$$\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \varepsilon$$

$$\underset{\approx}{\mathcal{H}} \quad \underset{\sim}{V}^{\pm} = \underset{\sim}{V}^{\pm} \varepsilon^{\pm}$$

$$\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} c_1^+ & c_1^- \\ c_2^+ & c_2^- \end{bmatrix} = \begin{bmatrix} c_1^+ & c_1^- \\ c_2^+ & c_2^- \end{bmatrix} \begin{bmatrix} \varepsilon^+ & 0 \\ 0 & \varepsilon^- \end{bmatrix}$$

$$\boxed{\underset{=}{\mathcal{H}}} \quad \underset{\approx}{V} = \underset{\approx}{V} \quad \underset{=}{\varepsilon}$$

known

unknown eigenvectors

unknown eigenvalues

eigenvalues

$$(\underset{=}{\mathcal{H}} - \varepsilon)\, \underset{\sim}{V} = \underset{\sim}{0}$$

$$\boxed{\det(\mathcal{H} - \varepsilon) = 0}$$ to have solutions

$$\begin{vmatrix} -2-\varepsilon & -1 \\ -1 & -2-\varepsilon \end{vmatrix} = 0$$

the det. of a 2×2 matrix

$$\begin{vmatrix} a & b \\ b & c \end{vmatrix} \equiv ac - b^2$$

$$\Rightarrow (-2-\varepsilon)^2 - (-1)^2 = 0$$

$$(\varepsilon+2)^2 - 1 = 0$$

$$\Leftrightarrow (\varepsilon+2-1)(\varepsilon+2+1) = 0$$

$$\Leftrightarrow (\varepsilon+1)(\varepsilon+3) = 0$$

$$\Rightarrow \varepsilon = \begin{cases} -1 \\ -3 \end{cases}$$

given the 2 e'values of our 2×2 matrix, we can now solve for the 2 e'vectors

$$(\underset{\approx}{\mathcal{H}} - \varepsilon)\underset{\sim}{v} = 0$$

for $\varepsilon = -3$:

$$\left(\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\right)\begin{bmatrix} c_1^- \\ c_2^- \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} c_1^- \\ c_2^- \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$c_1^- - c_2^- = 0 \quad \Rightarrow c_1^- = c_2^- = c^-$$

$$-c_1^- + c_2^- = 0 \quad \Rightarrow c_1^- = c_2^-$$

$$\Rightarrow \text{e'vector is } \begin{bmatrix} c^- \\ c^- \end{bmatrix}$$

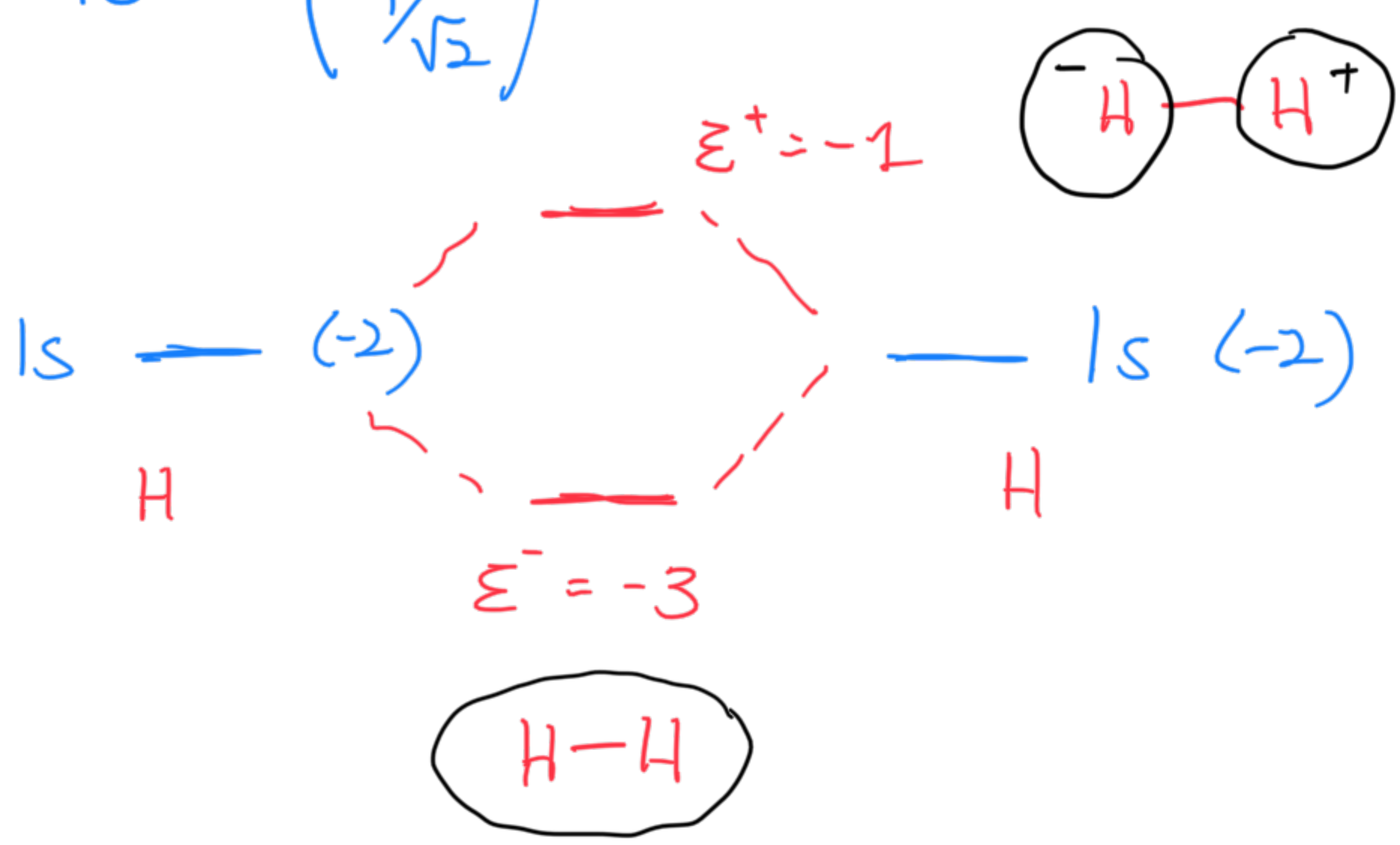
fix the value of c^- by normalization

$$\underset{\sim}{V}^T \cdot \underset{\sim}{V} = 1$$

$$\begin{bmatrix} c^- & c^- \end{bmatrix} \begin{bmatrix} c^- \\ c^- \end{bmatrix} = 1$$

$$2(c^-)^2 = 1 \quad \Rightarrow \quad c^- = \frac{1}{\sqrt{2}}$$

$$\underset{\sim}{V}^- = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \qquad \text{for } \varepsilon^- = -3$$



For the higher energy eigenvalue $\varepsilon^+ = -1$, we get

$$\underset{\sim}{V}^+ = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$