Multi-Agent Systems

Homework Assignment 3
MSc AI, VU

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3 Sequential Games with Perfect Information

3.1 Reduced centipede game

Consider a sequential 2-player game with the following game-tree: at each decision node the associated player needs to decide whether to continue (c) or stop (s). The tree (including utilities) and the players' rationality are common knowledge. Notice that the utility for both players increases along the game tree - and this is known to both players (perfect and complete information).

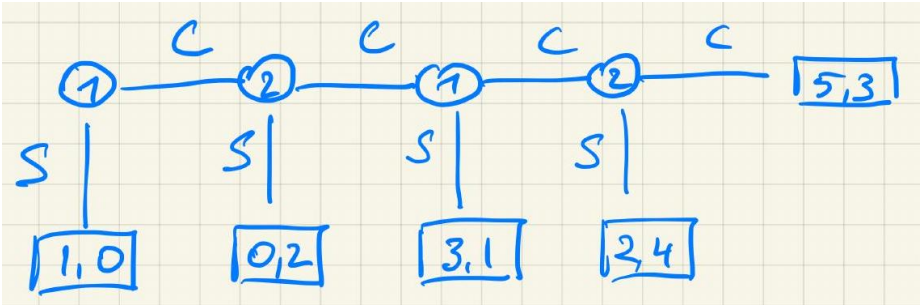


Figure 1: Shortened version of the centipede game.

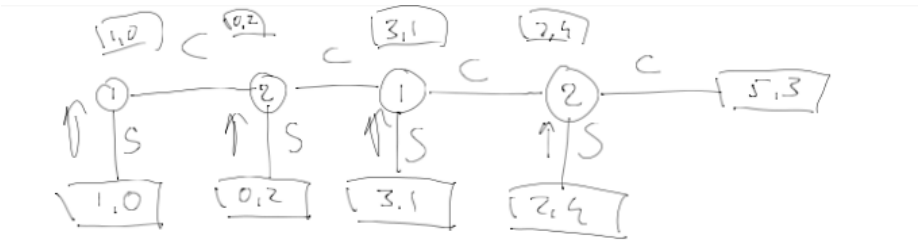
Questions

1. Use backward induction to predict the (rational) outcome of this game. Does it make sense to you?

   The rational outcome would be (1, 0). Player 1 will always stop at the first step, since player 2 will otherwise stop at the second step. Because player 2 knows player 1 will stop etc. This game makes no sense, because the player will always stop and there is never any indifference.

2. Write the normal form for this game and find all Nash equilibria in pure strategies (PNEs).

   There is a total of 6 NEs.



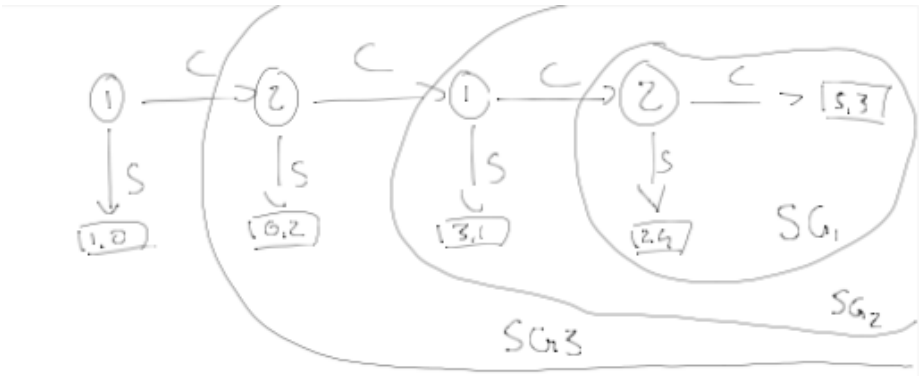
Player 1: $\{S_1, C_1\} \times \{S_3, C_3\} = \{S_1S_3, S_1C_3, C_1S_3, C_1C_3\}$

Player 2: $\{S_2, C_2\} \times \{S_4, C_4\} = \{S_2S_4, S_2C_4, C_2S_4, C_2C_4\}$

	S_2S_4	S_2C_4	C_2S_4	C_2C_4
S_1S_3	1) 1,0	2) 1,0	1,0	1,0
S_1C_3	3) 1,0	4) 1,0	1,0	1,0
C_1S_3	0,2	0,2	5) 3,1	3,1
C_1C_3	0,2	0,2	6) 2,4	5,3

3. List all subgames and determine which of these PNEs are also subgame-perfect?

   See the figure.



SG1: Player S chooses S_4 (NE)

SG2:

	S_4	C_4
S_3	3,1	3,1
C_3	2,4	5,3

{(S_3, S_4), (C_3, C_4)}

SG3:

	S_2S_4	S_2C_4	C_2S_4	C_2C_4
S_3	0,2	0,2	3,1	3,1
C_3	0,2	0,2	2,4	5,3

$S_1S_3S_2S_4 \rightarrow S_4$ S_3S_4 $S_3S_2S_4$

$S_1S_3S_2C_4 \rightarrow C_4$

$S_1C_3S_2S_4 \rightarrow S_4$ ~~C_3S_4~~

$S_1C_3S_2C_4 \rightarrow C_4$

$C_1S_3C_2S_4 \rightarrow S_4$ S_3S_4 ~~$S_3C_2S_4$~~

$C_1C_3C_2C_4 \rightarrow C_4$

3.2 Boss and stealing employee

A boss notices that one of her employees has been stealing company material lately. The material was not all that valuable, so she is inclined to let it pass, preferring to keep the employee around rather than firing him and having to hire and retrain a replacement. Nevertheless she wants the stealing to stop.

She is therefore thinking to issue a warning at the next company meeting:

the next person caught stealing company property will be fired immediately. She envisages the following game tree with pay-offs (see fig below).

1. Analyse this game using backward induction.

   See figure.

2. What are the pure actions for the two players (boss and employee)? Construct the normal form matrix.

   See figure

3. Use this matrix to identify all the pure Nash equilibria of the normal form game.

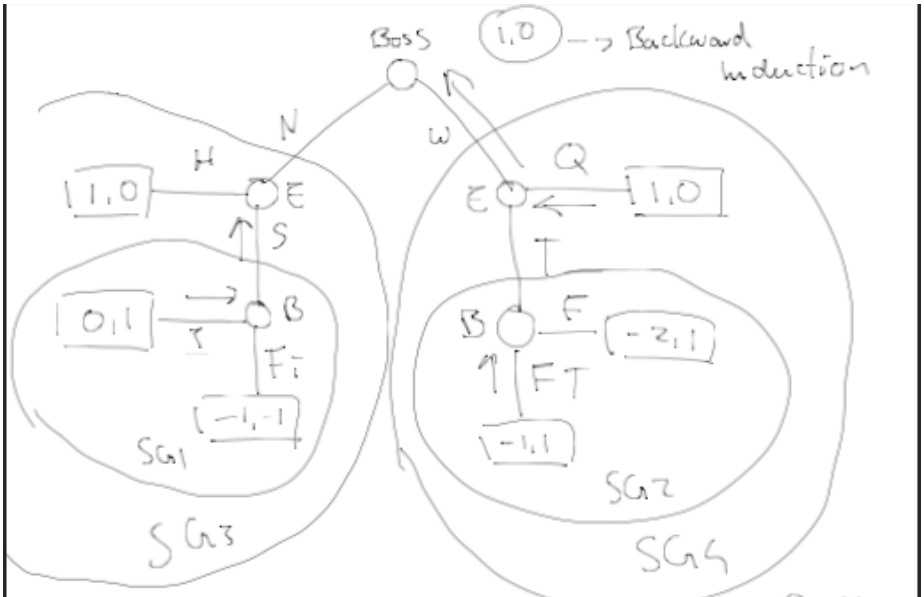
   All NE are given with a star.

4. Determine the subgame-perfect equilibrium (equilibria?) by eliminating all the Nash equilibria that fail to induce a NE in subgames.

   There is only 1 SPNE.

5. Compare to the solution based on backward induction.

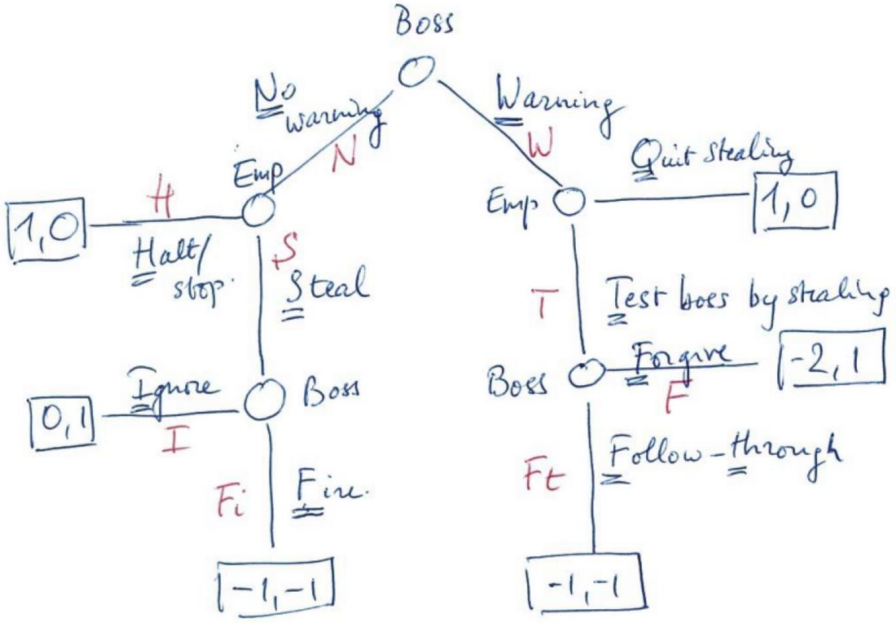
   The solution is exactly the same.



$\mathcal{R} = NE$

	HQ	HT	SQ	ST
NIF	1,0	1,0	0,1	0,1
NIF_t	1,0	1,0	0,1	0,1
NF_iF	1,0	1,0	-1,-1	-1,-1
NF_iF_t	1,0	1,0	-1,-1	-1,-1
WIF	1,0	-2,1	1,0	-2,1
WIF_t	1,0	-1,1	1,0	-1,-1
WF_iF	1,0	-2,1	1,0	-2,1
WF_iF_t	1,0	-1,-1	1,0	-1,-1

$\{(NIF_t), (SQ)\} \rightarrow SGPE$


Boss
No warning
N
Warning
W
Emp
H
1,0
Halt/ stop
S
Steal
Quit stealing
Emp
1,0
T
Test boss by stealing
Boss
Forgive
-2,1
F
Ignore
I
0,1
Boss
Fi
Fire
Ft
Follow-through
-1,-1
-1,-1