Online Newton Step for Portfolio Selection with Side Information

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***Abstract*—Online portfolio selection, as a research hotspot in financial signal processing, has been widely studied with machine learning perspective in recent years. Online Newton Step (ONS), which generates portfolios with low-complexity online convex optimization and achieves the same asymptotic wealth as the Best Constant-Rebalanced Portfolio in hindsight, is one promising algorithm among various portfolio selection strategies. However, ONS does not consider the downside risk, which leads to large investment loss in some market environments. To overcome this limitation, this paper proposes a novel portfolio selection method, namely ONS with Side Information (ONS-SI), which incorporates ONS with the side information derived from the market data, to reduce the investment risk. The performance of ONS-SI is evaluated on the Chinese A-share market. Experiment results show that the proposed ONS-SI achieves higher wealth and lower downside risk than ONS.**

Keywords-portfolio selection; side information; online newton step

I. INTRODUCTION

In the past decades, online Portfolio Selection (PS) has attracted increasing interests from the researchers with different research background, including machine learning [1]-[3], computer science [4], [5], information theory [6]-[8] and computational finance [9]-[11]. The major target of online PS is to achieve certain financial goal in the long run by sequentially distributing the wealth on a set of financial instruments.

Early studies on PS can trace back to the mean variance theory [9], which focuses on single period PS to achieve a tradeoff between the expected return and risk. In contrast, capital growth theory (or Kelly investment) [7] focused on multiple-period PS and tries to maximize the expected log return. Thus, Kelly's model is more suitable for online PS problem, and has been a baseline for most online PS algorithms.

The Buy And Hold (BAH) [17] and Constant-Rebalanced Portfolio (CRP) [10] are two common benchmarks in the PS problem. BAH strategy chooses a portfolio to buy assets at the beginning and holds the position until the end of the investment. The Uniform BAH is usual regarded as the market strategy. Another special BAH strategy is Best Stock in hindsight, which only buys the best assets. CRP is a more profitable benchmark, which keeps a fixed proportion of total wealth on each asset in every period. Similarly, Uniform CRP is one of the first to think about. Another special CRP, the Best CRP in hindsight, which has all the market information in advance, is the optimal CRP. Cover and Thomas [13] proved that the Best CRP perform as well as or better than any other portfolio in a i.i.d. market. Although the real market is usually not a i.i.d. market, experiment result shows that Best CRP can achieve greater wealth than other benchmarks. Thus, Best CRP is usually applied as the standard benchmark to evaluate the performance of an online PS algorithm. Hazan et al. [14], [15] evaluated the performance of an online PS algorithm with respect to the $regret$, which is defined by the relative difference between the logarithmic growth ratio it can achieve over the entire period and that achieved by Best CRP.

First, Cover [10] proposed Universal Portfolio (UP) algorithm and proved that UP can asymptotically achieve the same expected log return as the Best CRP without making any statistical assumptions on the assets' returns. The $regret$ achieved by UP is $O(\log n)$. However, the computational complexity of UP is exponential in the number of assets, which makes it computationally inefficient to deal with large portfolios. Later, Helmbold et al. [12] presented the Exponentiated Gradient (EG) algorithm, which has acceptable computational complexity, with $O(\sqrt{n})$ $regret$ under the no-junk-bond assumption [15]. Thus, EG is sub-optimal compared with UP algorithm. Hazan et al. [14], [15] proposed the Online Newton Step (ONS) algorithm, which is not only efficient but also optimal with respect to UP algorithm. The $regret$ achieved by ONS is $O(\log n)$, under the no-junk-bond assumption, and degenerates to $O(\sqrt{n})$ without the assumption. ONS lays basis on follow the leader algorithm, which chooses the Best CRP so far for the next period [15]. Moreover, ONS exploits the low-complexity newton method, which can take advantage of second-order information, to solve the optimization problem.

Besides, Cover [6] took the side information into consideration to adjust and update the portfolio and presented an UP with Side Information algorithm. However, the UP with Side Information algorithm is still computationally inefficient. Following Cover's ideas, Helmbold [12], Fagiuoli [16] and Bean [8] exploited the side information in their own algorithms.

However, the strategies mentioned above don't consider the downside risk during the investment. Although the wealth achieved by ONS has theoretic guarantee, ONS still has high downside risk during practical deployment.

Considering the fluctuating market environments, the market can be divided into different states. Therefore, we introduce an investment strategy motived by Cover [6], namely Online Newton Step with Side Information (ONS-SI), which exploits the market states as the side information to reduce the downside risk. Experiments result show that ONS-SI can achieve higher wealth and lower downside risk than ONS.

The remainder of this paper is organized as follows. Section II introduces the online PS problem. Then, Section III describes the strategy we propose, and Section IV presents the result of experiments on real financial market. Finally, Section V concludes this paper.

II. PRELIMINARIES

We consider the online PS problem of investment in a financial market with m assets for n trading periods. On each period t, the change of assets prices is denoted by a price relative vector $\mathbf{r}_t = (\mathbf{r}_t(1), \cdots, \mathbf{r}_t(m)) \in \mathbb{R}_+^m$. The ith entry $\mathbf{r}_t(i)$ denotes the ratio of the closing price of asset i on period t to the closing price on period $(t-1)$. The portfolio we choose at period t is denoted by $\mathbf{p}_t = (\mathbf{p}_t(1), \cdots, \mathbf{p}_t(m)) \in \mathbb{R}^m$, where i th entry $\mathbf{p}_t(i)$ denotes the proportion of wealth invested in asset i. We assume that only long position is allowed in the investment. Thus, $\mathbf{p}_t$ is a point in the m-dimensional simplex Δ_m, where $\Delta_m = \{\mathbf{p} \in \mathbb{R}^m : \sum_{i=1}^m \mathbf{p}(i) = 1, \mathbf{p}(i) \geq 0\}$. For convenience, we denote $\mathbf{r}^n = (\mathbf{r}_1, \cdots, \mathbf{r}_n)$ as the market sequence of price relative vectors and $\mathbf{p}^n = (\mathbf{p}_1, \cdots, \mathbf{p}_n)$ as the sequence of portfolios for n periods.

If the investor chooses a portfolio $\mathbf{p}_t$ at the beginning of period t, his wealth will change by a factor of $w_t = \mathbf{p}_t^T \mathbf{r}_t$ at the end of the period. Based on the past market sequence $\mathbf{r}^t = (\mathbf{r}_1, \cdots, \mathbf{r}_t)$, the investor needs to choose a new portfolio $\mathbf{p}_{t+1}$ for period $(t+1)$, and reinvest his wealth on the assets. Under this setup, the wealth achieved by the sequence of portfolios $\mathbf{p}^n$ after n periods is $W_n = W_n(\mathbf{p}^n, \mathbf{r}^n) = W_0 \prod_{t=1}^n \mathbf{p}_t^T \mathbf{r}_t$, where W_0 is the initial wealth. For convenience, we set $W_0 = 1$. We denote $G_n(\mathbf{p}^n, \mathbf{r}^n) = \log(W_n) = \sum_{t=1}^n \log(\mathbf{p}_t^T \mathbf{r}_t)$ as the logarithmic growth ratio of a PS strategy. A CRP strategy chooses a fix portfolio $\mathbf{p}$ for each period and achieves a wealth of $W_n = \prod_{t=1}^n \mathbf{p}^T \mathbf{r}_t$ and logarithmic growth ratio $G_n(\mathbf{p}^n, \mathbf{r}^n) = G_n(\mathbf{p}, \mathbf{r}^n) = \sum_{t=1}^n \log(\mathbf{p}^T \mathbf{r}_t)$. The Best CRP in hindsight $\mathbf{p}^*$ is the portfolio which achieves the maximum logarithmic growth ratio $G_n(\mathbf{p}, \mathbf{r}^n)$, that is, $\mathbf{p}^* = \arg\max_{\mathbf{p} \in \Delta_m} G_n(\mathbf{p}, \mathbf{r}^n)$. Following Agarwal et al. [15], the $regret$ of an online algorithm is defined by $regret \triangleq \sum_{t=1}^n \log(\mathbf{r}_t^T \mathbf{p}^*) - \sum_{t=1}^n \log(\mathbf{p}_t^T \mathbf{r}_t)$.

Note that scaling $\mathbf{r}_t$ by a factor have no effect on the $regret$. Thus, Agarwal et al. [15] assume that for all t, $\mathbf{r}_t$ is scaled so that $\max_i \mathbf{r}_t(i) = 1$. Besides, in this paper, we also make the no-junk-bond assumption [15], which means that all the $\mathbf{r}_t(i) \geq \alpha > 0$, where α is called the market variability parameter.

Moreover, we assume that the investor can buy and sell the required quantities of assets at closing price every period, and the PS strategy has no impact on the market behavior [17].

A. Side Information

In the above model, only the market data sequence $\mathbf{r}^n$ is utilized for PS. In practice, we can obtain side information about the state of the market, which can be used for adjusting the portfolio. Following Cover and Ordentlich [6], we present the side information as an integer y taking on values in a finite set $K = \{1, \cdots, k\}$. Thus, the sequence of side information state is denoted by $y^n = (y_1, \cdots, y_n)$, where $y_t \in K = \{1, \cdots, k\}$ for $t = 1, \cdots, n$. Note that, the side information state y_t is available at the beginning of each trading period [6]. Thus, at the start of period t, we use the past market sequence $\mathbf{r}^{t-1}$ and the side information sequence y^t to produce a portfolio $\mathbf{p}_t = \mathbf{p}_t(\mathbf{r}^{t-1}, y^t)$ for current period. Following the idea of Cover and Ordentlich [6], the market sequence $\mathbf{r}^n$ can be segmented into k subsequences based on the corresponding side information. The investor can handle each subsequence separately to run his strategy, and the final logarithmic growth ratio on $\mathbf{r}^n$ is the sum of each logarithmic growth ratio generated by the k subsequences.

There are many ways to get the sequence of side information states. For example, the side information can be the indicator of the assets which have the best performance so far [6]. The side information can be a causal function of the past market performance [12], or the indicator of the best investment each period with the knowledge of the future information. We note that the side information state can only be learned from the past performance of the market [12], since we have no information about the future.

B. Transaction Cost

As a practical setting, transaction cost must be paid for reallocating the wealth. Based on [1], [3], [8], [17], we assume a symmetric transaction cost proportion c, $0 \leq c < 1$, for both buy and sell, which means c dollar is paid for the trade of every 1 dollar. Hence, the transaction cost for reallocating the total wealth to the portfolio $\mathbf{p}_{t+1}$ at the end of period t is given by $C_t = w_t c \sum_{i=1}^m |\mathbf{p}_{t+1}(i) - \bar{\mathbf{p}}_t(i)|$, where $\bar{\mathbf{p}}_t = \frac{1}{\mathbf{p}_t^T \mathbf{r}_t} [\mathbf{p}_t(1)\mathbf{r}_t(1), \cdots, \mathbf{p}_t(m)\mathbf{r}_t(m)]^T$ is the proportion of m assets before reallocating. Thus, the final wealth is calculated as $W_n^c = w_n \prod_{t=1}^{n-1} (w_t - C_t)$.

III. THE PROPOSED ONS WITH SIDE INFORMATION ALGORITHM

We first introduce the ONS algorithm. ONS includes three parameters η, β and δ [15]. The portfolio generated by ONS is given by $\mathbf{p}_t = \arg\max_{\mathbf{p} \in \Delta_m} \sum_{\tau=1}^{t-1} f_\tau(\mathbf{p}) - \frac{\beta}{2} \|\mathbf{p}\|^2$, where $f_t(\mathbf{p}) \triangleq \log(\mathbf{p}_t^T \mathbf{r}_t) + \nabla_t^T (\mathbf{p} - \mathbf{p}_t) - \frac{\beta}{2} [\nabla_t^T (\mathbf{p} - \mathbf{p}_t)]^2$, $\nabla_t = \nabla[\log(\mathbf{p}_t^T \mathbf{r}_t)] = \frac{\mathbf{r}_t}{\mathbf{p}_t^T \mathbf{r}_t}$ and $\beta = \frac{\alpha}{8\sqrt{m}}$.

The performance of ONS is proved to be, asymptotically, as well as the Best CRP [15]. However, ONS algorithm does not consider the risk in the market. The investor suffers from risk due to the volatility in the market. The market can be divided into two states, i.e., bull market and bear market. It is easy to gain wealth in the bull market, since it is a rising market. Traditional trading strategies use many technical analysis methods [6], such as moving average of one composite index, to track the trend of the market. The moving average of price x_t is given by

$$\bar{x}_t \triangleq \begin{cases} \dfrac{1}{t}\displaystyle\sum_{\tau=1}^{t} x_\tau & d > t \geq 1 \\ \dfrac{1}{d}\displaystyle\sum_{\tau=t-d+1}^{t} x_\tau & t \geq d \end{cases},$$

where d is the window size.

In this paper, we assume the side information y_t has $k = 2$ states, according to whether the market index exceeds its moving average at period $(t-1)$, i.e.,

$$y_t = \begin{cases} 1 & x_{t-1} > \bar{x}_{t-1} \\ 2 & x_{t-1} \leq \bar{x}_{t-1} \end{cases},$$

where x_t represents one composite index associated with the market and $\bar{x}_t$ is the moving average of x_t. For initialization, we set $y_1 = 2$. When $y_t = 2$, the market has high probability to go down, so we sell assets, hold cash and wait for the next bull market. Hence, the proposed ONS-SI algorithm can be demonstrated in **Algorithm 1**.

To sum up, the proposed ONS-SI exploits the side information to improve the final wealth and reduce the downside risk, with no increase on the computational complexity compared with ONS.

IV. Experiments

In this section, we evaluate the proposed ONS-SI algorithm on two real datasets obtained from the Chinese A-share market. The two datasets are formed by top 50 liquid stocks selected from constituents of the CSI 300 Index and CSI Smallcap 500 Index, respectively, as summarized in TABLE I.[1]

In our experiments, we compared the proposed ONS-SI with ONS, where the parameters $\eta = 0$, $\beta = 1$, $\delta = \frac{1}{8}$ are set as recommended by [15], as well as other PS strategies, such as Uniform BAH [17], Best Stock [12], Uniform CRP [15], Best CRP [10] and EG (with the parameter suggested by [12]). We note that Uniform BAH, Uniform CRP, EG, ONS and ONS-SI are causal strategies, while Best Stock and Best CRP can only be determined in hindsight. On each dataset, the side information state of ONS-SI is generated by the moving average of the corresponding Index. Besides, we set the transaction cost proportion as $c = 0.1\%$ for all causal strategies, and set the window size $d = 20$ in calculating moving average. TABLE II lists the abbreviations used in the experiments.

The performance evaluation is conducted with respect to the following several indicators. The first indicator is the final wealth W_n^c at the end of the investment. An equivalent indicator is Annualized Percentage Yield (APY), which is given by $\text{APY} = \sqrt[D/n]{W_n^c} - 1$, where $D = 250$ is the number of trading periods per year. We also use two other criteria, i.e., Maximum DrawDown (MDD) and annualized Sharpe Ratio (SR) to evaluate the downside risk and risk-adjusted return, respectively. The SR is given by $\text{SR} = \frac{\text{APY} - R_f}{\sigma_p}$, where R_f is the risk-free return (fixed as 4% throughout experiments) and σ_p is the annualized standard deviation of daily return.

Algorithm 1. ONS-SI($\mathbf{r}^n, y^n, c, \eta, \beta, \delta$)

Input: market sequence $\mathbf{r}^n$, side information sequence y^n, transaction cost proportion c, parameters η, β and δ

Output: final wealth W_n^c

Initialization: $\tilde{\mathbf{p}}_1 = \frac{1}{m}\mathbf{1}$, $\bar{\mathbf{p}}_0 = \mathbf{0}$, $W_0^c = 1$

for $t = 1, \cdots, n$ **do**

Receive side information state y_t

Play the portfolio:

$$\hat{\mathbf{p}}_t = \begin{cases} \tilde{\mathbf{p}}_t & y_t = 1 \\ \mathbf{0} & y_t = 2 \end{cases}$$

Pay the transaction cost:

$$W_t^c = W_{t-1}^c \left(1 - c\sum_{i=1}^{m} |\hat{\mathbf{p}}_t(i) - \bar{\mathbf{p}}_{t-1}(i)|\right)$$

Receive price relative vector $\mathbf{r}_t$

The wealth changes by a factor w_t:

$$w_t = \begin{cases} \mathbf{p}_t^T\mathbf{r}_t & y_t = 1 \\ 1 & y_t = 2 \end{cases}$$

Update the wealth $W_t^c = W_t^c w_t$

The proportion $\bar{\mathbf{p}}_t = \frac{1}{w_t}[\mathbf{p}_t(1)\mathbf{r}_t(1), \cdots, \mathbf{p}_t(m)\mathbf{r}_t(m)]^T$

Update ONS portfolio

$$\tilde{\mathbf{p}}_{t+1} = (1-\eta)\prod\nolimits_{\Delta_m}^{\mathbf{A}_t}(\delta\mathbf{A}_t^{-1}\mathbf{b}_t) + \frac{\eta}{m}\mathbf{1},$$

where $\mathbf{b}_t = \left(1 + \frac{1}{\beta}\right)\sum_{\tau=1}^{t}\nabla_\tau$, $\mathbf{A}_t = \sum_{\tau=1}^{t}\nabla_\tau\nabla_\tau^T + \mathbf{I}_m$, and $\prod_{\Delta_m}^{\mathbf{A}_t}$ is the projection in the norm induced by $\mathbf{A}_t$, i.e.,

$$\prod\nolimits_{\Delta_m}^{\mathbf{A}_t}(\mathbf{y}) = \arg\min_{\mathbf{p}\in\Delta_m}(\mathbf{y}-\mathbf{p})^T\mathbf{A}_t(\mathbf{y}-\mathbf{p})$$

end for

TABLE I. Summary of two datasets

Dataset	Time Frame	Days	Assets	Index

[1] The constituents of the two Index are available at http://www.csindex.com.cn/zh-CN, and the stocks data can be downloaded from https://hk.finance.yahoo.com.

1	8/4/2005-26/5/2017	2949	50	CSI 300 Index
2	15/1/2007-2/6/2017	2523	50	CSI Smallcap 500 Index

A. *Evaluation of Final Wealth*

First of all, the final wealth achieved by different strategies are summarized in TABLE II. It is observed that the final wealth achieved by ONS-SI is higher than all the other causal strategies on both datasets. Best CRP achieves greater wealth than other three benchmarks, i.e., Uniform BAH, Best Stock and Uniform CRP, which coincides the analysis in Section I. It is worth noting that both ONS and ONS-SI beat the Best CRP on Dataset 2.

TABLE II. WEALTH ACHIEVED BY VARIOUS STRATEGIES ON THE TWO DATASETS

Strategy	Abbreviation	Dataset 1	Dataset 2
Uniform BAH	UBAH	8.99	3.77
Best Stock in hindsight	Best Stock	39.01	13.46
Uniform CRP	UCRP	12.48	6.04
Best CRP in hindsight	BCRP	**47.55**	17.17
Exponentiated Gradient	EG	12.27	5.90
Online Newton Step	ONS	17.83	21.16
ONS with Side Information	ONS-SI	22.20	**23.42**

B. *Performance vs. Portfolio Size*

Following Agarwal et al. [15], we evaluate the impact of the portfolio size m on the performance of different PS strategies, by varying m from 5 to 40. For each m and each dataset, the PS strategies are applied on m stocks which are randomly selected from the dataset, and 100 trials are conducted to calculate the mean APY, mean SR and mean MDD. The results are shown in Figure 1 and Figure 2.

Several conclusions can be drawn from the results. First, ONS-SI achieves higher APY and higher SR than other causal strategies regardless of the portfolio size. Moreover, among all the strategies, ONS-SI achieves the highest SR and the lowest MDD on both two datasets. The MDD of ONS-SI is much lower than that of other strategies, which means the downside risk of ONS-SI is much lower than others. Second, as the number of stocks increases, the performance of all strategies improves in different degree in terms of APY, SR and MDD.

C. *Probability Density of APY, SR and MDD*

For further comparison between ONS-SI and ONS algorithms, abundant experiments are conducted to get the approximate Probability Density Function (PDF) of APY, SR and MDD achieved by the two algorithms. 5000 trials are conducted to approximate the PDF, with each trial consisting of 20 random stocks on the corresponding dataset. According to the evaluation results in Figure 3 and Figure 4, ONS-SI achieves higher APY, higher SR and much lower MDD than ONS, which validate that the superior performance of the proposed ONS-SI is not achieved just by luck.

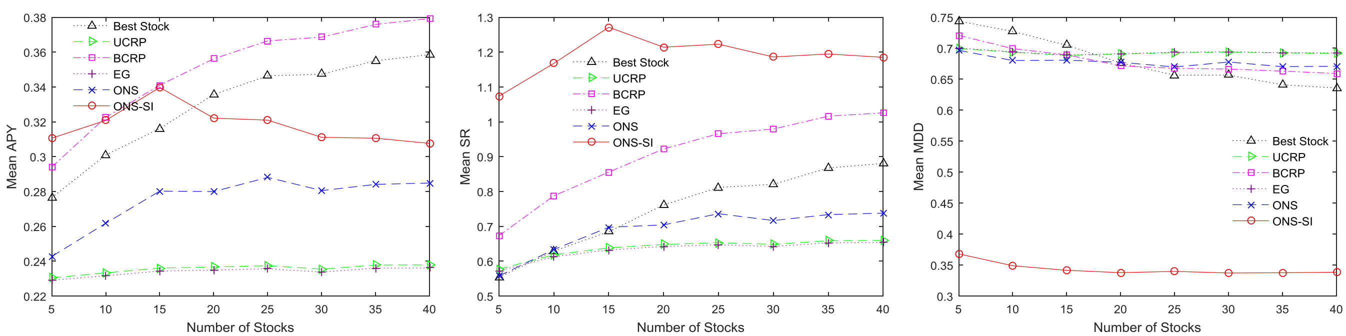

Figure 1. Performance vs. Portfolio Size on Dataset 1

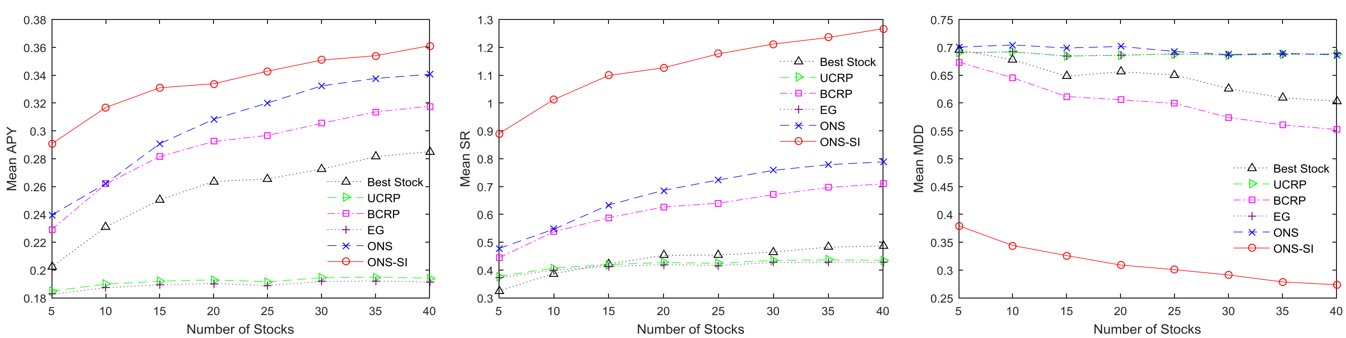

Figure 2. Performance vs. Portfolio Size on Dataset 2

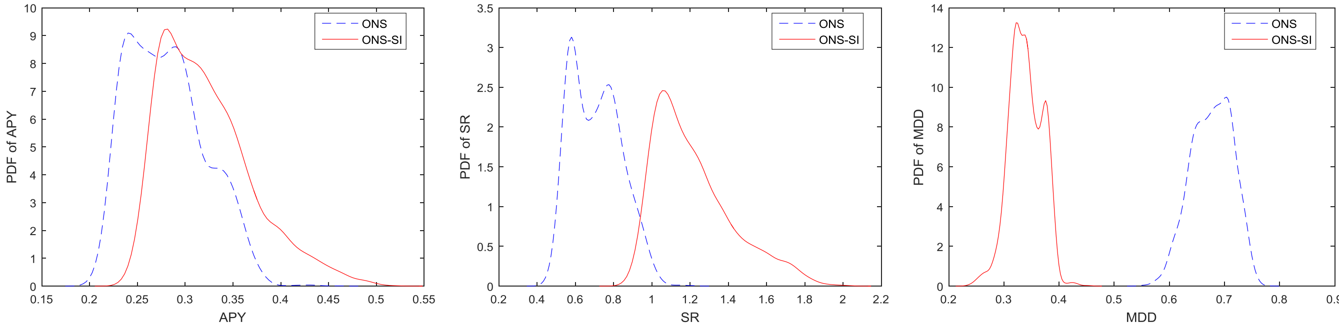

Figure 3. PDF of APY, SR and MDD with $m = 20$ on Dataset 1

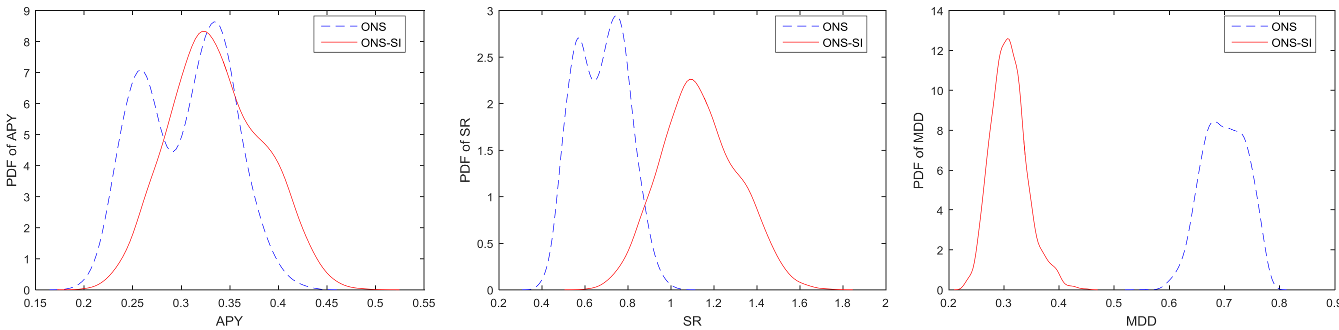

Figure 4. PDF of APY, SR and MDD with $m = 20$ on Dataset 2

V. Conclusion

This paper proposed an online portfolio selection strategy, namely ONS-SI, by incorporating the side information with the conventional ONS algorithm. The basic idea of the proposed ONS-SI is to exploit the side information to reduce the investment risk. Experiments showed that ONS-SI can improve the final wealth and reduce the downside risk than ONS. Future work will exploit other meaningful side information as well as find the theoretical explanation.

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