BENG MECHANICAL ENGINEERING FT

Pin-Jointed Truss

Experiment AM1.5

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This experiment looks at a simple pin-jointed truss under load and the corresponding forces in each member. This experiment aims to introduce pin-jointed trusses and the calculations that accompany them. This will be done by analysing the strain in members of a pin-jointed truss under varying load conditions. This experiment shows a linear relation between the load on the pin-jointed truss and the strain or force in individual members where the strain and force increases as the load increases.

[1] Introduction and Background

A truss is defined as "a simple structure composed of slender members joined together at their end points" [1]. Despite its simplicity the truss is used widely within engineering: from the framework of buildings and airplanes to the supports of many modern bridges. Because every beam is connected by pins allowing the beams to freely rotate about the joint the forces can only act parallel to the beam they act on, meaning that there are no shear forces. This particular structure allows for a simple mathematical model, which through the use of a few basic equations you can calculate all forces acting on the structure when under an external load. Because each beam itself is in equilibrium the forces at each end can be considered a couple of forces, acting with equal magnitude but opposite direction [2]. This fact makes analysis of the compressive or tensile forces for the purposes of calculating stress in the beam, or analysing stress to calculate the forces acting on the beam, relatively simple. This experiment is going to look at how the strain and force in members of a pin-jointed truss are effected by a varying load.

[2] Aims and Objectives

Working through this experiment and the accompanying mathematics and data analysis will introduce the idea of a pin-jointed truss as well as develop a better understanding of how to calculate the internal forces in the pin-jointed truss under an external load from a simple mathematical model. In addition this experiment aims to assist in understanding how to use strain gauges to measure the deflection of beams and how to use these strains as well as the Young's Modulus of the beam and the beams diameter, to calculate the force in the beams. Looking at these forces will demonstrate the accuracy of the initial theoretical model when compared to the experimental demonstration of the pin-jointed truss.

[3] Method

All measurements or calculations of force are in N with an error of $\pm 0.05\ N$ and all measures of distance are in mm to an accuracy of $\pm 0.005\ mm$.

Framework- TecQuipment STR8 Pin-Jointed Framework (TecQuipment Ltd., Nottingham, UK)

Test Frame- TecQuipment STR1 Universal Test Frame (TecQuipment Ltd., Nottingham, UK)

Digital Strain Display- TecQuipment Digital Strain Display (TecQuipment Ltd., Nottingham, UK)

Display- TecQuipment STR1A Digital Force Display (TecQuipment Ltd., Nottingham, UK)

Load Cell- TecQuipment STR8A 500N Load Cell (TecQuipment Ltd., Nottingham, UK)

[i] Experiment- Varying Load on the Pin-Jointed Truss

Set up the framework in the test frame as it is set up in **Figure 1**, attaching strain gauges to each beam as labelled. Connect strain gauges 1 through 7 to the digital strain display, using the same numbered ports. Connect the display and load cell. Using the display set the load cell to $0\ N$. Take measurements of strain in each beam. Increase the load by 100 N then take further measurements of strain in each beam, repeat this until the load reaches 500 N when the final measurement will be taken. Note each of these measurements of strain in **Table 1**. Measure the diameter of the beam using a micrometer. Use **Equation 4**, **Equation 5** and **Equation 6** to derive **Equation 7**, **Equation 8**, **Equation 9** and **Equation 10** (See Appendix). To calculate the theoretical force in each beam use **Equation 7**, **Equation 8**, **Equation 9**, **Equation 10** and the fact that $W = 500\ N$. To calculate the experimental force in each beam, first use **Equation 1** and the fact that the Young's Modulus of the steel is $210\ GPa$ [3] to calculate the stress on each beam. Then use **Equation 2** to calculate the area of the beam. Then use **Equation 3** to calculate the force in each beam. This is to be done at a load of $500\ N$. Note the theoretical and experimental values for force in each beam in **Table 2**.

[ii] Data Analysis

Results from **Table 1** were plotted onto the graph in **Figure 2**. Percentage error between theoretical and experimental forces from **Table 2** was calculated using **Equation 11**. Lines of best fit were obtained using the MATLAB function *polyfit* [5] and where obtained to a first order polynomial.

[4] <u>Results</u>

	True Member Strain (με)						
Load (N)	**Beam 1**	**Beam 2**	**Beam 3**	**Beam 4**	**Beam 5**	**Beam 6**	**Beam 7**
0	-22	-23	-26	-36	-15	-15	-43
100	-31	-33	-34	-31	-10	-6	-33
200	-41	-44	-45	-26	-6	4	-23
300	-50	-53	-54	-21	-1	14	-13
400	-59	-64	-65	-16	4	24	-3
500	-69	-73	-64	-11	9	34	7

Table 1: Results from the Experiment

Beam	**Experimental Force (N)**	**Theoretical Force (N)**	**Percentage Difference (%)**
1	-288.45	-288.7	0.087
2	-306.86	-288.7	6.290
3	-294.58	-288.7	2.037
4	153.43	144.3	6.327
5	147.29	144.3	2.072
6	300.72	288.7	4.163
7	306.86	288.7	6.290

Table 2: Comparison of Theoretical and Experimental Forces

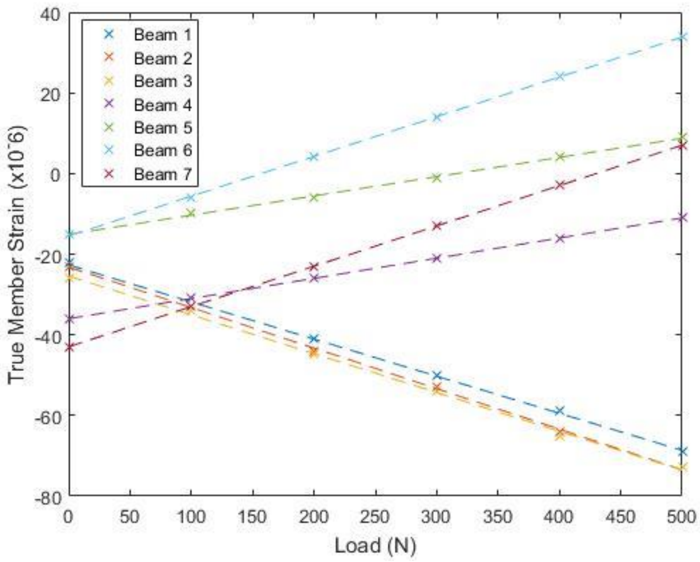

Figure 2: A Graph of Strain against Load on Each Beam

[5] Discussion

Looking at the data presented in **Table 1** it can be seen that the magnitude of the strain on each beam increased as the load increased, regardless of if the beam was in tension or compression. By using **Equation 1**, **Equation 2** and **Equation 3** it can be shown that the magnitude of the force also increases as load increases. **Table 2** shows that the variation between theoretical and actual values of force on each beam are very similar, varying by less than $6.5\ \%$. Looking at **Figure 2** it can be seen that the relationship between strain and load is linear under both tension and compression. This is further backed up by the relationship derived using **Equation 1**, **Equation 2** and **Equation 3**; which is $\varepsilon = \frac{F}{E \times A}$. This equation is a linear equation which explains the relationship between force and strain as $\varepsilon = F \times C$ where C is a constant which in this case represents $\frac{1}{E \times A}$. The largest percentage difference between the theoretical and actual results was $6.290\ \%$ which can be compared to the experimental uncertainty which can be calculated using **Equation 12**. The experimental uncertainty is $7.325\ \%$, as the difference between the theoretical values and experimental values is less than the experimental uncertainty it can be said that this experiment is accurate. This analysis of member strain shows that even under zero load the members are under strain, and by finding the initial strain and then using the fact that the gradient of the line is $\frac{1}{E \times A}$ where the force is the force given when using **Equation 7**, **Equation 8**, **Equation 9** or **Equation 10** for the corresponding beam. This allowed for a quick calculation of the maximum force the beams could support before the stress is at the limits of what the pin-jointed truss is designed for. This experiment shows that the strain in a beam is proportional to the force acting on that beam and inversely proportional to the Young's Modulus and cross-sectional area of the beam.

[6] Conclusion

In conclusion, this experiment shows that as the load on a pin-jointed increases the magnitudes of both strain and force increase. It has also shown that the relationship between strain and load is linear.

[7] References

[1] R. C. Hibbeler. 2014. **Statics and Mechanics of Materials**. 4^{th} Edition. Chapter 5: Structural Analysis. Page 243. Pearson; London.

[2] J. Awrejcewicz. 2012. **Classical Mechanics: Kinematics and Statics**. Chapter 2: Statics. Page 32. Springer; New York.

[3] School of Engineering, Birmingham. **Experiment AM1.5- Pin-Jointed Truss**. University of Birmingham; Birmingham.

[4] Lab Group A1. **Experiment AM1.5- Pin-Jointed Truss**. Experiment Conducted 17^{th} February 2017.

[5] MATLAB (MathWorks, Inc, Natick, USA)

[8] Appendix

Equation 1: $E = \frac{\sigma}{\varepsilon}$ $\quad Youngs\ Modulus = \frac{Stress}{Strain}$

Equation 2: $A = \frac{\pi d^2}{4}$ $\quad Area = \frac{\pi \times Diameter^2}{4}$

Equation 3: $\sigma = \frac{F}{A}$ $\quad Stress = \frac{Force}{Area}$

Equation 4: $\sum H = 0 \quad Sum\ of\ Horizontal\ Forces = 0$

Equation 5: $\sum V = 0 \quad Sum\ of\ Vertical\ Forces = 0$

Equation 6: $\sum M = 0 \quad Sum\ of\ Moments = 0$

Equation 7: $Force\ in\ Beam\ 1\ \&\ 3 = \frac{-W}{\sqrt{3}}$

Equation 8: $Force\ in\ Beam\ 4\ \&\ 5 = \frac{W}{2\sqrt{3}}$

Equation 9: $Force\ in\ Beam\ 2 = \frac{-W}{\sqrt{3}}$

Equation 10: $Force\ in\ Beam\ 6\ \&\ 7 = \frac{W}{\sqrt{3}}$

Equation 11: $Percentage\ Error = \frac{|Theoretical - Experimental|}{|Theoretical|} \times 100$

Equation 12: $Experimental\ Uncertainty = Uncertainty\ in\ Strain + Uncertainty\ in\ Force + Uncertainty\ in\ Diameter$

Sample Calculations

Equation 1:

$$E = \frac{\sigma}{\varepsilon} \quad \therefore \quad \sigma = E \times \varepsilon = 210 \times 10^9 \times -47 \times 10^{-6} = -9870000\ Pa$$

Equation 2:

$$Area = \frac{\pi \times (6.1 \times 10^{-3})^2}{4} = 2.922 \times 10^{-5}\ m^2$$

Equation 3:

$$\sigma = \frac{F}{A} \quad \therefore \quad F = \sigma \times A = -9870000 \times 2.922 \times 10^{-5} = -288.4\ N$$

Equation 4, 5 and 6 to derive Equation 7:

$$Looking\ at\ F_1\ (See\ Figure\ 1){:}\ Taking\ Moments\ about\ F_2$$

$$Letting\ the\ length\ of\ a\ rod\ be\ l; \quad l \times W = 2l \times F_1 \quad \therefore \quad F_1 = 2W$$

$$\sum V = 0 \quad \therefore \quad Force\ in\ Beam\ 1\ vertically = -2W$$

$$Total\ force\ in\ Beam\ 1 = \frac{-2W}{\sin(60)} = \frac{-W}{\sqrt{3}}$$

Equation 11:

$$\frac{|-288.45 - -288.7|}{|-288.7|} \times 100 = 0.087\ \%$$

Equation 12:

$$Experimental\ Uncertainty = \frac{0.05}{7} \times 100 + \frac{0.5}{500} \times 100 + \frac{0.005}{6.1} \times 100 = 7.325\ \%$$

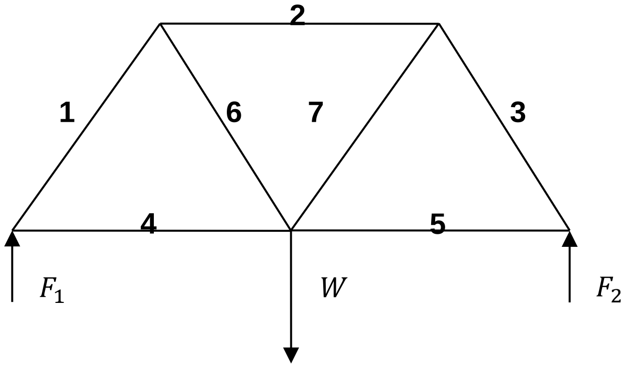

Figure 1: The Layout of the Framework