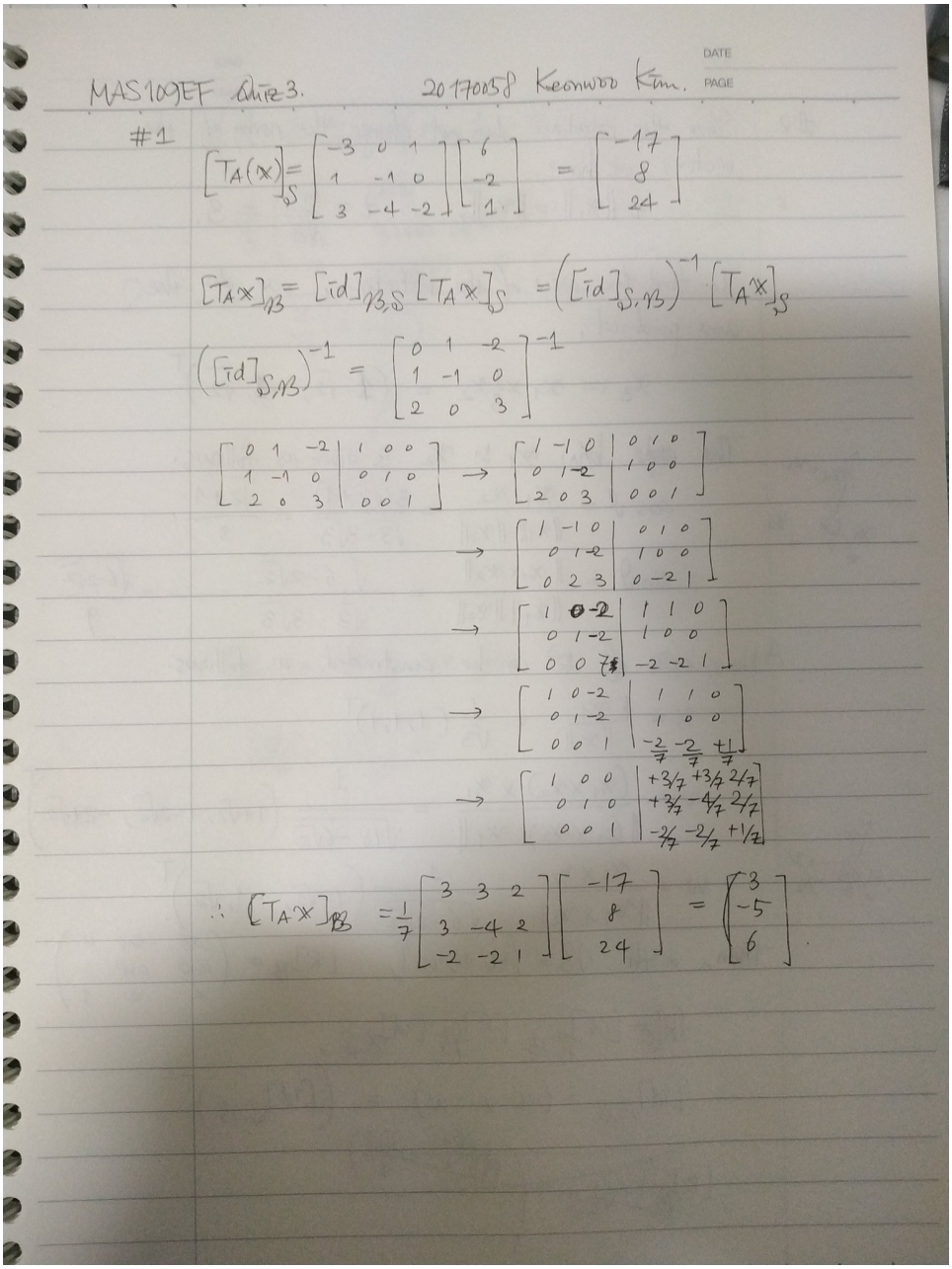
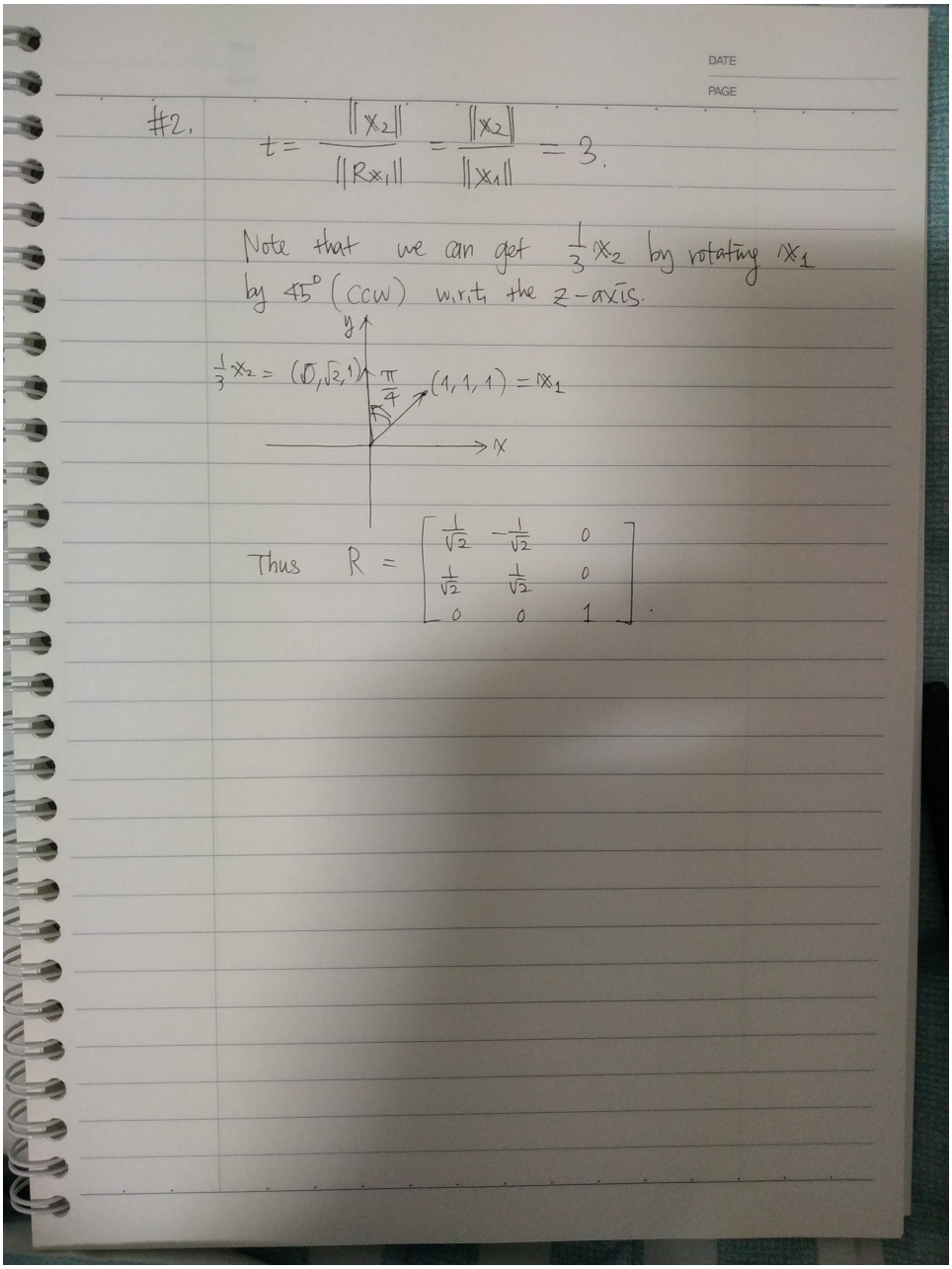
20170058 Keonwoo Kim · April 14, 2020
MAS109(E,F) Introduction to Linear Algebra

Quiz 3

Problems





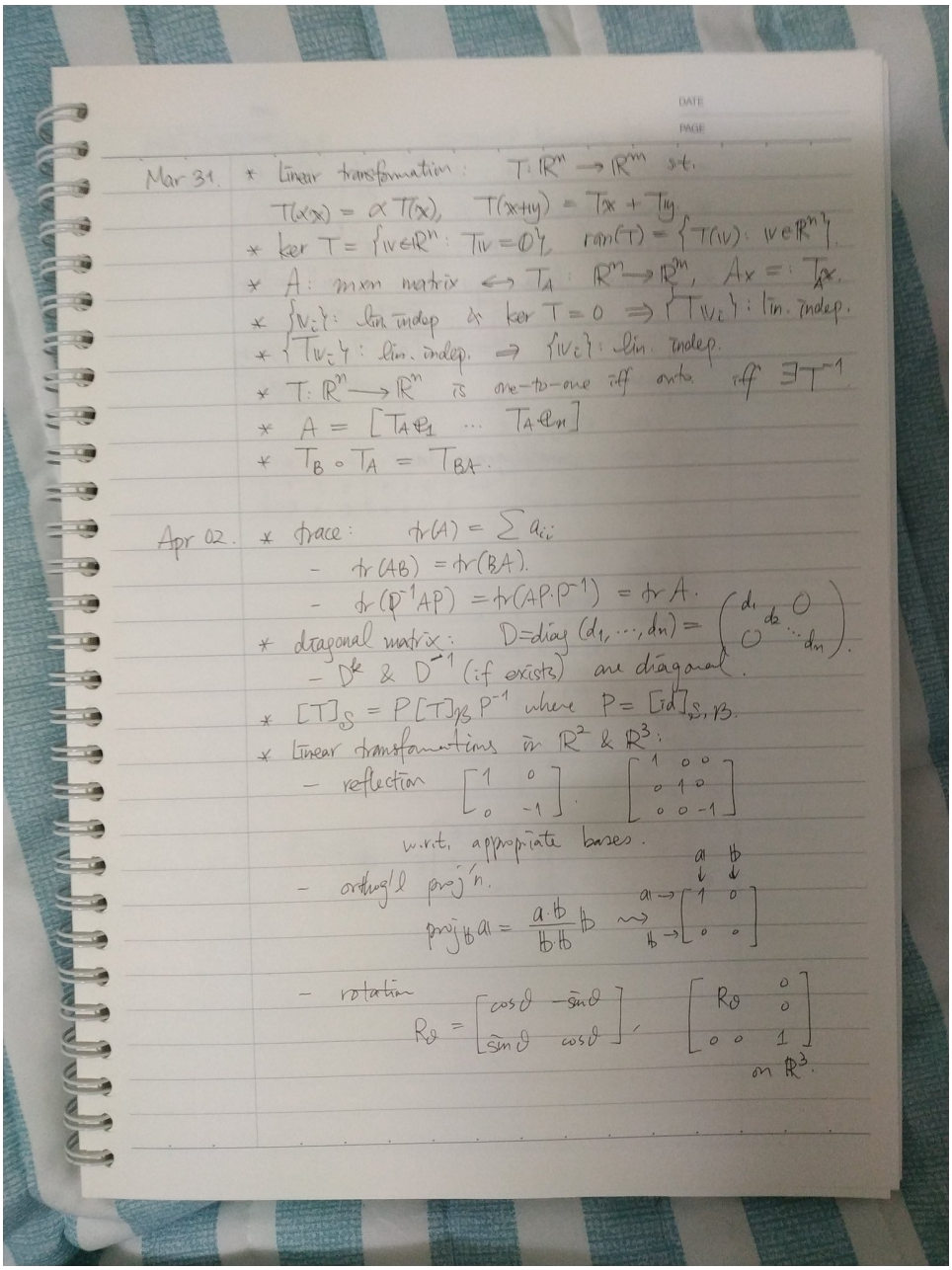
#2.

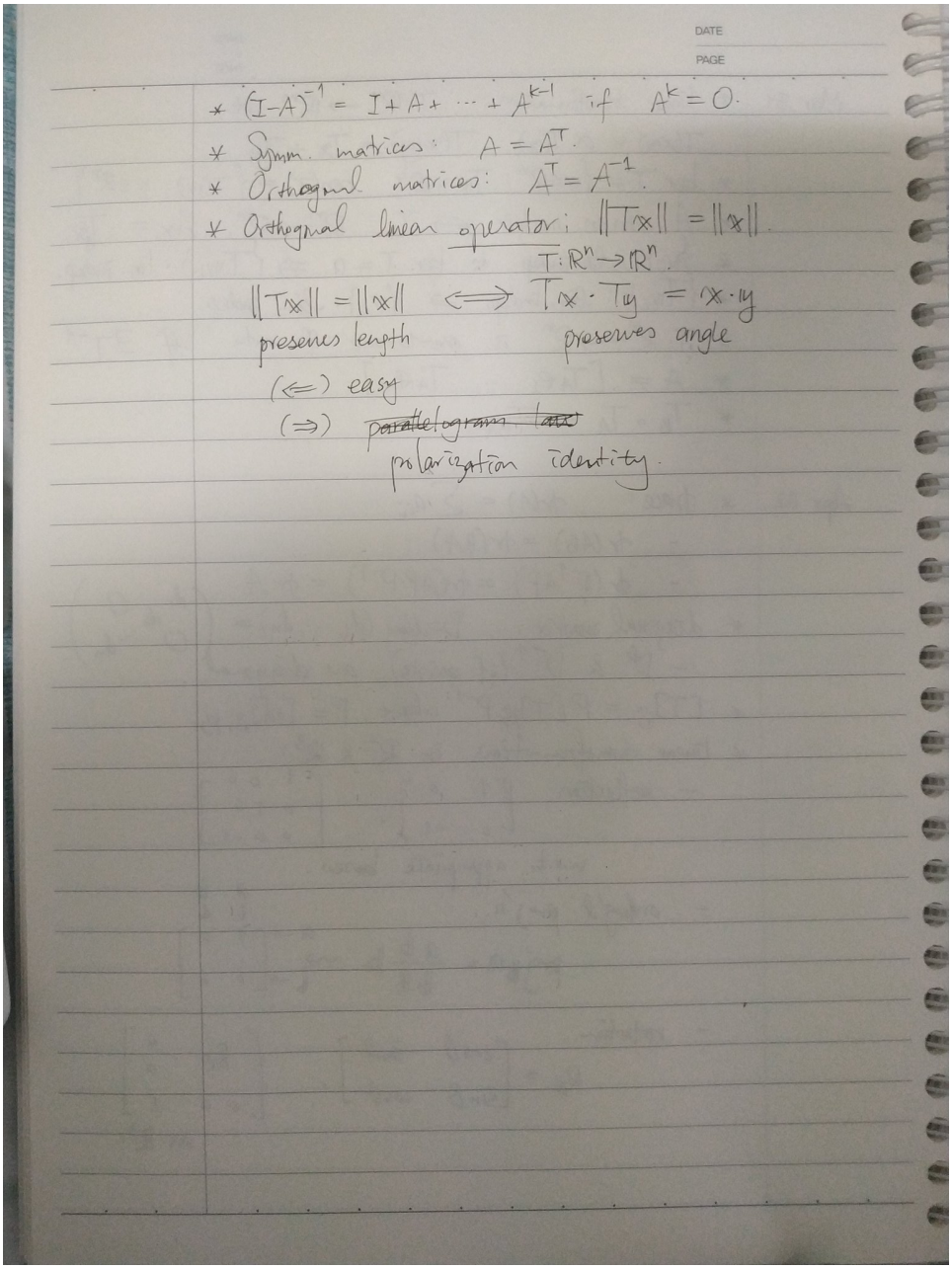
$$t = \frac{\|x_2\|}{\|Rx_1\|} = \frac{\|x_2\|}{\|x_1\|} = 3.$$

Note that we can get $\frac{1}{3}x_2$ by rotating x_1 by 45° (ccw) w.r.t. the z-axis.

Thus $R = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Summary


Mar 31.
* Linear transformation: $T: \mathbb{R}^n \to \mathbb{R}^m$ s.t.
$T(\alpha x) = \alpha T(x)$, $T(x+y) = Tx + Ty$
* $\ker T = \{v \in \mathbb{R}^n : Tv = 0\}$, $\mathrm{ran}(T) = \{T(v) : v \in \mathbb{R}^n\}$
* A: $m \times n$ matrix $\leftrightarrow$ $T_A : \mathbb{R}^n \to \mathbb{R}^m$, $Ax =: T_A x$.
* $\{v_i\}$: lin. indep. & $\ker T = 0 \Rightarrow \{Tv_i\}$: lin. indep.
* $\{Tv_i\}$: lin. indep. $\Rightarrow \{v_i\}$: lin. indep.
* $T: \mathbb{R}^n \to \mathbb{R}^n$ is one-to-one iff onto iff $\exists T^{-1}$
* $A = [T_A e_1 \ \cdots \ T_A e_n]$
* $T_B \circ T_A = T_{BA}$.
Apr 02.
* trace: $\mathrm{tr}(A) = \sum a_{ii}$
- $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.
- $\mathrm{tr}(P^{-1}AP) = \mathrm{tr}(AP \cdot P^{-1}) = \mathrm{tr}\, A$.
* diagonal matrix: $D = \mathrm{diag}(d_1, \cdots, d_n) = \begin{pmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & \ddots \\ & & & d_n \end{pmatrix}$.
- D^k & D^{-1} (if exists) are diagonal.
* $[T]_\delta = P[T]_\beta P^{-1}$ where $P = [\mathrm{id}]_{\delta,\beta}$.
* Linear transformations in $\mathbb{R}^2$ & $\mathbb{R}^3$:
- reflection $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
w.r.t. appropriate bases.
- orthog'l proj'n.
$\mathrm{proj}_b a = \frac{a \cdot b}{b \cdot b} b$ $\rightsquigarrow$ $\begin{matrix} a \to \\ b \to \end{matrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (a, b)
- rotation
$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $\begin{bmatrix} R_\theta & 0 \\ 0 & 1 \end{bmatrix}$ on $\mathbb{R}^3$.


DATE
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* $(I-A)^{-1} = I + A + \cdots + A^{k-1}$ if $A^k = O$.
* Symm. matrices: $A = A^T$.
* Orthogonal matrices: $A^T = A^{-1}$.
* Orthogonal linear operator: $\|Tx\| = \|x\|$.
$T: \mathbb{R}^n \to \mathbb{R}^n$.
$\|Tx\| = \|x\| \iff Tx \cdot Ty = x \cdot y$
preserves length
preserves angle
($\Leftarrow$) easy
($\Rightarrow$) ~~parallelogram law~~
polarization identity.