



Ultimate consumption risk and investment-based stock returns[☆]



Hankil Kang^a, Jangkoo Kang^b, Changjun Lee^{c,*}

^a Securities & Derivatives R&D Center, Korea Exchange, Busan, Republic of Korea

^b Graduate School of Finance & Accounting, College of Business, Korea Advanced Institute of Science and Technology (KAIST), Seoul, Republic of Korea

^c College of Business, Hankuk University of Foreign Studies, 107, Imun-ro, Dongdaemoon-Gu, Seoul, Republic of Korea

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ABSTRACT

Motivated by recent works documenting that the returns formed on real investment predict aggregate economic activities, we study whether the ultimate consumption model proposed by Parker and Julliard (2005) explains the cross-section of investment-based stock returns. We find that the ultimate consumption model with horizons from 3 to 4 years outperforms the contemporaneous consumption model. The linearized model's performance is better than that of the Fama-French three-factor model and comparable to that of the Chen-Roll-Ross model. The explanatory power of the ultimate consumption model arises from the close business-cycle relationship between the ultimate consumption growth and the investment-based returns.

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1. Introduction

Among the empirical asset-pricing models, the Fama and French (1993) three-factor model has been widely used for risk adjustment since it explains most CAPM-related anomalies (Fama & French, 1996). Recent empirical studies, however, document stock market anomalies which the Fama-French three-factor model cannot explain (see Hou, Xue, and Zhang (2015) and reference therein). One challenge is to understand the negative relationship between real investment (and asset growth) and subsequent stock returns. For example, Titman, Wei, and Xie (2004) report that firms with high capital investments earn low subsequent returns. Xing (2008) finds that firms with low investment growth rates have higher returns than those with high investment growth rates. In addition, Cooper, Gulen, and Schill (2008) show that the growth in total assets has strong predictive power for future stock returns.

Given that one central function of capital market is to price real investment accurately, examining whether corporate investment is fairly priced in the cross-section of stock returns is an important research question. The literature, however, has not thoroughly investigated the driving force behind the negative relationship between firm's investment and equity returns. A few exceptions include the work of Cooper and Priestley (2011), the four-factor model proposed by Hou et al. (2015), and the five-factor model of Fama and French (2015). Cooper and Priestley (2011) show that the five-factor model

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* Corresponding author.

E-mail addresses: hkangfree@gmail.com (H. Kang), jkkang@business.kaist.ac.kr (J. Kang), leechangjun@hufs.ac.kr (C. Lee).

of Chen, Roll, and Ross (1986) explains the return spread between low and high asset growth portfolios. Hou et al. (2015), and Fama and French (2015) suggest asset pricing models which explain a broad set of stock market anomalies including asset growth anomaly. Despite outstanding empirical performance, previously proposed empirical models have relatively many factors compared with the Fama-French three-factor model. Although one may include additional factors to explain anomalous stock return behavior, one should be cautious about models with many factors.

In this paper, we focus on the *one factor* consumption-based asset pricing models to overcome the shortcomings of the prior works. Although the consumption-based model has suffered from its poor empirical performance (Breedeen, Gibbons, & Litzenberger, 1989; Cochrane, 1996; Mankiw & Shapiro, 1986), researchers have paid considerable attention to the consumption-based models. One reason is that consumption-based models are general in that any factor model can be regarded as a specialization of consumption-based model documented by Cochrane (2005). In addition, Cochrane (2008, p. 267) expresses the importance of consumption-based models: “At some level, the consumption-based models must be right if economics is to have any hope of describing the stock markets.” Therefore, a promising research direction is to improve our understanding of consumption-based models rather than to propose alternative asset pricing models.

Among various consumption-based models, we are particularly interested in the ultimate consumption-based model proposed by Parker and Julliard (2005). Our choice of specific consumption-based model is motivated by recent empirical studies which document that the return spread between low and high investment portfolios predict aggregate economic activities including future consumption growth. Cooper and Priestley (2011) find that return spread between low and high asset growth contains information about industrial production growth and GDP growth. Wang (2013) reports that the return spread between low and high investment-to-asset portfolios is a powerful predictor for future GDP growth. Most importantly, Min, Kang, and Lee (2017) show that the zero-cost portfolios that take a long position in low-investment stocks and a short position on high-investment stocks predict future consumption growth. Note that the success of the ultimate consumption model of Parker and Julliard (2005) in explaining the cross-section of 25 size and book-to-market sorted portfolios arises from the predictive power of Fama-French factors for future consumption growth. Likewise, we expect the ultimate consumption model explain well the cross-section of asset growth portfolios given that return spread between low and high investment portfolios predict future consumption growth.

First, using the generalized method moment (GMM) cross-sectional estimator, we evaluate the empirical performance of the one-factor ultimate consumption model in explaining the 10 investment-sorted portfolios and 25 size-investment sorted portfolios. As documented in the previous literature, we find that the performance of contemporaneous consumption CAPM is disappointing in that its adjusted R^2 is near zero, and the estimated pricing error is economically large and statistically significant. In a sharp contrast, we observe the substantially improved performance of the consumption model when we increase the horizon S to measure the consumption growth. Focusing on the 10 investment-sorted portfolios, the largest cross-sectional R^2 of 0.671 occurs with horizon of four years ($S = 15$) for equal-weighted portfolios, and we observe the highest R^2 of 0.727 at $S = 9$ for value-weighted portfolios. In addition, the average difference between the sample equity returns and the model-implied returns has its quarterly absolute value under 1% and statistically insignificant when $S \geq 9$. Therefore, the remarkable increase in cross-sectional explanatory power in horizons from $S = 9$ to $S = 15$ indicates that the ultimate consumption risk with business-cycle horizon is closely connected to the cross-section of investment-sorted returns. In sum, the ultimate consumption model explains well the cross-section of investment-based returns.

Our result is inconsistent with the finding of Parker and Julliard (2005) who find the hump-shaped explanatory power with its peak at $S = 11$ using the 25 size and book-to-market sorted portfolios. Then, why does the best-performing horizon differ across test assets? One possible explanation is to consider the business-cycle consumption risk by Bandi and Tamoni (2017). They decompose the consumption growth into its subcomponents based on their persistence. In their model, the business-cycle components, with periodicity between 2 and 8 years, play an important role in explaining the cross-section of returns. Therefore, we do not restrict ourselves to define the best-performing horizon, which may differ from asset to asset, and focus on the “business-cycle” frequency consumption shocks as the potential driving force behind the success of the ultimate consumption model.

Second, we evaluate the ability of linearized ultimate consumption model to explain the lowest (highest) investment portfolio returns and the return spread between the two portfolios. We then compare the empirical performance of the ultimate consumption model with the two well-known asset pricing models, which are the Fama-French three-factor model and the Chen-Roll-Ross model. For ultimate consumption model, we find that the proportion of actual return spread explained by the expected return increases with the horizon S . When equal-weighted portfolios are employed, the ultimate consumption models with $S = 11$ and $S = 15$ capture 27% and 48% of the actual premium, while the Fama-French three-factor model explains 36% of the actual spread between the lowest and highest investment portfolios. Finally, the Chen-Roll-Ross model explains 67% of the actual return spread.

At a first glance, the ultimate consumption model performs worse than the competing asset pricing models. However, that is not the case for the following reasons. First, the ultimate consumption model better explains the actual return of the two extreme portfolios than any other models. When we use the 10 equal-weighted investment-sorted portfolios, the actual quarterly return of the lowest investment portfolio is 5.9% and it is 2.3% for the highest investment portfolio. We find that the expected returns of the ultimate consumption model with $S = 15$ are 4.3% for the lowest investment portfolio and 2.6% for the highest investment portfolio, respectively. In a sharp contrast, the Fama-French model desperately fails in that its expected returns for the lowest and highest investment portfolios are -3.9% and -5.1% . In the Chen-Roll-Ross model, the

expected returns are 2.5% and 0.1%, which are far from the actual returns. Second, the empirical performance of ultimate consumption model is striking in that the ultimate consumption model is single factor model. It may not be surprising that a model with five factors can explain the cross-section of 10 portfolios.

To understand the driving force behind the empirical success of the ultimate consumption model, we borrow an argument from Parker and Julliard (2005). That is, if the return spread between the lowest and highest investment portfolios predicts the future consumption, the covariance of the investment-based portfolios and the ultimate consumption would increase with the horizon, implying better performance of the ultimate consumption model. Indeed, in predictive regressions of the long-horizon consumption growth on the equal-weighted return spread, the regression coefficient is marginally significant for $S \geq 11$, and the regression R^2 is almost monotonically increasing with S attaining its maximum of 2.8% at $S = 15$. Therefore, the explanatory power of the ultimate consumption model can be justified by the close business-cycle relationship between the ultimate consumption growth and the investment-based returns.

Researchers often study return-on-equity sorted portfolios when they examine investment-sorted portfolios. For example, Hou et al. (2015) and Fama and French (2015) address both investment and profitability as important factors to explain the cross-section of stock returns. In this paper, however, we do not report the results from the return-on-equity sorted portfolios for the following reasons. First, previous studies have documented that the return spread sorted on profitability is not consistent with risk-based story (Stambaugh, Yu, & Yuan, 2012; Wang & Yu, 2013). Second, and more important, previous literature has documented that returns on ROE are not related with future GDP growth and future consumption growth (Min et al., 2017; Wang, 2013). This is very important given that the explanatory power of the ultimate consumption model in Parker and Julliard (2005) comes from the fact that current SMB and HML have forecasting power for future consumption growth. Consistent with our conjecture, we indeed find poor performance of ultimate consumption CAPM when return-on-equity portfolios are used as test assets. In the GMM estimation, negative risk aversion parameters (γ) and negative adjusted R^2 s are observed for some cases. In addition, the model-implied return of low return-on-equity portfolio is always higher than that of high return-on-equity portfolio, and this occurs because the estimated consumption beta of low return-on-equity portfolio is higher than that of high return-on-equity portfolio.

Our work adds to the extensive literature on the long-run risk in consumption-based asset pricing model (Bansal, Dittmar, & Lundblad, 2005; Bansal & Yaron, 2004; Hansen, Heaton, & Li, 2008; Parker & Julliard, 2005; Piazzesi & Schneider, 2006). In addition, our empirical work on the consumption-based explanations of the cross-section of stock returns is related with the studies with Campbell and Cochrane (1999), Lettau and Ludvigson (2001), Yogo (2006), Jagannathan and Wang (2007), and Savov (2011).

Another contribution of our study is that we find linkage between financial markets and the real economy. As Cochrane (2008) expresses, at some level, financial markets should be related to the macroeconomy since the risk premium in financial assets eventually reflects aggregate macroeconomic risks. Therefore, our empirical finding that the investment-based returns are related to aggregate consumption is quite intriguing.

The remainder of this paper is organized as follows. Section 2 describes the data and empirical methodology used in this paper. Section 3 reports our empirical evidence and following discussions. Section 4 concludes.

2. Data and empirical methodology

2.1. Data

For test assets, we use quarterly returns on 10 investment-sorted portfolios and 25 size-investment sorted portfolios.¹ Following Fama and French (2015), we use asset growth, the percentage growth of total asset (compustat item AT), as the investment of a firm. Cooper et al. (2008) first employ this measure to examine the relationship between corporate investment and the cross-section of stock returns. Cooper and Priestley (2011) and Hou et al. (2015) also use the same definition in measuring the investment of a firm. Since we extend our ultimate consumption model to the horizon $S = 15$ (four years), we discard the last 15 quarters from the available sample. As a result, the sample period for test assets is from 1963:Q3 to 2010:Q1. We use returns of 3-month Treasury bills as the risk-free rate of the quarter. Table 1 describes the returns of our test assets. In Panel A, we report the mean quarterly returns of the 10 investment-sorted portfolios. The mean returns show the clear pattern in that the expected returns decrease with investment. The pattern is stronger in the equal-weighted portfolios, and the difference between the lowest and the highest investment portfolios is 3.61% per quarter and it is statistically significant. Panel B shows the mean returns of 25 size-investment double-sorted portfolios. For each size quintile, we observe that the expected returns tend to decrease as investment increases. The negative association between investment and expected return is consistent with results in previous studies.

Table 2 shows summary statistics of our explanatory variables. We use real per capita consumption data from the National Income and Product Accounts (NIPA). We make the standard “end-of-period” assumption that consumption during period t takes place at the end of the period and our sample period for consumption is from 1963:Q2 to 2013:Q4. Following Parker and Julliard (2005), we use nondurable consumption expenditure as our consumption measure. As shown in Panel A of Table 2, the quarterly consumption growth rate is on average 0.35% and its volatility is 0.76% during our sample period.

¹ We obtain the portfolio returns from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 1

Summary Statistics of Test Assets. Panel A presents the mean quarterly percent returns of 10 investment-sorted portfolios. Panel B shows the mean quarterly returns of 25 size-investment double-sorted portfolios. $p(dif)$ shows the p-value of the test whether the mean lowest-minus-highest investment portfolio return is different from zero. The sample period is from 1963:Q3 to 2010:Q1 (183 quarters).

Panel A: 10 investment-sorted portfolios												
Investment	Low	2	3	4	5	6	7	8	9	High	dif	$p(dif)$
Equal-weighted	5.93	4.98	4.54	4.25	4.16	4.03	4.15	3.94	3.57	2.32	3.61	0.000
Value-weighted	3.61	3.54	3.11	2.97	2.89	2.82	3.06	2.71	2.94	2.21	1.40	0.003

Panel B: 25 size-investment double-sorted portfolios									
Equal-weighted	Investment	1	2	3	4	5	(1–5)	$p(dif)$	
Size	1	6.18	5.09	4.71	4.50	3.03	3.15	0.00	
	2	4.14	4.17	4.23	4.23	2.63	1.51	0.00	
	3	4.28	4.23	3.73	3.96	2.63	1.66	0.00	
	4	3.70	3.56	3.57	3.54	2.86	0.83	0.08	
	5	3.65	3.13	3.13	2.90	2.33	1.32	0.01	
Value-weighted	Investment	1	2	3	4	5	(1–5)	$p(dif)$	
Size	1	4.53	4.36	4.31	4.14	2.60	1.93	0.00	
	2	4.12	4.02	4.08	4.05	2.82	1.30	0.00	
	3	3.97	4.09	3.62	3.81	2.81	1.15	0.01	
	4	3.62	3.44	3.45	3.54	2.94	0.68	0.16	
	5	3.35	2.81	2.66	2.68	2.52	0.83	0.09	

Table 2

Summary Statistics of Explanatory Variables. Panel A shows consumption growth rate during various horizons. For example, “15” means consumptions growth rate with the horizon of four years. We use real per capita consumption data from the National Income and Product Accounts (NIPA), and our sample period of consumption is from 1963:Q2 to 2013:Q4. Panel B presents the mean and standard deviation of the Fama-French factors. The market factor (MKT) is the value-weighted excess returns of NYSE, AMEX, and NASDAQ ordinary common stocks. The size factor (SMB) is the return spread between small and big stock portfolios, and the book-to-market factor (HML) is the return difference between high and low book-to-market stock portfolios. Panel C displays the summary statistics of the Chen-Roll-Ross macroeconomic factors. Marginal production (MP) is the log growth of the index of industry production from the Federal Reserve Bank of St. Louis. Unexpected inflation (UI) is defined as $UI_t = I_t - E[I_t|t-1]$, and change of expected inflation (DEI) is $DEI_t = E[I_{t+1}|t] - E[I_t|t-1]$. Inflation is measured as the log difference of consumer price index from the Federal Reserve Bank of St. Louis. The expected inflation is calculated as $E[I_t|t-1] = r_{ft} - E[RHO_t|t-1]$, where r_{ft} is one-month Treasury bill rate and $RHO_t = r_{ft} - I_t$ is the *ex post* real return on Treasury bills. RHO_t is modeled as ARIMA (0,1,1) process as $RHO_t - RHO_{t-1} = u_t + \theta u_{t-1}$ and the *ex ante* real rate, $E[RHO_t|t-1]$, is calculated as $E[RHO_t|t-1] = (r_{ft-1} - I_{t-1}) + u_t + \theta u_{t-1}$. Term spread (UTS) is the difference between the yields of a 10-year and a 1-year government bonds. Default spread (UPR) is the difference between the yields on Moody's Baa and Aaa corporate bonds. The sample period of the Fama-French factors and the Chen-Roll-Ross factors is from 1963:Q3 to 2010:Q1.

Panel A: Consumption growth									
S	0	1	3	5	7	9	11	13	15
MEAN	0.35	0.70	1.41	2.10	2.77	3.43	4.05	4.67	5.29
STD	0.76	1.18	1.88	2.44	2.89	3.21	3.44	3.64	3.83
Panel B: Fama-French factors									
			MKT		SMB			HML	
MEAN			1.30		0.77			1.25	
STD			8.61		5.70			5.91	
Panel C: Chen-Roll-Ross factors									
		MP	UI		DEI		UTS		UPR
MEAN	0.64		0.00		0.00		2.70		3.12
STD	1.75		0.50		0.16		3.43		1.42

We compare the empirical performance of ultimate consumption model with the Fama-French three-factor model and the Chen-Roll-Ross model. Since [Fama and French \(2015\)](#) and [Hou et al. \(2015\)](#) use the investment-based excess returns as one of the risk factors, we do not include those models here, because our purpose is to explain the investment-based returns themselves. We obtain the Fama-French factors from Kenneth French's website, and the detailed construction of the factors is described in [Fama and French \(1993\)](#). The market factor (MKT) is the value-weighted excess returns of NYSE, AMEX, and NASDAQ ordinary common stocks. The size factor (SMB) is the return spread between small and big stock portfolios, and the book-to-market factor (HML) is the return difference between high and low book-to-market stock portfolios. During our sample period, the mean quarterly returns of MKT, SMB, and HML are 1.30% (t-statistic of 2.03), 0.77% (t-statistic of 1.83), and 1.25% (t-statistic of 2.85), respectively. We compare the empirical performance of our model with that of the Chen-Roll-Ross model because [Cooper and Priestley \(2011\)](#) show that the investment-based anomalies are well explained by the loadings on the Chen-Roll-Ross factors. We follow [Cooper and Priestley \(2011\)](#) to construct the Chen-Roll-Ross macroeconomic factors. Specifically, we define marginal production (MP) as the log growth of the index of industry production from

the Federal Reserve Bank of St. Louis. Unexpected inflation (UI) is defined as $UI_t = I_t - E[I_t|t-1]$, and change of expected inflation (DEI) is $DEI_t = E[I_{t+1}|t] - E[I_t|t-1]$. Inflation is measured as the log difference of consumer price index from the Federal Reserve Bank of St. Louis. The expected inflation is calculated as $E[I_t|t-1] = r_{ft} - E[RHO_t|t-1]$, where r_{ft} is one-month Treasury bill rate, and $RHO_t = r_{ft} - I_t$ is the *ex post* real return on Treasury bills. Following Fama and Gibbons (1984), RHO_t is modeled as ARIMA(0,1,1) process as $RHO_t - RHO_{t-1} = u_t + \theta u_{t-1}$, and the *ex ante* real rate, $E[RHO_t|t-1]$, is calculated as $E[RHO_t|t-1] = (r_{ft-1} - I_{t-1}) + u_t + \theta u_{t-1}$. Term spread (UTS) is the difference between the yields of a 10-year and a 1-year government bonds. Default spread (UPR) is the difference between the yields on Moody's Baa and Aaa corporate bonds. Panel C of Table 2 displays summary statistics of the Chen-Roll-Ross factors. Although we report the summary statistics of original macroeconomic factors, we use mimicking portfolios for our asset pricing tests. We follow Cooper and Priestley (2011) to construct mimicking portfolios of the factors from 10 equal-weighted size portfolios, 10 equal-weighted book-to-market portfolios, 10 value-weighted momentum portfolios, and 10 equal-weighted investment portfolios.

2.2. Empirical methodology

2.2.1. The ultimate consumption model

To introduce the ultimate consumption model in Parker and Julliard (2005), we start from contemporaneous consumption-based model. The model implies the Euler equation as follows:

$$E_t \left[\delta \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^e \right] = 0 \quad (1)$$

where C_t is the consumption expenditure at time t , $u(\cdot)$ denotes the utility function, δ is the time-discount factor, and R_{t+1}^e is the excess return of any asset. The stochastic discount factor (SDF) in this model is defined as $m_{t+1} = \delta \frac{u'(C_{t+1})}{u'(C_t)}$, and the expected excess return is $E[R_{t+1}^e] = -\frac{Cov(R_{t+1}^e, m_{t+1})}{E(m_{t+1})}$.

Parker and Julliard (2005) exploit the consumption Euler equation for the risk-free rate from time $t+1$ to $t+1+S$ to yield the following relation:

$$u'(C_{t+1}) = \delta E_{t+1} [R_{t+1,t+1+S}^f u'(C_{t+1+S})] \quad (2)$$

When we define $m_{t+1}^S = R_{t+1,t+1+S}^f u'(C_{t+1+S})/u'(C_t)$ and substitute Eq. (2) into Eq. (1), we end up with the following relation of expected excess return and consumption growth to the far future.

$$E[R_{t+1}^e] = -\frac{Cov(R_{t+1}^e, m_{t+1}^S)}{E(m_{t+1}^S)} \quad (3)$$

Following Parker and Julliard (2005), we name $-Cov(R_{t+1}^e, m_{t+1}^S)$ as the *ultimate* consumption risk, and we assume the power utility function $u(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma}$.

Using the first-order log linearization of Lettau and Ludvigson (2001), we examine the performance of linearized consumption CAPM. With this approximation, the linearized version of stochastic discount factor has the following form:

$$m_{t+1}^S = R_{t+1,t+1+S}^f - \gamma_S R_{t+1,t+1+S}^f \Delta c_{t+1+S} \quad (4)$$

where $\Delta c_{t+1+S} = \ln \left(\frac{C_{t+1+S}}{C_t} \right)$. If we further assume that $R_{t+1,t+1+S}^f$ is constant over time, the ultimate consumption model can be viewed as a one-factor model with the ultimate consumption growth as the unique factor.

2.2.2. Estimation methodology

Following Parker and Julliard (2005) and Yogo (2006), we use a slightly modified version of moment restrictions implied by Eq. (3). Taking this approach has a number of benefits. First, it allows us to compare our results directly to those of Parker and Julliard (2005). Second, it shows the model's performance in two dimensions: (1) whether it explains the overall level of equity premium and (2) whether it explains the cross-section of the test assets. With the power utility function, the stochastic discount factor is $m_t^S = R_{t,t+S}^f \left(\frac{C_{t+S}}{C_{t-1}} \right)^{-\gamma}$. The moment condition for the GMM estimation is expressed as follows:

$$E[g(R_t^e, C_{t+S}, C_{t-1}, \mu_S, \gamma_S, \alpha_S)] = \begin{bmatrix} R_t^e - \alpha_S 1_N + \frac{(m_t^S - \mu_S) R_t^e}{\mu_S} \\ m_t^S - \mu_S \end{bmatrix} \quad (5)$$

where α_S captures the average difference of the sample equity returns and the model-implied returns. The last moment allows us to compare the pricing models with different S s under a similar criterion.

We estimate the model above with two types of GMM. First, the GMM with a prespecified weighting matrix uses a diagonal matrix that has ones in the diagonals except for the last element and a very large weight for the last moment. Under this method, the GMM prices the exact portfolios we use in the test, not the combinations of the portfolios. To examine the

model's performance, we use the distance measure in Jagannathan and Wang (1996) and Hansen and Jagannathan (1997), which is called the Hansen-Jagannathan distance. We follow the appendix of Parker and Julliard (2005) to evaluate this measure. Second, we also report the efficient GMM which we re-estimate the weighting matrix until convergence. In this case, we use Hansen (1982)'s J -test to evaluate the model's performance.

To test the linearized model's performance, we use two approaches. First, we adopt the standard cross-sectional regression of Fama and MacBeth (1973). We examine whether the cross-sectional coefficient on the ultimate consumption beta is positive and significant when we increase the horizon S to measure the consumption growth. Second, we use the methodology used in Liu and Zhang (2008) and Cooper and Priestley (2011). More specifically, we construct the risk premium of the linear factor pricing models from a broad collection of test assets. After we build the risk premium of the factor models, we calculate the expected return of a portfolio P as follows:

$$E[R_P] = \widehat{\lambda}_F' \widehat{\beta}_F \quad (6)$$

where $\widehat{\beta}_F$ is the vector of factor loadings on the asset pricing factors from the time-series regression, and $\widehat{\lambda}_F$ is the vector of risk premiums. By doing this, we estimate the model-implied spread of low and high investment portfolios. We examine how the actual spread can be explained by the factor models.

3. Empirical evidence

3.1. Performance of the ultimate consumption model

Tables 3 and 4 illustrate the GMM estimation results with 10 equal and value weighted investment portfolios, respectively. Panel A shows the results when we use a prespecified weighting matrix, and Panel B presents the results with the efficient GMM. We evaluate cross-sectional R^2 as follows:

$$R^2 = 1 - \frac{\text{Var}(E_T[R_i^e] - \widehat{R}_i^e)}{\text{Var}(E_T[R_i^e])} \quad (7)$$

where $E_T[R_i^e] = \frac{1}{T} \sum_{t=1}^T R_{i,t}^e$ and $\widehat{R}_i^e = \widehat{\alpha}_S - \frac{E_T \left[\left(\widehat{m}_t^S - \widehat{\mu}_S \right) R_{i,t}^e \right]}{\widehat{\mu}_S}$. We report estimated values of α_S and γ_S with their GMM standard errors in parentheses. In Panels A and B, we show the Hansen-Jagannathan distance and the Hansen J -statistic with their p -values in square brackets.

As documented in the previous literature, the performance of contemporaneous consumption CAPM with $S = 0$ is disappointing for the following two reasons. First, its R^2 is near zero or even negative in the efficient GMM, which means that contemporaneous consumption CAPM does not help explain the cross-section of excess returns of investment-based portfolios. Second, the term α_0 has the value near 2% and statistically significant. This is close to the cross-sectional average of excess returns, implying the model's poor performance in capturing the average level of equity premium.

We observe the substantially improved performance of the model when we increase the horizon S to measure the consumption growth. First, in Table 3, the cross-sectional R^2 increases to 0.671 when $S = 15$. In Table 4, we observe the highest R^2 when $S = 9$ with the highest value of 0.727. Although these results do not exactly replicate those of Parker and Julliard (2005) with 25 size and book-to-market sorted portfolios, the remarkable increase in cross-sectional explanatory power in horizons from $S = 9$ to $S = 15$ shows that the ultimate consumption risk with business-cycle horizon is closely connected to the cross-section of investment-sorted returns. Second, when S is greater than 5 quarters, α_S , which captures the average difference between the sample equity returns and the model-implied returns, has its quarterly absolute value under 1% and statistically insignificant. Compared to the contemporaneous consumption CCAPM, this shows that the ultimate consumption risk explains the overall level of equity premium.

The Hansen-Jagannathan distance in Panel A of Tables 3 and 4 rejects the consumption CAPM for all values of S when we use prespecified weighting matrix for GMM estimation. In Panel B of Table 3, the model is not rejected in 5% significance level only when $S = 9$. In Panel B of Table 4, the ultimate consumption CAPM is not rejected in 10% significance level when $S \geq 3$. Overall, although we mainly observe rejections of models, the Hansen-Jagannathan distance and the J -statistic have decreasing pattern when we increase S , which implies the superior performance of the ultimate model.

We also perform the same GMM estimation with 25 equal and value weighted size-investment portfolios in Tables A1 and A2 of the Appendix. The patterns of the results repeat those in Tables 3 and 4. Focusing on the results from a prespecified weighting matrix, the ultimate consumption model attains the highest R^2 when $S = 15$ when 25 equal-weighted portfolios are used.

In sum, the ultimate consumption model performs better than the contemporaneous consumption model in explaining the cross-section of investment-based returns. Given the striking empirical performance, we further investigate the performance of the linearized model with simple regression approaches.

Table 3

GMM Estimation Results – Equal-Weighted Portfolios. This table displays the GMM estimation results of the ultimate consumption models from Eqs. (4) and (5) with 10 equal-weighted investment-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the parentheses are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen–Jagannathan distance and its p-value is calculated from 10,000 simulations. The column J shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2010:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
<i>S</i>	R^2	α	γ	HJ	R^2	α	γ	<i>J</i>
0	0.000	0.027 (0.008)	5.002 (82.881)	0.383 [0.001]	−0.033	0.019 (0.007)	57.764 (51.205)	40.910 [0.000]
1	0.003	0.028 (0.01)	0.792 (35.722)	0.384 [0.000]	−0.071	0.014 (0.009)	33.898 (19.683)	42.066 [0.000]
3	0.059	0.016 (0.015)	19.038 (27.776)	0.342 [0.000]	−0.012	0.012 (0.011)	30.733 (15.665)	30.982 [0.000]
5	0.164	−0.006 (0.015)	30.282 (13.337)	0.279 [0.000]	0.145	0.021 (0.01)	32.012 (9.556)	22.021 [0.005]
7	0.125	0.003 (0.013)	20.432 (11.971)	0.323 [0.000]	0.053	0.022 (0.01)	24.146 (8.728)	24.875 [0.002]
9	0.478	−0.009 (0.013)	40.591 (12.871)	0.198 [0.000]	0.448	0.027 (0.008)	33.248 (8.432)	15.380 [0.052]
11	0.498	−0.005 (0.012)	42.078 (16.840)	0.214 [0.000]	0.438	0.022 (0.008)	34.600 (9.037)	15.902 [0.044]
13	0.535	0.003 (0.009)	42.476 (18.151)	0.204 [0.000]	0.496	0.021 (0.007)	42.425 (10.166)	18.904 [0.015]
15	0.671	0.006 (0.011)	48.951 (22.590)	0.197 [0.000]	0.650	0.025 (0.007)	41.244 (10.403)	21.156 [0.007]

Table 4

GMM Estimation Results – Value-Weighted Portfolios. This table displays the GMM estimation results of the ultimate consumption models from Eqs. (4) and (5) with 10 value-weighted investment-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the parentheses are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen–Jagannathan distance and its p-value is calculated from 10,000 simulations. The column J shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2010:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
<i>S</i>	R^2	α	γ	HJ	R^2	α	γ	<i>J</i>
0	0.052	0.022 (0.006)	−49.824 (53.022)	0.146 [0.008]	−0.117	0.016 (0.006)	42.098 (32.823)	18.919 [0.015]
1	0.106	0.024 (0.007)	−28.772 (32.389)	0.139 [0.000]	−0.342	0.013 (0.007)	23.558 (19.549)	17.849 [0.022]
3	0.002	0.018 (0.008)	−3.901 (25.636)	0.129 [0.000]	−0.150	0.009 (0.009)	24.655 (14.017)	13.073 [0.109]
5	0.023	0.022 (0.007)	−12.174 (19.667)	0.149 [0.000]	−0.137	0.008 (0.007)	20.308 (8.654)	12.334 [0.137]
7	0.287	−0.002 (0.009)	22.849 (11.448)	0.065 [0.000]	0.285	0.007 (0.007)	21.724 (7.602)	9.322 [0.316]
9	0.727	0.003 (0.009)	30.941 (10.425)	0.051 [0.000]	0.662	0.010 (0.006)	24.129 (7.782)	6.617 [0.578]
11	0.615	0.004 (0.009)	30.983 (13.166)	0.073 [0.000]	0.510	0.009 (0.006)	20.343 (8.344)	9.311 [0.317]
13	0.437	0.005 (0.009)	31.283 (19.237)	0.068 [0.000]	0.404	0.006 (0.006)	24.614 (11.982)	7.437 [0.490]
15	0.282	0.007 (0.009)	29.304 (17.455)	0.083 [0.000]	0.245	0.007 (0.006)	21.953 (11.803)	10.027 [0.263]

3.2. Performance of the linearized model

In this subsection, we examine the linearized ultimate consumption model's performance to price the investment-based portfolios and compare it to that of the other well-known asset pricing models.

We first conduct the Fama-MacBeth regressions for the linearized model. First, we perform the time-series regressions for the ultimate consumption models as follows:

$$R_t^{ei} = \alpha^i + \beta_{\Delta c_{t+S}}^i \Delta c_{t+S} + \eta_t^i \quad \text{for } i = 1, 2, \dots, N \quad (8)$$

In the second-pass regressions, we use the betas from the time-series regression and perform cross-sectional regressions as follows:

$$E[R_t^{ei}] = \lambda_0 + \lambda_S \beta_{\Delta c_{t+S}}^i + \epsilon_S^i \quad (9)$$

In Table 5, we report the cross-sectional coefficients, λ_S , with their Fama-MacBeth t -statistics, the adjusted t -statistics in Shanken (1992), and the cross-sectional R^2 s. Panels A, B, C, and D exhibit the results when the test assets are 10 equal and value weighted investment-based portfolios and 25 equal and value weighted size-investment portfolios.

The results in Table 5 show that the ultimate consumption models perform better than the contemporaneous consumption model. Although we cannot firmly argue that the higher cross-sectional R^2 in cross-sectional regression is a robust measure to examine the model's performance following Lewellen, Nagel, and Shanken (2010), the cross-sectional R^2 attains its maximum at $S = 11$ in Panels C and D. The size of the risk premium of consumption beta increases with the horizon S . The risk premiums are significant when $S = 11$ in Panels A, C, and D based on the Fama-MacBeth t -statistics. However, the Shanken (1992) t -statistics are not significant in any cases, which implies that the linearized model's performance is not perfectly successful in explaining the cross-section of returns. This result is consistent with the finding of Lettau and Ludvigson (2001) who document that the Shanken correction to the t -statistics is large when we use macroeconomic factors as explanatory variables. Overall, the results from the classical Fama-MacBeth regression provide evidence that the ultimate consumption model also outperforms the contemporaneous consumption model in the log-linearized version.

3.3. Expected versus realized returns

We now focus on the ability of asset pricing models to explain the spread between the low and high investment portfolios. First, we construct the risk premiums from the Fama-MacBeth regressions using 40 test assets. Following Cooper and Priestley (2011), the 40 test assets include 10 equal-weighted size, 10 equal-weighted book-to-market, 10 value-weighted momentum, and 10 equal-weighted investment portfolios. The returns of test assets are from the Kenneth French's website. The factor pricing models include the ultimate consumption model, the Fama-French three-factor model, and the Chen-Roll-Ross model. In estimating the risk premiums, we assume that the risk premium is constant over time, and perform the Fama-MacBeth full sample regressions following Griffin, Ji, and Martin (2003).

Table 6 shows the estimated risk premiums from the 40 portfolios. Panels A, B, and C display the results for the ultimate consumption model, the Fama-French three-factor model, and the Chen-Roll-Ross model, respectively. We report the

Table 5

Cross-sectional regression results of Linearized Ultimate Consumption Models. This table displays the results of the following cross-sectional regressions. $E[R_t^{ei}] = \lambda_0 + \lambda_S \beta_{\Delta c_{t+S}}^i + \epsilon_S^i$, where $\beta_{\Delta c_{t+S}}^i$ is the loading from the time-series regressions $R_t^{ei} = \alpha^i + \beta_{\Delta c_{t+S}}^i \Delta c_{t+S} + \eta_t^i$. We report the estimated coefficients λ_S , the Fama-MacBeth t -statistic in rows "FM t ", the Shanken t -statistics in rows "SH t ", and the cross-sectional R^2 s. Panels A and B show the results with 10 equal and value weighted investment portfolios, respectively. Panels C and D show the results with 25 equal and value weighted size-investment portfolios, respectively. The sample period is from 1963:Q3 to 2010:Q1.

S	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
<i>Panel A: 10 equal-weighted investment-sorted portfolios</i>																
coeff	0.003	0.002	0.006	0.008	0.006	0.010	0.013	0.015	0.030	0.032	0.041	0.055	0.054	0.058	0.070	0.073
FM t	1.206	0.451	0.958	1.081	0.726	0.926	1.090	0.987	1.587	1.532	1.680	2.086	2.024	2.359	3.703	4.549
SH t	1.017	0.439	0.843	0.955	0.685	0.829	0.928	0.838	1.037	1.004	0.978	1.021	1.036	1.186	1.664	2.036
R^2	0.080	0.011	0.049	0.063	0.028	0.045	0.061	0.049	0.118	0.108	0.120	0.150	0.122	0.096	0.138	0.151
<i>Panel B: 10 value-weighted investment-sorted portfolios</i>																
coeff	0.000	-0.002	-0.003	-0.002	-0.003	-0.004	0.000	0.006	0.010	0.014	0.017	0.019	0.018	0.019	0.019	0.015
FM t	0.089	-0.747	-0.798	-0.410	-0.498	-0.487	0.041	0.546	0.998	1.289	1.476	1.672	1.496	1.628	1.595	1.093
SH t	0.089	-0.722	-0.762	-0.404	-0.488	-0.476	0.041	0.531	0.924	1.149	1.274	1.432	1.312	1.415	1.397	1.013
R^2	0.001	0.078	0.081	0.021	0.031	0.027	0.000	0.026	0.057	0.083	0.092	0.090	0.069	0.069	0.062	0.033
<i>Panel C: 25 equal-weighted size-investment sorted portfolios</i>																
coeff	0.005	0.006	0.010	0.013	0.015	0.020	0.025	0.030	0.041	0.045	0.053	0.063	0.071	0.086	0.092	0.092
FM t	2.176	1.712	1.638	1.903	1.801	1.992	2.179	2.243	2.547	2.571	2.668	2.832	2.861	3.092	3.419	3.476
SH t	1.507	1.338	1.250	1.387	1.359	1.383	1.428	1.389	1.361	1.346	1.293	1.246	1.188	1.122	1.219	1.278
R^2	0.341	0.211	0.147	0.257	0.250	0.307	0.384	0.415	0.489	0.506	0.536	0.542	0.513	0.470	0.441	0.421
<i>Panel D: 25 value-weighted size-investment sorted portfolios</i>																
coeff	0.002	0.003	0.003	0.005	0.006	0.009	0.015	0.020	0.030	0.033	0.039	0.047	0.053	0.066	0.068	0.067
FM t	1.149	0.800	0.509	0.765	0.797	1.004	1.328	1.548	1.966	2.012	2.151	2.359	2.412	2.737	3.048	3.042
SH t	1.036	0.761	0.495	0.723	0.752	0.901	1.104	1.186	1.285	1.288	1.282	1.289	1.254	1.244	1.391	1.455
R^2	0.129	0.063	0.020	0.053	0.063	0.096	0.172	0.235	0.319	0.343	0.387	0.397	0.375	0.363	0.347	0.326

Table 6

Estimates of Risk Premiums. We estimate the risk premiums of the risk factors in three asset pricing models: the consumption CAPM, the Fama-French three-factor model, the Chen-Roll-Ross model. We estimate the factor loadings from time-series regressions using 40 test assets in the first stage. In the second stage, we use those loadings to estimate the risk premiums. For the test assets, we use 10 equal-weighted size portfolios, 10 equal-weighted book-to-market portfolios, 10 value-weighted momentum portfolios, and 10 equal-weighted investment portfolios. We report the estimated risk premium coefficients from the second stage. In parentheses, we display the Fama-MacBeth t -statistics.

S	λ_0	Δc					R^2
<i>Panel A: Ultimate consumption CAPM</i>							
0	0.017 (1.840)	0.008 (1.623)					0.199
3	0.017 (1.955)	0.014 (1.674)					0.178
7	0.019 (2.267)	0.018 (1.581)					0.155
11	0.016 (1.594)	0.037 (2.063)					0.358
15	0.011 (0.905)	0.057 (1.961)					0.485
<i>Panel B: Fama-French three-factor model</i>							
	λ_0	MKT	SMB	HML			R^2
	0.082 (5.130)	−0.056 (−3.225)	0.011 (2.101)	0.013 (2.310)			0.654
<i>Panel C: Chen-Roll-Ross model</i>							
	λ_0	MP	UI	DEI	UTS	UPR	R^2
	0.029 (2.718)	0.037 (4.207)	0.003 (0.697)	0.000 (0.186)	0.041 (2.869)	−0.017 (−3.425)	0.898

cross-sectional loadings ($\hat{\lambda}_S$), the pricing error (λ_0), the Fama-MacBeth t -statistics in parentheses, and the cross-sectional R^2 . Although our goal is not to explain the cross-section of 40 portfolios, several features are worth highlighting. First, in Panel A, the estimated λ_0 is insignificant except for the case of $S = 7$, meaning that the ultimate consumption model does well in explaining the average level of stock portfolios. Second, even though the risk premiums on SMB and HML are positively significant, the Fama-French three-factor model does not explain the cross-section of 40 portfolios given that the estimated pricing error is 8.2% per quarter and statistically significant. Third, the risk premiums of Chen-Roll-Ross factors show a similar pattern as in [Cooper and Priestley \(2011\)](#). Again, the estimated intercept is 2.9% per quarter and it is statistically significant indicating that the Chen-Roll-Ross model cannot capture the zero-beta rate.

Having estimated the risk premiums of the asset pricing factors, we now study whether the competing asset pricing models can explain the lowest (highest) investment portfolio returns and the return spread between the two portfolios. We estimate the factor loadings of each portfolio from the time-series regressions, and multiply the risk premium estimates in [Table 6](#) to obtain the expected returns as in Eq. (6). [Table 7](#) shows the results with the equal-weighted portfolios. We report the factor loadings, actual returns, explained returns, and the proportion of explained and actual returns for the lowest and highest decile portfolios, and their differences.

In Panel A, we report the actual and model implied returns of the lowest (highest) investment decile portfolio. For the sake of brevity, we focus on the contemporaneous consumption model with $S = 0$, and the ultimate consumption model of horizon $S = 11, 15$ since the omitted horizons exhibit a monotonic pattern. We find that although the factor loadings of the lowest and highest portfolios on the consumption growth are not significantly different in every case, the loadings on the lowest decile are always greater than the highest decile. More importantly, the proportion of actual return spread explained by the expected return increases with the horizon S . The ultimate consumption models with $S = 11$ and $S = 15$ capture 27% and 48% of the actual premium, respectively.

In Panel B, we display the performance of the Fama-French three-factor model. The negative value of market beta, and positive values of the SMB and HML betas in low-high portfolio help explain the realized spread since the estimated risk premium is negative for MKT, and positive for both SMB and HML in [Table 6](#). As a result, the model explains 36% of the actual spread. However, the model fails to capture the returns of the extreme portfolios. The expected returns for the lowest and highest investment portfolios are −5.1% and −3.9%, while the actual returns for the portfolios are 5.9%, and 2.3%, respectively. This failure comes from the big negative premium of the market beta in [Table 6](#).

In Panel C, we study the Chen-Roll-Ross model. Combined with their signs of risk premiums in [Table 6](#), all but DEI loadings help explain the spread. The wrong direction of DEI is negligible because its estimated risk premium in [Table 6](#) is near zero. The Chen-Roll-Ross model explains 67% of the actual return spread. When we look at the extreme portfolios, however, the expected returns are 2.5% and 0.1%, which are far from the actual returns.

[Table 8](#) shows the results when 10 value-weighted investment sorted portfolios are employed. As we have found from equal-weighted portfolios, the ultimate consumption model better explains the actual return of the extreme portfolios than

Table 7

Average and Expected Return Spreads from Equal-Weighted Portfolios. This table displays the loadings on the mimicking portfolios of the asset pricing factors for the lowest and highest equal-weighted investment decile. $E[R_F^{25}] = \lambda_F' \beta$ where β is the vector of factor loadings from the time-series regression, and λ_F is the vector of risk premiums estimated in Table 6. Under the name of variables, we report the λ_F estimate, its t-statistics in parentheses. “Actual” column is the sample mean of the portfolio. “Explained” column displays the model-implied returns. “Proportion” column shows the ratio of the model-implied spread and actual spread. “dif” rows exhibit the difference of the estimates, or the difference of expected returns. In rows “p(dif)”, we report p-values with the null of equal coefficient in square brackets. Panels A, B, and C show the results from the ultimate consumption CAPM, Fama-French three-factor model, and Chen-Roll-Ross model, respectively.

Panel A: the ultimate consumption CAPM												
S	0				11				15			
	Δc	Actual	Explained	Proportion	Δc	Actual	Explained	Proportion	Δc	Actual	Explained	Proportion
Low	3.479 (2.023)	0.059	0.028		0.884 (2.621)	0.059	0.033		0.762 (2.533)	0.059	0.043	
High	3.300 (2.272)	0.023	0.027		0.623 (1.970)	0.023	0.023		0.457 (1.688)	0.023	0.026	
dif	0.179	0.036	0.001	0.040	0.260	0.036	0.010	0.267	0.305	0.036	0.017	0.481
p(dif)	[0.468]				[0.287]				[0.226]			
Panel B: the Fama-French three-factor model												
	MKT	SMB	HML						Actual	Explained	Proportion	
Low	1.102 (11.184)	1.694 (11.571)	0.363 (2.310)						0.059	−0.039		
High	1.147 (20.283)	1.363 (15.649)	−0.144 (−1.672)						0.023	−0.051		
dif	−0.046	0.331	0.507						0.036	0.013	0.356	
p(dif)	[0.657]	[0.027]	[0.003]									
Panel C: the Chen-Roll-Ross model												
	MP	UI	DEI	UTS	UPR				Actual	Explained	Proportion	
Low	0.727 (13.297)	0.046 (0.381)	4.529 (5.698)	0.541 (13.144)	1.463 (20.125)				0.059	0.025		
High	0.344 (6.721)	−1.206 (−10.164)	5.578 (14.252)	0.464 (21.059)	1.658 (24.093)				0.023	0.001		
dif	0.383	1.252	−1.049	0.077	−0.195				0.036	0.024	0.673	
p(dif)	[0.000]	[0.000]	[0.881]	[0.050]	[0.974]							

any other models in Table 8. The actual return of the lowest investment portfolio is 3.6% and it is 2.2% for the highest investment portfolio. We find that the expected returns of the ultimate consumption model with $S = 15$ are 2.3% for the lowest investment portfolio and 1.5% for the highest investment portfolio, respectively. In a sharp contrast, the Fama-French model desperately fails in that its explained returns for the lowest and highest investment portfolios are −5.3% and −6.6%. For the Chen-Roll-Ross model, the expected returns of low and high investment portfolios are 1.0% and −0.7%, which are far from the actual returns.

In sum, the ultimate consumption CAPM better explain the cross-section of investment sorted portfolios. The empirical performance of ultimate consumption model is striking in that the performance of ultimate consumption risk as a linear one-factor model is better than the Fama-French three-factor model, and comparable with the Chen-Roll-Ross five-factor model. It may not be surprising that a model with five factors can explain the cross-section of 10 portfolios. Therefore, one should be cautious about models with many factors.

3.4. Business-cycle frequency relation between consumption and investment-based returns

In this subsection, we examine the source behind the success of the ultimate consumption model. Parker and Julliard (2005) document that the explanatory power of the ultimate consumption model in pricing the 25 size and book-to-market portfolios arises from the predictive power of SMB and HML factors for future consumption. If the investment-based factor forecasts the future consumption, the covariance of the investment-based portfolios and the ultimate consumption would increase with the horizon, implying the better performance of the ultimate consumption model.

In Model 1 of Panel A (B) of Table 9 shows predictive regression results when the difference of the lowest and highest equal (value) weighted investment decile portfolios are used. For each S , we report the time-series regression coefficient in the first row, t-statistics in the second row, and R^2 . Focusing on Panel A, when $S = 1$, the coefficient is 0.007 with the regression R^2 of 0.2%. The regression coefficient is significant at the 10% significance level when $S \geq 11$, and it is significant at the 5% significant level when $S \geq 14$. The regression is almost monotonically increasing with S , with its maximum of 2.8% at $S = 15$.

When the elasticity of intertemporal substitution is equal to one, the ultimate consumption growth can be decomposed into two parts: the contemporaneous consumption growth and the Epstein-Zin long-run innovation. Therefore, Cochrane

Table 8

Average and Expected Return Spreads from Value-Weighted Portfolios. This table displays the loadings on the mimicking portfolios of the asset pricing factors for the lowest and highest value-weighted investment decile. $E[R_t^{dec}] = \lambda_F^T \beta$ where β is the vector of factor loadings from the time-series regression, and λ_F is the vector of risk premiums estimated in Table 6. Under the name of variables, we report the λ_F estimate, its t-statistics in parentheses. “Actual” column is the sample mean of the portfolio. “Explained” column displays the model-implied returns. “Proportion” column shows the ratio of the model-implied spread and actual spread. “dif” rows exhibit the difference of the estimates, or the difference of expected returns. In rows “p(dif)”, we report p-values with the null of equal coefficient in square brackets. Panels A, B, and C show the results from the ultimate consumption CAPM, Fama-French three-factor model, and Chen-Roll-Ross model, respectively.

Panel A: the ultimate consumption CAPM												
S	0				11				15			
	Δc	Actual	Explained	Proportion	Δc	Actual	Explained	Proportion	Δc	Actual	Explained	Proportion
Low	2.427 (2.214)	0.036	0.020		0.518 (2.119)	0.036	0.019		0.402 (1.876)	0.036	0.023	
High	2.492 (1.969)	0.022	0.020		0.287 (1.230)	0.022	0.011		0.264 (1.292)	0.022	0.015	
dif	−0.065	0.014	−0.001	−0.038	0.231	0.014	0.009	0.610	0.137	0.014	0.008	0.559
p(dif)	[0.515]				[0.248]				[0.322]			
Panel B: the Fama-French three-factor model												
	MKT	SMB	HML						Actual	Explained	Proportion	
Low	1.084 (22.502)	0.377 (4.439)	0.246 (3.877)						0.036	−0.053		
High	1.153 (35.713)	0.308 (5.360)	−0.391 (−6.749)						0.022	−0.066		
dif	−0.069	0.069	0.637						0.014	0.013	0.933	
p(dif)	[0.882]	[0.251]	[0.000]									
Panel C: the Chen-Roll-Ross model												
	MP	UI	DEI	UTS	UPR				Actual	Explained	Proportion	
Low	0.326 (7.145)	−1.471 (−13.651)	2.900 (6.009)	0.365 (14.181)	0.794 (11.976)				0.036	0.010		
High	0.187 (4.974)	−2.447 (−23.181)	5.756 (12.037)	0.282 (13.075)	1.140 (19.266)				0.022	−0.007		
dif	0.138	0.977	−2.856	0.084	−0.346				0.014	0.017	1.216	
p(dif)	[0.010]	[0.000]	[1.000]	[0.007]	[1.000]							

(2008) emphasizes that one should check the forecasting ability of the current returns when the target is $\sum_{i=2}^{S+2} \Delta c_{t+i-1,t+i}$, not the ultimate consumption $\sum_{i=2}^{S+1} \Delta c_{t+i-1,t+i}$, to see whether the forecasting ability of the current returns come from the predictability of contemporaneous consumption growth, not the long-run consumption growth. In Model 2 of each Panel of Table 9, we forecast the ultimate consumption growth starting from $t+2$ ($\sum_{i=2}^{S+2} \Delta c_{t+i-1,t+i}$). The investment factor is still useful in forecasting the consumption growth starting from $t+2$. Therefore, we argue that the relation between the investment-based return and future consumption growth comes from the long-run innovation in consumption growth, not from the contemporaneous consumption growth.² In Model 3 of each Panel of Table 9, we follow Parker and Julliard (2005) and perform the reverse regressions, which regress the investment factor on future ultimate consumption growth. We perform this to test the null hypothesis of zero correlation of the two variables. Not surprisingly, the results repeat the patterns in Model 1.

The fact that the investment factor predicts the future consumption growth, as shown here and in Min et al. (2017), justifies the superior performance of the ultimate consumption model. The R^2 in Table 9, however, does not exhibit the hump-shaped pattern as in Parker and Julliard (2005). Rather, the highest R^2 is obtained when $S = 15$ for the equal-weighted portfolios, while the horizon $S = 11$ has the highest R^2 for the value-weighted portfolios. Therefore, unlike the result from Parker and Julliard (2005), $S = 11$ may not be the most appropriate horizon in explaining the investment-based returns.

Why does the best-performing horizon differ across test assets? One possible explanation is to consider the business-cycle consumption risk by Bandi and Tamoni (2017). They decompose the consumption growth into its subcomponents based on their persistence. In their model, the business-cycle components, with periodicity between 2 and 8 years, play an important role in explaining the cross-section of returns. The emphasis on the business-cycle component may correspond to the small persistent component in Bansal and Yaron (2004). Taking the long-horizon consumption growth, which is called *aggregation*, can be a good way to eliminate the short-horizon components (Bandi, Perron, Tamoni, & Tebaldi, 2017). Therefore, we do not restrict ourselves to define the best-performing horizon, which may differ from asset to asset, and focus on the “business-cycle” frequency consumption shocks as the potential driving force behind the success of the ultimate consumption model.

² Cochrane (2008) also suggests using the weighted sum of future consumption growth $\sum_{i=2}^{S+2} \rho^i \Delta c_{t+i-1,t+i}$, which is a better proxy for the Epstein-Zin long-run consumption growth. When we use $\rho = 0.994$, the value from Bansal and Yaron (2004) on a quarterly basis, the results are virtually the same.

Table 9

Forecasting the Future Consumption by Investment-based Factors In Panel A (B), we construct the INV_{t+1} factor as the difference of the lowest and highest equal (value) weighted investment decile portfolios. In Model 1 of each Panel, we show the time-series regression coefficients, their t-statistics in parentheses, and R^2 s when we forecast the ultimate consumption growth ($\sum_{i=1}^{S+1} \Delta c_{t+i-1,t+i}$) with the INV_{t+1} factor. Model 2 of each Panel reports the results from the forecast regressions of $\sum_{i=2}^{S+2} \Delta c_{t+i-1,t+i}$ on the INV_{t+1} factor. Model 3 of each Panel reports the results of reverse regressions, which regress current INV_{t+1} factor on the future ultimate consumption growth $\sum_{i=2}^{S+2} \Delta c_{t+i-1,t+i}$.

S	Panel A: Equal-weighted portfolios						Panel B: Value-weighted portfolios					
	Model 1		Model 2		Model 3		Model 1		Model 2		Model 3	
	coeff	R^2	coeff	R^2	coeff	R^2	coeff	R^2	coeff	R^2	coeff	R^2
0	0.002 (0.227)	0.000	0.005 (0.583)	0.002	0.375 (0.576)	0.002	−0.001 (−0.089)	0.000	−0.007 (−0.667)	0.004	−0.508 (−0.685)	0.004
1	0.007 (0.516)	0.002	0.013 (1.181)	0.006	0.466 (1.169)	0.006	−0.008 (−0.452)	0.002	−0.006 (−0.405)	0.001	−0.178 (−0.410)	0.001
2	0.016 (0.940)	0.005	0.020 (1.345)	0.008	0.417 (1.291)	0.008	−0.007 (−0.311)	0.001	0.006 (0.316)	0.001	0.098 (0.313)	0.001
3	0.023 (1.154)	0.007	0.015 (0.792)	0.003	0.211 (0.821)	0.003	0.005 (0.184)	0.000	0.003 (0.148)	0.000	0.038 (0.148)	0.000
4	0.018 (0.718)	0.003	0.023 (1.010)	0.006	0.237 (1.103)	0.006	0.002 (0.079)	0.000	0.005 (0.189)	0.000	0.041 (0.191)	0.000
5	0.025 (0.927)	0.005	0.028 (1.062)	0.007	0.231 (1.209)	0.007	0.004 (0.124)	0.000	0.017 (0.572)	0.002	0.113 (0.606)	0.002
6	0.030 (0.974)	0.006	0.028 (0.946)	0.005	0.186 (1.056)	0.005	0.016 (0.467)	0.001	0.038 (1.164)	0.008	0.216 (1.334)	0.008
7	0.030 (0.875)	0.005	0.036 (1.219)	0.008	0.210 (1.457)	0.008	0.037 (1.018)	0.007	0.049 (1.509)	0.012	0.239 (1.777)	0.012
8	0.038 (1.115)	0.008	0.042 (1.352)	0.009	0.218 (1.718)	0.009	0.048 (1.349)	0.010	0.053 (1.522)	0.012	0.232 (1.773)	0.012
9	0.044 (1.240)	0.009	0.044 (1.410)	0.009	0.211 (1.759)	0.009	0.052 (1.403)	0.011	0.055 (1.541)	0.012	0.224 (1.730)	0.012
10	0.046 (1.286)	0.009	0.062 (1.986)	0.017	0.277 (2.561)	0.017	0.055 (1.460)	0.011	0.068 (1.879)	0.017	0.256 (1.977)	0.017
11	0.064 (1.787)	0.017	0.066 (2.050)	0.018	0.274 (2.629)	0.018	0.067 (1.800)	0.016	0.062 (1.680)	0.013	0.216 (1.722)	0.013
12	0.067 (1.882)	0.017	0.064 (1.944)	0.016	0.253 (2.489)	0.016	0.061 (1.625)	0.012	0.060 (1.690)	0.012	0.199 (1.696)	0.012
13	0.066 (1.804)	0.016	0.082 (2.341)	0.025	0.301 (2.867)	0.025	0.059 (1.660)	0.011	0.060 (1.585)	0.011	0.186 (1.593)	0.011
14	0.083 (2.225)	0.024	0.090 (2.529)	0.029	0.318 (3.026)	0.029	0.059 (1.573)	0.010	0.050 (1.296)	0.008	0.149 (1.297)	0.008
15	0.092 (2.435)	0.028	0.090 (2.421)	0.027	0.304 (2.875)	0.027	0.050 (1.270)	0.007	0.054 (1.352)	0.008	0.152 (1.358)	0.008

4. Conclusion

Recently, researchers report negative relationship between real investment and subsequent stock returns. Given that one central function of capital market is to price real investment accurately, examining whether corporate investment is fairly priced in the cross-section of stock returns is an important research question. Motivated by recent works which document that the return spread between low and high investment portfolios predict aggregate economic activities including future consumption growth, we study whether the ultimate consumption risk can explain the cross-section of investment-based portfolio returns.

We find that the ultimate consumption model with horizons from $S = 9$ to $S = 15$ performs better than the contemporaneous consumption model in explaining the cross-section of investment-based returns. In addition, the ultimate consumption model performs better than the Fama-French three-factor model or the Chen-Roll-Ross model in that the ultimate consumption model better explains the actual return of the lowest and highest portfolios. Moreover, the empirical performance of ultimate consumption model is striking in that the ultimate consumption model is single factor model. We interpret that the explanatory power of the ultimate consumption model arises from the close business-cycle relationship between the ultimate consumption growth and the investment-based returns. In sum, the present study shed some light on our understanding of negative relationship between real investment and subsequent stock returns.

Appendix

This appendix includes the GMM estimation results using 25 equal and value weighted size-investment double-sorted portfolios.

Table A1

GMM Estimation Results – 25 Equal-Weighted Portfolios This table displays the GMM estimation results of the ultimate consumption models from Eqs. (4) and (5) with 25 equal-weighted size-investment double-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the brackets are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen-Jagannathan distance and its p-value is calculated from 10,000 simulations. The column *J* shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2010:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
S	R^2	α	γ	HJ	R^2	α	γ	<i>J</i>
0	0.141	0.009 (0.016)	100.426 (78.127)	0.838 [0.000]	0.085	0.026 (0.006)	45.388 (23.535)	68.613 [0.000]
1	0.045	0.016 (0.011)	18.540 (32.624)	0.762 [0.000]	0.046	0.020 (0.005)	20.447 (13.899)	69.441 [0.000]
3	0.114	0.013 (0.012)	18.448 (23.262)	0.800 [0.000]	−0.074	0.025 (0.005)	−6.985 (11.872)	70.005 [0.000]
5	0.344	−0.003 (0.014)	27.277 (12.944)	0.960 [0.000]	0.210	0.025 (0.005)	4.215 (6.722)	98.898 [0.000]
7	0.368	−0.001 (0.012)	22.276 (10.297)	0.919 [0.000]	0.007	0.024 (0.005)	−3.300 (6.736)	77.099 [0.000]
9	0.574	0.005 (0.01)	25.115 (8.493)	0.894 [0.000]	−0.028	0.025 (0.005)	−1.951 (6.402)	71.659 [0.000]
11	0.630	0.002 (0.01)	30.221 (11.879)	0.831 [0.000]	0.552	0.021 (0.005)	19.802 (5.419)	66.100 [0.000]
13	0.643	0.004 (0.009)	37.245 (18.381)	0.886 [0.000]	0.602	0.016 (0.005)	29.662 (7.061)	59.514 [0.000]
15	0.727	0.007 (0.010)	46.580 (28.243)	0.836 [0.000]	0.656	0.018 (0.005)	31.869 (7.710)	56.332 [0.000]

Table A2

GMM Estimation Results – 25 Value-Weighted Portfolios This table displays the GMM estimation results of the ultimate consumption models from Eqs. (4) and (5) with 25 value-weighted size-investment double-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the brackets are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen-Jagannathan distance and its p-value is calculated from 10,000 simulations. The column *J* shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2010:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
S	R^2	α	γ	HJ	R^2	α	γ	<i>J</i>
0	0.063	0.015 (0.008)	47.314 (63.013)	0.554 [0.001]	−0.033	0.022 (0.005)	−10.754 (25.985)	56.072 [0.000]
1	0.022	0.017 (0.009)	10.979 (33.469)	0.525 [0.000]	0.015	0.021 (0.005)	4.964 (15.51)	55.289 [0.000]
3	0.058	0.015 (0.011)	13.091 (26.086)	0.553 [0.000]	−0.205	0.024 (0.005)	−20.344 (10.837)	53.327 [0.000]
5	0.267	0.002 (0.011)	22.062 (14.649)	0.628 [0.000]	−0.217	0.024 (0.005)	−12.128 (8.732)	53.957 [0.000]
7	0.348	0.004 (0.008)	17.726 (10.110)	0.605 [0.000]	−0.054	0.022 (0.005)	−2.300 (7.032)	55.836 [0.000]
9	0.528	0.008 (0.008)	19.916 (8.657)	0.618 [0.000]	0.087	0.021 (0.005)	2.222 (6.066)	56.430 [0.000]
11	0.587	0.007 (0.007)	22.946 (10.837)	0.644 [0.000]	0.117	0.021 (0.005)	2.981 (5.852)	57.030 [0.000]
13	0.552	0.007 (0.008)	28.250 (17.08)	0.702 [0.000]	0.091	0.021 (0.005)	2.999 (6.687)	57.455 [0.000]
15	0.555	0.009 (0.009)	34.331 (22.718)	0.732 [0.000]	0.275	0.019 (0.005)	9.957 (6.021)	58.332 [0.000]

References

- Bandi, F. M., & Tamoni, A. (2017). Business-cycle consumption risk and asset prices, Working Paper.
- Bandi, F. M., Perron, B., Tamoni, A., & Tebaldi, C. (2017). The scale of predictability, Working Paper.
- Bansal, R., Dittmar, R. F., & Lundblad, C. T. (2005). Consumption, dividends, and the cross section of equity returns. *Journal of Finance*, 60, 1639–1672.
- Bansal, R., & Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59, 1481–1509.
- Breeden, D. T., Gibbons, M. R., & Litzenberger, R. H. (1989). Empirical tests of the consumption-oriented CAPM. *Journal of Finance*, 44, 231–262.
- Campbell, J. Y., & Cochrane, J. H. (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 107, 205–251.
- Chen, N., Roll, R., & Ross, S. A. (1986). Economic forces and the stock market. *Journal of Business*, 59, 383–403.
- Cochrane, J. H. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy*, 104, 572–621.
- Cochrane, J. H. (2005). *Asset pricing*. Princeton University Press. revised edition.
- Cochrane, J. H. (2008). *Financial markets and the real economy*. Amsterdam: Elsevier.
- Cooper, M. J., Gulen, H., & Schill, M. J. (2008). Asset growth and the cross-section of stock returns. *Journal of Finance*, 63, 1609–1651.
- Cooper, I., & Priestley, R. (2011). Real investment and risk dynamics. *Journal of Financial Economics*, 101, 182–205.
- Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33, 3–56.
- Fama, E. F., & French, K. R. (1996). Multifactor explanations of asset pricing anomalies. *Journal of Finance*, 51, 55–84.
- Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116, 1–22.
- Fama, E. F., & Gibbons, M. R. (1984). A comparison of inflation forecasts. *Journal of Monetary Economics*, 13, 327–348.
- Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 81, 607–636.
- Griffin, J. M., Ji, X., & Martin, S. J. (2003). Momentum investing and business cycle risk: Evidence from pole to pole. *Journal of Finance*, 58, 2515–2547.
- Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica*, 50, 1029–1054.
- Hansen, L. P., Heaton, J. C., & Li, N. (2008). Consumption strikes back? Measuring long-run risk. *Journal of Political Economy*, 116, 260–302.
- Hansen, L. P., & Jagannathan, R. (1997). Assessing specification errors in stochastic discount factor models. *Journal of Finance*, 52, 557–590.
- Hou, K., Xue, C., & Zhang, L. (2015). Digesting anomalies: An investment approach. *Review of Financial Studies*, 28, 650–705.
- Jagannathan, R., & Wang, Z. (1996). The conditional CAPM and the cross-section of expected returns. *Journal of Finance*, 51, 3–53.
- Jagannathan, R., & Wang, Y. (2007). Lazy investors, discretionary consumption, and the cross-section of stock returns. *Journal of Finance*, 62, 1623–1661.
- Lettau, M., & Ludvigson, S. (2001). Resurrecting the (C) CAPM: A cross-sectional test when risk premia are time-varying. *Journal of Political Economy*, 109, 1238–1287.
- Lewellen, J., Nagel, S., & Shanken, J. (2010). A skeptical appraisal of asset pricing tests. *Journal of Financial Economics*, 96, 175–194.
- Liu, L. X., & Zhang, L. (2008). Momentum profits, factor pricing, and macroeconomic risk. *Review of Financial Studies*, 21, 2417–2448.
- Mankiw, G. N., & Shapiro, M. D. (1986). Risk and return: Consumption beta versus market beta. *Review of Economics and Statistics*, 68, 452–459.
- Min, B., Kang, J., & Lee, C. (2017). The q-factors and macroeconomic conditions: Asymmetric effects of the business cycles on long and short sides, Working Paper.
- Parker, J. A., & Julliard, C. (2005). Consumption risk and the cross section of expected returns. *Journal of Political Economy*, 113, 185–222.
- Piazzesi, M., & Schneider, M. (2006). Equilibrium yield curves. *NBER Macroeconomics Annual*, 21, 389–472.
- Savov, A. (2011). Asset pricing with garbage. *Journal of Finance*, 66, 177–201.
- Shanken, J. (1992). On the estimation of beta-pricing models. *Review of Financial Studies*, 5, 1–55.
- Stambaugh, R. F., Yu, J., & Yuan, Y. (2012). The short of it: investor sentiment and anomalies. *Journal of Financial Economics*, 104, 288–302.
- Titman, S., Wei, K. C. J., & Xie, F. (2004). Capital investments and stock returns. *Journal of Financial and Quantitative Analysis*, 39, 677–700.
- Wang, Z. (2013). Do the investment and return-on-equity factors proxy for economic risks? *Financial Management*, 42, 183–209.
- Wang, H., & Yu, J. (2013). Dissecting the profitability premium, Working Paper.
- Xing, Y. (2008). Interpreting the value effect through the q-theory: An empirical investigation. *Review of Financial Studies*, 21, 1767–1795.
- Yogo, M. (2006). A consumption-based explanation of expected stock returns. *Journal of Finance*, 61, 539–580.