



# Consumption growth predictability and asset prices<sup>☆</sup>

Tai-Yong Roh<sup>a</sup>, Changjun Lee<sup>b</sup>, Byoung-Kyu Min<sup>c,\*</sup>

<sup>a</sup> Advanced Institute of Finance and Economics, Liaoning University, Shenyang, China

<sup>b</sup> College of Business, Hankuk University of Foreign Studies, Seoul, South Korea

<sup>c</sup> Discipline of Finance, Business School, The University of Sydney, Sydney, Australia



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## ABSTRACT

We derive and test a consumption-based intertemporal asset pricing model in which an asset earns a risk premium if it performs poorly when expected future consumption growth deteriorates. The predictability of consumption growth combined with the recursive preference delivers news about future consumption growth an additional risk factor, in addition to news about current consumption growth. We model the consumption growth dynamics using a vector autoregressive (VAR) structure with a set of instrumental variables commonly used for forecasting future economic growth. Our VAR estimation provides strong empirical support for future consumption growth predictability. The cross-sectional test shows that the model explains reasonably well the dispersion in average excess returns of 25 portfolios sorted on size and book-to-market, as well as 25 portfolios sorted on size and long-term return reversal. Growth stocks and long-term winners underperform value stocks and long-term losers, respectively, because growth stocks and long-term winners hedge adverse changes in the future consumption growth opportunities.

## 1. Introduction

Understanding the cross-sectional dispersions in asset returns is a central topic in finance. In rational asset pricing models, the dispersion in expected returns across different assets should be determined by a corresponding dispersion in the co-movement of the asset return with a set of common risk factors. A key insight derived from the Consumption CAPM (CCAPM) of Lucas (1978) and Breeden (1979) is that assets that covary negatively with contemporaneous aggregate consumption growth (and thus, covary positively with the marginal utility of consumption) should earn a lower risk premium, since they offer a hedge for bad states when consumption growth is lower. Despite its intuitive appeal, little empirical evidence exists to support the standard CCAPM (Breeden et al., 1989).

This paper derives and tests a consumption-based intertemporal asset pricing model in which an asset earns a risk premium if it performs poorly when expected future consumption growth deteriorates. We begin with the recursive utility function developed by Epstein and Zin (1989) and Weil (1989), in which a pricing kernel consists of the consumption growth and the return on the wealth portfolio. Using the restriction implied by the aggregate budget constraint, we then substitute out the return on wealth portfolio for the current and future consumption growths. The intuition behind this substitution is that for a long-term investor an unexpected high current wealth return should be associated with an increased level of consumption today or an improvement in future consumption due to precautionary savings. As a consequence, we obtain a pricing kernel driven by two differently priced news components with

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\* Corresponding author.

E-mail address: [byoungkyu.min@sydney.edu.au](mailto:byoungkyu.min@sydney.edu.au) (B.-K. Min).

respect to consumption growth: revisions in current consumption growth (as in the standard CCAPM), and revisions in expected future consumption growth. The latter comes from allowing expected consumption growth to vary over time.

We model consumption growth dynamics using a vector autoregressive (VAR) structure with a set of instrumental variables commonly used for forecasting future economic growth.<sup>1</sup> Empirically, we find that future consumption growth is strongly predicted by current economic conditions. In particular, the default spread and relative T-bill rate strongly negatively predict future consumption growth, consistent with business-cycle-related interpretation; during economic recessions, when the default spread and the short-term interest rate are likely to be high, future consumption growth is expected to be low. This evidence is important since if the consumption growth rate cannot be predicted, news about future consumption growth will have zero risk price in our model. Thus, the model collapses to the standard CCAPM. Put differently, the risk prices on the suggested risk factors are not free parameters, but are rather determined by the importance of the instrumental variables in forecasting future consumption growth.

We estimate and test our consumption-based intertemporal asset pricing model (E-CCAPM) with 25 portfolios sorted on size and book-to-market and 25 portfolios sorted on size and long-term return reversal. The cross-sectional tests using generalized method of moments (GMM) show that the model explains a significant portion of the dispersion in average excess returns of the two test assets, with explanatory ratios varying between 65% and 79%. In addition, revisions in expected future consumption growth are significantly priced, and seem to drive most of the explanatory power in explaining the cross-section of average returns. In contrast, revisions in current consumption growth seem to play a secondary role. Finally, the suggested model compares favorably with the CAPM, Intertemporal CAPM (ICAPM), and standard CCAPM.

The two-factor consumption-based model does a good job of addressing the value premium anomaly, explaining more than half of the realized value premium. For instance, the realized return on the value-minus-growth portfolio for the smallest quintile is 2.64% per quarter and the expected return from the model is 2.05% per quarter; the model thus explains 78% of the realized value premium. According to our model, value stocks, on average, outperform growth stocks, because value stocks have greater exposure to adverse changes in the future consumption growth opportunities.

The presented model also provides an explanation of the long-term return reversal anomaly. A majority of the realized return on a hedge portfolio that buys long-term losers and shorts long-term winners can be explained by the model. On average, stocks with low long-term past returns (long-term losers) outperform stocks with high long-term past returns (long-term winners), because long-term losers perform poorly when expected future consumption growth deteriorates.

Why do value (long-term loser) firms have greater loadings on the news about future consumption growth than growth (long-term winner) firms? One possible explanation is that typical value firms – stocks with low prices relative to their book values – have suffered a sequence of terrible shocks to which growth firms are less exposed. Similarly, firms exhibiting consistent stock price erosion for several years (long-term losers) are likely to experience a long sequence of negative cash flow shocks, whereas firms that see consistent stock price appreciation for several years (long-term winners) are likely to experience a long sequence of positive cash flow shocks. To support this argument, we calculate a direct cash-flow shock measure for each book-to-market (long-term reversal) quintile portfolio following an approach proposed by Campbell et al. (2010). If a portfolio has suffered a sequence of terrible shocks, its profitability in the near future will be severely lower. Indeed, for horizons from two to five years, we find that value (long-term loser) firms tend to have lower cash-flow fundamental than do the growth (long-term winner) firms.

The explanatory power of the model holds under a battery of robustness checks. First, alternative vector autoregression (VAR) specifications are used to estimate revisions in current and future consumption growths. Second, the model is tested in alternative sets of test assets, by adding industry or bond portfolios. The model is also tested simultaneously on value and long-term return reversal portfolios. Finally, the model is estimated in expected return-beta form. Results obtained from each robustness check are substantially similar to those for the benchmark test.

Our work is related to recent literature on the long-run risk model of Bansal and Yaron (2004). Both our model and the long-run risk model study the asset pricing implications of time-varying consumption growth rate combined with the recursive utility function of Epstein and Zin (1989). The model presented in this paper, however, differs from the long-run risk model in several important ways. First, our model shares the assumption of the long-run risk model that future consumption growth is predictable. Our model, however, does not rely on the highly persistent component in the consumption growth. The persistent predictable component in the aggregate consumption growth is necessary for the long-run risk model to explain various asset market phenomena. Some have criticized this assumption, however, questioning whether the consumption data indeed has a highly persistent component (e.g., Beeler and Campbell, 2012). Second, the cross-sectional implication of the long-run risk model is that dispersion in expected returns across assets is determined by a corresponding dispersion in the cash flow (dividends) beta (i.e., the covariance between dividends and consumption growth). Bansal et al. (2005, 2009) directly explore this implication. In contrast, we study the counterpart returns-based consumption beta (i.e., the covariance between returns and consumption growth).

Performing a persistence-based decomposition of the time series of consumption growth, Ortu et al. (2013) find that consumption growth contains predictable components, and provide empirical evidence that components that correspond to business-cycle frequencies are correlated with the term spread and default spread. Our paper is related to their work in that their empirical finding supports our assumption that future consumption growth is predicted by business-cycle variables and provides some clues for choosing state variables in our VAR framework. The ability of these variables to predict consumption growth is a key requirement of our model; indeed, we find that default spread strongly negatively predicts future consumption growth.

Finally, our work is related to Fang (2004) and Hansen et al. (2008) which obtain the pricing kernel similar to ours in that news about future consumption growth affects the marginal utility of wealth. Our main innovation relative to these studies is not the

<sup>1</sup> The VAR approach is commonly used for estimating news (unobservable) components of variables in the asset pricing literature (Campbell, 1996; Campbell and Vuolteenaho, 2004; Petkova, 2006; Campbell et al., 2018).

derivation, but empirical implementations of the pricing kernel to explain the cross-section of asset returns. First, we force our model to explain a broader cross-section of asset returns than the prior studies, which essentially focus on explaining the value premiums. We evaluate whether revisions in expected future consumption growth can help to explain a broad set of test portfolios including long-term return reversals, industry, momentum, and bond portfolios. The use of these alternative test portfolios addresses the concern raised by [Lewellen et al. \(2010\)](#) that asset pricing tests have relied exclusively on size and book-to-market sorted portfolios.

Second, we explore a potential explanation for the superior performance of our consumption-based model (E-CCAPM) relative to the ICAPM. We show that the E-CCAPM, which substitutes out the market return, better explains the cross-section of asset returns than does the ICAPM, which substitutes out consumption growth. Unlike the prior studies, we ask what accounts for the superior performance of the E-CCAPM over the ICAPM. We propose that, if human wealth is a significant component of the wealth portfolio, changes in the wealth portfolio are better summarized by changes in consumption than by the market return. We conduct a simulation study that shows the superior performance of the E-CCAPM relative to the ICAPM could be partly driven by the omitted human capital problem.

The remainder of this paper proceeds as follows. Section 2 describes the development of a consumption-based intertemporal asset pricing model. Section 3 presents a method for estimating two types of news components in consumption growth, and explains empirical procedure for testing the asset pricing model. Section 4 presents VAR estimates of the dynamic process for consumption growth as well as the results of the cross-sectional asset pricing test. Section 5 reports a set of robustness exercises in which we vary our basic VAR specifications for the dynamics of consumption growth, and consider alternative test portfolios. Finally, Section 6 summarizes the findings and offers a conclusion.

## 2. Theoretical background

Consider an agent who maximizes his [Epstein and Zin \(1989, 1991\)](#) objective function

$$U_t = \left[ (1 - \delta) C_t^{1-1/\psi} + \delta \left( E_t \left[ U_{t+1}^{1-\gamma} \right] \right)^{1/\theta} \right]^{\frac{1}{1-1/\psi}}, \quad (1)$$

subject to the intertemporal budget constraint

$$W_{t+1} = R_{c,t+1} (W_t - C_t), \quad (2)$$

where  $C_t$  is consumption,  $W_t$  is total wealth,  $R_{c,t+1}$  is the gross return on a claim to aggregate consumption (i.e., the wealth portfolio),  $\psi$  is the elasticity of intertemporal substitution,  $\gamma$  is the coefficient of relative risk aversion,  $\delta$  is the time discount factor, and  $\theta$  is defined as  $\frac{(1-\gamma)}{(1-\frac{1}{\psi})}$ .

From Eqs. (1) and (2), an Euler equation for asset  $i$  is obtained:

$$E_t \left[ \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left( \frac{1}{R_{c,t+1}} \right)^{1-\theta} R_{i,t+1} \right] = 1. \quad (3)$$

Thus, the log stochastic discount factor or pricing kernel is equal to

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{c,t+1}, \quad (4)$$

where  $\Delta c_{t+1} \equiv \log \left( \frac{C_{t+1}}{C_t} \right)$  and  $r_{c,t+1} \equiv \log (R_{c,t+1})$  denote the log consumption growth and the log return on total wealth, respectively.<sup>2</sup> Subtracting  $E_t (m_{t+1})$  from both sides yields

$$m_{t+1} - E_t(m_{t+1}) = -\frac{\theta}{\psi} [\Delta c_{t+1} - E_t(\Delta c_{t+1})] - (1 - \theta) [r_{c,t+1} - E_t(r_{c,t+1})]. \quad (5)$$

Eq. (5) implies that shock to the pricing kernel is a linear combination of current unexpected consumption growth and current unexpected return on the wealth portfolio. Using Eq. (5), one might be tempted to specify an asset pricing model in which shock to the consumption growth and shock to return on the wealth portfolio are two risk factors. Consumption and wealth portfolio, however, do not move independently; news about consumption growth should affect the wealth return since the aggregate budget constraint must hold ([Cochrane, 2008](#)).

In order to overcome this potential model misspecification, we want to express the stochastic discount factor as a function of consumption growth only.<sup>3</sup> To do so, we use the restriction implied from the aggregate budget constraint.

[Campbell \(1993\)](#) linearizes the budget constraint and uses the Euler equation to derive a relation between expected consumption growth and expected return on the wealth portfolio. We adopt this relation and obtain an expression for unexpected return on the wealth portfolio as a function of innovations to current and future consumption growths. Thus, an expression with only consumption growth appears in the pricing kernel.

<sup>2</sup> Hereafter, lowercase letters denote the logs of uppercase letters.

<sup>3</sup> Of course, the stochastic discount factor can be expressed as a function of market wealth instead of consumption. This alternative specification leads to the ICAPM of [Campbell \(1993\)](#), a cousin of our consumption-based model.

Log-linearizing the budget constraint around the steady state, and assuming the consumption–wealth ratio to be stationary, in the sense that  $\lim_{j \rightarrow \infty} \rho^j (c_{t+j} - w_{t+j})$  exists, we obtain

$$r_{c,t+1} - E_t(r_{c,t+1}) = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta c_{t+1+j} - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j r_{c,t+1+j}, \quad (6)$$

where  $\rho$  is defined as  $1 - e^{E(c-w)}$ .

Campbell (1993) derives a linear relation between expected consumption growth and expected return on the wealth portfolio under the assumption that consumption and returns are conditionally homoskedastic and jointly log normal, such that

$$E_t(r_{c,t+1}) = \mu_c + \frac{1}{\psi} E_t(\Delta c_{t+1}), \quad (7)$$

where  $\mu_c$  is a constant that includes the variance and covariance terms for innovation to consumption and the wealth return. We substitute expected return on the wealth portfolio  $E_t(r_{c,t+1})$  into Eq. (6) to obtain an expression with only consumption growth on the right-hand side:

$$r_{c,t+1} - E_t(r_{c,t+1}) = \Delta c_{t+1} - E_t(\Delta c_{t+1}) + \left(1 - \frac{1}{\psi}\right) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}. \quad (8)$$

Eq. (8) states that, in the long-run, news about return on the wealth portfolio is entirely determined by news about consumption growth. The intuition is that for a long-term investor unexpected high current wealth return should be associated with either an increased level of consumption today or an improvement in future consumption due to precautionary savings.

Substituting Eq. (8) into (5), we derive the (log) stochastic discount factor:

$$m_{t+1} - E_t(m_{t+1}) = -\gamma [\Delta c_{t+1} - E_t(\Delta c_{t+1})] - \left(\gamma - \frac{1}{\psi}\right) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}, \quad (9)$$

which makes use of the fact that  $\left(\frac{\theta}{\psi} + 1 - \frac{1}{\psi}\right) = \gamma$ , and  $(1 - \theta)\left(1 - \frac{1}{\psi}\right) = \gamma - \frac{1}{\psi}$ . Fang (2004) and Hansen et al. (2008) also derive the pricing kernel in a manner similar to Eq. (9). Note that news about *future* consumption growth appears in the *current* stochastic discount factor. This is because a long-term investor cares about expected future consumption. Changes in expected future consumption affect the current marginal rate of substitution through intertemporal consumption-smoothing. As a special case, when investors behave myopically ( $\theta = 1$ ), news about future consumption growth drops out from the log pricing kernel, because investors do not adjust current consumption through consumption-smoothing.

In equilibrium, expected return on any asset  $i$  should be determined by its covariance with the stochastic discount factor in the economy:

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = -Cov(r_{i,t+1}, m_{t+1}), \quad (10)$$

where  $\frac{\sigma_i^2}{2}$  is a Jensen inequality adjustment arising from the lognormal model. Eq. (9) is then substituted into Eq. (10) to obtain

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \gamma Cov[r_{i,t+1}, \Delta c_{t+1} - E_t(\Delta c_{t+1})] + \left(\gamma - \frac{1}{\psi}\right) Cov\left[r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}\right]. \quad (11)$$

In addition to the current consumption growth shock, as in the standard CCAPM of Breeden (1979), shocks to variables that predict future consumption growth represent a new risk factor. This new risk factor arises when expected consumption growth is allowed to be time-varying. The key implication of this specification is that the representative agent demands a compensation for holding assets that perform poorly when expected future consumption growth deteriorates. Note that the two different consumption growth shocks receive different prices of risk. Thus, assets should have differing exposure to the two types of shocks. In the special case where  $\psi = \infty$ , innovations in long-run consumption growth have the same price of risk as the current consumption innovation, since there is a one-for-one relation between current and future consumption innovations.

Following Campbell and Vuolteenaho (2004) and Campbell et al. (2018), we use simple expected returns,  $E(R_{i,t+1} - R_{f,t+1})$ , on the left-hand side of Eq. (11) instead of log returns,  $E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2}$ . Both expectations are the same in the lognormal model. In addition, results can be compared directly with previous empirical studies by using simple returns. This modification yields

$$E(R_{i,t+1} - R_{f,t+1}) = \gamma Cov[r_{i,t+1}, \Delta c_{t+1} - E_t(\Delta c_{t+1})] + \left(\gamma - \frac{1}{\psi}\right) Cov\left[r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}\right]. \quad (12)$$

Eq. (12) represents the consumption-based two-factor model set forth in this paper.

### 3. Data and empirical methodology

The consumption growth dynamics and the econometric framework used to estimate and evaluate an asset pricing model are set forth below.

### 3.1. Econometric specification

We adopt the VAR approach of [Campbell \(1993, 1996\)](#) to specify a process for the time-series dynamics of consumption growth. The (log) consumption growth rate is the first element of a state vector  $z_{t+1}$ . The other elements of  $z_{t+1}$  are state variables that are known to the market by the end of period  $t+1$  and that are relevant in forecasting future consumption growth. We assume that vector  $z_{t+1}$  is generated by a first-order VAR model<sup>4</sup>:

$$z_{t+1} = a + \Gamma z_t + u_{t+1}. \quad (13)$$

In this framework, current and future consumption growth news are both linear functions of the VAR shock ( $u_{t+1}$ ):

$$N_C \equiv \Delta c_{t+1} - E_t(\Delta c_{t+1}) = e1' u_{t+1}, \quad (14)$$

$$\begin{aligned} N_{LR} &\equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \delta^j \Delta c_{t+1+j} \\ &= e1' \rho \Gamma (I - \rho \Gamma)^{-1} u_{t+1} = \lambda' u_{t+1}. \end{aligned} \quad (15)$$

Here,  $e1$  is a vector with the first element equal to 1 and all others equal to zero,  $\Gamma$  is the VAR coefficient matrix, and  $\lambda' \equiv e1' \rho \Gamma (I - \rho \Gamma)^{-1}$  is a function that captures the long-run significance of each individual VAR shock to the expectation of consumption growth. We follow [Campbell and Vuolteenaho \(2004\)](#) and assume an annual consumption–wealth ratio of 5%, implying that  $\rho = 0.95^{1/4}$  quarterly.<sup>5</sup>

In order to empirically implement the VAR approach, it is necessary to specify the identity of the state variables. We choose a set of state variables that have been shown in the literature to be theoretically or empirically useful in forecasting future growth. Specifically, we select aggregate dividend yield (*DIV*), default spread (*DEF*), relative T-bill rate (*RREL*), and term spread (*TERM*), all of which are widely used to predict business cycles ([Stock and Watson, 2003](#)).

Aggregate dividend yield, computed as the sum of dividends over the last one-year divided by the price level of the CRSP value-weighted portfolio, is helpful in predicting future growth ([Fama and French, 1988](#)). In the simple log-linear representative agent model of [Campbell and Shiller \(1988\)](#), the dividend yield embodies rational discounted forecasts of dividend growth rate and discount rate, making it an appropriate state variable to use for predicting future economic growth.

The default spread is also known to be a strong predictor of real economic activity. [Stock and Watson \(1989\)](#) find that the default spread is a potent predictor of the output growth in the postwar period. When an economic downturn is expected, investors tend to buy higher quality securities and sell lower quality securities. This leads to higher prices and lower yields for the higher quality securities relative to lower quality ones. As a result, the default spread increases when economic recessions are expected. We measure the default spread as the difference between the yield spread between Moody's BAA and AAA corporate bonds ([Fama and French, 1989](#)).

The short-term interest rate is also used as a predictor of output growth. For instance, [Bernanke and Blinder \(1992\)](#) find that the federal funds rate holds significant predictive content for real output growth. The standard explanation for why the short-term interest rate can forecast future output growth is that the short rate is an appropriate measure of monetary policy; monetary tightening results in high short-term interest rates, and these high short rates in turn produce an economic slowdown. Following [Campbell \(1996\)](#), we detrend the short-term interest rate by taking the difference between the three-month Treasury bill rate and its moving average over the previous one-year. We call this term the relative T-bill rate.

Finally, the term spread, measured as the difference between ten-year government bond yields and three-month government bond yields, is shown to contain important information about fluctuations in the business cycle. For instance, [Harvey \(1988\)](#) shows that the term spread is able to predict future consumption growth. [Estrella and Hardouvelis \(1991\)](#) document strong evidence that the term spread contains predictive content for the output growth. The values of the term spread are high during recessions and low during expansion. Thus, the term spread tends to be higher when future growth is expected to be low.

Data on bond yields are obtained from the FRED database, available from the website of the St. Louis Federal Reserve Bank. Consumption data are obtained from National Income and Product Accounts and we deflate it by the consumer price index (CPI) and total population in order to compute per capital real consumption. [Table 1](#) provides descriptive statistics and contemporaneous correlations of the VAR state variables. The average (log) consumption growth over the sample period is 0.54% per quarter with a standard deviation of 0.57%. From the first-order autocorrelation coefficients (AR(1)), we see that both *DIV* and *DEF* are quite persistent: *DIV* has an AR(1) coefficient of 0.965, and *DEF* has an AR(1) coefficient of 0.865. In the correlation matrix, we see that consumption growth rate is positively correlated with *RREL* and *TERM*, and negatively correlated with *DIV* and *DEF*. Among the state variables, *DIV* and *DEF* exhibit a high positive correlation, with a correlation coefficient of 44%. Conversely, *RREL* and *TERM* exhibit a high negative correlation, with a correlation coefficient of −53%.

<sup>4</sup> Any  $P$  order VAR with  $P > 1$  can be stacked into first-order VAR if the state vector is expanded by including lagged state variables, with  $\Gamma$  denoting the VAR companion matrix.

<sup>5</sup> We also allow discount rates to vary within a range of 0 to 10% ( $\rho = 0.9^{1/4}$  to 1) and find no significant difference in results.

**Table 1**

Descriptive statistics for VAR state variables. The table reports the descriptive statistics of the VAR state variables used to predict to future consumption growth.  $\Delta c_t$  is the log consumption growth.  $DIV_t$  is the aggregate dividend yield, computed as the sum of dividends over the last one-year divided by the price level of the CRSP value-weighted portfolio.  $DEF_t$  is the default spread, measured as the difference between the yield spread between Moody's BAA and AAA corporate bonds.  $RREL_t$  is the relative T-bill rate, calculated as the difference between the three-month Treasury bill rate and its moving average over the previous one-year.  $TERM_t$  is the term spread, measured as the difference between ten-year government bonds yields and three-month government bonds yields. The sample period is from 1963:Q3 to 2010:Q4.

Panel A					
Variables	Mean	Std.	Min.	Max.	AR(1)
$\Delta c_t$	0.005	0.006	−0.018	0.021	0.480
$DIV_t$	0.030	0.011	0.011	0.055	0.965
$DEF_t$	0.010	0.005	0.003	0.034	0.865
$RREL_t$	0.000	0.009	−0.041	0.036	0.452
$TERM_t$	0.015	0.013	−0.027	0.044	0.806

  

Panel B					
	$\Delta c_t$	$DIV_t$	$DEF_t$	$RREL_t$	$TERM_t$
$\Delta c_t$	1.00	−0.10	−0.36	0.20	0.02
$DIV_t$		1.00	0.44	0.04	−0.08
$DEF_t$			1.00	−0.33	0.28
$RREL_t$				1.00	−0.53
$TERM_t$					1.00

### 3.2. Portfolios

We use two sets of equity portfolios to estimate and evaluate the asset pricing models. The first group is comprised of 25 portfolios sorted on both size and book-to-market (SBM25). The second group contains 25 portfolios sorted on both size and long-term return reversal (SLTR25). SLTR25 are constructed from the intersection of five portfolios formed on size and five portfolios formed on past returns (13 to 60 months before the portfolio formation month). These portfolios are made by Fama and French (1996) to capture the reversal of long-term returns, a CAPM anomaly documented by De Bondt and Thaler (1985); stocks with low long-term past returns (long-term losers) tend to have higher future average returns, while stocks with high long-term past returns (long-term winners) tend to have lower future average returns. In response to the criticism of Lewellen et al. (2010) that most asset pricing tests rely exclusively on the SBM25 portfolios, we further consider 30 industry portfolios and 10 momentum portfolios as alternative test assets. We compound the monthly returns to compute quarterly portfolio returns. We subtract the return on the three-month T-bill to calculate the portfolio excess returns. Finally, we use the CPI inflation rate to obtain the ex post real portfolio. All portfolio return data are obtained from Kenneth French's website. The return on the three-month T-bill rate and the seasonally adjusted CPI are obtained from the FRED database.

### 3.3. Econometric methodology

We estimate our specification by a first-stage GMM procedure (see Hansen, 1982). The first-stage estimation uses equally weighted moments, conceptually equivalent to an ordinary least-squares (OLS) cross-sectional regression of average excess returns on factor covariances. This procedure allows for an evaluation of whether our consumption-based model can explain the returns of a set of economically interesting portfolios (e.g., value premium or long-term return reversal effect).

Let  $R_{t+1}^e = (R_{1,t+1} - R_{f,t+1}, \dots, R_{N,t+1} - R_{f,t+1})'$  and  $f_{t+1} = (N_{C,t+1}, N_{LR,t+1})'$  be the observation of the vector of  $N$  excess returns on test assets at time  $t + 1$ , and the vector of  $K$  factors, respectively. The GMM system has  $N + K$  moment conditions, where the first  $N$  sample moments correspond to the pricing errors for each of the  $N$  test portfolios:

$$\begin{bmatrix} R_{t+1}^e - R_{t+1}^e (f_{t+1} - \mu_f)' b \\ f_{t+1} - \mu_f \end{bmatrix} = 0, \quad (16)$$

where  $b$  denotes the factor prices of risk and  $\mu_f$  denotes the means of factors. The last  $K$  moment conditions in the system above allow us to estimate the factor means; thus the estimated factor risk prices ( $b$ ) take into account for the estimation error in the factor means, as in Cochrane (2005, Chapter 13) and Yogo (2006).

The over-identifying restrictions of the model can be tested using Hansen's 1982  $J$ -test, and is given by

$$T \hat{\alpha}' \hat{S}_N^{-1} \hat{\alpha} \sim \chi^2(N - K), \quad (17)$$

where  $\hat{\alpha}$  denotes the pricing errors associated with the  $N$  test assets and  $\hat{S}_N$  represents the block of the spectral density matrix associated with the  $N$  pricing errors. The degree of over-identification is  $N - K$  ( $N$  moments and  $K$  parameters to estimate). The  $J$ -test evaluates the null hypothesis that the pricing errors across the  $N$  test assets are jointly equal to zero. This test is conceptually similar to the GRS test (Gibbons et al., 1989), since the test statistic is a quadratic form in the vector of pricing errors (Cochrane, 2005).



In addition to the formal test statistic (17), we compute two simpler and more robust goodness-of-fit measures to evaluate the overall pricing ability of the model. The first measure is the cross-sectional OLS  $R^2$ :

$$R^2 = 1 - \frac{\sum_{i=1}^N \tilde{\alpha}_i^2}{\sum_{i=1}^N \tilde{R}_i^2}, \quad (18)$$

where  $\tilde{R}_i = \frac{1}{T} \sum_{t=0}^{T-1} (R_{i,t+1}^e) - \frac{1}{N} \sum_{i=1}^N \left[ \frac{1}{T} \sum_{t=0}^{T-1} (R_{i,t+1}^e) \right]$  represents the (cross-sectionally) demeaned average excess returns,  $\hat{\alpha}_i$  denotes the pricing errors, and  $\tilde{\alpha}_i$  denotes the (cross-sectionally) demeaned pricing errors. The cross-sectional  $R^2$  measures the proportion of the cross-sectional variance of average excess returns explained by the model.<sup>6</sup>

The second measure is the mean absolute pricing error (MAE):

$$MAE = \frac{1}{N} \sum_{i=1}^N |\hat{\alpha}_i| \quad (19)$$

### 3.4. Alternative asset pricing models

We also consider alternative asset pricing models to compare the empirical performance of the suggested model. We choose the CAPM, ICAPM, and CCAPM as alternative models based on their similarity to our model in terms of theoretical background and empirical methodology for estimating risk factors.<sup>7</sup>

The first alternative model is the CAPM of Sharpe (1964) and Lintner (1965). The model's expected return-covariance representation can be written as

$$E(R_{i,t+1} - R_{f,t+1}) = \gamma Cov[r_{i,t+1}, r_{c,t+1} - E_t(r_{c,t+1})]. \quad (20)$$

Here, the return on the wealth portfolio is the sole risk factor. As a proxy for the wealth return, we use the return on the CRSP value-weighted portfolio.

The second model with which we compare the performance of our consumption-based model is the ICAPM of Campbell (1993). In this model, the risk factors are news about current wealth return and news about future wealth returns, representing the time-varying investment opportunity set. The corresponding pricing equation can be represented as

$$E(R_{i,t+1} - R_{f,t+1}) = \gamma Cov[r_{i,t+1}, r_{c,t+1} - E_t(r_{c,t+1})] + (\gamma - 1) Cov\left[r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{c,t+1+j}\right]. \quad (21)$$

Following Campbell (1993, 1996) and Campbell and Vuolteenaho (2004), we use a first-order VAR model to estimate two news components with respect to the return on the wealth portfolio. By adopting the same econometric methodology used to estimate news components with respect to the consumption growth, we can more directly compare our model with the ICAPM. We consider a state vector that includes the market (excess) return and the same set of instrumental variables considered above: the dividend yield, default spread, relative T-bill rate, and term spread.<sup>8</sup>

The third alternative model is the CCAPM of Lucas (1978) and Breeden (1979), which is obtained as a special case of Eq. (12) by imposing  $\theta = 1$ :

$$E(R_{i,t+1} - R_{f,t+1}) = \gamma Cov[r_{i,t+1}, \Delta c_{t+1} - E_t(\Delta c_{t+1})]. \quad (22)$$

The contemporaneous consumption growth news,  $\Delta c_{t+1} - E_t(\Delta c_{t+1})$ , is estimated from our VAR specification as in Eq. (14).

## 4. Empirical results

### 4.1. VAR Estimation

In order to identify two different types of news about consumption growth, it is necessary that the set of state variables have predictable power. It is important, therefore, to examine whether the chosen VAR state variables considered here (*DIV*, *DEF*, *RREL*, and *TERM*) are actually able to predict future consumption growth. Table 2 reports the VAR estimation results. In the first row of Table 2, we see that the set of state variables have some ability to predict future consumption growth. Consumption growth, *DEF*, and *RREL* are statistically significant at the 5% level. The signs of the coefficients of these variables are consistent with the business-cycle-related interpretation, as discussed in Section 3.1. For instance, during economic recessions, when the default spread (*DEF*) is high, future consumption growth is expected to be low. The adjusted  $R^2$  of the consumption growth forecasting regression is 28%, indicating that the variables considered here demonstrate predictable power. The remaining rows in Table 2 show the dynamics of the VAR state variables. Each state variable is predicted not only by its own lag, but also by the other lagged state variable(s). For example, term spread is predicted by the lagged value of all other state variables (except the dividend yield).

<sup>6</sup> The cross-sectional  $R^2$  measure defined here follows Campbell and Vuolteenaho (2004), Yogo (2006), and Maio (2013). This measure can have a negative value for poorly fitting models estimated under the constraint that the zero-beta rate equals the risk-free rate.

<sup>7</sup> ICAPM is related to our consumption-based model, since time variation in the investment opportunity set must eventually affect consumption at some horizon because the aggregate budget constraint must hold. The CAPM and CCAPM are nested models of the ICAPM and our model, respectively.

<sup>8</sup> This set of instrumental variables is widely used for predicting future market return as well. Petkova (2006) uses the same set of variables to implement the ICAPM. Petkova and Zhang (2005) use this set of variables to estimate the expected market risk premium.

**Table 2**

VAR parameter estimates. The table shows the estimate results for a first-order VAR model where the state variables are a constant, the log consumption growth rate ( $\Delta c_t$ ), dividend yield ( $DIV_t$ ), default spread ( $DEF_t$ ), relative T-bill rate ( $RREL_t$ ), and term spread ( $TERM_t$ ). Each set of five rows corresponds to a different dependent variable. The first six columns report coefficients and  $t$ -value of the six explanatory variables, and the remaining column shows  $R^2$ . The sample period is from 1963:Q3 to 2010:Q4.

	$\Delta c_t$	$DIV_t$	$DEF_t$	$RREL_t$	$TERM_t$	Constant	$R^2$
$\Delta c_{t+1}$	0.47 (6.85)	0.04 (1.00)	−0.21 (−2.20)	−0.15 (−2.96)	0.05 (1.39)	0.33 (2.52)	0.28
$DIV_{t+1}$	−0.01 (−0.37)	0.97 (45.68)	−0.03 (−0.47)	0.04 (1.32)	−0.02 (−0.81)	0.14 (1.85)	0.94
$DEF_{t+1}$	−0.09 (−2.65)	0.04 (2.43)	0.80 (17.27)	0.01 (0.51)	−0.01 (−0.48)	0.14 (2.27)	0.76
$RREL_{t+1}$	0.26 (2.36)	0.01 (0.18)	−0.15 (−0.96)	0.43 (5.49)	0.05 (1.03)	−0.11 (−0.53)	0.23
$TERM_{t+1}$	−0.18 (−1.66)	−0.06 (−1.04)	0.37 (2.45)	0.14 (1.85)	0.82 (16.00)	0.17 (0.82)	0.67

**Table 3**

Current and future consumption growth innovations. The table reports the properties of current consumption growth shock (CCS) and future consumption growth shock (FCS) estimated from the VAR model of Table 2. The upper-left section of the table reports the covariance matrix of the two consumption growth shocks. The upper-right section reports the correlation matrix of the two shocks with standard deviations on the diagonal. The lower-left section reports the correlation of shocks to individual state variables with the consumption growth shocks. The lower-right section reports the column vectors that map the state variable shocks to consumption growth shocks.  $\Delta c_t$  is the log consumption growth.  $DIV_t$  is the aggregate dividend yield.  $DEF_t$  is the default spread.  $RREL_t$  is the relative T-bill rate.  $TERM_t$  is the term spread. The sample period is from 1963:Q3 to 2010:Q4.

News covariance	CCS	FCS	News corr/std	CCS	FCS
CCS	0.23	0.15	CCS	0.48	0.54
FCS	0.15	0.34	FCS	0.54	0.59
Shock correlations	CCS	FCS	Functions	CCS	FCS
$\Delta c_t$	1.00	0.54	$\Delta c_t$	1.00	0.71
$DIV_t$	−0.12	−0.21	$DIV_t$	0.00	0.05
$DEF_t$	−0.33	−0.35	$DEF_t$	0.00	−0.85
$RREL_t$	0.20	−0.59	$RREL_t$	0.00	−0.36
$TERM_t$	−0.10	0.69	$TERM_t$	0.00	0.34

The results in Table 2 clearly show that consumption growth is indeed predictable. This evidence is important, since if the consumption growth cannot be predicted, news about future consumption growth will have zero risk price in our model, as we can see from Eq. (12). Thus, the model collapses to the standard CCAPM. Our evidence is consistent with Bansal et al. (2007), among others, who provide an evidence on the predictability of consumption growth by running a univariate regression of annual consumption growth on a constant, the price–dividend ratio, and the risk-free rate.<sup>9</sup>

Table 3 summarizes the characteristics of the implied current and future shocks to consumption growth. The top panel reports the variances, covariances, standard deviations, and correlations of two consumption growth shocks. The standard deviation of news about future consumption growth is slightly larger than that of news about current consumption growth. In addition, shock to future consumption growth is positively correlated with shock to current consumption growth, indicating that the prospects of long-run growth opportunities rise when consumption growth rises.

The bottom panel of Table 3 shows the correlations between innovations in the VAR state variables and the two types of consumption growth shocks. Innovations to the term spread are negatively correlated with shock to the current consumption growth, and positively correlated with shock to future consumption growth. This suggests that positive shocks to the steepness of the yield curve are associated with a contemporaneous bad economic condition, but in the long-run are associated with positive growth opportunities. Similarly, unexpected decreases in the short-term interest rate are associated with a contemporaneous economic downturn, but are also associated with a positive impact on revisions of future consumption growth. Table 3 also reports the coefficients that map innovations of state variables to news about current and future consumption growths. It is evident that innovations to default spread and consumption growth are the most important determinants of news about future consumption growth.

#### 4.2. Estimation of factor risk premia

Table 4 reports the estimation and evaluation results for the CAPM, ICAPM, CCAPM, and suggested consumption-based intertemporal asset pricing model (E-CCAPM). The test assets are the 25 portfolios sorted on size and book-to-market (Panel A)

<sup>9</sup> An incomplete list of recent papers that present evidence of predictability of the consumption growth includes Parker and Julliard (2005), Hansen et al. (2008), and Malloy et al. (2009).



**Table 4**

Estimation of factor risk premia using GMM. The table reports the estimation and evaluation results for the CAPM, Intertemporal CAPM (ICAPM), Consumption CAPM (CCAPM), and Extended CCAPM with news about future consumption growth (E-CCAPM). The testing portfolios are the 25 size/book-to-market portfolios (SBM25, Panel A) and the 25 size/long term return reversal portfolios (SLTR25, Panel B). The estimation procedure is first-stage GMM. CMS and LRMS are the risk prices associated with news about current and future stock returns, respectively. CCS and FCS are the risk prices associated with news about current and future consumption growth, respectively. Standard deviations are reported in parentheses. MAE refers to the mean absolute pricing error (in %), and  $R^2$  is 1 minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns. Bootstrapped 95% confidence intervals associated with  $R^2$  are also reported. The  $p$ -values for the  $J$ -test (test of over-identifying restrictions) are reported in parentheses. The sample period is from 1963:Q3 to 2010:Q4.

Panel A: SBM25				
Factor price	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.43 (0.93)	−14.60 (3.38)		
LRMS		−27.72 (5.54)		
CCS			92.98 (29.39)	−208.99 (57.72)
FCS				243.14 (49.54)
MAE (%)	0.61	0.43	0.44	0.27
$R^2$	−0.17	0.48	0.27	0.79
C.I of $R^2$	[−0.62, 0.76]	[−0.28, 0.82]	[−0.59, 0.74]	[−2.27, 0.73]
$J$ -test	78.75 (0.00)	83.11 (0.00)	75.16 (0.00)	45.04 (0.00)
Panel B: SLTR25				
Factor price	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.19 (0.93)	8.87 (3.34)		
LRMS		10.58 (5.25)		
CCS			55.68 (33.56)	−202.67 (60.64)
FCS				222.87 (45.31)
MAE (%)	0.46	0.39	0.42	0.25
$R^2$	0.17	0.36	0.35	0.73
C.I of $R^2$	[−1.56, 0.72]	[−0.99, 0.80]	[−1.87, 0.73]	[−4.18, 0.73]
$J$ -test	62.93 (0.00)	62.24 (0.00)	57.80 (0.00)	49.88 (0.00)

and the 25 portfolios sorted on size and long-term return reversal (Panel B). In the test of CAPM using the SBM25 portfolios, the estimated market risk price is 2.43 and is significant (1% level). The average pricing error is 0.61% per quarter, and the model is strongly rejected by the  $J$ -test statistic ( $p$ -value = 0.0%). In addition, the cross-sectional  $R^2$  is negative (−17%), indicating that the CAPM performs worse than a model with only a constant factor.

The results for the ICAPM show that the model explains 48% of the cross-section variation in average excess returns, and the average pricing error is 43% per quarter. This fit represents a modest improvement relative to the CAPM. The two covariance risk prices, however, are both negative, inconsistent with theory prediction. The estimate for the risk price associated with the market excess return is −14.60 and significant (1% level), whereas the estimate for the risk price associated with news about future stock return is −27.72 and significant (1% level). The fit of the CCAPM is slightly worse than the ICAPM; the CCAPM yields an explanatory ratio of 27% and an average pricing error of 0.44% per quarter. The point estimate for the risk price of consumption (estimate of the relative risk aversion) is 92.98 and significant (1% level). This high estimate of relative risk aversion stems from the fact that consumption growth is insufficiently volatile and is less correlated with stock returns.

For the suggested consumption-based model, the results show that the E-CCAPM explains well the SBM25 portfolios; the 79% of the cross-sectional variation in average excess returns is explained by the model. Further, an average pricing error is relatively small; it is 0.27% per quarter. The risk price estimate for news about future consumption growth is positive and significant (1% level), while the risk price estimate for news about current consumption growth is negative and significant (1% level).

Using the relation between the risk prices and preference parameters in Eqs. (12), we back out the implied preference parameters by  $\gamma = \lambda_1$  and  $\psi = 1/(\lambda_1 - \lambda_2)$  and their 95% confidence intervals, where  $\lambda_1$  and  $\lambda_2$  are the risk price estimates for current and long-run consumption shocks, respectively. The results show that for SBM25 portfolios, the estimate of the elasticity of intertemporal substitution (EIS),  $\psi$ , is −0.002 with the confidence interval of [−0.0040, −0.0015]. The small magnitude of this estimate, being close to zero, is consistent with the literature. For instance, Yogo (2006) obtains an EIS estimate of 0.002, and Da et al. (2016) obtain EIS estimates ranging from −0.0040 to 0.0069. For SBM25 portfolios, the estimate of the relative risk aversion coefficient,  $\gamma$ ,

is implausibly negative, however, at  $-209$  with the confidence interval of  $[-322, -96]$ .<sup>10</sup> This clearly being beyond the scope of a reasonable range for a representative investor's risk aversion, constitutes a caveat of our proposed model.<sup>11</sup>

The results for the test using the SLTR25 portfolios (Panel B) show that the fit of the CAPM improves in relation to the test using the SBM25 portfolios, with a positive cross-sectional coefficient of determination of 17%. The average pricing error is 0.46% per quarter, which is still economically large. The point estimate for market price of risk is close to the corresponding estimate in the test using the SBM25 portfolios. As in the case with SBM25 portfolios, the ICAPM outperforms the CAPM in pricing the SLTR25 portfolios, with a coefficient of determination of 36% and an average pricing error of 39% per quarter. In addition, in the test with SLTR25 portfolios, the estimates of risk prices for market excess return and news about future market return become both positive and significant (1% level).

In the test with the SLTR25 portfolios, the fit of the CCAPM is comparable to the test with the SBM25 portfolios. The estimate for the relative risk aversion becomes nearly half (55.68) of the corresponding value in the test with the SBM25 portfolios, but is only marginally significant (10% level). The consumption-based model with news about future consumption growth as an additional factor performs well in pricing the SLTR25 portfolios, with an  $R^2$  estimate of 73% and an average pricing error of 25% per quarter. The estimates of risk prices for the two consumption risk factors are very stable across the test assets.

In addition, Table 4 shows the confidence intervals for the competing models.<sup>12</sup> For our model, the 95% confidence interval for  $R^2$  is  $[-2.27, 0.73]$  with SBM25 and  $[-4.18, 0.73]$  with SLTR25. The evaluated  $R^2$ s of 0.79 for SBM25 and 0.73 for SLTR25 are thus not less than the upper bounds of the confidence intervals. In contrast, for the alternative models the evaluated  $R^2$ s are within the corresponding 95% confidence intervals. It suggests that the E-CCAPM explains the cross-section of SBM25 and SLTR25 portfolios better than a model with a constant. However, it is important to note that the results do not imply that our model outperforms the alternative models. Further, our results regarding the confidence intervals for  $R^2$ s should be interpreted with caution since the  $R^2$  estimates of the E-CCAPM in later analysis are often within the 95% confidence intervals when we use the alternative VAR specifications (Table 9) and alternative test assets (Table 10).

The lower bounds of the confidence interval for  $R^2$  of the E-CCAPM are extremely negative values ( $-2.27$  or  $-4.18$ ) and much lower than those of the alternative models. In an attempt to understand what causes these large negative  $R^2$  estimates, we re-estimate the model without imposing the restriction that the zero-beta rate equals the risk-free rate using the bootstrapped samples with the 5th percentiles of the  $R^2$  estimates. In these bootstrapped samples, the excess zero-beta estimates are economically large and statistically significant, indicating that the E-CCAPM cannot match the zero-beta rate. When we force the model to match the zero-beta rate, the  $R^2$  becomes large negative value. It suggests that the E-CCAPM fails to match the zero-beta rate by a big margin for these bootstrapped samples.<sup>13</sup> Further, from a statistical point of view, the wider confidence intervals of  $R^2$  for the E-CCAPM implies that the in-sample  $R^2$  estimates for the E-CCAPM is relatively more imprecisely estimated than the corresponding estimates for the alternative models. As such, comparing the in-sample  $R^2$  estimates across models should be interpreted with caution.

The results of the evaluation for the considered asset pricing models in Table 4 can be summarized in the following ways. First, the suggested consumption-based model is able to explain a significant portion of the dispersion in average excess returns of the SBM25 portfolios and also the SLTR25 portfolios. Second, news about future consumption growth is significantly priced, and seems to drive most of the explanatory power in explaining the cross-section of average returns, while news about current consumption growth seems to play a secondary role. Third, our model specification compares favorably with the CAPM, ICAPM, and CCAPM.

#### 4.3. Expected versus realized return

Although both  $R^2$  and MAE represent the overall fit of the model, it would be interesting to examine how much of the observed magnitude of the value premium (the average value-minus-growth return) and the profit for the long-term reversal strategy (the average long-term losers-minus-winners return) can be explained by our two-factor model. If our consumption-based model is sufficient to explain the value premium and the long-term reversal anomaly, then the difference between the realized return and estimated expected return (i.e., the pricing error) should be indistinguishable from zero.

Expected return is obtained from Eq. (12) with the estimated risk prices reported in Table 4. Panel A of Table 5 reports the results for the value premium within each size quintile. The first column ( $\bar{R}_{V_{MG}}$ ) and second column ( $E[R]_{V_{MG}}$ ) show realized and expected returns on the value-minus-growth portfolios, respectively. The third column ( $\alpha_{V_{MG}}$ ) reports pricing errors, and the fourth column ( $t(\alpha_{V_{MG}})$ ) reports the  $t$ -statistics corresponding the null hypothesis that the pricing error is zero. The fifth column ( $E[R]_{V_{MG}}/\bar{R}_{V_{MG}}$ ) shows the ratio of expected to realized value premium.

<sup>10</sup> For SLTR25 portfolios, the EIS estimate is  $-0.002$  with the confidence interval of  $[-0.0040, -0.0015]$ , and RRA estimate is  $-203$  with the confidence interval of  $[-322, -84]$ .

<sup>11</sup> In our model, relative risk aversion equals the risk price of current consumption shock (CCS), which suggests that the implausible negative estimate of risk aversion arises from the fact that current consumption shock is negatively priced in the presence of long-run consumption shock. This is because the current consumption shocks beta goes in the wrong direction to explain the value premium, which will be shown in Table 6. The negative relation between the betas of CCS and average returns, in turn, yields a negative price of its risk.

<sup>12</sup> We estimate the confidence intervals for cross-sectional  $R^2$  statistics using a bootstrap-based procedure. We follow Lioui and Maio (2014) because their GMM estimation is similar to ours, enabling us to directly apply their procedure without modification.

<sup>13</sup> It is worthwhile to note that the poor performance of the E-CCAPM in matching the zero-beta rate is obtained for only some of the bootstrapped samples. In our original sample data, the excess zero-beta estimates are economically small and statistically insignificant; it is 0.3% per quarter with  $t$ -value of 0.35 (estimation with SBM25) and 0.4% per quarter with  $t$ -value of 0.36 (estimation with SLTR25). Thus, the E-CCAPM can match the zero-beta rate and are not misspecified when it comes to pricing original sample data.

**Table 5**

Realized returns versus expected returns. The table reports realized returns, expected returns, and pricing errors for long-short portfolios within each size quintile. The assets are the 25 size/book-to-market portfolios (SBM25, Panel A) and the 25 size/long term return reversal portfolios (SLTR25, Panel B). Column  $\bar{R}_{VMG}$  ( $\bar{R}_{LMW}$ ) reports realized return on the value-minus-growth (long-term losers-minus-winners) portfolios. Column  $E[R]_{VMG}$  ( $E[R]_{LMW}$ ) reports expected returns on the value-minus-growth (long-term losers-minus-winners) portfolios implied from the E-CCAPM. Column  $\alpha$  denotes pricing error, defined as the differences between realized and expected return.  $t(\alpha)$  reports the  $t$ -statistics for  $\alpha$ . The sample period is from 1963:Q3 to 2010:Q4.

Panel A: SBM25					
	$\bar{R}_{VMG}$	$E[R]_{VMG}$	$\alpha_{VMG}$	$t(\alpha_{VMG})$	$E[R_{VMG}]/\bar{R}_{VMG}$
S1	2.64	2.05	0.59	1.06	78%
S2	1.69	1.72	−0.04	−0.05	102%
S3	1.81	0.77	1.04	1.52	42%
S4	0.80	1.50	−0.70	−0.98	187%
S5	0.43	0.68	−0.26	−0.32	161%

  

Panel B: SLTR25					
	$\bar{R}_{LMW}$	$E[R]_{LMW}$	$\alpha_{LMW}$	$t(\alpha_{LMW})$	$E[R_{LMW}]/\bar{R}_{LMW}$
S1	2.01	1.15	0.85	1.01	57%
S2	1.49	1.54	−0.05	−0.06	103%
S3	1.05	0.44	0.61	0.68	42%
S4	1.06	1.32	−0.26	−0.27	125%
S5	1.10	0.60	0.50	0.53	55%

Panel A shows that the value premium is stronger for small firms (2.64% per quarter) than big firms (0.43% per quarter), a stylized fact that poses a challenge to asset pricing models. The pricing errors (the difference between realized and expected returns on the value-minus-growth stocks) within each size quintile are all insignificant. For example, the largest pricing error occurs in medium size firms and is insignificant ( $t = 1.52$ ). In addition, the pricing errors show no systematic pattern across size quintiles. In other words, pricing errors do not show monotonically decreasing patterns across size quintiles, as is the case when other asset pricing models (CAPM, ICAPM, and CCAPM) are used (not reported here). Most importantly, our consumption-based two-factor model explains more than half of the realized value premium. For example, the realized return on the value-minus-growth portfolio for the smallest quintile is 2.64% per quarter and the expected return from the model is 2.05% per quarter, thus the model explains 78% of the realized value premium.

Panel B reports realized returns, expected returns, and pricing errors for the long-term losers-minus-winners portfolios within each size quintile. Long-term losers earn higher average returns than long-term winners. The realized return on the losers-minus-winners is 2.01% per quarter in the smallest size quintile and 1.10% per quarter in the largest size quintile. The proposed consumption-based model does a good job of explaining profits for the long-term reversal strategies. Predicted returns from the model are not significantly different from realized returns, evidenced by the fact that the pricing errors for each size quintile are all insignificant. For instance, the model predicts the expected profits for the long-term reversal strategy to be 1.15% per quarter for small stocks, which is 57% of the realized profit of 2.01% per quarter. Further, the point estimates of pricing errors are negative for two long-short portfolios among five hedge portfolios.

Overall, the results in Table 5 show that the average return spreads between value and growth stocks (value premium) and between long-term losers and winners (long-term reversal profits) can be explained by the suggested consumption-based model. The average portion of the realized return spread that can be explained by the model is more than 50%, and pricing errors are statistically insignificant for all size quintile.

#### 4.4. Factor betas

In order to understand which factor drives the explanatory power of the suggested consumption-based model for the cross-section of stock returns, we examine whether there is a systematic pattern in the risk exposures (betas) of the two different consumption growth shocks. Put differently, we want to study which factor's betas can match the value premium (return difference between value and growth stocks) and the long-term return reversal effect (return difference between long-term losers and winners). Panel A of Table 6 reports the (quarterly) average excess returns for the SBM25 portfolios and the betas for news about current and future consumption growths. The last column reports the difference in betas between value (BM5) and growth (BM1) quintiles (BM5-BM1) within each size quintile, while "Mean" stands for the average difference (BM5-BM1) across all size quintiles.

Reading across the rows of the table, average returns increase in book-to-market quintile within each size quintile, confirming the well-known value premium in our sample. For the case of news about current consumption growth, the growth stocks have higher betas than value stocks, with the exception of the fifth size quintile. The average difference for the current consumption growth shock, BM5-BM1, is −1.47. Furthermore, the betas with respect to current consumption growth exhibit an approximate *u*-shaped pattern; betas of current consumption risk decrease from the first to fourth book-to-market quintile and increase from the fourth to the fifth book-to-market quintile. These results indicate that current consumption growth risk seems not to be systematically related to the value premium and confirms why the standard CCAPM performs poorly in pricing the size/book-to-market portfolios.

**Table 6**

Factor betas. The table reports the average excess returns of test assets and the betas of the excess returns on the factors in the E-CCAPM estimated by GMM. The testing portfolios are the 25 size/book-to-market portfolios (SBM25, Panel A) and the 25 size/long term return reversal portfolios (SLTR25, Panel B). The factors are news about current consumption growth (CCS) and future consumption growth (FCS). BM1 (LTR1) denotes the lowest BM (LTR) quintile. The column labeled BM5-BM1 denotes the spread in beta estimates between the largest and lowest BM quintiles. The column labeled LTR1-LTR5 denotes the spread in beta estimates between the lowest and largest LTR quintiles. “Mean” is the respective average across size quintiles. The sample period is from 1963:Q3 to 2010:Q4.

Panel A: SBM25						
Average excess return (%)						
	BM1	BM2	BM3	BM4	BM5	BM5-BM1
S1	0.90	2.45	2.55	3.05	3.54	2.64
S2	1.36	2.12	2.77	2.76	3.05	1.69
S3	1.35	2.19	2.26	2.55	3.16	1.81
S4	1.65	1.57	1.97	2.43	2.45	0.80
S5	1.16	1.32	1.19	1.42	1.59	0.43
Mean						1.47
Betas on CCS						
	BM1	BM2	BM3	BM4	BM5	BM5-BM1
S1	7.22	5.57	3.54	3.17	3.93	−3.29
S2	4.29	2.99	2.21	1.68	2.28	−2.01
S3	3.38	2.41	1.93	1.39	1.77	−1.62
S4	3.09	2.62	1.72	0.45	2.10	−1.00
S5	1.66	0.69	1.76	0.92	2.20	0.54
Mean						−1.47
Betas on FCS						
	BM1	BM2	BM3	BM4	BM5	BM5-BM1
S1	3.42	4.05	4.95	4.78	6.32	2.90
S2	2.81	4.01	4.46	5.18	5.39	2.58
S3	3.16	3.83	3.84	4.80	4.13	0.97
S4	2.66	2.84	4.05	4.81	5.05	2.39
S5	1.89	1.55	1.10	2.93	3.11	1.22
Mean						2.01
Panel B: SLTR25						
Average excess return (%)						
	LTR1	LTR2	LTR3	LTR4	LTR5	LTR1-LTR5
S1	3.43	2.66	2.94	2.46	1.43	2.01
S2	3.43	2.45	2.77	2.72	1.94	1.49
S3	2.98	2.63	2.34	2.27	1.93	1.05
S4	2.66	2.03	2.11	2.03	1.59	1.06
S5	2.09	1.69	1.45	1.24	0.99	1.10
Mean						1.34
Betas on CCS						
	LTR1	LTR2	LTR3	LTR4	LTR5	LTR1-LTR5
S1	5.44	2.90	2.23	2.23	4.15	1.29
S2	3.67	2.03	1.28	1.78	3.84	−0.18
S3	3.73	1.41	0.61	2.04	3.35	0.38
S4	2.34	0.91	0.55	1.32	3.54	−1.19
S5	2.36	1.06	0.92	0.66	2.36	0.00
Mean						0.06
Betas on FCS						
	LTR1	LTR2	LTR3	LTR4	LTR5	LTR1-LTR5
S1	6.54	5.29	4.82	5.01	4.54	2.00
S2	6.21	4.92	4.11	4.50	3.67	2.54
S3	4.51	4.78	4.00	3.67	3.78	0.72
S4	4.93	3.99	3.94	3.46	2.85	2.08
S5	2.82	3.04	1.64	1.40	1.86	0.96
Mean						1.66

In contrast to the current consumption growth beta, the pattern of the long-run consumption growth beta closely matches that of the returns. Value stocks always have greater loadings on the news about future consumption growth than growth stocks within each size quintile, and the relation is approximately monotonic. In the case of future consumption growth shock, we have an average positive difference in factor loadings between value and growth stocks of 2.01. Overall, our results suggest that the value premium

can be explained by a corresponding dispersion in betas associated with future consumption risk. In other words, revisions in expected future consumption growth is the key factor for the suggested model's explanation of the value premium anomaly.

In Panel B of Table 6, we perform the previous analysis again for the long-term return reversal effect and report the (quarterly) average excess returns and the factor betas for the SLTR25 portfolios. Results show that the average returns increase from long-term winners (LTR5) to long-term losers (LTR1). The average return on long-term losers minus long-term winners, LTR1-LTR5, is 1.34% per quarter. As in the SBM25 portfolios, the current consumption risk betas show no meaningful dispersion across long-term return reversal portfolios within each size quintile. The average difference in betas for the current consumption growth beta, LTR1-LTR5, is close to zero, with a value of 0.06.

In the case of future consumption growth, the betas for long-term losers are significantly more positive than those for long-term winners, with an average difference of 1.66. As such, past losers are more sensitive to shocks to future consumption growth opportunities. These results show that the factor that allows the consumption-based model to explain the long-term return reversal anomaly is essentially news about future consumption growth. Put differently, the evidence on factor betas explains why the suggested consumption two-beta model can explain the long-term return reversal effect significantly better than can the standard CCAPM.<sup>14</sup>

Why do value (long-term loser) firms have greater exposure than growth (long-term winner) firms? One possible explanation is that typical value firms – stocks with low prices relative to their book values – have suffered a sequence of terrible shocks to which growth firms are less exposed. Similarly, firms that exhibit consistent stock price erosion for several years (long-term losers) are likely to experience a long sequence of negative cash flow shocks, whereas firms that see consistent stock price appreciation for several years (long-term winners) are likely to experience a long sequence of positive cash flow shocks.

We explore this argument using the approach proposed by Campbell et al. (2010), whose framework enables us to calculate direct cash-flow shocks. Following their approach, we calculate a direct cash-flow shock measure for each book-to-market (long-term reversal) quintile portfolio  $i$  ( $N_{i,CF,t+1}$ ) as

$$N_{i,CF,t+1} = \sum_{k=1}^K \rho^k roe_{i,t+k}, \quad (23)$$

where  $roe_{i,t+k}$  is the log of return on equity for portfolio  $i$ . We consider horizons ( $K$ ) from two to five years, and examine how choice of horizon affects the results. To construct earnings series, we compute portfolio level book-equity by aggregating firm level book-equity using Compustat data. Combined with constructed portfolio level book-equity, we generate earnings series using the clean-surplus relation,

$$BE_t - BE_{t-1} = X_t - D_t^{net}, \quad (24)$$

where book value today is equal to book value last year plus clean-surplus earnings  $X_t$  less (net) dividends. With an appropriate adjustment for equity offerings,  $X_t$  can be computed as

$$X_t = \left[ \frac{(1 + R_t) ME_{t-1} - D_t}{ME_t} \right] \times BE_t - BE_{t-1} + D_t, \quad (25)$$

where  $D_t$  is gross dividends computed from CRSP.

Panel A of Table 7 reports the averages of portfolio level cash-flow shocks for each book-to-market quintile. If a portfolio has suffered a sequence of terrible shocks, its profitability in the near future will be severely lower. Indeed, for any  $K$  considered, the cash-flow fundamental tends to be lower for value than for growth firms. In Panel B, we calculate the cash-flow shocks for each long-term reversal quintile. We find that, for any  $K$  considered, the average of cash-flow shocks is larger for long-term losers than for long-term winners. As such, long-term losers should have higher risk exposure to news about future prospects of consumption growth than long-term winners.<sup>15</sup>

#### 4.5. Discussion

We substitute out the market return to obtain the pricing kernel as a function of consumption growth only, as shown in Eq. (9). In contrast, Campbell (1993) proposes the ICAPM by getting rid of consumption growth in favor of the market return. We show that the E-CCAPM, which substitutes out the market return, better explains the cross-section of asset returns than does the ICAPM, which substitutes out consumption growth. The important question is what accounts for the superior performance of the E-CCAPM over the ICAPM? One potential explanation could be that if human wealth is a significant component of the wealth portfolio, changes in the wealth portfolio are better summarized by changes in consumption than by the market return. If the wealth portfolio consists of human as well as financial wealth, using a return on financial wealth portfolio (i.e., the market return) as a proxy for the return on the

<sup>14</sup> Following the approach of Patton and Timmermann (2010), we perform the monotonic relation test for the future consumption growth betas. Since it is a joint test for a monotonic relation, it is relatively difficult to reject the null hypothesis of no monotonic relation compared to the test that considers only the two betas for a given quintile. Nevertheless, for some size quintiles the monotonic relation is confirmed. For SBM25, the null hypothesis is rejected for size quintiles 2 and 4, and for SLTR25, we reject the null hypothesis for size quintile 4.

<sup>15</sup> For additional analysis, we also estimate cash-flow betas for each BM (LTR) quintile since revisions in expectations of cash flows is an important determinant of stock returns. For horizons from two to five years, we find that value (long-term loser) stocks cash flows have higher betas with respect to the markets cash-flow news (these unreported results are available upon request). We thus show value premium and long-term return reversal to be explained by the difference in the exposure of assets cash flows to the markets ROE. These additional results from cash-flow betas complement our earlier empirical evidence based on return betas.

**Table 7**

Cash flow shocks. The table reports the averages of cash-flow shocks for book-to-market quintile portfolios (Panel A) and long-term return reversal quintile portfolios (Panel B), formed each year. Following [Campbell et al. \(2010\)](#), we calculate a direct cash-flow shock measure for each quintile portfolio  $i$  ( $N_{i,CF,t+1}$ ) as  $N_{i,CF,t+1} = \sum_{k=1}^K \rho^{k-1} roe_{i,t+k}$ , where  $roe_{i,t+k}$  is the log of real profitability for portfolio  $i$ . We consider horizons ( $K$ ) from two to five years. The sample period is from 1963 to 2010.

Panel A: BM quintiles					
	BM1	BM2	BM3	BM4	BM5
K = 2	0.51	0.31	0.25	0.19	0.06
K = 3	0.67	0.43	0.35	0.27	0.10
K = 4	0.82	0.54	0.44	0.35	0.15
K = 5	0.95	0.65	0.53	0.42	0.20
Panel B: LTR quintiles					
	LTR1	LTR2	LTR3	LTR4	LTR5
K = 2	0.12	0.17	0.20	0.21	0.27
K = 3	0.19	0.26	0.29	0.31	0.37
K = 4	0.25	0.33	0.38	0.39	0.46
K = 5	0.32	0.41	0.45	0.48	0.55

wealth portfolio could, as [Roll \(1977\)](#) points out, constitute a potential model misspecification. The intertemporal budget constraint, on the other hand, dictates that any shock to wealth, including human wealth, must appear in consumption. An agent subject to a positive human wealth shock due to an increase in labor income and/or decrease in the discount rate of human wealth, for instance, will adjust consumption spending. This positive human wealth shock may not be reflected, however, in the return on the financial wealth portfolio.<sup>16</sup>

We conduct a simulation study in a manner similar to [Campbell and Cochrane \(2000\)](#) to show that the superior performance of the E-CCAPM relative to the ICAPM could be partly driven by the omitted human capital problem.<sup>17</sup> In particular, we consider a true model where return on wealth portfolio consists of both financial wealth and human capital. In artificial data simulated under the assumed true model data-generating process, we evaluate the pricing abilities of two alternative models, specifically, the E-CCAPM and ICAPM. If consumption summarizes the total wealth portfolio better than financial wealth portfolio, E-CCAPMs pricing ability should be better in this simulated economy.

We model the return on the wealth portfolio as a weighted average of returns on human capital,  $r_{y,t+1}$ , and financial wealth,  $r_{d,t+1}$ :

$$r_{c,t+1} = (1 - \omega)r_{d,t+1} + \omega r_{y,t+1}, \quad (26)$$

where  $\omega$  is the share of human wealth in total wealth. We further assume that the expected return on human capital is linear in state variables, following [Bansal et al. \(2014, BKS hereafter\)](#). The innovations to a pricing kernel in the null model,  $m_{t+1}^N$ , can be written as

$$\begin{aligned} m_{t+1}^N - E_t(m_{t+1}^N) = & -\gamma(1 - \omega) [r_{d,t+1} - E_t(r_{d,t+1})] - (\gamma - 1)(1 - \omega) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} \\ & - \gamma\omega [r_{y,t+1} - E_t(r_{y,t+1})] - (\gamma - 1)\omega (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{y,t+1+j}. \end{aligned} \quad (27)$$

Eq. (27) suggests that the shock to the pricing kernel is driven by current and future shocks to return on financial wealth as well as human capital. [Appendix](#) details the derivation of the pricing kernel as well as our simulation procedures.

Our estimation uses the VAR approach to estimate conditional expectations as well as innovations to the variables considered. In a true model, state variables in the VAR include consumption growth, labor income growth, market return, and the four instrumental variables (*DIV*, *DEF*, *RREL*, and *TERM*) used in our estimation. We term this true model “A True Model with Human Capital” (Case 1 in the [Appendix](#)).

We then consider two alternative models, the E-CCAPM and ICAPM. In the E-CCAPM, the shock to the pricing kernel is driven by current and future consumption growth shocks (see Eq. (9)). To be consistent with the VAR specification used in our empirical estimation, state variables for the E-CCAPM include consumption growth and the four instrumental variables. In the ICAPM, the shock to the pricing kernel is determined by current and future shocks to the return on financial wealth (see Eq. (21)). State variables for the ICAPM include the market return and the four instrumental variables, again, consistent with our estimation.

<sup>16</sup> To address the omitted human capital problem, an alternative approach in the ICAPM framework is to model explicitly returns on both financial and human wealth. However, this approach faces challenge that only the cash flow component of human wealth returns is observable; the discount rate component is not. For instance, [Campbell \(1996\)](#) aims to accommodate human wealth using labor income growth, but assumes that the discount rate of human wealth is the same as the stock market discount rate. As pointed out by [Cochrane \(2008\)](#), however, most variation in human wealth is likely to be driven by changes in the discount rate rather than in cash flow.

<sup>17</sup> [Campbell and Cochrane \(2000\)](#) consider the external habit model of [Campbell and Cochrane \(1999\)](#) as a true model. Even in a simulated economy where return is generated by the consumption-based model, they show that the return-based model outperforms the consumption-based model.



**Table 8**

A simulation-based study: Maximum pricing errors from approximate asset pricing models. The table reports the maximum expected return pricing error per unit of standard deviation under the two approximate asset pricing models (E-CCAPM and ICAPM) examined in our study, by following the simulation-based approach of Campbell and Cochrane (2000). We set artificial economy under a VAR framework consisting of 7 state variables (real consumption growth, real labor income growth, real market return, and the four forecasting variables), combined with reasonable set of risk aversion ( $\gamma$ ) and EIS parameter values (6 cases). We consider two case of null (true) models: “A True Model with Human Capital” (Panel A) and “A True Model without Human Capital” (Panel B). Results are based on 190 quarters of 1000 bootstrapped samples.

Panel A: A true model with human capital		
( $\gamma$ , EIS)	Alternative Model 1: E-CCAPM	Alternative Model 2: ICAPM
( $\gamma$ , EIS) = (7.5, 2.5)	0.081	0.312
( $\gamma$ , EIS) = (5, 2.5)	0.055	0.203
( $\gamma$ , EIS) = (7.5, 5)	0.080	0.313
( $\gamma$ , EIS) = (5, 5)	0.054	0.205
( $\gamma$ , EIS) = (7.5, 1.5)	0.082	0.310
( $\gamma$ , EIS) = (10, 1.5)	0.108	0.444
Panel B: A true model without human capital		
( $\gamma$ , EIS)	Alternative Model 1: E-CCAPM	Alternative Model 2: ICAPM
( $\gamma$ , EIS) = (7.5, 2.5)	0.372	0.075
( $\gamma$ , EIS) = (5, 2.5)	0.248	0.046
( $\gamma$ , EIS) = (7.5, 5)	0.371	0.076
( $\gamma$ , EIS) = (5, 5)	0.248	0.047
( $\gamma$ , EIS) = (7.5, 1.5)	0.372	0.075
( $\gamma$ , EIS) = (10, 1.5)	0.514	0.113

Next, we follow simulation procedures of Campbell and Cochrane (2000). We use a bootstrapping method to simulate 190 quarterly observations for 1000 bootstrapped samples. To set parameters associated with the dynamics of state variables, we use parameters estimated from the actual data over our sample period. We use various combinations of the EIS and risk aversion parameters by referring to the related literature. We set portfolio weight on human wealth equal to 0.8.<sup>18</sup>

To measure the degree of misspecification arising from the use of ICAPM or E-CCAPM rather than the null model, we use a metric that measures the maximum pricing error that the false models produce, which is Hansen and Jagannathan (1997) distance measure. We obtain the distribution of the maximum pricing error for the bootstrapped samples. Details of the simulation procedures are provided in the Appendix.

Panel A of Table 8 shows the maximum pricing errors in artificial data from the true model, in which the return on the wealth portfolio consists of both financial wealth and human capital. The results reveal that maximum pricing error is much lower for the E-CCAPM than for the ICAPM, consistent with our empirical estimation results that show pricing ability to be better for the E-CCAPM than for the ICAPM (as reported in Section 4.2).

To further demonstrate that the superior performance of the E-CCAPM relative to the ICAPM is likely attributable to the omitted human capital problem, we consider an alternative artificial economy in which total wealth has only a financial wealth component (Case 2 in the Appendix). The null model can be derived by simply letting  $\omega = 0$  in Eq. (27). Thus, the shock to the pricing kernel in Case 2 is given by

$$m_{t+1}^N = -\gamma(1-\omega) [r_{d,t+1} - E_t(r_{d,t+1})] - (\gamma-1)(1-\omega) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j}. \quad (28)$$

The null model of Case 2 is termed “A True Model without Human Capital.” Note that state variables for ICAPM are different from those for the true model. As such, ICAPM is still considered a false model.

Panel B of Table 8 reports the maximum pricing errors in artificial data from the true model in which the return on the wealth portfolio is assumed to be the return on financial wealth. In a sharp contrast to the results in Panel A, the ICAPM performs better than our model when the null model does not include human capital in the wealth portfolio. This result suggests that the superior performance of the E-CCAPM in Case 1 is not mechanical, and that the human capital component does, indeed, plays a significant role in the differential pricing abilities of the E-CCAPM and ICAPM.

## 5. Additional results

In this section, we conduct several robustness checks on our main results. Alternative vector autoregression (VAR) specifications are used to estimate revisions in current and future consumption growths. Next, we test the model while including industry (or bond) portfolios in the test assets. The model is also tested simultaneously on value and long-term return reversal portfolios. Finally, the model is estimated in expected return-beta form. Results obtained from each robustness check are substantially similar to those for the benchmark test.

<sup>18</sup> Our results are quantitatively robust to the use of different weights for the human wealth component.

**Table 9**

Estimation of factor risk premia: Alternative VAR specifications. The table reports the estimation and evaluation results for the Extended CCAPM, based on alternative VAR specifications. The first VAR specification follows Campbell (1996) (column labeled as C), in which (i) the dividend yield on the CRSP value-weighted index, (ii) the relative T-bill rate, and (iii) the yield spread between long- and short-term government bonds are state variables. The second VAR specification follows Campbell and Vuolteenaho (2004) (column labeled as CV), in which (i) the price–earnings ratio (measured as the ratio of the S&P 500 price index to a ten-year moving average of S&P 500 earnings), (ii) the yield spread between long-term and short-term bonds, and (iii) the value spread are state variables. The third VAR specification follows Campbell et al. (2018) (column labeled as CGPT), in which (i) the price–earnings ratio, (ii) the term yield spread, (iii) the value spread, and (iv) the default spread are state variables. The testing portfolios are the 25 size/book-to-market portfolios (SBM25, Panel A) and the 25 size/long term return reversal portfolios (SLTR25, Panel B). The estimation procedure is first-stage GMM. CCS and FCS are the risk prices associated with news about current and future consumption growth, respectively. Standard deviations are reported in parentheses. The  $p$ -values for the  $J$ -test (test of over-identifying restrictions) are reported in parentheses. The sample period is from 1963:Q3 to 2010:Q4.

Panel A: SBM25			
Factor price	C	CV	CGPT
CCS	−229.57 (125.68)	−286.69 (153.82)	−282.61 (160.44)
FCS	322.49 (116.31)	282.26 (119.21)	275.40 (113.57)
MAE (%)	0.30	0.33	0.30
R <sup>2</sup>	0.77	0.69	0.74
C.I of R <sup>2</sup>	[−0.09, 0.88]	[−0.05, 0.89]	[−0.05, 0.88]
$J$ -test	44.51 (0.00)	26.22 (0.29)	26.88 (0.26)
Panel B: SLTR25			
Factor price	C	CV	CGPT
CCS	−142.06 (81.03)	−209.16 (110.45)	−211.83 (114.81)
FCS	256.77 (94.93)	232.99 (86.09)	230.00 (84.63)
MAE (%)	0.28	0.28	0.27
R <sup>2</sup>	0.65	0.65	0.67
C.I of R <sup>2</sup>	[−3.84, 0.71]	[−3.92, 0.75]	[−3.49, 0.73]
$J$ -test	51.48 (0.00)	48.30 (0.00)	51.39 (0.00)

### 5.1. Alternative VAR specifications

In the main analysis, we choose the aggregate dividend yield, default spread, relative T-bill rate, and term spread as the VAR state variables for estimating revisions in current and future consumption growths. Since our results could be sensitive to the choice of VAR state variables, it is important to conduct robustness tests using alternative VAR specifications.

We consider three sets of alternative VAR specifications previously used in the empirical asset pricing literature. The first VAR specification follows Campbell (1996), in which the state variables include (i) the aggregate dividend yield, (ii) the relative T-bill rate, and (iii) the term yield spread. Excluding the default spread from our benchmark VAR specification yields the first alternative specification. The second VAR specification follows Campbell and Vuolteenaho (2004), in which the state variables include (i) the price–earnings ratio (measured as the ratio of the S&P 500 price index to a ten-year moving average of S&P 500 earnings), (ii) the yield spread between long-term and short-term bonds, and (iii) the value spread. The third VAR specification follows Campbell et al. (2018), in which state variables include (i) the price–earnings ratio, (ii) the term yield spread, (iii) the value spread, and (iv) the default spread. Put differently, replacing the dividend yield and relative T-bill rate with the price–earnings ratio and value spread, respectively, delivers the third specification.

Table 9 presents the estimation results. The results show that the  $R^2$  estimates for the consumption-based model are comparable to the corresponding estimates in the benchmark test. Specifically, in the test with SBM25, the  $R^2$  estimates vary between 69% (using the Campbell and Vuolteenaho (2004) specification) and 77% (using the Campbell (1996) specification). The corresponding estimates in the test with SLTR25 vary between 65% (using the Campbell (1996) specification) and 67% (using the Campbell et al. (2018) specification). The estimates for news about future consumption growth risk price have greater magnitudes than the corresponding estimates in the benchmark test, and are statistically significant at the 1% level in all cases. On the other hand, the estimates for news about contemporaneous consumption growth risk price are negatively significant at the 5% level in all cases, consistent with the benchmark test. In sum, the estimation results of the model are robust to alternative VAR specifications.

### 5.2. Alternative test assets

One way to improve empirical tests is to expand the set of test portfolios as suggested by Lewellen et al. (2010). First, we add industry portfolios because explaining the cross-section of industry portfolios is a challenge for asset pricing models given that industry

**Table 10**

Estimation of factor risk premia: Alternative portfolios. The table reports the estimation and evaluation results for the Extended CCAPM using an alternative set of test portfolios. The first test assets consist of 25 size/book-to-market portfolios, and 30 Fama–French industry portfolios (SBM25 + Industry30, Panel A). The second test assets consist of 25 size/long term return reversal portfolios, and 30 Fama–French industry portfolios (SLTR25 + Industry30, Panel B). The third test assets consist of 10 size portfolios, 10 book-to-market portfolios, and 10 long term return reversal portfolios (S10 + BM10 + LTR10, Panel C). The fourth test assets consist of 25 size/book-to-market portfolios and 7 bond portfolios (SBM25 + Bond7, Panel D). The estimation procedure is first-stage GMM. CCS and FCS are the risk prices associated with news about current and future consumption growth, respectively. Standard deviations are reported in parentheses. MAE refers to the mean absolute pricing error (in %), and  $R^2$  is 1 minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns. Bootstrapped 95% confidence intervals associated with  $R^2$  are also reported. The  $p$ -values for the  $J$ -test (test of over-identifying restrictions) are reported in parentheses. The sample period is from 1963:Q3 to 2010:Q4.

Panel A: SBM25 + Industry30				
Factor price	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.28 (0.90)	−5.35 (2.33)		
LRMS		−9.12 (2.96)		
CCS			113.49 (37.44)	−159.75 (74.64)
FCS				181.42 (56.75)
MAE (%)	0.53	0.45	0.46	0.32
$R^2$	−0.08	0.24	0.13	0.57
C.I of $R^2$	[−0.03, 0.77]	[0.13, 0.78]	[−2.10, 0.77]	[−0.25, 0.78]
$J$ -test	104.60 (0.00)	121.01 (0.00)	126.55 (0.00)	119.31 (0.00)
Panel B: SLTR25 + Industry30				
Factor price	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.42 (0.93)	−2.10 (2.60)		
LRMS		−5.48 (3.25)		
CCS			120.94 (39.57)	−145.51 (66.55)
FCS				176.39 (55.08)
MAE (%)	0.50	0.46	0.47	0.31
$R^2$	0.09	0.19	0.15	0.56
C.I of $R^2$	[−0.23, 0.76]	[0.03, 0.76]	[−2.87, 0.74]	[−0.30, 0.77]
$J$ -test	91.27 (0.00)	100.29 (0.00)	122.64 (0.00)	99.91 (0.00)
Panel C: S10 + BM10 + LTR10				
Factor price	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.34 (0.94)	7.97 (5.52)		
LRMS		9.56 (9.10)		
CCS			146.79 (54.99)	−166.63 (51.51)
FCS				224.70 (47.60)
MAE (%)	0.30	0.29	0.29	0.19
$R^2$	0.25	0.37	0.38	0.71
C.I of $R^2$	[−1.72, 0.74]	[−2.13, 0.73]	[−1.14, 0.80]	[−1.70, 0.79]
$J$ -test	24.39 (0.71)	24.73 (0.64)	26.21 (0.61)	25.06 (0.62)

(continued on next page)

portfolios are not based on price variables. We consider 25 portfolios sorted on size and book-to-market (SBM25) combined with the 30 industry portfolios, and 25 portfolios sorted on size and long-term return reversal (SLTR25) plus 30 industry portfolios. Panels A and B of Table 10 report the estimation results. Although inclusion of industry portfolios generally weakens the explanatory power of models from benchmark test, the risk price estimate for news about future consumption growth is positive and statistically significant.

The second alternative test portfolios consist of 10 portfolios sorted on size (S10), 10 portfolios sorted on book-to-market (B10), and 10 portfolios sorted on long-term return reversal (LTR10), a total of 30 portfolios. This empirical test allows us to examine whether our model can explain simultaneously the value premium and the long-term return reversal. Panel C of Table 10 presents

Table 10 (continued).

Panel D: SBM25 + Bond7				
Factor price	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.32 (0.84)	−7.37 (2.59)		
LRMS		−15.40 (3.89)		
CCS			110.20 (33.62)	−145.51 (111.94)
FCS				216.70 (88.25)
MAE (%)	0.52	0.41	0.43	0.26
R <sup>2</sup>	0.52	0.74	0.66	0.88
C.I of R <sup>2</sup>	[0.54, 0.91]	[0.60, 0.93]	[−0.23, 0.89]	[−0.09, 0.88]
J-test	99.33 (0.00)	102.82 (0.00)	110.58 (0.00)	63.74 (0.00)

the results. The fit of the model is essentially unchanged from the benchmark test, with an explanatory ratio of 71% and an average pricing error of 0.19% per quarter. The risk prices for two consumption growth shocks are close to the corresponding estimates in the benchmark case and are significant at the 1% level. The explanatory powers of alternative asset pricing models considered (CAPM, ICAPM, and CCAPM) are similar to the benchmark test, with a cross-sectional  $R^2$  that ranges from 25% to 38%.

The third alternative test assets are the SBM25 portfolios plus seven Treasury bond portfolios (Bond7) with average maturities of 1, 2, 5, 7, 10, 20, and 30 years (a total of 32 portfolios).<sup>19</sup> Adding bond portfolios to equity portfolios allows us to evaluate whether our model can jointly explain the cross-section of stock and bond returns.<sup>20</sup> Panel D reports the results. As before, the risk price estimates for the innovations about future and current consumption growths are positive and negative, respectively, but only the future consumption growth factor is priced (1% level).

Overall, these results show that news about future consumption growth help to explain a broad set of test portfolios including industry portfolios. In addition, our model does a good job of explaining simultaneously the value premium and the long-term return reversal effect, as well as jointly pricing stock and bond returns.<sup>21</sup>

### 5.3. Beta representation

We test the model in expected return-beta form by using the Fama and Macbeth (1973) two-pass methodology. In the first step, we run time-series multiple regressions to estimate the factor betas for each test asset:

$$R_{i,t+1}^e = \delta_i + \beta_i' f_{t+1} + \epsilon_{i,t+1}, \quad (29)$$

where  $R_{i,t+1}^e$  is excess return for test asset  $i$ ,  $f_{t+1}$  is the vector of  $K$  factors, and  $\beta_i$  is the vector of (factor) betas of asset  $i$ . In the second step, we run a cross-sectional regression of average excess returns on the factor betas to estimate the (beta) risk premiums:

$$\bar{R}_i^e = \lambda' \beta_i + \alpha_i, \quad (30)$$

where  $\bar{R}_i^e$  is the sample average excess return for test asset  $i$ ,  $\lambda$  is the (beta) risk premiums, and  $\alpha_i$  is the pricing error. We do not include an intercept in the cross-sectional regression to impose a restriction that zero-beta rate equals to the risk-free rate. Under this restriction, the model is forced to explain the unconditional equity premium as well as the value premium (or the long-term return reversal effect) (Campbell and Vuolteenaho, 2004; Campbell et al., 2018).<sup>22</sup> The critical difference between the two-pass time-series/cross-sectional regression approach and GMM estimation is that the factor betas from the two-pass regression approach are multiple-regression betas, while the betas from GMM estimation are single-regression betas (Cochrane, 2005). Multiple-regression betas take into account correlation among the factors. Thus, the estimated risk prices may have different signs when the factors are significantly correlated.

Table 11 presents the estimation results. The explanatory power of the consumption-based model is preserved in the test of the beta representation, since the cross-sectional  $R^2$  and MAE estimates are very similar for both sets of test assets. Once again, the point estimates for the risk price associated with news about future consumption growth is positive and significant (at the 1% level), while those associated with news about contemporaneous consumption growth is negative and significant (at the 1% level).<sup>23</sup> With the

<sup>19</sup> Bond portfolios are available from CRSP. We compute bond portfolio excess returns by subtracting the return on three-month T-bill.

<sup>20</sup> Malloy et al. (2009) and Maio and Santa-Clara (2012) also evaluate asset pricing models over the joint cross-section of stock and bond portfolios.

<sup>21</sup> We also examine whether our model can explain momentum profits. We test our model with the 10 portfolios sorted on momentum. We find that our model does not explain well the cross-section of returns sorted on momentum. The betas on CCS and FCS do not exhibit a systematic pattern across the momentum portfolios, resulting in neither CCS nor FCS being significantly priced in these portfolios.

<sup>22</sup> Alternatively, we can include an intercept in the cross-sectional regression. This specification corresponds to an unrestricted zero-beta rate, following Black (1972). By construction, this specification allows a model to have greater explanatory power.

<sup>23</sup> The scale of risk premiums reported here are different from those in the benchmark test. Table 11 reports the beta risk premium ( $\lambda$  in Eq. (30)), while Table 4 reports the factor prices of risk ( $b$  in Eq. (16)).

**Table 11**

Estimation of factor risk premia: Expected return-beta representation. The table reports the estimation and evaluation results for the CAPM, Intertemporal CAPM (ICAPM), Consumption CAPM (CCAPM), and Extended CCAPM with news about future consumption growth (E-CCAPM). The testing portfolios are the 25 size/book-to-market portfolios (SBM25, Panel A) and the 25 size/long term return reversal portfolios (SLTR25, Panel B). The estimation procedure is the time-series/cross-sectional regressions approach. CMS and LRMS are the beta risk premia associated with news about current and future stock returns, respectively. CCS and FCS are the beta risk premia associated with news about current and future consumption growth, respectively. Standard deviations are reported in parentheses. MAE refers to the mean absolute pricing error (in %), and  $R^2$  is 1 minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns. The  $p$ -values for the  $J$ -test (test of over-identifying restrictions) are reported in parentheses. The sample period is from 1963:Q3 to 2010:Q4.

Panel A: SBM25				
Risk Premium	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	1.85 (0.68)	−6.82 (5.20)		
LRMS		−6.98 (4.06)		
CCS			0.38 (0.14)	−0.46 (0.20)
FCS				0.89 (0.19)
MAE (%)	0.66	0.67	0.61	0.27
$R^2$	−0.34	−0.25	−0.34	0.76
Panel B: SLTR25				
Risk Premium	CAPM	ICAPM	CCAPM	E-CCAPM
CMS	2.03 (0.68)	7.00 (4.60)		
LRMS		3.97 (3.48)		
CCS			0.43 (0.14)	−0.45 (0.15)
FCS				0.88 (0.18)
MAE (%)	0.48	0.45	0.55	0.25
$R^2$	0.09	0.13	−0.06	0.69

exception of the proposed model, the explanatory power of the models considered (CAPM, ICAPM, and CCAPM) becomes weaker than in the benchmark test. This poor performance could be attributed to the zero-beta restriction of our specification. Untabulated results show that when we include an intercept in the cross-sectional regression, the explanatory ratios significantly increase. These results indicate that the model with future consumption risk is not misspecified in pricing the book-to-market and long-term return reversal portfolios, while there are relevant missing risk factors in alternative asset pricing models. Overall, the estimation results of the suggested model are consistent with those in the test of the expected return-covariance representation.

#### 5.4. An alternative asset pricing model

Recent studies that attribute the failure of the consumption CAPM to its static specification between risk and return propose a conditional version of the consumption CAPM. A conditional CCAPM proposed by Lettau and Ludvigson (2001, LL hereafter), in which the variable *cay*, a proxy for movement in the consumption–wealth ratio, serves as a conditioning variable, performs well in explaining the value premium. In this subsection, we compare the empirical performance of the E-CCAPM with that of Lettau and Ludvigson (2001). We conduct a cross-sectional test using GMM for the LL (2001) model.<sup>24</sup> We find that the LL (2001) model has some explanatory power for SBM25; the interaction term of consumption growth and *cay* is positive and statistically significant. In terms of the MAE and  $R^2$ , the performance of the LL (2001) model is comparable to that of our model. The explanatory power of the LL (2001) model, however, significantly weakens for SLTR25; the estimated risk price of the interaction term is not statistically significant, and the cross-sectional  $R^2$  is low. The E-CCAPM compares favorably with Lettau and Ludvigson's conditional CCAPM in pricing both the SBM25 and SLTR25 portfolios.

## 6. Conclusion

In this paper, we derive and test a consumption-based intertemporal asset pricing model in which an asset earns a risk premium if it performs poorly when the expected future consumption growth deteriorates. The predictability of consumption growth combined with the recursive preference delivers revisions in expected future consumption growth an additional risk factor, in addition to revisions

<sup>24</sup> These results, unreported to conserve space, are available from the authors upon request.

in current consumption growth. This paper contributes to the literature by empirically demonstrating that the consumption-based model can be dramatically improved if we consider implications of time variation in expected future consumption growth for asset pricing.

We model the consumption growth dynamics using a VAR structure with a set of instrumental variables commonly used for forecasting future economic growth. Empirically, we find that future consumption growth is strongly predicted by current economic conditions. This evidence is important, since if the consumption growth rate cannot be predicted, news about future consumption growth will have zero risk price. Thus, the model collapses to the standard CCAPM.

We estimate and test our consumption-based intertemporal asset pricing model with 25 portfolios sorted on size and book-to-market and 25 portfolios sorted on size and long-term return reversal. The cross-sectional tests using GMM show that the model explains a significant portion of the dispersion in average excess returns of the two test assets, with explanatory ratios varying between 65% and 79%. In addition, revisions in expected future consumption growth is significantly priced, and seems to drive most of the explanatory power in explaining the cross-section of average returns, while revisions in current consumption growth seems to play a secondary role. Finally, the suggested model compares favorably with the CAPM, ICAPM, and standard CCAPM.

The E-CCAPM has, however, some limitations. The estimates of the relative risk aversion coefficient are implausibly negative. Further, the  $R^2$  estimates in some robustness tests are not outside their 95% confidence intervals. Finally, the confidence intervals of  $R^2$  for the E-CCAPM are wider, indicating that the in-sample  $R^2$  estimates for the E-CCAPM are relatively more imprecisely estimated.

To overcome these limitations, the E-CCAPM can be extended by allowing the volatility of consumption growth to be time-varying. Following Campbell (1993, 1996), we maintain the assumption of constant consumption growth volatility. Given the evidence of time variation in consumption growth volatility, it seems natural to study their asset pricing implications. With the heteroskedasticity assumption on consumption growth, revisions in expected future consumption growth volatility will appear as an additional risk factor. Extending the model along this dimension and examining the empirical performance of the extended model could be interesting.

## Appendix. A simulation-based study comparing performance between ECCAPM and ICAPM

### A.1. Case 1: A true model with human capital

A representative agent has the recursive utility function of Epstein and Zin (1989) and Weil (1989),

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{c,t+1}, \quad (\text{A.1})$$

$$m_{t+1} - E_t(m_{t+1}) = -\frac{\theta}{\psi} [\Delta c_{t+1} - E_t(\Delta c_{t+1})] - (1 - \theta) [r_{c,t+1} - E_t(r_{c,t+1})]. \quad (\text{A.2})$$

where  $r_{c,t+1}$  is the return on a claim to aggregate consumption. We distinguish between the return on a claim to aggregate consumption,  $r_{c,t+1}$  and return on the aggregate dividend claim,  $r_{d,t+1}$ . The first is the return on the wealth portfolio, and the latter is the return on the market portfolio.

In a true model, state variables in the VAR include consumption growth, labor income growth, market return, and four instrumental variables ( $DIV$ ,  $DEF$ ,  $RREL$ ,  $TERM$ ). These seven state variables are assumed to follow a first-order VAR model,

$$X_{t+1}^N = A + BX_t^N + u_{t+1}^N, \quad (\text{A.3})$$

where  $X_t^N = (\Delta c_t, \Delta y_t, r_{d,t}, DIV_t, DEF_t, RREL_t, TERM_t)$ .

We assume that the wealth portfolio return is a linear combination of dividend-income portfolio return,  $r_{d,t+1}$ , and human-income portfolio return,  $r_{y,t+1}$ :

$$r_{c,t+1} = (1 - \omega) r_{d,t+1} + \omega r_{y,t+1}. \quad (\text{A.4})$$

where  $X_t^N$  are state variables in the system. We further assume that consumption growth and wealth returns are conditionally homoscedastic and jointly log-normal. By log-linearizing the budget constraint around the steady state, and assuming the consumption–wealth ratio to be stationary, the pricing kernel is obtained as,

$$\begin{aligned} m_{t+1}^N - E_t(m_{t+1}^N) &= -\gamma [r_{c,t+1} - E_t(r_{c,t+1})] - (\gamma - 1) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{c,t+1+j} \\ &= -\gamma(1 - \omega) [r_{d,t+1} - E_t(r_{d,t+1})] - (\gamma - 1)(1 - \omega) (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} \\ &\quad - \gamma\omega [r_{y,t+1} - E_t(r_{y,t+1})] - (\gamma - 1)\omega (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{y,t+1+j}. \end{aligned} \quad (\text{A.5})$$

The true stochastic discount factor is thus a function of the market return (return on financial wealth) and human capital return. The terms associated with the market return in the pricing kernel can be extracted from the VAR directly,

$$r_{d,t+1} - E_t(r_{d,t+1}) = e_1' u_{t+1}^N, \quad (\text{A.6})$$



$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} = e3' \rho B(I - \rho B)^{-1} u_{t+1}^N. \quad (\text{A.7})$$

However, the terms associated with the human capital return in the pricing kernel cannot be extracted from the VAR directly, as only labor income, not the human capital return, is observable in the data. To extract the terms associated with the human capital return using empirical data, we assume that the expected return to labor income is linear in the state variables by following BKS (2014),

$$E_t(r_{y,t+1}) = \alpha + b' X_t^N. \quad (\text{A.8})$$

The terms associated with the human capital return are given by:

$$r_{y,t+1} - E_t(r_{y,t+1}) = e2' (I + \rho B(I - \rho B)^{-1}) u_{t+1}^N - b' \rho (I - \rho B)^{-1} u_{t+1}^N, \quad (\text{A.9})$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{y,t+1+j} = b' B^{-1} \rho B(I - \rho B)^{-1} u_{t+1}^N = b' \rho (I - \rho B)^{-1} u_{t+1}^N. \quad (\text{A.10})$$

The pricing kernel with unknown value of  $b$  (the vector of the expected labor return loadings) is specified as:

$$\begin{aligned} m_{t+1}^N - E_t(m_{t+1}^N) &= -\gamma(1-\omega)e1' u_{t+1}^N - \gamma\omega(e2' (I + \rho B(I - \rho B)^{-1}) u_{t+1}^N - b' \rho (I - \rho B)^{-1} u_{t+1}^N) \\ &\quad - (\gamma-1)(1-\omega)e3' \rho B(I - \rho B)^{-1} u_{t+1}^N - (\gamma-1)\omega b' \rho (I - \rho B)^{-1} u_{t+1}^N. \end{aligned} \quad (\text{A.11})$$

To complete the specification of the true stochastic discount factor, we need to derive the unknown value of  $b$ , which we solve numerically by backing out the implied consumption shock equation:

$$\Delta c_{t+1} - E_t(\Delta c_{t+1}) = [r_{c,t+1} - E_t(r_{c,t+1})] + (1-\psi)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{c,t+1+j}. \quad (\text{A.12})$$

Using assumption (A.4), the RHS of Eq. (A.12) can be split into the terms associated with the market return and terms associated with the human capital return:

$$\begin{aligned} \Delta c_{t+1} - E_t(\Delta c_{t+1}) &= (1-\omega)[r_{d,t+1} - E_t(r_{d,t+1})] + (1-\psi)(1-\omega)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} \\ &\quad + \omega[r_{y,t+1} - E_t(r_{y,t+1})] + (1-\psi)\omega(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{y,t+1+j}. \end{aligned} \quad (\text{A.13})$$

Using the terms extracted from the VAR, Eq. (A.13) is expressed as:

$$\begin{aligned} e1' u_{t+1}^N &= (1-\omega)e3' u_{t+1}^N + \omega(e2' (I + \rho B(I - \rho B)^{-1}) u_{t+1}^N - b' \rho (I - \rho B)^{-1} u_{t+1}^N) \\ &\quad + (1-\psi)(1-\omega)e3' \rho B(I - \rho B)^{-1} u_{t+1}^N + (1-\psi)\omega b' \rho (I - \rho B)^{-1} u_{t+1}^N. \end{aligned} \quad (\text{A.14})$$

Since Eq. (A.14) holds for any  $u_{t+1}^N$ , finally,  $b$  should satisfy the following equation,

$$\begin{aligned} e1' &= (1-\omega)e3' + \omega(e2' (I + \rho B(I - \rho B)^{-1}) - b' \rho (I - \rho B)^{-1}) \\ &\quad + (1-\psi)(1-\omega)e3' \rho B(I - \rho B)^{-1} + (1-\psi)\omega b' \rho (I - \rho B)^{-1}, \end{aligned} \quad (\text{A.15})$$

where  $b$  is the solution of Eq. (A.15).

#### A.1.1. Alternative model 1: E-CCAPM

E-CCAPM is set by expressing the pricing kernel as a function of consumption growth by substituting out the wealth portfolio returns. The state variables consist of consumption and 4 variables ( $DIV$ ,  $DEF$ ,  $RREL$ ,  $TERM$ ). In other words,  $X_t^B = (\Delta c_t, DIV_t, DEF_t, RREL_t, TERM_t)$ .

By rearranging the pricing kernel to be expressed as a function of consumption growth, the pricing kernel for ECCAPM is:

$$m_{t+1}^B - E_t(m_{t+1}^B) = -\gamma[\Delta c_{t+1} - E_t(\Delta c_{t+1})] - (\gamma-1)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}. \quad (\text{A.16})$$

The VAR framework is:

$$X_{t+1}^B = E + F X_t^B + u_{t+1}^B. \quad (\text{A.17})$$

Each term in the pricing kernel is given by:

$$\Delta c_{t+1} - E_t(\Delta c_{t+1}) = e1' u_{t+1}^B, \quad (\text{A.18})$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} = e1' \rho F(I - \rho F)^{-1} u_{t+1}^B. \quad (\text{A.19})$$

### A.1.2. Alternative model 2: ICAPM

ICAPM is set by making two assumptions in our model economy. The first is that  $r_{c,t+1} = r_{d,t+1}$ , and the second is that state variables consist of market return and four variables ( $DIV$ ,  $DEF$ ,  $RREL$ ,  $TERM$ ). In other words,  $X_t^A = (r_{d,t}, DIV_t, DEF_t, RREL_t, TERM_t)$ .

The VAR framework is,

$$X_{t+1}^A = C + DX_t^A + u_{t+1}^A. \quad (A.20)$$

Each term in the pricing kernel is given by:

$$r_{d,t+1} - E_t(r_{d,t+1}) = e1'u_{t+1}^A, \quad (A.21)$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} = e1' \rho D(I - \rho D)^{-1} u_{t+1}^A. \quad (A.22)$$

### A.2. Case 2: True model without human capital

In the second case, we set a true stochastic discount factor is a function of only the market return. By log-linearizing the budget constraint around the steady state, and assuming the consumption–wealth ratio to be stationary, the pricing kernel under our model is:

$$\begin{aligned} m_{t+1}^N - E_t(m_{t+1}^N) &= -\gamma [r_{c,t+1} - E_t(r_{c,t+1})] - (\gamma - 1)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{c,t+1+j} \\ &= -\gamma(1 - \omega) [r_{d,t+1} - E_t(r_{d,t+1})] - (\gamma - 1)(1 - \omega)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} \\ &\quad - \gamma\omega [r_{y,t+1} - E_t(r_{y,t+1})] - (\gamma - 1)\omega(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{y,t+1+j}. \end{aligned} \quad (A.23)$$

ICAPM with no human capital implies  $\omega = 0$ :

$$m_{t+1}^N - E_t(m_{t+1}^N) = -\gamma [r_{d,t+1} - E_t(r_{d,t+1})] - (\gamma - 1)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j}. \quad (A.24)$$

Therefore, only the market return appears in a true stochastic discount factor. Under the VAR framework specified above, each term in the pricing kernel is given by:

$$r_{d,t+1} - E_t(r_{d,t+1}) = e3'u_{t+1}^N, \quad (A.25)$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} = e3' \rho B(I - \rho B)^{-1} u_{t+1}^N. \quad (A.26)$$

#### A.2.1. Alternative model 1: E-CCAPM

E-CCAPM is set by expressing the pricing kernel as a function of consumption growth by substituting out the wealth portfolio returns. The state variables consist of consumption and 4 variables ( $DIV$ ,  $DEF$ ,  $RREL$ ,  $TERM$ ). In other words,  $X_t^B = (\Delta c_t, DIV_t, DEF_t, RREL_t, TERM_t)$ .

By rearranging the pricing kernel to be expressed as a function of consumption growth, the pricing kernel for ECCAPM is:

$$m_{t+1}^B - E_t(m_{t+1}^B) = -\gamma [\Delta c_{t+1} - E_t(\Delta c_{t+1})] - (\gamma - 1)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}. \quad (A.27)$$

The VAR framework is:

$$X_{t+1}^B = E + FX_t^B + u_{t+1}^B. \quad (A.28)$$

Each term in the pricing kernel is given by:

$$\Delta c_{t+1} - E_t(\Delta c_{t+1}) = e1'u_{t+1}^B, \quad (A.29)$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} = e1' \rho F(I - \rho F)^{-1} u_{t+1}^B. \quad (A.30)$$

#### A.2.2. Alternative model 2: ICAPM

ICAPM is set by making two assumptions in our model economy. The first is that  $r_{c,t+1} = r_{d,t+1}$ , and the second is that state variables consist of market return and 4 variables ( $DIV$ ,  $DEF$ ,  $RREL$ ,  $TERM$ ). In other words,  $X_t^A = (r_{d,t}, DIV_t, DEF_t, RREL_t, TERM_t)$ .

The VAR framework is,

$$X_{t+1}^A = C + DX_t^A + u_{t+1}^A. \quad (A.31)$$

Each term in the pricing kernel is given by:

$$r_{d,t+1} - E_t(r_{d,t+1}) = e1'u_{t+1}^A, \quad (\text{A.32})$$

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{d,t+1+j} = e1' \rho D(I - \rho D)^{-1} u_{t+1}^A. \quad (\text{A.33})$$

### A.3. Maximum pricing errors

By following Hansen and Jagannathan (1997) and Campbell and Cochrane (2000), the maximum pricing error is given by ,

$$\max_{\theta} \frac{|E^Y(\theta) - E(\theta)|}{\sigma(\theta)} = \frac{\sigma(M - Y)}{E(M)}, \quad (\text{A.34})$$

where  $Y$  is false discount factor and  $M$  is the true discount factor. In our setting,  $M = M^N$  and  $Y = M^A$  or  $M^B$ .

### A.4. Simulation setup

1. Using empirical data consisting of  $X_t^e = (\Delta c_t, \Delta y_t, r_{d,t}, DIV_t, DEF_t, RREL_t, TERM_t)$ , we estimate  $X_{t+1}^e = A^e + B^e X_t^e + u_{t+1}^e$  and get  $u_{t+1}^e$ ,  $A^e$ ,  $B^e$ .
2. We bootstrap  $u_t^e$  and construct 1000 190-quarter simulated time series of  $X_t^{N,i}$  ( $i = 1, 2, 3, \dots, 1000$ ) by using  $A^e$ ,  $B^e$  and  $X_1^e$ .
3. We estimate the VAR model with the bootstrapped time series  $X_t^{N,i}$  ( $i = 1, 2, 3, \dots, 1000$ ) and get  $A^i$ ,  $B^i$ , and  $u_{t+1}^i$  ( $i = 1, 2, 3, \dots, 1000$ ).
4. We construct  $M_t^{N,i}$  using  $A^i$ ,  $B^i$ , and  $u_t^{N,i}$ .
5. We construct time series  $X_t^{A,i} = (r_{d,t}^i, DIV_t^i, DEF_t^i, RREL_t^i, TERM_t^i)$ , and  $X_t^{B,i} = (r_{d,t}^i, DIV_t^i, DEF_t^i, RREL_t^i, TERM_t^i)$  by taking a subset of  $X_t^{N,i}$ .
6. We estimate the VAR model ( $X_{t+1}^{A,i} = C^i + D^i X_t^{A,i} + u_{t+1}^{A,i}$ ) with the bootstrapped time series  $X_{t+1}^{A,i}$  and get  $C^i$ ,  $D^i$  and  $u_{t+1}^{A,i}$  ( $i = 1, 2, 3, \dots, 1000$ ).
7. We estimate the VAR model ( $X_{t+1}^{B,i} = E^i + F^i X_t^{B,i} + u_{t+1}^{B,i}$ ) with the bootstrapped time series  $X_{t+1}^{B,i}$  and get  $E^i$ ,  $F^i$  and  $u_{t+1}^{B,i}$  ( $i = 1, 2, 3, \dots, 1000$ ).
8. We construct  $M_t^{A,i}$  through 190-quarter lengths of simulation using  $C^i$ ,  $D^i$  and  $u_t^{A,i}$ .
9. We construct  $M_t^{B,i}$  through 190-quarter lengths of simulation using  $E^i$ ,  $F^i$  and  $u_t^{B,i}$ .
10. We calculate maximum pricing errors, given by  $\frac{\sigma(M_t^{N,i} - M_t^{A,i})}{E(M_t^{N,i})}$  and  $\frac{\sigma(M_t^{N,i} - M_t^{B,i})}{E(M_t^{N,i})}$ , for each  $i$ .
11. We take averages of the maximum pricing errors for ICAPM ( $M_t^A$ ) and ECCAPM ( $M_t^B$ ).

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