BOSONS

Hamiltonian of N D-dimensional bosons in harmonic oscillator potential with hard-sphere interaction ($\hbar = m = 1$):

$$\hat{H} = \sum_{i=1}^{N} \Big\{ -\frac{1}{2}\nabla_i^2 + \frac{1}{2}(\omega_1^2 r_{i,1}^2 + \cdots + \omega_D^2 r_{i,D}^2) \Big\} + \sum_{i<j}^{N} V_{int}(\vec{r_i}, \vec{r_j}), \tag{1}$$

$$V_{int}(\vec{r_i}, \vec{r_j}) = \begin{cases} \infty, & r_{ij} \leq a \\ 0, & r_{ij} > a \end{cases} \tag{2}$$

Store the squares of the harmonic oscillator frequencies in a vector of size D so we can write $\hat{H}$ as:

$$\hat{H} = \frac{1}{2}\sum_{i=1}^{N} \Big\{ -\nabla_i^2 + \vec{\omega^2} \cdot \vec{r_i^2} \Big\}, \tag{3}$$

$$\vec{\omega^2} = \vec{\omega} \ \% \ \vec{\omega} = \langle \omega_1^2, \cdots, \omega_D^2 \rangle \tag{4}$$

$$\vec{r_i^2} = \vec{r_i} \ \% \ \vec{r_i} = \langle r_{i,1}^2, \cdots, r_{i,D}^2 \rangle \tag{5}$$

$$\tag{6}$$

The interaction is taken care of by forcing the trial wavefunction to vanish if any two bosons become separated by a distance less than a. Trial wavefunction:

$$\Psi_T(\vec{R}) = \Big\{ \prod_{i=1}^{N} \exp(-\vec{\alpha} \cdot \vec{r_i^2}) \Big\} \Big\{ \prod_{i<j}^{N} \exp\left(\ln\left(1 - \frac{a}{r_{ij}}\right)\right) \Big\} = \exp\left(p(\vec{\alpha}, \vec{R}) + q(\vec{R})\right) \tag{7}$$

$$p(\vec{\alpha}, \vec{R}) = -\sum_{i=1}^{N} \vec{\alpha} \cdot \vec{r_i^2} \tag{8}$$

$$q(\vec{R}) = \sum_{i<j}^{N} \ln\left(1 - \frac{a}{r_{ij}}\right) \tag{9}$$

The local energy is defined as

$$E_L(\vec{R}) = \frac{1}{\Psi_T} \hat{H} \Psi_T \tag{10}$$

We need the following derivatives to obtain the analytical form of the local energy:

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\begin{aligned}
\frac{1}{\Psi_T}\nabla_i^2\Psi_T &= \nabla_i^2 p + \nabla_i^2 q + (\nabla_i p + \nabla_i q)^2 \\
\nabla_i p &= -2\vec{\alpha} \,\%\, \vec{r_i} \\
\nabla_i^2 p &= -2\vec{\alpha} \cdot \vec{\mathbb{1}} \\
\nabla_i q &= \sum_{j\neq i}^{N} \frac{a}{r_{ij}^2(r_{ij}-a)}(\vec{r_i}-\vec{r_j}) \\
\nabla_i^2 q &= \nabla_i \cdot \left(\sum_{j\neq i}^{N} \frac{a}{r_{ij}^2(r_{ij}-a)}(\vec{r_i}-\vec{r_j}) \right) \\
&= \sum_{d=1}^{D} \frac{\mathrm{d}}{\mathrm{d}r_{i,d}} \left(\sum_{j\neq i}^{N} \frac{a}{r_{ij}^2(r_{ij}-a)}(r_{i,d}-r_{j,d}) \right) \\
&= \sum_{j\neq i}^{N} \sum_{d=1}^{D} \frac{\mathrm{d}}{\mathrm{d}r_{i,d}} \left(\frac{a}{r_{ij}^2(r_{ij}-a)}(r_{i,d}-r_{j,d}) \right) \\
&= \sum_{j\neq i}^{N} \sum_{d=1}^{D} \left(-\frac{a(r_{i,d}-r_{j,d})^2}{r_{ij}^4(r_{ij}-a)^2}(3r_{ij}-2a) + \frac{a}{r_{ij}^2(r_{ij}-a)} \right)
\end{aligned}
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