MSD. □

3. The Experiments and the Results

The subjects in our experiments were students, university professors, and practitioners. The students are graduate and undergraduate students in the business schools at UCLA, the University of Washington, and the Hebrew University. The university professors are faculty in the business and economics departments at Baruch College, UCLA, and the Hebrew University. The practitioners are financial analysts, mutual funds managers, and portfolio managers.

The subjects were asked to answer a questionnaire with one or more tasks. In each task the subjects had to choose between two investments denoted by F and G. We have conducted three experiments, with questionnaires as given in Tables 1, 2, and 3. All probabilities given in the experiments are relatively large ($p \geq 0.25$),[9] hence it is unlikely that subjective probability distortion plays an important role in the decision-making process.[10] Since we have no certain outcome, the certainty effect is neutralized.

Experiment 1: Design

Experiment 1 is designed to investigate which type of preference class (risk aversion, S-shaped functions, or reverse S-shaped functions) best describes individual choice under risk. Subjects were presented with four tasks, as given in Table 1. In Task I, G dominates F by PSD, but F dominates G by MSD, as shown by Figure 3. Thus, G dominates F for all value functions $V \in \mathbf{V}_{KT}$ because Condition (3) holds with F and G in switched roles. At the same time, Condition (4) holds, and hence F dominates G for all reverse S-shaped functions $V \in \mathbf{V}_{M}$. Thus, Task I is a head-to-head competition between these two alternative theories. Note that neither prospect dominates the other by SSD.

Task IV is designed to test whether risk aversion provides a good description of behavior. In this task, G dominates F by SSD. In the remaining two tasks, Tasks II and III, one prospect dominates the other by FSD. These tasks are "controls" designed to test whether the subjects comprehend the experimental setup, and whether they are rational in the sense of preferring more money to less money.

The subjects participating in Experiment 1 were 132 students, 66 professors, and 62 practitioners, for a total of 260 subjects.

Experiment 1: Results

The results of the experiment are given in Table 1b. As there are no significant differences between the results

[9] Except in Task 3 of Experiment 1, which tests for FSD, and is a "control" task in our experiment.

[10] It is found experimentally that small probabilities, e.g., those related to the chance of winning a lottery, are subjectively distorted by subjects. However, this distortion typically takes place for extreme probabilities, and not for the moderate probabilities employed here.

Table 1a Experiment 1: The Choices Presented to the Subjects

Suppose that you decided to invest $10,000 either in stock F or in stock G. Which stock would you choose, F, or G, when it is given that the *dollar gain or loss* one month from now will be as follows.

TASK I:

F		G	
Gain or loss	Probability	Gain or loss	Probability
−3,000	1/2	−6,000	1/4
4,500	1/2	3,000	3/4

Please write F for G: ☐

TASK II:
Which would you prefer, F or G, if the dollar gain or loss one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
−500	1/3	−500	1/2
+2,500	2/3	2,500	1/2

Please write F for G: ☐

TASK III:
Which would you prefer, F or G, if the dollar gain or loss one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
+500	3/10	−500	1/10
+2,000	3/10	0	1/10
+5,000	4/10	+500	1/10
		+1,000	2/10
		+2,000	1/10
		+5,000	4/10

Please write F or G: ☐

TASK IV:
Which would you prefer, F or G, if the dollar gain or loss one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
−500	1/4	0	1/2
+500	1/4		
+1,000	1/4	+1,500	1/2
+2,000	1/4		

Please write F or G: ☐

Table 1b The Results of Experiment 1*

Task No.	F	G	Indifferent	Total
I ($G \succ_{PSD} F$, $F \succ_{MSD} G$)	71	27	2	100
II ($F \succ_{FSD} G$)	96	4	0	100
III ($F \succ_{FSD} G$)	82	18	0	100
IV ($G \succ_{SSD} F$)	47	51	2	100

Number of subjects: 260

*Numbers in the tables are in percent, rounded to the nearest integer. The notations $\succ_{FSD}$, $\succ_{SSD}$, $\succ_{PSD}$, and $\succ_{MSD}$ indicate dominance by FSD, SSD, PSD, and MSD, respectively.

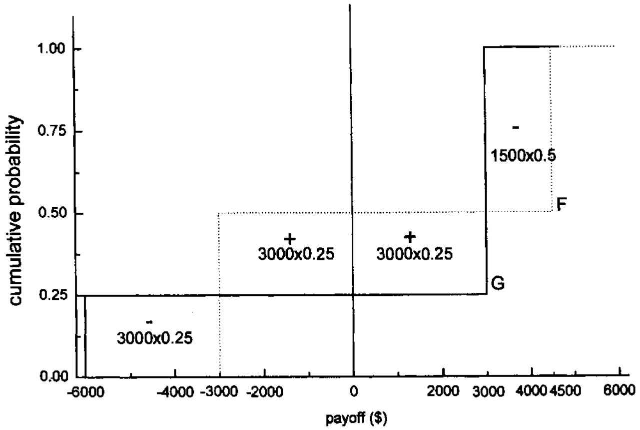
obtained in different institutions, or across the subject categories, we report only the aggregate results.

In Tasks II and III F dominates G by FSD because $F(x) \le G(x)$ for all values x. Indeed, in all groups of subjects we find a strong preference for F, the FSD dominating prospect (see Task II and Task III). In Task II there is an obvious FSD superiority of F over G, and therefore we are not surprised with the experimental findings. In Task III the FSD preference of F over G is a little less transparent in comparison to Task II; still, almost all subjects identified the superior FSD investment in this task. Given that investors are rational, and always prefer more money to less money, the findings in Tasks II and III are expected regardless of the investors' specific preferences (at least with our sophisticated group of subjects, where we do not expect to find many errors in the decision making). This is because FSD is a decision rule which corresponds to all types of nondecreasing utility or value functions.[11]

The findings of Tasks II and III neither confirm nor reject prospect theory, since there is FSD dominance

[11] In their 1979 paper, Kahneman and Tversky propose possible distortion in probabilities by the decision makers. Such a distortion may imply that an option which is inferior to the other by FSD (with the objective probabilities) may be selected and the superior FSD option may be rejected. Kahneman and Tversky realize that FSD dominance is a cornerstone for the decision-making process, hence suggested in their 1992 paper the cumulative version of prospect theory such that if F dominates G with the objective probabilities, the dominance will not be violated even after the probability distortion (see also Quiggin 1982 and Yaari 1987). Indeed, FSD is sometimes used as one of the axioms of expected utility theory (see Fishburn 1982 and Levy 1998).

Figure 3 **Task I of Experiment 1 – *G* Dominates *F* by PSD, *F* Dominates *G* by MSD**



of F over G, and therefore a subject with any non-decreasing utility or value function should prefer F to G, whether he has a "standard" concave utility function, an S-shaped value function, or a reverse S-shaped function. Thus, any choice of G over F is due to irrationality or due to a possible human error. Fortunately, we did not find many such errors, indicating that the subjects were not answering the questionnaire irrationally or arbitrarily. This point is important for the interpretation of the results of Tasks I and IV, which are the focus of this experiment.

In Task I neither F nor G dominates the other by FSD or by SSD (see Figure 3). However, G dominates F by PSD, thus we would expect any subject with S-shaped preferences to select Prospect G in Task I. At the same time, F dominates G by MSD. Thus Task I constitutes a head-to-head race between two competing theories about preferences: the prospect theory S-shaped function and the Markowitz reverse S-shaped function. The striking result is that the majority of subjects (71%) preferred prospect F, strongly contradicting the S-shaped function suggested by prospect theory, and supporting the reverse S-shaped function hypothesis. It is interesting to note that the preference of F over G in Task I is even stronger among the more sophisticated investors—practitioners (79%) and faculty (86%) (for brevity's sake not reported in the table).

The highest proportion rejecting at least one segment of the S-shaped value function in the Kahneman and Tversky experiment is 20% (see Kahneman and Tversky 1979, p. 268). We have on the aggregate 71% of the choices rejecting the S-shaped function. A statistical test reveals that the difference in the proportion of subjects rejecting the S-shaped function in these two studies is highly significant with a z-statistic of 8.59.[12]

[12] The sample statistic is:

$$z = \frac{\bar{p}_{E1} - \bar{p}_{KT}}{\sqrt{\frac{p^* q^*}{N_{E1}} + \frac{p^* q^*}{N_{KT}}}},$$

where $\bar{p}_{E1}$ and $\bar{p}_{KT}$ are the sample mean proportions rejecting the S-shaped function in Experiment 1 (Task I) and the Kahneman and Tversky experiment, respectively, N_{E1} and N_{KT} are the number of observations in these two experiments, p^* is the overall sample proportion:

$$p^* = \frac{\bar{p}_{E1} N_{E1} + \bar{p}_{KT} N_{KT}}{N_{E1} + N_{KT}},$$

and $q^* = 1 - p^*$. For Task I of Experiment 1 we obtain:

$$z = \frac{0.71 - 0.20}{\sqrt{\frac{0.57 \cdot 0.43}{260} + \frac{0.57 \cdot 0.43}{95}}} = 8.59.$$

The results in Task I constitute a rejection of the S-shaped function because they imply that *at least* 71% of the subjects do not have this type of preference.[13] This result lends support to the Markowitz reverse S-shaped function hypothesis, yet proving this hypothesis is much more difficult than rejecting the S-shape hypothesis. This is because one could claim that individuals are driven to choose F by preferences other than a reverse S-shaped function. For example, it is possible that subjects are risk averse, and while F does not dominate G by SSD, it could be that most subjects have higher expected utility under F. One could also suspect that the subjects (explicitly or implicitly) employ the Markowitz (1952a, 1959, 1987) well-known mean-variance rule, because,

$$E_F(x) = E_G(x) = 750 \quad \text{and} \quad \sigma_F(x) = 3750 < \sigma_G(x) = 3897$$

hence, F is slightly preferable to G by the mean-variance rule. Thus, the subjects may employ the mean-variance rule even if it is not justified in this specific case because the distributions are not normal; see Tobin (1958) and Hanoch and Levy (1969).

Task IV is designed to address these issues. In Task IV, there is no FSD and no PSD or MSD dominance, but G dominates F by SSD. Thus, if subjects are risk-averse we would expect strong support for G. We find that only about half of the subjects selected Prospect G.[14] Thus, almost half of the subjects made choices contradicting risk aversion, casting grave doubt on the ability of preferences which are concave throughout (as in Figure 1a) to describe the subjects' behavior. This contradiction of risk aversion is consistent with the findings of Levy and Levy (2001).

The results of Task IV also casts doubt on the hypothesis that the subjects employed the mean-variance rule in their decision making. We tend to reject this hypothesis for the following reason: In Task IV, Prospect G dominates prospect F not only by SSD but also by the mean-variance rule:

$$E_F(x) = E_G(x) = 750 \qquad \sigma_F(x) = 901 > \sigma_G(x) = 750.$$

Moreover, the mean-variance dominance of G over F is much more pronounced in Task IV than the mean-variance dominance of F over G in Task I. If the subjects employ the mean-variance rule (even if there is no theoretical justification to employ this rule here, as suggested above), we would expect to find a strong preference for Prospect G in Task IV. This did not occur, and approximately half of the subjects preferred Prospect F in Task IV. Thus, the decision making of the subjects is not driven by the mean-variance rule.

Experiment 2: Design

Experiment 2 is designed to create another head-to-head competition between the prospect theory S-shaped function and the Markowitz reverse S-shaped function. The subjects in this experiment were 84 business school students. Table 2a provides the task in this experiment. It is straightforward to verify that F dominates G by PSD, but G dominates F by MSD. The task in this experiment is different than Task I of Experiment 1 in two respects: First, it is more complex. In this experiment there are four possible outcomes for each prospect, rather than the two outcomes of Task I in Experiment 1. Second, in Experiment 2 all the outcomes are equally likely. This is an attractive feature, because it makes any subjective probability distortion very unlikely.[15]

Experiment 2: Results

The results of Experiment 2 are given in Table 2b. The table reveals that 62% of the subjects chose Prospect G, which is inferior by PSD but dominant by MSD. Thus, we can state that *at least* 62% of the choices are inconsistent with prospect theory. While this result again rejects the S-shaped function and

[13] We stress "at least" because any individual with S-shaped preferences should prefer G, but choosing G does not imply that preferences are necessarily S-shaped.

[14] 71% of the faculty preferred G in Task IV. It is interesting that while the faculty (most of whom are conscious of the normative theory) chose G, most students (59%) who decide based on "gut feeling" prefer F.

[15] This point is made by Quiggin (1982). In addition, any subjective transformation performed directly on the probabilities (as in prospect theory) will still attach an equal probability weight to each outcome. This is also true of Viscusi's (1989) prospective reference theory with a symmetric reference point.

Table 2a The Choices Presented to the Subjects

Suppose that you decided to invest $10,000 either in stock F or in stock G. Which stock would you choose, F, or G, when it is given that the *dollar gain or loss* one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
−1,600	1/4	−1,000	1/4
−200	1/4	−800	1/4
1,200	1/4	800	1/4
1,600	1/4	2,000	1/4

Please write F or G: ☐

Table 2b The Results of Experiment 2*

	F	G	Indifferent	Total
($F \succ_{PSD} G$, $G \succ_{MSD} F$)	38%	62%	0%	100%

Number of subjects: 84.

*Numbers in the tables are in percent, rounded to the nearest integer. The notations $\succ_{PSD}$ and $\succ_{MSD}$ indicate dominance by PSD and MSD, respectively.

lends support to the reverse S-shaped function, it is somewhat less dramatic than the 71% who chose the PSD-inferior (and MSD-dominant) prospect in Task I of Experiment 1. This difference may be due to the increased complexity of the task in Experiment 2 relative to Task I of Experiment 1. Indeed, a similar drop (from 96% to 82%) is observed in the choices conforming with FSD as the complexity of the tasks was increased (see Tasks II and III in Experiment 1). Yet the proportion rejecting the S-shaped function here is significantly larger than in the Kahneman and Tversky (1979) experiment, with a z-value of 5.72 (see Footnote 12).

The strong results rejecting the prospect theory S-shaped function that was revealed in Experiments 1 and 2 raise the following questions: How do these results fit with the wide support documented for prospect theory? Is there perhaps some special attribute to the subject population in our experiments which systematically biases our results? To answer these questions, we have conducted a third experiment, in which the same group of subjects is presented with tasks identical to the original Kahneman and Tversky certainty equivalent tasks, and also with a task involving mixed and uncertain prospects. By this procedure we can pinpoint whether having prospects with mixed outcomes is of crucial importance.

Experiment 3: Design

The tasks in Experiment 3 are given in Table 3a. Tasks I and II are precise replications of the tasks in the Kahneman and Tversky (1979) experiment (see p. 268). Task I compares a positive prospect with a certain positive outcome, and Task II compares a negative prospect with a certain negative outcome. Task III involves two uncertain prospects with mixed outcomes. In this third task, Prospect G dominates by PSD, but Prospect F dominates by MSD. The subjects participating in this experiment were 51 practitioners and 129 business school students.

Table 3a The Choices Presented to the Subjects

Task I: Imagine that you face the following two alternatives and you must choose one of them. Which one would you select?

F		G	
Gain or loss	Probability	Gain or loss	Probability
4,000	0.80	3,000	1
0	0.20		

Please write F for G: ☐

TASK II: Imagine that you face the following two alternatives and you must choose one of them. Which would you select?

F		G	
Gain or loss	Probability	Gain or loss	Probability
−4,000	0.80	−3,000	1
0	0.20		

Please write F for G: ☐

TASK III: Suppose that you decided to invest $10,000 either in stock F or in Stock G. Which stock would you choose, F, or G, when it is given that the *dollar gain or loss* one month from now will be as follows:

F		G	
Gain or loss	Probability	Gain or loss	Probability
−1,500	1/2	−3,000	1/4
4,500	1/2	3,000	3/4

Table 3b The results of Experiment 3

Task	F	G	Indifferent	Total
I	19%	81%	0%	100%
II	69%	30%	1%	100%
III ($G \succ_{PSD} F, F \succ_{MSD} G$)	76%	23%	1%	100%

Number of subjects: 180.

Numbers in the tables are in percent, rounded to the nearest integer. The notations $\succ_{PSD}$ and $\succ_{MSD}$ indicate dominance by PSD and MSD, respectively.

Experiment 3: Results

Table 3b reports the results of Experiment 3. Because there are no significant differences between the subject populations, we report only the aggregate results. In Task I we find that 81% of the choices are consistent with risk aversion for gains. In Task II, 69% of the choices are consistent with risk seeking for losses. Thus, the results in these two tasks are very similar to Kahneman and Tversky's results, albeit our results in the negative domain are somewhat weaker (Kahneman and Tversky obtain 80% and 92%, respectively, see Kahneman and Tversky 1979, Table 1).

The striking result is related to Task III. Here, 76% of the subjects chose *F*, the prospect which is inferior by PSD![16] The same subjects which seem to support risk aversion for gains and risk seeking for losses when they are confronted with nonmixed gambles in the certainty equivalent framework, reject the S-shaped function when mixed and uncertain prospects are compared. Only 17% of the subjects made choices consistent with the S-shaped function throughout: a choice of *G* in Task I, *F* in Task II, and *G* in Task III. This leads us to believe that the support found in previous studies for the S-shaped function may be due to the use of nonmixed gambles and the certainty equivalent framework, with one certain outcome. As these constitute very special and atypical situations, we believe that the results in these tasks may be biased due to their special setup or to the well-known "certainty effect".

Note that in Task III, *F* dominates *G* by MSD. Thus, when the two competing theories of the S-shaped function and the reverse S-shaped function race head to head, 76% of the choices conform with MSD and contradict PSD.

[16] This is significantly larger than the proportion of subjects rejecting the S-shaped value function in the Kahneman and Tversky experiment, with a z-value of 8.92

4. Concluding Remarks

Prospect theory is a paradigm challenging expected utility theory. The four main components of prospect theory are: Individuals make decisions based on change of wealth rather than total wealth; preferences are described by an S-shaped value/utility function, $V(x)$, with an inflection point at $x = 0$; individuals distort small probabilities; and the "framing" of alternatives affects individuals' choices. The main justification for prospect theory is the results of experimental studies in which the subjects had to declare the certainty equivalent of *either* negative *or* positive bets, bets which virtually do not exist in practice, and certainly are not common in the financial markets. This unrealistic "framing" of the bets, and possible biases induced by the "certainty effect" could account for the support for the S-shaped value function which is found in previous experiments, and which is rejected in our experiment.

In our experiment, we employ stochastic dominance investment criteria. Because the stochastic dominance criteria are invariant to the initial wealth, we can focus on change of wealth rather than total wealth. We deal with relatively large probabilities ($p \geq 0.25$), hence it is very unlikely that probability distortion plays an important role in our experiment. Furthermore, in our study we have prospects with no certain outcome, hence the certainty effect is irrelevant here. Neutralizing these three factors allows us to focus on the preference and to examine whether it is indeed S-shaped, as claimed by prospect theory.

In the previous experiments the certainty equivalent approach was employed, hence, in order to characterize the properties of the value function there was no practical choice but to use nonmixed bets (either negative or positive). The stochastic dominance approach allows us to employ mixed bets, which are much more typical of real investment situations. For the first time, we use a recently developed investment criterion called Prospect Stochastic Dominance (PSD), and a criterion which is developed here, called Markowitz Stochastic Dominance (MSD).

Using the PSD criterion and mixed bets, based on the experimental results of three distinct groups of subjects (students, university professors, and practitioners), we conclude that the S-shaped preference is rejected. Moreover, employing the MSD criterion reveals support for the reverse S-shaped value function, which was suggested by Markowitz (1952b). To be more specific, in tasks where one prospect dominates the other by PSD but an opposite dominance holds for MSD, three experiments revealed that 71%, 62%, and 76% of the subjects preferred the MSD dominating prospect. Moreover, this preference is even stronger with the more sophisticated investors: 79% of the practitioners and 86% of the professors preferred F over G, where G is the PSD dominating option! Thus, we reject the prospect theory S-shaped function.

It is interesting to note that when faced with outcomes restricted either to the positive domain or to the negative domain, as in the certainty equivalent approach, most subjects make choices according to prospect theory. However, the same subjects reject the PSD superior option with mixed bets. This leads us to believe that the support for the S-shaped function as obtained by Kahneman and Tversky is actually due to the well-known certainty effect and does not represent investors' preferences in a realistic setting of mixed bets.

Obviously, different individuals may have preferences of different types, i.e., some may be characterized by risk aversion, others by an S-shaped value function, etc. Some individuals may not even be consistent in their decision making. Thus, one cannot hope to find a preference class which perfectly characterizes all individuals. However, our experimental evidence indicates that the best characterization of subjects' behavior is given by the class of reverse S-shaped functions, as suggested by Markowitz, which is just the opposite of the S-shaped function advocated by prospect theory.

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Appendix A. A Proof of the Markowitz Stochastic Dominance (MSD) Criterion

THEOREM 1. *Let $V(x) \in \mathbf{V}_M$ where $\mathbf{V}_M$ is the class of all continuously and twice differentiable Markowitz utility functions such that $V' \geq 0$ for all x, with $V'' \leq 0$ for $x < 0$ and $V'' \geq 0$ for $x > 0$. Then F dominates G for all $V \in \mathbf{V}_M$ if and only if:*

$$\int_{-\infty}^{y} [G(t) - F(t)]\, dt \geq 0 \quad \text{for all } y < 0 \quad \text{and}$$

$$\int_{x}^{\infty} [G(t) - F(t)]\, dt \geq 0 \quad \text{for all } x > 0.$$

PROOF. Let us first formulate our proof in terms of change of wealth, x, rather than total wealth, $w + x$, and then show that the dominance is invariant to the value w. For convenience, assume that the outcomes of Prospects F and G have lower and upper bounds a and b. One can later generalize the result to the case of unbounded prospects by taking the limits $a \to -\infty$ $b \to \infty$. Define:

$$\Delta \equiv E_F V(x) - E_G V(x) = \int_a^b V(x)\, dF(x) - \int_a^b V(x)\, dG(x).$$

Integrating by parts, we have

$$\Delta = [F(x) - G(x)]V(x)\Big|_a^b - \int_a^b (F(x) - G(x))V'(x)\, dx.$$

As $F(b) = G(b) = 1$ and $F(a) = G(a) = 0$, we have:

$$\begin{aligned} \Delta &= \int_a^b [G(x) - F(x)]V'(x)\, dx \\ &= \int_a^0 [G(y) - F(y)]V'(y)\, dy + \int_0^b [G(x) - F(x)]V'(x)\, dx, \end{aligned}$$

where we use the notation y for variable values in the negative domain. Integrating once again by parts, the two terms on the right-hand side yield:

$$\begin{aligned} \Delta = {} & V'(y) \int_a^y (G(t) - F(t))\, dt \Big|_a^0 - \int_a^0 V''(y) \int_a^y (G(t) - F(t))\, dt\, dy \\ & + V'(x) \int_0^x (G(t) - F(t))\, dt \Big|_0^b - \int_0^b V''(x) \int_0^x (G(t) - F(t))\, dt\, dx. \end{aligned}$$

As some of the terms (i.e., the cases $y = a$, and $x = 0$) are equal to zero, Δ can be rewritten as:

$$\begin{aligned} \Delta = {} & V'(0) \int_a^0 [G(t) - F(t)]\, dt - \int_a^0 V''(y) \int_a^y (G(t) - F(t))\, dt\, dy \\ & + V'(b) \int_0^b [G(t) - F(t)]\, dt - \int_0^b V''(x) \int_0^x (G(t) - F(t))\, dt\, dx. \end{aligned}$$

Because $V' \geq 0$ and $V'' \leq 0$ for $y < 0$, the condition $\int_a^y [G(t) - F(t)]\, dt \geq 0$ ensures that the first two terms on the right-hand side of Δ are nonnegative. (Note that we assume that the utility function is twice differentiable, and that $V' \geq 0$ for all x. If the utility function is not differentiable at a given point x_0, approximations can be used without altering the results.) One is tempted to believe that for

$x > 0$, the condition for dominance should be $\int_0^x [G(t)-F(t)]\,dt \le 0$, because at this range $V'' \ge 0$. This condition indeed guarantees that the fourth term on the right-hand side is nonnegative, but the third term can be negative, hence it is not guaranteed that $\Delta \ge 0$. Indeed a different condition is needed to obtain the dominance. To see this, let us rewrite the fourth term as follows:

$$\begin{aligned}
&-\int_0^b V''(x)\int_0^x [G(t)-F(t)]\,dt\,dx \\
&=-\int_0^b V''(x)\int_0^b [G(t)-F(t)]\,dt\,dx+\int_0^b V''(x)\int_x^b [G(t)-F(t)]\,dt\,dx \\
&=-\int_0^b [G(t)-F(t)]\,dt\int_0^b V''(x)\,dx+\int_0^b V''(x)\int_x^b [G(t)-F(t)]\,dt\,dx, \\
&=-\int_0^b [G(t)-F(t)]\,dt[V'(x)]_0^b+\int_0^b V''(x)\int_x^b [G(t)-F(t)]\,dt\,dx, \\
&=-V'(b)\int_0^b [G(t)-F(t)]\,dt+V'(0)\int_0^b [G(t)-F(t)]\,dt \\
&\quad+\int_0^b V''(x)\int_x^b [G(t)-F(t)]\,dt\,dx.
\end{aligned}$$

Substituting these three terms instead of the fourth term on the right-hand side of Δ yields

$$\begin{aligned}
\Delta = {}& V'(0)\int_a^b [G(t)-F(t)]\,dt-\int_a^0 V''(y)\int_a^y [G(t)-F(t)]\,dt\,dy \\
&+\int_0^b V''(x)\int_x^b [G(t)-F(t)]\,dt\,dx.
\end{aligned}$$

Because by the theorem conditions

$$\int_a^y [G(t)-F(t)]\,dt \ge 0 \quad \text{for } y<0 \quad \text{and}$$

$$\int_x^b [G(t)-F(t)]\,dt > 0 \quad \text{for } x>0,$$

we can conclude that $\int_a^b [G(t)-F(t)]\,dt \ge 0$. Thus, the first term on the right-hand side of Δ is nonnegative. Because $V'' \le 0$ for $y < 0$ and $V'' \ge 0$ for $x > 0$, the conditions of the theorem guarantee that $\Delta \ge 0$.

Finally, note that if the utility function is $V(w+x)$ and the inflection point is at $x = 0$, the proof is kept unchanged because $F(w+x)$ and $G(w+x)$ are simply shifted to the right by w with no change in the area enclosed between F and G.

Necessity. It can be easily shown that if $\int_a^{x_0} [G(t)-F(t)]\,dt < 0$ for some $x_0 < 0$, then there is some $\mathbf{V} \in \mathbf{V}_M$ for which $\Delta_0 < 0$. To show this, employ the same necessity proof of Hanoch and Levy (1969) for second-degree stochastic dominance. By a similar argument, one can show that $\int_x^b [G(t)-F(t)]\,dt > 0$ for $x > 0$ is also a necessary condition for MSD dominance.

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