This document is meant to help me organize my thoughts. I kept losing track of indices and factors while doing this by hand. I want expressions which can be easily and efficiently implemented.

Index convention:

$$
\begin{aligned}
a, b, c, d, \ldots &= \text{particles}, \\
i, j, k, l, \ldots &= \text{holes}, \\
p, q, r, s, \ldots &= \text{either}.
\end{aligned}
$$

One- and two- body operators have the following form:

$$
A^{(1)} = \sum_{pq} A_{pq} : a_p^\dagger a_q :, \tag{1}
$$

$$
A^{(2)} = \frac{1}{4} \sum_{pqrs} A_{pqrs} : a_p^\dagger a_q^\dagger a_s a_r :, \tag{2}
$$

where the colons indicate normal ordering. I want to write a function that returns the zero-, one-, and two-body components of the commutator

$$
[A^{(1)} + A^{(2)}, B^{(1)} + B^{(2)}]. \tag{3}
$$

I will name these components $C^{(0)}$, $C^{(1)}$, and $C^{(2)}$, respectively. Note: the three-body component will be omitted. Since this commutator is equivalent to the following sum of commutators

$$
[A^{(1)}, B^{(1)}] + [A^{(1)}, B^{(2)}] + [A^{(2)}, B^{(1)}] + [A^{(2)}, B^{(2)}], \tag{4}
$$

we can use the fundamental commutator expressions to evaluate each term. For reference, I will write the relevant expressions here:

$$
[A^{(1)}, B^{(1)}]^{(0)} = \sum_{pq} (n_p - n_q) A_{pq} B_{qp}, \tag{5}
$$

$$
[A^{(1)}, B^{(1)}]^{(1)} = \sum_{pq} \sum_{r} : a_p^\dagger a_q : (A_{pr} B_{rq} - B_{pr} A_{rq}), \tag{6}
$$

$$
[A^{(1)}, B^{(2)}]^{(1)} = \sum_{pq} \sum_{rs} : a_p^\dagger a_q : (n_r - n_s) A_{rs} B_{sprq}, \tag{7}
$$

$$
[A^{(1)}, B^{(2)}]^{(2)} = \frac{1}{4} \sum_{pqrs} \sum_{t} : a_p^\dagger a_q^\dagger a_s a_r : \{(1 - P_{pq}) A_{pt} B_{tqrs} - (1 - P_{rs}) A_{tr} B_{pqts}\}, \tag{8}
$$

$$
[A^{(2)}, B^{(1)}]^{(1)} = -[B^{(1)}, A^{(2)}]^{(1)} = \sum_{pq} \sum_{rs} : a_p^\dagger a_q : (n_s - n_r) B_{rs} A_{sprq}, \tag{9}
$$

$$
[A^{(2)}, B^{(1)}]^{(2)} = -[B^{(1)}, A^{(2)}]^{(2)} = \frac{1}{4} \sum_{pqrs} \sum_{t} : a_p^\dagger a_q^\dagger a_s a_r : \{(1 - P_{rs}) B_{tr} A_{pqts} - (1 - P_{pq}) B_{pt} A_{tqrs}\}, \tag{10}
$$

$$
[A^{(2)}, B^{(2)}]^{(0)} = \frac{1}{4} \sum_{pqrs} n_p n_q \bar{n}_r \bar{n}_s (A_{pqrs} B_{rspq} - B_{pqrs} A_{rspq}), \tag{11}
$$

$$
[A^{(2)}, B^{(2)}]^{(1)} = \frac{1}{2} \sum_{pq} \sum_{rst} : a_p^\dagger a_q : (\bar{n}_r \bar{n}_s n_t + n_r n_s \bar{n}_t)(A_{tprs} B_{rstq} - B_{tprs} A_{rstq}), \tag{12}
$$

$$
[A^{(2)}, B^{(2)}]^{(2)} = \frac{1}{4} \sum_{pqrs} \sum_{tu} : a_p^\dagger a_q^\dagger a_s a_r : \{\frac{1}{2}(A_{pqtu} B_{turs} - B_{pqtu} A_{turs})(1 - n_t - n_u) \tag{13}
$$

$$
+ (n_t - n_u)(1 - P_{ij})(1 - P_{kl}) A_{tpur} B_{uqts}\} \tag{14}
$$

Now, we can simplify this a bit because many of the terms in the summations vanish. Using the index notation mentioned above, we have

$$C^{(0)} = \sum_{pq} (n_p - n_q) A_{pq} B_{qp} + \frac{1}{4} \sum_{pqrs} n_p n_q \bar{n}_r \bar{n}_s (A_{pqrs} B_{rspq} - B_{pqrs} A_{rspq}), \tag{15}$$