LECTURE 1

COMP 3760 – Fall 2019

About me

- Tom Magliery
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- Best way to find me?
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My history

- BS (x2) in Math and Computer Science, Master of Computer Science
- Worked for one company for 17(!) years, but finally have escaped to BCIT
- Lived in Florida, Kansas, Illinois, Texas, and British Columbia
- My home page (c.1994) is older than some? Most? All? of you
  - *Please, not "all"...*

N C S A

MOSAIC

X Window System • Microsoft Windows • Macintosh

(Ungraded) quiz

- (HANDOUT)

Course objectives

SWBAT:

- Discuss the importance of algorithms in the problem-solving process.
- Choose the appropriate data structure or container for modeling a given problem.
- Describe, implement, and use common data structures and algorithms.
- Design and implement new algorithms using several techniques e.g. Divide and Conquer, Greedy, Dynamic programming, Graph techniques, etc.
- Argue the correctness of their algorithms.
- Analyze pseudo-code using the Big-Oh notation.
- Deduce the complexity of a program by running different experiments.
- Discuss the computational efficiency of the principal algorithms for sorting, searching, and hashing.

Textbook

- Introduction to The Design and Analysis of Algorithms, 3rd Ed.
  - *Author: Anany Levitin*

Pre-requisites

- Java (COMP 2526)
  - *You need to be able to program in Java*
  - *I am not going to teach you Java*
  - *You can use any IDE you want*
  - *You are expected to write "good code" as you learned previously*
- Discrete math (COMP 2121)
- Pseudocode

Grading

Criteria	%	Comments
Lab Assignments	25	Weekly assignments, 11 assignments in total
Quizzes	20	Held in lab throughout the term, 5 quizzes in total
Midterm	25	
Final Exam	30	

Labs and quizzes

- Labs are done during lab time
- Mandatory attendance in labs
- Quizzes are also done in lab time

Tips for success

- Practice!
- Keep up with the material
- Interrupt me ANY time in class

MATH REVIEW

Review topics

- Logarithm
- Floor and Celling
- Counting
  - *Permutations*
  - *Subsets*
- Summation

(Ungraded) pre-test

- Please install the Kahoot app or go to www.kahoot.it

Logarithms

- Mostly what you need to know:
  - $log_b n = e$
  - *just means:* $b^e = n$

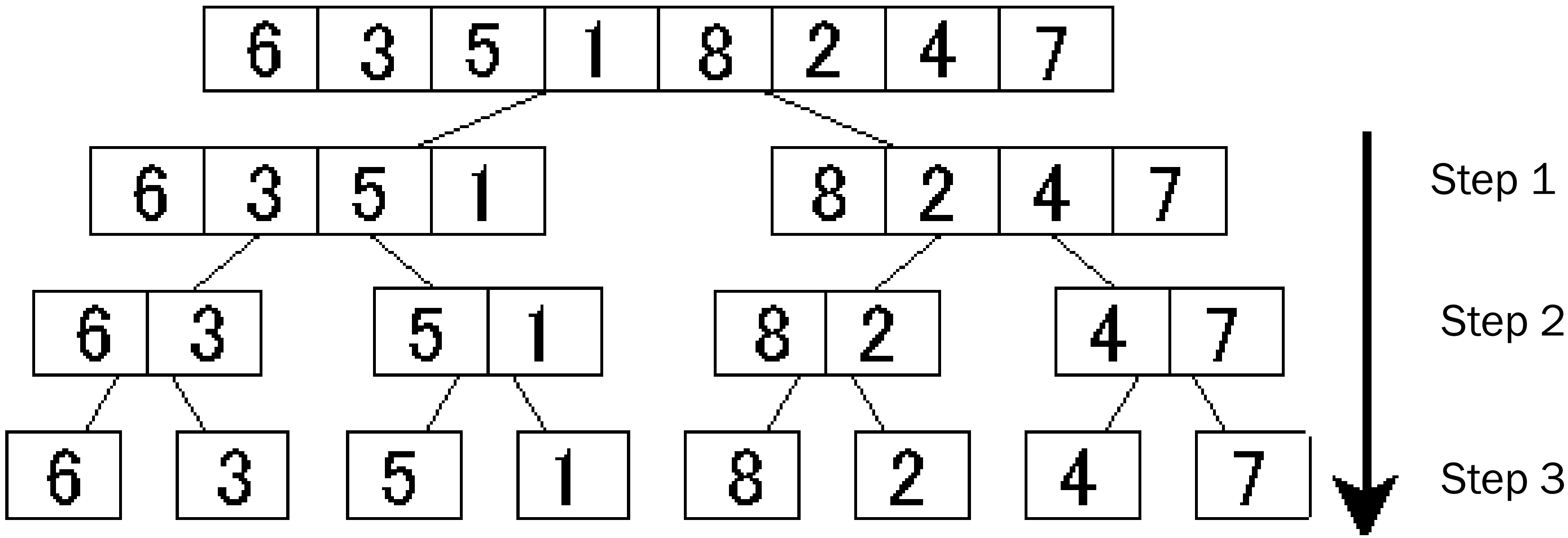
- So these are the same question:
  - $log_2 16=?$
  - $16 = 2^?$

- In words:
  - *"What is log base 2 of 16?"*
  - *"What power of 2 gives 16?"*

When we'll see logarithms

- The most common time to use:
  - *Start with n items*
  - *Divide the group in half at each step*
  - *How many steps does it take to get down to one?*

Example



$$\log_2 8 = 3$$

Floor and ceiling

- If *x* is not a whole number, these are useful:

  $\lceil x \rceil$ *= The closest whole number above x (the* ***ceiling*** *of x)*

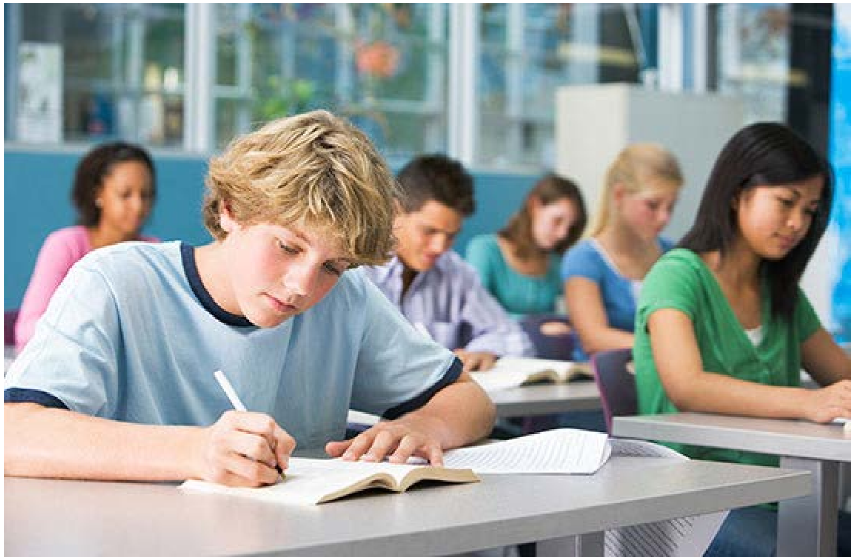
  $\lfloor x \rfloor$ *= The closest whole number below x (the* ***floor*** *of x)*

So: $\lceil \log 38 \rceil$ = 6

$\lfloor \log 38 \rfloor$ = 5

Counting

- Sometimes, we need to count things
- Example



In how many different ways could students sit on the chairs in a class?

Counting

- The trick when counting is this:
  - *Divide the problem into a sequence of independent choices*
  - *See how many options there are for each choice*
  - *Multiply those number together*

Counting permutations

- A permutation is an arrangement in which order matters. ABC differs from BCA
- How many permutations are there on a collection of 3 items, A, B, C?
- ABC, ACB, BAC, BCA, CAB, CBA

Permutations

- Suppose you have n items: $A_1, \ldots, A_n$
- Then you have n independent choices:

  ....

- Count the # of options for each choice

  n 1

- Multiply together:

  $n*(n-1)*\ldots*1 = n!$ permutations

Subsets

- Given a set of 3 items {a, b, c}, how many different subsets can we make?

- Subsets are:

  {a, b, c}, {a, b}, {b, c}, {a, c}, {a}, {b}, {c}, {}

Subsets

- Suppose you have n things: A_1, ..., A_n
- Then you have n items (choices) to consider:

  A_1 .... A_n

- You have 2 options for each item (in/out)

  i/o .... i/o

- Multiply together:

  *2*2*…*2 (n times) = 2^n subsets*

Summations

- We use compact notation for summations

End by putting this value for i.

f(i) is just a function… Think of it as a formula that has an i in it.

$$\sum_{i=1}^{n} f(i)$$

Start by putting this value for i.

Put all values in between … and add them up.

- So this is really just a shorthand for:

$$f(1) + f(2) + f(3) + \ldots + f(n)$$

Example

- Evaluate this expression: $$\sum_{i=1}^{4}(2+i^2)$$

- Start with i=1, end with i=4…

$$(2+1^2)+(2+2^2)+(2+3^2)+(2+4^2)$$

- Now you just have numbers… so you can add

$$= 3+6+11+18$$

$$= 38\ .$$

Sum of a constant

$$\sum_{i=1}^{n} C$$

- What it means:

  $$\underbrace{C + C + \ldots + C}_{\text{(n times)}}$$

  – *So:*

  $$\sum_{i=1}^{n} C = nC$$

Sum of a constant

$$\sum_{i=1}^{n} n$$

- What it means:

  $$\underbrace{n + n + \ldots + n}_{\text{(n times)}}$$

  - *So:*

    $$\sum_{i=1}^{n} \quad = n^2$$

Changing the start and end

- We don’t always start from 1 and end at n
- What is this sum:

$$\sum_{i=m}^{n} c = \underbrace{c + c + \cdots + c}_{(n-m+1)\text{ times}}$$

$$\sum_{i=m}^{n} c = (n-m+1) * c$$

Question

- What is this sum? $$\sum_{i=0}^{n} 1$$

- Careful... before we had i=1

$$\sum_{i=0}^{n} 1 = \underbrace{1 + 1 + \cdots + 1}_{(n-0+1)\text{ times}} = (n+1) * 1 = n + 1$$

Sums of sums

- Sometimes you have a sum with two parts added together:

$$\sum_{n=s}^{t} [f(n) + g(n)]$$

- You can just break it into two sums:

$$\sum_{n=s}^{t} f(n) + \sum_{n=s}^{t} g(n)$$

Constant rule

- You can actually move the constant in front for any sum
- RULE:

$$\sum_{n=s}^{t} C \cdot f(n) = C \cdot \sum_{n=s}^{t} f(n), \text{ where } C \text{ is a constant}$$

More summation rules

- There are many more summation rules in the appendix of your text.
- Important examples:

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \; .$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \; .$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \; .$$

Practice problems

- Try to evaluate these:

$$\sum_{i=0}^{3}(5+\sqrt{4^i})$$

$$\sum_{i=1}^{100}(4+3i)$$

Solution 1

$$\sum_{i=0}^{3}(5+\sqrt{4^i}) = (5+\sqrt{4^0}) + (5+\sqrt{4^1}) + (5+\sqrt{4^2}) + (5+\sqrt{4^3})$$

$$= (5+\sqrt{1}) + (5+\sqrt{4}) + (5+\sqrt{16}) + (5+\sqrt{64})$$

$$= (5+1) + (5+2) + (5+4) + (5+8)$$

$$= 6 + 7 + 9 + 13$$

$$= 35 \ .$$

Solution 2

$$\sum_{i=1}^{100}(4+3i)=\sum_{i=1}^{100}4+\sum_{i=1}^{100}3i$$

$$=\sum_{i=1}^{100}4+3\left(\sum_{i=1}^{100}i\right)$$

$$=4(100)+3\left\{\frac{100(100+1)}{2}\right\}$$

$$=400+15{,}150$$

$$=15{,}550\ .$$

Sums of summations

- We will often see things like this:

$$\sum_{j=1}^{i}\sum_{k=j}^{n} 1$$

- What does this mean?
  - *It means you have a sum of sums*
    - (NOT two sums multiplied)
  - *In order to solve it… you work from the inside out.*

Sum of summations

- In this example: $$\sum_{j=1}^{i}\sum_{k=j}^{n} 1 = \sum_{j=1}^{i}(n-j+1)$$

- Now you can divide into three sums and solve:

$$\sum_{j=1}^{i} n - \sum_{j=1}^{i} j + \sum_{j=1}^{i} 1 = \mathrm{n} * \mathrm{i} - \frac{\mathrm{i} * (\mathrm{i}+1)}{2} + \mathrm{i}$$

We will solve this kind of sum often in the first part of the course… so make sure you understand how to do it.

ALGORITHM EFFICIENCY

Section objectives

SWBAT:

- State a definition of the term "algorithm"
- Explain the difference between "time efficiency" and "space efficiency"
- Determine the "basic operation" for a given algorithm represented in pseudocode
- Determine a formula for the number of times that any step in an algorithm will be performed, as a function of N (the size of the input to the algorithm)

Why do we care about algorithms?

Some reasons we care

- Algorithms are at the core of computer programming
- There are many important, standard algorithms
- We want to design new algorithms and analyze their efficiency

Algorithm origin

The word “algorithm” derived from the name of Persian mathematician Abdallāh Muḥammad ibn Mūsā al-Khwārizmī

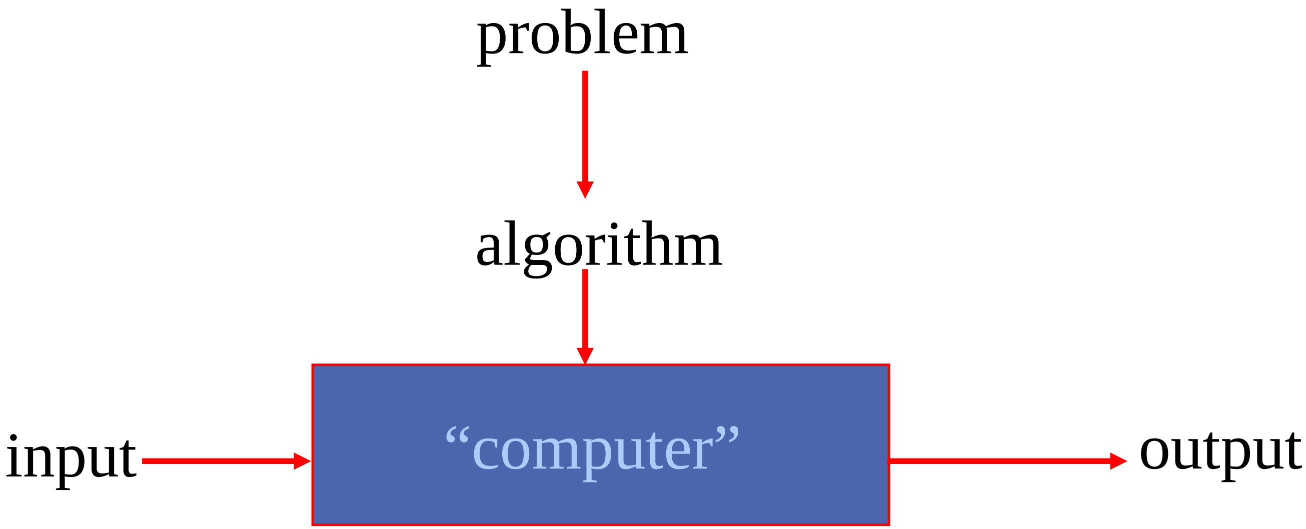
(The word algebra also comes from this name)

What is an Algorithm?

- One definition:

  An algorithm is a sequence of ***unambiguous instructions*** *for solving a problem.*

  i.e: for obtaining a required output for any ***legitimate input*** *in a* ***finite amount of time***



Key points

- Each step is precise
- There can be more than one algorithm for the same problem

Example

- Here is a pseudocode algorithm:

```
Algo: find( A[0…n-1] )
   m ← A[0]
   for i ← 1 to n-1 do
       if A[i] > m
           m ← A[i]
   return m
```

- What does it do?

It finds the largest element of an array

Time Efficiency

- Is *find* a time-efficient algorithm?

- Seems good
  - *To find the largest, you need to check each array element exactly once*

```
Algo: find( A[0…n-1] )
   m ← A[0]
   for i ← 1 to n-1 do
       if A[i] > m
          m ← A[i]
   return m
```

Space Efficiency

- Is *find* a space-efficient algorithm? (amount of memory)
- Again… it seems reasonable
  - *One temp variable introduced*

```
Algo: find( A[0…n-1] )
   m ← A[0]
   for i ← 1 to n-1 do
       if A[i] > m
          m ← A[i]
   return m
```

Example

- What if you knew that the array A were already sorted?
- Is *find* still efficient?
- Could you think of a better algorithm?

```
Algo: find( A[0…n-1] )
   m ← A[0]
   for i ← 1 to n-1 do
       if A[i] > m
          m ← A[i]
   return m
```

Why do we care?

- Think about computing the n^{th} Fibonacci number:
  - *0, 1, 1, 2, 3, 5, 8, 13, …*

First algorithm

```
Algo: fib( n )
   if n ≤ 1
      return n
   else
      return fib( n-1 ) + fib( n-2 )
```

Java implementation

```
public static int fib(int n)  {
   if (n<=1)
      return n;
   else
      return ( fib(n-1) + fib(n-2) );
}
```

Why do we care, Part 2

- Now look at a different algorithm

Second algorithm

```
Algo: fib2( n )
   F[0] ← 0; F[1] ← 1;
   for i ← 2 to n do
      F[i] ← F[i-1] + F[i-2]
   return F[n]
```

```
public static int fib2(int n) {

   int[] f = new int[n+1];

   f[0] = 0;
   f[1] = 1;
   for (int i=2; i<=n; i++)
       f[i] = f[i-1] + f[i-2];
   return f[n];
}
```

Difference

- First approach
  - *Recursively calls the Fib function over and over again*

- Second approach
  - *Stores successive results so we don't have to re-compute them …*

- Second approach is much much faster.
  - *For n = 30*
  - *Running time of first approach = 5957 microseconds*
  - *Running time of second approach = 7 microseconds*

So?

- Fib is a basic example of why we care about algorithm efficiency

- A well thought out algorithm can run much faster

- There can be big variation in efficiency

How to determine efficiency

- Could do it experimentally
  - *i.e. Write a bunch of implementations, see which one is fastest*
- Problem?
  - *Time consuming and expensive*
  - *It is not accurate*

- *Want to estimate efficiency before writing code*

How to Determine Efficiency

- What we know:

1. *Running time (efficiency) of an algorithm depends on the input size*

2. *The total execution time for any algorithm depends on the number of instructions executed*

Example

- Remember this algorithm:

```
1. Algo: find( A[0…n-1] )
2.     m ← A[0]
3.     for i ← 1 to n-1 do
4.        if A[i] > m
5.           m ← A[i]
6.     return m
```

for n=3

stmt	#times
1	0
2	1
3	2
4	2
5	2
6	1

- How many instructions are executed if n=3?

$$f(3) = 1 + 3*(3-1) + 1$$

Example

- Remember this algorithm:

```
1. Algo: find( A[0…n-1] )
2.     m ← A[0]
3.     for i ← 1 to n-1 do
4.        if A[i] > m
5.           m ← A[i]
6.     return m
```

for n=8

stmt	#times
1	0
2	1
3	7
4	7
5	7
6	1

- What about n=8?
  - f(8) = 1 + 3*(8-1) + 1
  - For input size n, the running time is

$$f(n) = 1 + 3*(n-1) + 1$$

Basic operations

- Which instruction in *find* gets executed the most?

```
1. Algo: find( A[0…n-1] )
2.      m ← A[0]
3.      for i ← 1 to n-1 do
4.          if A[i] > m
5.              m ← A[i]
6.      return m
```

stmt	(n=3) #times	(n=10) #times	(n=100) #times
1	0	0	0
2	1	1	1
3	2	9	99
4	2	9	99
5	2	9	99
6	1	1	1

- We define the basic operation of an algorithm as the statement that gets executed most frequently

Basic operations

This is the fundamental concept we use to analyze algorithmic efficiency:

count the number of basic operations executed for an input of size n

- Using this idea, we would say for *find*

  $f(n) = n-1$

- We don't count instructions that are not basic operations

Example 1

- Consider this algorithm:

```
1. Mystery1(n)  // n > 0
2.   S ← 0
3.   for i ← 1 to n do
4.       S ← S + i * i
5.   return S
```

1. What does this algorithm do?

   Calculates: $1^2 + 2^2 + 3^2 + \ldots + n^2$

2. What is the basic operation?

   It's line 4
   (multiplication/addition… doesn't matter)

3. How many times is the basic operation executed for input size n?

```
Mystery(n)  // n > 0
  S ← 0
  for i ← 1 to n do
      S ← S + i * i
  return S
```

How many times?

- Count operations each time in loop
  - *1st time: 1 op,*
  - *2nd time: 1 op, ...*
  - *nth time: 1 op*
- So you have a sum

$$\sum_{i=1}^{n} 1$$

- What does this equal?

  1+ 1 + 1 ... + 1 (n times)

  = n

Example 2

- Consider this algorithm:

```
1. Mystery2(A[0..n-1][0..n-1])  // n > 0
2.  S ← 0
3.  for i ← 0 to n-1 do
4.     for j← 0 to n-1 do
5.          S ← S + A[i][j];
6.  return S
```

1. What does this algorithm do?

   Calculates sum of the elements in array A

2. What is the basic operation?

   Addition on line 5

3. How many times is the basic operation executed for input size n?

Example 2

```
Mystery2(A[0..n-1][0..n-1])  // n > 0
 S ← 0
 for i ← 0 to n-1 do
     for j← 0 to n-1 do
           S ← S + A[i][j];
 return S
```

- The outer loop
  - *i goes from 0 to n-1*
  - *So we have*

$$\sum_{i=0}^{n-1} (something)$$

Example 2

```
Mystery2(A[0..n-1][0..n-1])  // n > 0
 S ← 0
 for i ← 0 to n-1 do
     for j← 0 to n-1 do
          S ← S + A[i][j];
 return S
```

- The inner loop:
  - *j goes from 0 to n-1*
  - *At each iteration, we do one basic operation*
  - *So for the inner loop we have*

$$\sum_{j=0}^{n-1} 1$$

  - *We do this for each iteration of the outer loop*

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1$$

Simplifying the sum

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1$$

- The inner summation is:

$$\sum_{j=0}^{n-1} 1 = 1 + 1 + \ldots + 1 = n$$

- So the outer summation is:

$$\sum_{i=0}^{n-1} n = n + n + \ldots + n = n^2$$

Example 3

```
Loops(A[0..n-1])
 for i ← 1 to n-1 do
   v ← A[i]
   j ← i-1
   while j≥0 and A[j]>v do
      A[j+1] ← A[j]
      j ← j-1
   A[j+1] ← v
```

- What does this algorithm do?
- What is the basic operation?
- How many times is the operation executed for input size n?

What does it do?

```
Loops(A[0..n-1])
 for i ← 1 to n-1 do
   v ← A[i]
   j ← i-1
   while j≥0 and A[j]>v do
      A[j+1] ← A[j]
      j ← j-1
   A[j+1] ← v
```

5	**2**	4	6	1	3
2	5	**4**	6	1	3
2	4	5	**6**	1	3
2	4	5	6	**1**	3
1	2	4	5	6	**3**
1	2	3	4	5	6

Basic operation

```
Loops(A[0..n-1])
  for i ← 1 to n-1 do
    v ← A[i]
    j ← i-1
    while j≥0 and A[j]>v do
      A[j+1] ← A[j]
      j ← j-1
    A[j+1] ← v
```

- Two options:
  - *There are variable assignments and comparisons*
  - *Most people would say the basic operation is the key comparison A[j]>v*
  - *Why?*
    - It is really the key thing being checked in each loop

Example 3 analysis

```
Loops(A[0..n-1])
 for i ← 1 to n-1 do
   v ← A[i]
   j ← i-1
   while j≥0 and A[j]>v do
      A[j+1] ← A[j]
      j ← j-1
   A[j+1] ← v
```

- Look at outer loop first
- There is a variable *i* getting incremented from *1* up to *n-1*
- So we have:

$$\sum_{i=1}^{n-1} (something)$$

```
Loops(A[0..n-1])
 for i ← 1 to n-1 do
   v ← A[i]
   j ← i-1
   while j≥0 and A[j]>v do
      A[j+1] ← A[j]
      j ← j-1
   A[j+1] ← v
```

Example 3 analysis

- The inner loop:
  - *j goes from i-1 down to 0*
  - *At each iteration, we do one basic operation*
  - *Mathematically, the number of steps is:* $\sum_{j=0}^{i-1} 1$
- We do this for each iteration of the outer loop
- So the total number of basic operations is:

$$\sum_{i=1}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplifying the sum

- We know: $$\sum_{j=0}^{i-1} 1 = i$$
- So: $$\sum_{i=1}^{n-1} \sum_{j=0}^{i-1} 1 = \sum_{i=1}^{n-1} i$$
- Which equals: $$\frac{(n-1)n}{2}$$

(we just showed this… and it is in appendix A)

Two main areas of interest in this course

- How to design algorithms

- How to analyze algorithm efficiency
  - *Time/space efficiency*

Algorithm design techniques

- Brute force
- Divide and conquer
- Decrease and conquer
- Transform and conquer
- Space and time tradeoffs
- Greedy approach
- Dynamic programming
- Iterative improvement
- Backtracking
- Branch and bound

Important problem types

- Sorting
- Searching
- String processing
- Graph problems
- Combinatorial problems
- Numerical problems

Try it/homework

- Chapter 1.1 page 8, question 5
- Chapter 1.2 page 18, question 9
- Chapter 1.3 page 23, question 1