20170058 **Keonwoo Kim** · April 3, 2020
MAS109(E,F) Introduction to Linear Algebra

Quiz 1

Problems

MAS109EF Quiz 1. S#: 20170058 Name: Keonwoo Kim

#1. The given linear system can be transformed into the following form:

$$\left[\begin{array}{ccc|c} 7 & 1 & 8 & 20 \\ 3 & 2 & -8 & 5 \\ 2 & 1 & 5 & 12 \\ 1 & 1 & -2 & 4 \end{array}\right]$$

as an augmented matrix. By a series of elementary row operations on the augmented matrix above,

$$\left[\begin{array}{ccc|c} 7 & 1 & 8 & 20 \\ 3 & 2 & -8 & 5 \\ 2 & 1 & 5 & 12 \\ 1 & 1 & -2 & 4 \end{array}\right] \xrightarrow{E(1,4)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 3 & 2 & -8 & 5 \\ 2 & 1 & 5 & 12 \\ 7 & 1 & +8 & 20 \end{array}\right]$$

$$\xrightarrow[\longrightarrow]{\longrightarrow}{}^{\!\!\!\!E(1,2;-3),\,E(1,3;-2),\,E(1,4;-7)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & -1 & -2 & -7 \\ 0 & -1 & 9 & 4 \\ 0 & -6 & 22 & -8 \end{array}\right]$$

$$\xrightarrow{E(2;-1)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & +1 & +2 & +7 \\ 0 & -1 & 9 & 4 \\ 0 & -6 & 22 & -8 \end{array}\right]$$

$$\xrightarrow{E(2,1;-1),\,E(2,3;1),\,E(2,4;6)} \left[\begin{array}{ccc|c} 1 & 0 & -4 & -3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 11 & 11 \\ 0 & 0 & 34 & 34 \end{array}\right]$$

$$\xrightarrow{E(3;\frac{1}{11})} \left[\begin{array}{ccc|c} 1 & 0 & -4 & -3 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 34 & 34 \end{array}\right]$$

$$\xrightarrow{E(3,1;4),\,E(3,2;-2),\,E(3,4;-34)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Thus the given system of linear equations is consistent, with the unique solution $X = [7 \quad 5 \quad 1]^T$.

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#2. The given system can be translated into the following augmented matrix:

$$\left[\begin{array}{cccc|c} 2 & 3 & 1 & 2 & 5 \\ 1 & 1 & 4 & 5 & 2 \\ 3 & -1 & -4 & 3 & 6 \end{array}\right]$$

By a series of elementary row operations,

$$\left[\begin{array}{cccc|c} 2 & 3 & 1 & 2 & 5 \\ 1 & 1 & 4 & 5 & 2 \\ 3 & -1 & -4 & 3 & 6 \end{array}\right] \xrightarrow{E(1,2)} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 5 & 2 \\ 2 & 3 & 1 & 2 & 5 \\ 3 & -1 & -4 & 3 & 6 \end{array}\right]$$

$$\xrightarrow{E(1,2;-2),\ E(1,3;-3)} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 5 & 2 \\ 0 & 1 & -7 & -8 & 1 \\ 0 & -4 & -16 & -12 & 0 \end{array}\right]$$

$$\xrightarrow{E(3;-\frac{1}{4})} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 5 & 2 \\ 0 & 1 & -7 & -8 & 1 \\ 0 & 1 & 4 & 3 & 0 \end{array}\right]$$

$$\xrightarrow{E(2,3)} \left[\begin{array}{cccc|c} 1 & 1 & 4 & 5 & 2 \\ 0 & 1 & 4 & 3 & 0 \\ 0 & 1 & -7 & -8 & 1 \end{array}\right]$$

$$\xrightarrow{E(2,1;-1),\ E(2,3;-1)} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & -11 & -11 & 1 \end{array}\right]$$

$$\xrightarrow{E(3;-\frac{1}{11})} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 4 & 3 & 0 \\ 0 & 0 & 1 & 1 & -1/11 \end{array}\right]$$

$$\xrightarrow{E(3,2;-4)} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & 0 & -1 & 4/11 \\ 0 & 0 & 1 & 1 & -1/11 \end{array}\right].$$

with an unspecified variable x_4. Thus, the set of all solutions to the given system is:

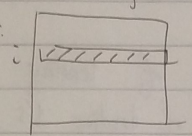
$$\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + 2x_4 = 2,\ x_2 - x_4 = 4/11,\ x_3 + x_4 = -1/11\}$$

$$= \left\{\left(-2t+2,\ \frac{4}{11}+t,\ -\frac{1}{11}-t,\ t\right) : t \in \mathbb{R}\right\}.$$

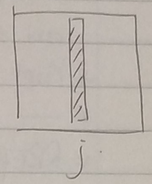
Summary

Mar 17. * Matrix: rectangular array of real numbers.

i^{th} Row:



j^{th} column:



* vector = an $n \times 1$ matrix
* operations in $\mathbb{R}^n$: elementwise addition & scalar multiplication.

  dot product: $\mathbb{x} \cdot \mathbb{y} := \mathbb{x}^T \mathbb{y}$.

  $\mathbb{u} \cdot \mathbb{v} = 0 \iff \mathbb{u}$ is orthogonal to $\mathbb{v}$.
* Matrix multiplication $A = [C_1 \cdots C_n]$. $B = \begin{bmatrix} R_1' \\ \vdots \\ R_n' \end{bmatrix}$.

  $AB = C_1 R_1' + \cdots + C_n R_n'$.

  $AB = \left(\sum_j a_{ij} b_{jk} \right)_{ik}$
* Matrix as a linear map: $A: \mathbb{x} \mapsto A\mathbb{x}$.

Mar 19. * System of linear equations. $A\mathbb{x} = \mathbb{b}$

$A = (a_{ij})$: $1 \le i \le m$, $1 \le j \le n$

$$A\mathbb{x} = \begin{bmatrix} a_{11}x_1 + \cdots + a_{1n}x_n \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n \end{bmatrix} = \mathbb{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}.$$

$\Updownarrow$ 1-1.

* Augmented matrix $[A \mid \mathbb{b}]$.

* $A\mathbb{x} = \mathbb{b}$ & $A'\mathbb{x} = \mathbb{b}'$ have the same set of solutions if one can transform $[A \mid \mathbb{b}]$ into $[A' \mid \mathbb{b}']$ with elementary row ops.

  ↳ multiply row by a nonzero const, interchange rows, add a multiple of a row to another one.

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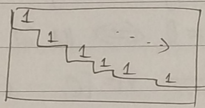
* Gaussian elimination to make a matrix be in a (reduced) row echelon form.
* Pivot position: position of leading 1's.
* — column: the column index of leading 1's.
* RREF = Row echelon form + each leading 1 is the unique nonzero entry in its column.

zero rows are at bottom.
first nonzero entry is 1 in each nonzero row.



leading 1's go right from top to bottom.

* $A\mathbb{x} = \mathbb{b}$ is consistent if it has a solution.
* otherwise, inconsistent
* $A\mathbb{x} = \mathbb{0}$: homogeneous linear system.
  $A\mathbb{x} = \mathbb{b}$ ($\mathbb{b} \neq \mathbb{0}$) : inhomogeneous lin. sys.
* $A\mathbb{x} = \mathbb{0}$ has $\mathbb{x} = \mathbb{0}$ as a solution, called "trivial soln".
  Other solutions which $A\mathbb{x} = \mathbb{0}$ may has are called nontrivial solutions.