7.3 – 7.5 Hypothesis Testing of μ

In statistics, a <u>hypothesis</u> is a claim or statement about a population parameter.

Example – the mean mass of a bag of chips is 43g.

A <u>hypothesis test</u> (or test of significance) is a standard procedure for testing a claim about a property of a population.

Hypothesis testing is based on the following principle:

If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is probably not correct.

We will do a few examples of formal hypothesis tests and along the way we will learn technique and terminology.

eg - Assumption: $\mu \geq 43g$

Observed event: in a sample size $n=50$, $\bar{x} = 42.5g$

Q: if $\mu = 43g$, what is the probability that a random sample of size 50 would have $\bar{x} \leq 42.5g$?

If it is "very small", we conclude $\mu < 43g$.

Otherwise, we were just unlucky.

The significance α gives us a measure of what we mean by "very small".

Traditional Method for μ (two-tailed)

1.) **Assume that the amount of time it takes to send a 10 Mb file across a network is normally distributed. At $\propto = 0.05$ test the claim that the mean length of time to transfer a 10 Mb file across a network is 12.44 seconds by sending a 10 Mb file across a network at 20 random times, finding a mean of 13.22 seconds and a standard deviation of 2.65 seconds.**

Step 1: We begin a hypothesis test by stating the claim that is being tested. The claim is a statement about a population parameter – in this case, __μ__.

$H_0: \mu = 12.44s$

$H_1: \mu \neq 12.44s$

null hypothesis - contains equality $=, \leq, \geq$

alternative hypothesis - does not contain equality $\neq, <, >$

If $\bar{x}$ is significantly different from 12.44s then we have reason to believe that $\mu \neq 12.44$. Otherwise, the difference was probably due to chance.

Step 2: Find the *test statistic*, t_{test}. This measures how many standard deviations our sample statistic is from the claimed value of the population parameter, assuming our claim is correct.

$$t_{test} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{13.22 - 12.44}{\frac{2.65}{\sqrt{20}}}$$

$$= 1.316$$

If $\mu = 12.44s$, then our sample mean was 1.316 standard deviations larger.

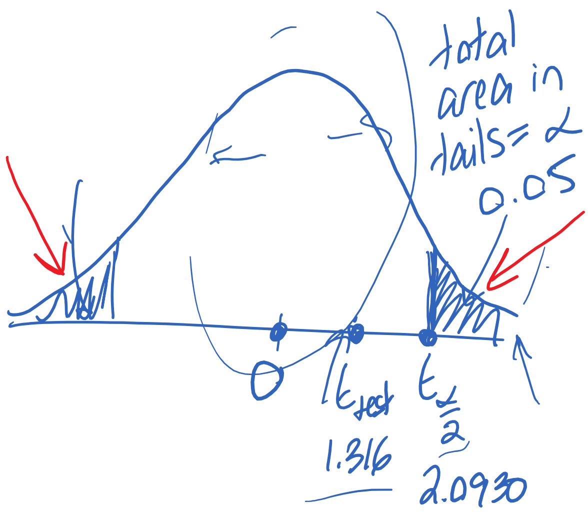
(not unusual!)

Step 3. Using the t-table, find $t_{\alpha/2}$. This gives us the boundary between the sample statistics that would lead us to reject the claim, and the sample statistics that would lead us not to reject it.

area in 2 tails = 0.05

df = n - 1 = 20 - 1 = 19

$t_{\frac{\alpha}{2}} = 2.0930$



Step 4. Draw a conclusion.

$|t_{test}| < |t_{\frac{\alpha}{2}}| \Rightarrow$ fail to reject H_0

At $\alpha = 0.05$, we do not have enough evidence to reject the claim that the mean transfer time for a 10MB file is 12.44s

Traditional Method for μ (right-tailed)

2.) At $\alpha = 0.05$ test the claim that the mean ~~length of time to~~ transfer a 10 Mb file across a network is greater than 12.44 seconds by sending a 10 Mb file across a ~~network 131~~ times, finding a mean of 13.22 seconds. Assume a population standard deviation of 2.65 seconds.

$H_0: \mu \leq 12.44$

$H_1: \mu > 12.44$

We know σ, so we use the z-table.

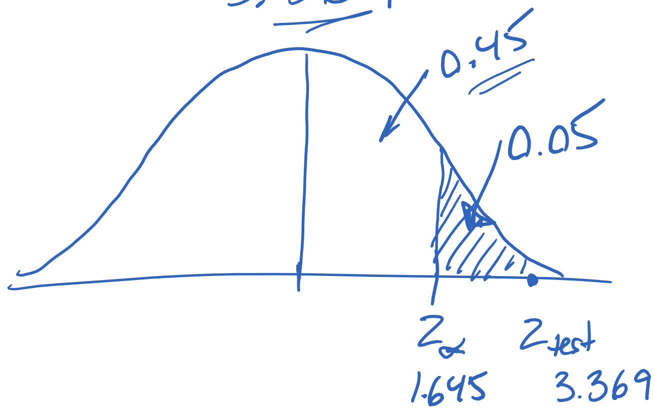
$$Z_{test} = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{13.22 - 12.44}{\frac{2.65}{\sqrt{131}}}$$

$$= 3.369$$

$\alpha = 0.05$

From z-table,

$Z_\alpha = 1.645$



$Z_{test} > Z_\alpha \Rightarrow$ reject H_0

At $\alpha = 0.05$, we have sufficient evidence that the mean time to transfer a 10MB file is greater than 12.44 s.

P-value Method for μ (right-tailed)

probability

The p-value is the probability of obtaining a value for the test statistic that is as extreme or more extreme than the value observed.

3.) At $\boxed{\alpha = 0.05}$ test the claim that the mean length of time to transfer a 10 Mb file across a network is greater than 12.44 seconds by sending a 10 Mb file across a network 131 times, finding a mean of 13.22 seconds. Assume a population standard deviation of 2.65 seconds.

σ

use z-table

$$H_0: \mu \le 12.44$$

$$H_1: \mu > 12.44$$

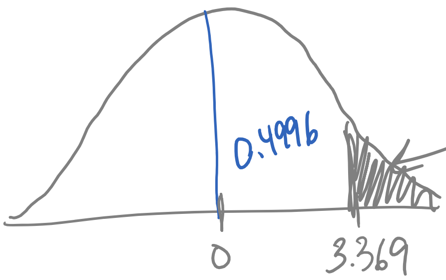
$$z_{test} = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{\sigma}{\sqrt{n}}} = \frac{13.22 - 12.44}{\frac{2.65}{\sqrt{131}}} = 3.369$$

The P-value is $P(\bar{x} > 13.22)$ assuming $\mu = 12.44$

Equivalently: $p = P(Z > 3.369)$

$$= 0.5 - 0.4996$$

$$= 0.0004$$



$P < \alpha \Rightarrow$ reject H_0

At $\alpha = 0.05$, we have sufficient evidence that the mean transfer time for a 10MB file is greater than 12.44 s.

What if we're wrong? Possible Errors in Hypothesis Testing – <u>Type I and Type II</u>

Type I Error - the mistake of rejecting the null hypothesis when it is actually true.

Type II Error - The mistake of failing to reject the null hypothesis when it is actually false.

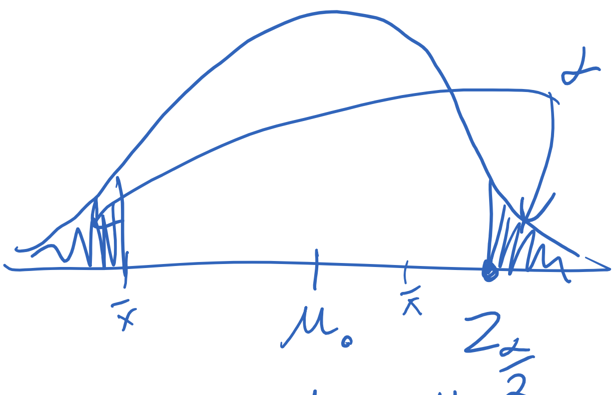
True State of Nature

		The null hypothesis is true	The null hypothesis is false
Decision	We decide to reject the null hypothesis	Type I error (rejecting a true null hypothesis) α	Correct decision
	We fail to reject the null hypothesis	Correct decision	Type II error (accepting a false null hypothesis) β

- **It is often up to the individual testing the claim to select an appropriate significance level α, which is the probability of a type I error**

How can we reduce the probability of making a Type I error?

If H_0 is true, then $\bar{x}$'s are distributed normally centred at μ_0

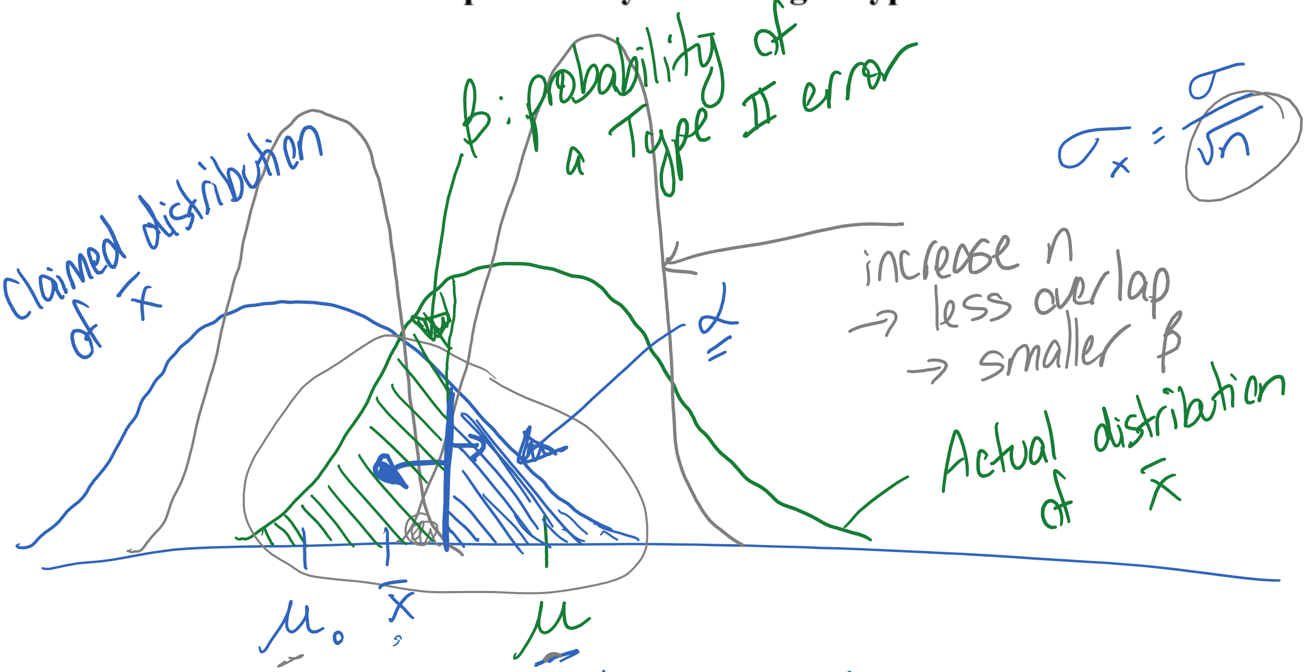


We reject H_0 if $\bar{x}$ is in the shaded region

Probability : α

We can reduce the probability of making a Type I error by reducing α.

How can we reduce the probability of making a Type II error?



We can reduce the probability of a Type II error by increasing α

OR: increase n (sample size)

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This week

- Lab 9 – normality testing and confidence intervals
- HW2 – due Friday if you emailed me
- Quiz 4 – Friday → Monday (CLT + CI

practice problems on LH