



Brief paper

Iterative learning impedance control for rehabilitation robots driven by series elastic actuators[☆]

Xiang Li^a, Yun-Hui Liu^a, Haoyong Yu^{b,*}

^a Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong

^b Department of Biomedical Engineering, National University of Singapore, Singapore 117583, Singapore

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ABSTRACT

Existing control techniques for rehabilitation robots commonly ignore robot dynamics by assuming a perfect inner control loop or are limited to rigid-joint robots. The dynamic stability of compliantly-actuated rehabilitation robots, consisting of the dynamics of both robot and compliant actuator, is not theoretically grounded. This paper presents an iterative learning impedance controller for rehabilitation robots driven by series elastic actuators (SEAs), where the control objective is specified as a desired impedance model. The desired impedance model is achieved in an iterative manner, which suits the repeating nature of patients' task through therapeutic process and also guarantees the transient performance of robot. The stability of the overall system is rigorously proved with Lyapunov methods by taking into account both the robot and actuator dynamics. Experimental results are presented to illustrate the performance of the proposed iterative control scheme.

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1. Introduction

Stroke has become one of the leading causes of adult disability with the growing aging population in developed countries. Rehabilitation through physical therapy is the main treatment for such patients to regain maximum function (Hallett, 1999), and it has been shown that repeated and concurrent happening of human intentions to move is the key to recovery (Jenkins & Merzenich, 1987). The conventional manually assisted training is labor-intensive and physically demanding (Krebs, Hogan, Aisen, & Volpe, 1998). This motivates the development of rehabilitation robots, which have the advantages of low labor intensity and high repetition. Various rehabilitation robots have been developed over these years, including the Lower Extremity Powered Exoskeleton (LOPES) for gait rehabilitation (Veneman, Kruidhof, Hekman, Ekelkamp, Van Asseldonk, & Van Der Kooij, 2007), the treadmill-based exoskeleton (Lokomat) (Colombo, Joerg, Schreier, & Dietz, 2000) and the assistive device (EXPOS) (Kong & Jeon, 2006) for

lower-limb rehabilitation, the finger rehabilitation robot (Agarwal, Fox, Yun, O'Malley, & Deshpande, 2015), and also the upper-limb rehabilitation robot (Li, Pan, Chen, & Yu, 2017b). Using series elastic actuators (SEAs) (Pratt & Williamson, 1995) is a popular solution and has become a mainstream in rehabilitation robots (Agarwal et al., 2015; Li et al., 2017b; Veneman et al., 2007), because of its several attractive features such as high force control accuracy, low output impedance, and tolerance to shocks (Paine, Oh, & Sentis, 2014).

While the use of SEA makes the hardware relatively safe, a safe control strategy is also required for the software of rehabilitation robots. Several robotic control strategies have been reported in the literature to realize the paradigm of "Assist-As-Need (AAN)" for rehabilitation (Duschau-Wicke, Zitzewitz, Caprez, Lunenburger, & Riener, 2000; Veneman et al., 2007). That is, the robot supplies the appropriate assistive force that a patient needs to accomplish tasks by assessing the performance of patients in real-time. These results commonly ignore the robot dynamics, by assuming a perfect inner control loop. Therefore, the stability of the closed-loop system is not theoretically grounded. In parallel, a class of control schemes (Hogan, 1985), known as impedance control, has been proposed for interaction control of different robotic systems (Albu-Schaffer, Ott, & Hirzinger, 2007; Cheah & Wang, 1998), which regulates a dynamic relationship between the desired trajectory and the interaction force and thus guarantees the stability of the closed-loop system. Among them, the iterative learning impedance controllers (Cheah & Wang, 1998) were proposed to achieve the

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* Corresponding author.

E-mail addresses: xiangli@mae.cuhk.edu.hk (X. Li), yhliu@mae.cuhk.edu.hk (Y.-H. Liu), biehyh@nus.edu.sg (H. Yu).

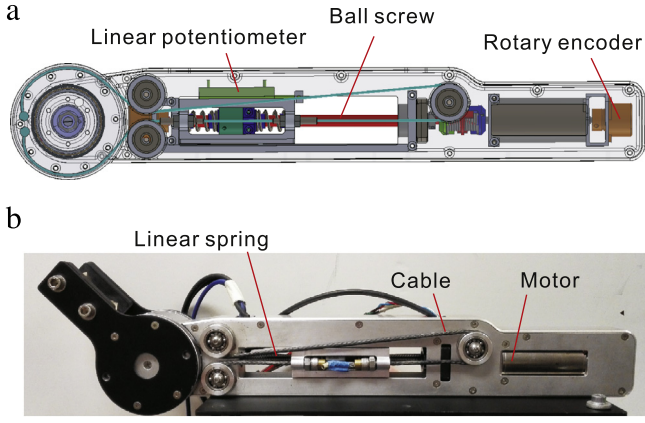


Fig. 1. Mechanical structure of SEA: (a) Principle of the actuator design (CAD model); (b) Prototype.

desired impedance model as actions are repeated, but they are applicable to rigid-joint robots without the coupling dynamics between robot and actuator. The iterative control explores and utilizes the information contained in repetitive actions, which is capable of achieving better transient tracking performance and attenuating repetitive disturbance (Arimoto, Kawamura, & Miyazaki, 1984). The iterative learning controller was proposed for rehabilitation robots in Freeman, Rogers, Hughes, Burridge, and Meadmore (2012), with the integration of functional electrical stimulation (FES) and robotics, but it does not consider the robot dynamics either.

In general, a rehabilitation robot driven by SEAs can be modeled as a high-order system (Petit, Dietrich, & Albu-Schaffer, 2015) consisting of both the rigid-link subsystem and the actuator subsystem. Finding a solution to stabilize both subsystems is not trivial. While several multi-modal controllers have been reported for SEA-driven robots in our previous works (Li, Pan, Chen, & Yu, 2017a; Li et al., 2017b), this paper considers the problem of iterative impedance control for rehabilitation robots. In particular, a desired impedance model (instead of stiffness only in Li et al. (2017a, 2017b)) is designed in the proposed controller such that more parameters can be specified (e.g. inertia, damping, stiffness) to suit stroked patients with different background during rehabilitation. The proposed controller also takes advantages of repetitive tasks through therapeutic process and iteratively achieves the desired impedance model with better transient performance. In addition, the proposed controller is able to stabilize both the rigid-link subsystem and the actuator subsystem, and the stability of the closed-loop system is rigorously proved with Lyapunov methods. Experimental results on the setup of an upper-limb rehabilitation robot are presented to demonstrate the performance of the proposed controller.

2. Background

2.1. Dynamic model of SEA-driven robot

A SEA is developed by placing an elastic element between motor and external load in the actuator (Pratt & Williamson, 1995). Fig. 1 illustrates a compact SEA (Li et al., 2017a), which is of high fidelity of force control, low output impedance, and large force range and bandwidth.

Consider a rehabilitation robot driven by SEAs. Let $\mathbf{q} \in \mathbb{R}^n$ denote the joint configurations, the dynamic model of SEA-driven rehabilitation robots can be described as (Albu-Schaffer et al., 2007; Li et al., 2017a; 2017b; Petit et al., 2015)

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) + \mathbf{d}_r + \boldsymbol{\tau}_e, \quad (1)$$

$$\mathbf{B}\ddot{\boldsymbol{\theta}} + \mathbf{K}(\boldsymbol{\theta} - \mathbf{q}) = \mathbf{d}_a + \boldsymbol{\tau}, \quad (2)$$

where $\boldsymbol{\theta} \in \mathbb{R}^n$ is the vector of motor rotor shaft positions, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n \times n}$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^n$, and $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^n$ denote the inertia matrix, the centripetal and Coriolis torque, and the gravitational torque of the robot respectively, $\mathbf{K} \in \mathbb{R}^{n \times n}$ denotes the stiffness of the SEA, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ represents the interaction torque, $\mathbf{B} \in \mathbb{R}^{n \times n}$ is the inertia matrix of actuator, $\boldsymbol{\tau} \in \mathbb{R}^n$ denotes the input torque exerted on the actuator, and \mathbf{d}_r , $\mathbf{d}_a \in \mathbb{R}^n$ represent unmodeled disturbances at the rigid-link side and the actuator side respectively. Note that $\boldsymbol{\tau}_e = \mathbf{J}^T(\mathbf{q})\mathbf{f}_e$, where $\mathbf{f}_e \in \mathbb{R}^m$ denotes the interaction force between human and robot, and $\mathbf{J}(\mathbf{q}) \in \mathbb{R}^{m \times n}$ is the Jacobian matrix from joint space to task space (e.g. Cartesian space).

It is assumed in this paper that the dynamic model is well defined or can be identified with sufficient accuracy such that $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, \mathbf{K} and \mathbf{B} are known. Some important properties of the dynamic model include (Arimoto, 1996; Cheah & Li, 2015): (i) The matrices $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$, and $\mathbf{g}(\mathbf{q})$ are bounded; (ii) The matrix $\mathbf{M}(\mathbf{q})$ is symmetric and positive definite; (iii) The matrices \mathbf{B} and \mathbf{K} are diagonal and positive definite; (iv) The matrix $\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew-symmetric.

2.2. Impedance control

The impedance control problem is formulated in terms of a reference trajectory and a desired dynamic relationship between the position error and the interaction force (Hogan, 1985). In this paper, the desired impedance model is defined in joint space as $\mathbf{M}_d(\ddot{\mathbf{q}} - \ddot{\mathbf{q}}_d) + \mathbf{C}_d(\dot{\mathbf{q}} - \dot{\mathbf{q}}_d) + \mathbf{K}_d(\mathbf{q} - \mathbf{q}_d) = \boldsymbol{\tau}_e$, where $\mathbf{q}_d \in \mathbb{R}^n$ is the vector of the desired joint angles which are time-varying, \mathbf{M}_d , \mathbf{C}_d , $\mathbf{K}_d \in \mathbb{R}^{n \times n}$ denote the desired inertia, the desired damping, the desired stiffness matrices respectively, which are diagonal and positive definite. Note that the interaction torque $\boldsymbol{\tau}_e$ is taken into account in the desired impedance model, and hence the interaction force \mathbf{f}_e is also considered in the subsequent development.

Let $\Delta\mathbf{q} = \mathbf{q} - \mathbf{q}_d$ represent the joint-space position error. Then, two matrices $\boldsymbol{\Gamma} \in \mathbb{R}^{n \times n}$ and $\boldsymbol{\Lambda} \in \mathbb{R}^{n \times n}$ and a vector $\boldsymbol{\tau}_l \in \mathbb{R}^n$ are defined as

$$\boldsymbol{\Lambda} + \boldsymbol{\Gamma} = \mathbf{M}_d^{-1}\mathbf{C}_d, \quad (3)$$

$$\boldsymbol{\Gamma}\boldsymbol{\Lambda} = \mathbf{M}_d^{-1}\mathbf{K}_d, \quad (4)$$

$$\dot{\boldsymbol{\tau}}_l + \boldsymbol{\Gamma}\boldsymbol{\tau}_l = \mathbf{M}_d^{-1}\boldsymbol{\tau}_e, \quad (5)$$

where both $\boldsymbol{\Lambda}$ and $\boldsymbol{\Gamma}$ are diagonal and positive definite. Eq. (5) describes a low-pass filter where $\boldsymbol{\tau}_e$ is the input signal and $\boldsymbol{\tau}_l$ is the output signal. Using (3)–(5), an augmented impedance error can be rewritten as

$$\begin{aligned} \ddot{\mathbf{w}} &= \Delta\ddot{\mathbf{q}} + (\boldsymbol{\Lambda} + \boldsymbol{\Gamma})\Delta\dot{\mathbf{q}} + \boldsymbol{\Gamma}\boldsymbol{\Lambda}\Delta\mathbf{q} - \dot{\boldsymbol{\tau}}_l - \boldsymbol{\Gamma}\boldsymbol{\tau}_l \\ &= \dot{\mathbf{z}} + \boldsymbol{\Gamma}\mathbf{z}, \end{aligned} \quad (6)$$

where $\mathbf{z} = \Delta\dot{\mathbf{q}} + \boldsymbol{\Lambda}\Delta\mathbf{q} - \boldsymbol{\tau}_l$ is the impedance vector. It can be derived that the convergence of $\ddot{\mathbf{w}} \rightarrow \mathbf{0}$ leads to the realization of the desired impedance model. From (6), the impedance vector \mathbf{z} can be treated as the low-pass-filtered signal of $\ddot{\mathbf{w}}$. In this paper, the control objective is formulated as $\mathbf{z} \rightarrow \mathbf{0}$, indicating the realization of the desired impedance model in the low-frequency range. This is reasonable for rehabilitation, as the frequency of patients' movement is usually low (e.g. <2 Hz). The formulation of $\mathbf{z} \rightarrow \mathbf{0}$ has been extensively used in Cheah and Wang (1998) and Wang and Cheah (1998), but those results are limited to rigid-joint robots. The overall SEA-driven robot, consisting of both the rigid-link subsystem and the actuator subsystem, is a high-order system, and finding a solution to stabilize both subsystems is not trivial, and neglecting the coupling dynamics is exposed to stability issues (Petit et al., 2015).

Remark 1. The parameters of the desired impedance model should be specified such that both Λ and Γ exist. In particular, given that $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_{\bar{n}})$, $\Gamma = \text{diag}(\gamma_1, \dots, \gamma_{\bar{n}})$, and $\mathbf{M}_d = \text{diag}(m_{d1}, \dots, m_{d\bar{n}})$, $\mathbf{C}_d = \text{diag}(c_{d1}, \dots, c_{d\bar{n}})$, $\mathbf{K}_d = \text{diag}(k_{d1}, \dots, k_{d\bar{n}})$ where λ_i , γ_i , m_{di} , c_{di} , and k_{di} ($i = 1, \dots, \bar{n}$) are positive constants, the parameters should be specified such that $c_{di}^2 \geq 4m_{di}k_{di}$. Then, the elements of Λ and Γ can be specified as:

$$\lambda_i = \frac{m_{di}^{-1}c_{di} \pm \sqrt{m_{di}^{-2}c_{di}^2 - 4m_{di}^{-1}k_{di}}}{2}, \quad (7)$$

$$\gamma_i = \frac{m_{di}^{-1}c_{di} \mp \sqrt{m_{di}^{-2}c_{di}^2 - 4m_{di}^{-1}k_{di}}}{2}. \quad (8)$$

Remark 2. In actual implementations, the parameters of the desired impedance model should also be initialized according to the assessment of patients' specific conditions (Zhang & Cheah, 2015). For example, if the patient is at early stage of the therapy and has minimal motion control, the desired impedance should be increased by setting larger \mathbf{M}_d , \mathbf{C}_d , \mathbf{K}_d , to allow the robot to take a lead and thus maximize its assistance; As the patient's motion control ability is being restored, the desired impedance could be decreased by setting smaller \mathbf{M}_d , \mathbf{C}_d , \mathbf{K}_d , to encourage patient's active motions and also minimize the robot's assistance.

3. Iterative learning impedance control

In this section, a new iterative learning impedance controller is proposed for SEA-driven rehabilitation robots, to guarantee the convergence of $\mathbf{z} \rightarrow \mathbf{0}$ as actions are repeated.

The development of the proposed controller follows a backstepping approach. First, (1) is rewritten as

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{K}_q \\ = \mathbf{K}\theta_d + \mathbf{K}\Delta\theta + \mathbf{d}_r + \boldsymbol{\tau}_e, \end{aligned} \quad (9)$$

where $\Delta\theta = \theta - \theta_d \in \mathbb{R}^{\bar{n}}$, and $\theta_d \in \mathbb{R}^{\bar{n}}$ represents a fictitious desired input. Then, the desired input θ_d is specified to stabilize the rigid-link subsystem (9) and achieve the impedance control task, while the control input $\boldsymbol{\tau}$ is developed to stabilize the actuator subsystem (2) such that the actual position of motor tracks the desired input, that is, $\theta \rightarrow \theta_d$.

The impedance vector is rewritten as

$$\mathbf{z} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r, \quad (10)$$

where $\dot{\mathbf{q}}_r = \dot{\mathbf{q}}_d - \Lambda\Delta\mathbf{q} + \boldsymbol{\tau}_l$ is a reference vector. By using (10), Eq. (9) can be rewritten as

$$\begin{aligned} \mathbf{M}(\mathbf{q})\dot{\mathbf{z}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{z} + \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r \\ + \mathbf{g}(\mathbf{q}) + \mathbf{K}_q \\ = \mathbf{K}\theta_d + \mathbf{K}\Delta\theta + \mathbf{d}_r + \boldsymbol{\tau}_e, \end{aligned} \quad (11)$$

where $\ddot{\mathbf{q}}_r$ is the time derivative of $\dot{\mathbf{q}}_r$.

Then, we can propose the desired input for the robot dynamics as

$$\begin{aligned} \theta_d = \mathbf{q} + \mathbf{K}^{-1}(-\mathbf{K}_z\mathbf{z} - k_r\text{sgn}(\mathbf{z}) - \boldsymbol{\tau}_e \\ + \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \mathbf{g}(\mathbf{q}) + \mathbf{m}), \end{aligned} \quad (12)$$

where $\mathbf{K}_z \in \mathbb{R}^{\bar{n} \times \bar{n}}$ is positive definite, k_r is a positive constant, and $\text{sgn}(\cdot)$ is a sign function (Li & Cheah, 2013). The term $-\mathbf{K}_z\mathbf{z}$ is to realize the impedance control task, the terms $-\boldsymbol{\tau}_e + \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \mathbf{g}(\mathbf{q})$ are to compensate the dynamics, and $\mathbf{m} \in \mathbb{R}^{\bar{n}}$ represents a feedforward learning control input which is updated by the following law

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \beta_m \mathbf{z}_k, \quad (13)$$

where β_m is a positive constant, and the subscript of k denotes the k th iteration.

Substituting (12) into (11), we obtain the dynamic equation of the rigid-link subsystem as

$$\begin{aligned} \mathbf{M}(\mathbf{q})\dot{\mathbf{z}} + (\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{K}_z)\mathbf{z} + k_r\text{sgn}(\mathbf{z}) \\ = \mathbf{K}\Delta\theta + \mathbf{d}_r + \mathbf{m}. \end{aligned} \quad (14)$$

Next, a sliding vector is proposed for the actuator dynamics as

$$\mathbf{s} = \dot{\theta} - \dot{\theta}_r = \dot{\theta} - \dot{\theta}_d + \alpha_\theta \Delta\theta, \quad (15)$$

where $\dot{\theta}_r = \dot{\theta}_d - \alpha_\theta \Delta\theta$ is another reference vector, $\dot{\theta}_d$ is the time derivative of θ_d in (12), and α_θ is a positive constant. By using the sliding vector (15), the actuator dynamics can be rewritten as

$$\mathbf{B}\dot{\mathbf{s}} + \mathbf{B}\ddot{\theta}_r + \mathbf{K}(\theta - \mathbf{q}) = \mathbf{d}_a + \boldsymbol{\tau}, \quad (16)$$

where $\ddot{\theta}_r$ is the time derivative of $\dot{\theta}_r$ as $\ddot{\theta}_r = \ddot{\theta}_d - \alpha_\theta \Delta\dot{\theta}$, where $\ddot{\theta}_d$ is the second-order time derivative of θ_d . Then, the control input $\boldsymbol{\tau}$ is proposed as

$$\begin{aligned} \boldsymbol{\tau} = \mathbf{K}(\theta - \mathbf{q}) - k_a\text{sgn}(\mathbf{s}) + \mathbf{B}\ddot{\theta}_r \\ - \mathbf{K}_s\mathbf{s} - \mathbf{K}_\theta\Delta\theta + \mathbf{n}, \end{aligned} \quad (17)$$

where $\mathbf{K}_s \in \mathbb{R}^{\bar{n} \times \bar{n}}$ and $\mathbf{K}_\theta \in \mathbb{R}^{\bar{n} \times \bar{n}}$ are positive definite matrices, and k_a is a positive constant. In (17), the terms of $\mathbf{K}(\theta - \mathbf{q})$ and $\mathbf{B}\ddot{\theta}_r$ represent the dynamic compensation, and the terms of $-\mathbf{K}_s\mathbf{s} - \mathbf{K}_\theta\Delta\theta$ include the velocity control and the position control. Then, the \mathbf{n} is the learning control input for the actuator subsystem, which is updated by

$$\mathbf{n}_{k+1} = \mathbf{n}_k - \beta_n \mathbf{s}_k, \quad (18)$$

where β_n is a positive constant.

Substituting (17) into (16), we have the dynamic equation of the actuator subsystem as

$$\mathbf{B}\dot{\mathbf{s}} + \mathbf{K}_s\mathbf{s} + k_a\text{sgn}(\mathbf{s}) + \mathbf{K}_\theta\Delta\theta = \mathbf{d}_a + \mathbf{n}. \quad (19)$$

The block diagram of the closed-loop system is shown in Fig. 2. We can now state the following theorem.

Theorem. Consider the learning control system given by (14) and (19). Let the control parameters be chosen such that the following conditions are satisfied:

- (C1) $\lambda_{\min}[\mathbf{K}_s] \geq \frac{\beta_n}{2}$,
- (C2) $\lambda_{\min}[\mathbf{K}_z] \geq \frac{\beta_m}{2}$,
- (C3) $\frac{2\alpha_\theta}{\beta_n} \lambda_{\min}[(\frac{2}{\beta_m}\mathbf{K}_z - \mathbf{I}_n)\mathbf{K}_\theta] \geq \frac{1}{\beta_m^2} \lambda_{\max}[\mathbf{K}^2]$,
- (C4) $k_r \geq B_r$,
- (C5) $k_a \geq B_a$,

where $\lambda_{\min}[\cdot]$ and $\lambda_{\max}[\cdot]$ represent the minimum and maximum eigenvalues respectively, B_r and B_a denote the upper bounds of \mathbf{d}_r and \mathbf{d}_a respectively. Then, a sequence of control inputs will be generated such that the impedance control task is achieved. That is

$$\mathbf{z}_k(t) \rightarrow \mathbf{0}, \quad (20)$$

for all $t \in [0, t_f]$ as $k \rightarrow \infty$, where t_f is the iteration period, and $\mathbf{I}_n \in \mathbb{R}^{\bar{n} \times \bar{n}}$ is an identity matrix.

Proof. See Appendix.

Remark 3. The control parameters should be set to guarantee both the performance and the stability of the closed-loop system. In particular, better tracking performance can be achieved by tuning \mathbf{K}_s and \mathbf{K}_z up, and smaller steady-state error can be achieved with

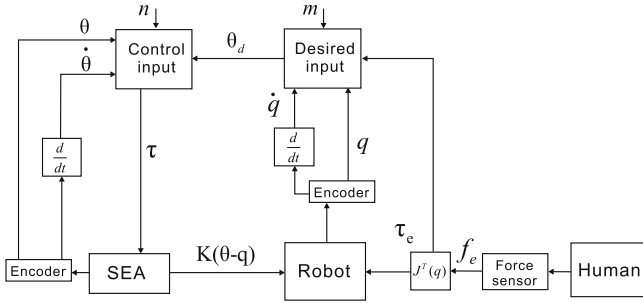


Fig. 2. A block diagram of the control structure.

higher α_θ . Faster convergence can be obtained by increasing β_n and β_m , but it should not be set too large such that (C1) and (C2) are satisfied. Then, (C3) is satisfied by setting K_θ large, and (C4) and (C5) can be satisfied by setting k_r and k_a large. The control parameters should be fine tuned together. Note that too large parameters should be avoided. Otherwise, the control input will be saturated and the measurement noises may be amplified.

Remark 4. The key novelty of the proposed controller is that two learning strategies (13) and (18) are developed to update feedforward control terms for the rigid-link subsystem (14) and the actuator subsystem (19) respectively. Note that m_k and n_k are updated in the next iteration by incorporating error information (i.e. z_k and s_k respectively). As the iterations go to infinity, both the impedance error and the tracking error reduce to zero, and the iterative control terms converge to cancel out unmodeled disturbances. Then, the ratio of m_k over θ_d and the ratio of n_k over τ start from zero and progressively increase, as the feedback control terms converge to zero, while the feedforward control terms are playing more dominant roles. Different from adaptive/robust control approaches, the proposed iterative learning controller explores and utilizes the information contained in repetitive actions of the rehabilitation, and taking into account the repetitive information can achieve better transient tracking performance and attenuate repetitive disturbance (Arimoto et al., 1984; Bristow, Tharayil, & Alleyne, 2006).

Remark 5. Note that the vector $\ddot{\theta}_r$ is a function of $\ddot{\theta}_d$. From (12), the desired input θ_d requires the information of \ddot{q}_r , which is a function of \ddot{q} , and hence θ_d is dependent on the third-order time derivative, i.e. \ddot{q} . That is, the high-order derivatives are required in the control input (17). To eliminate the requirement of high-order derivatives, a model-based observer can be developed to estimate the desired input by following (Li et al., 2017a) in a similar development.

4. Experiment

The proposed control method was implemented in the experimental setup of an upper-limb rehabilitation robot as shown in Fig. 3(a). The upper-limb rehabilitation robot mainly consists of a robot link driven by a SEA and a dSPACE control system. The subject is asked to put their forearm in the upright vertical position, and to make a cyclical flexion/extension movement around this position. A force sensor (ATI) is mounted on the end of the robot link and used to measure the interaction force. The SEA introduced in Section 2.1 is employed.

The desired trajectory was specified as a sinusoid as: $q_d = 0.26 \sin(\pi t)$ rad, where q_d denotes the desired joint angle. The parameters of the desired impedance model were set as: $M_d = 1$, $C_d = 15.4$, and $K_d = 6$, and the two matrices Γ and Λ were

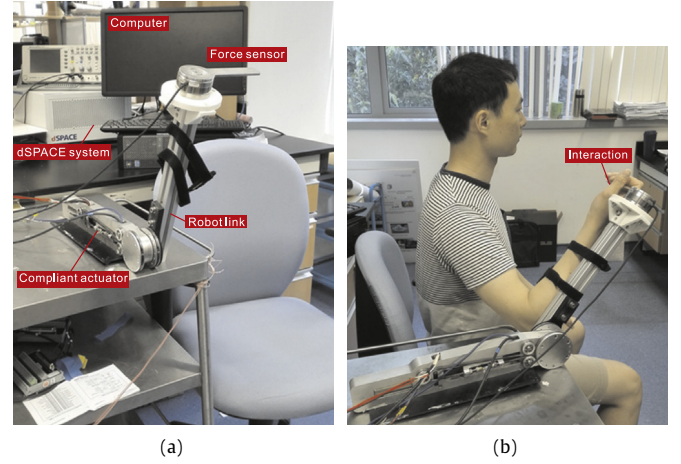


Fig. 3. An upper-limb rehabilitation robot. (a) The system consists of the SEA, the robot link, the force sensor, the dSPACE control system, and the computer; (b) The subject is asked to put his forearm in the upright vertical position.

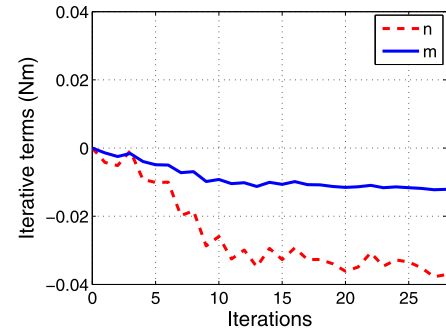


Fig. 4. Iterative control terms m and n .

specified as: $\Gamma = 15$ and $\Lambda = 0.4$, such that Eqs. (3)–(5) were satisfied. The parameters in the desired position input (12) and the control input (17) were set as: $K_z = 20$, $\alpha_\theta = 2$, $K_s = 2$, $K_\theta = 0.004$, $k_r = 0.02$, and $k_a = 0.04$. The parameters in the iteration update laws (13) and (18) were set as: $\beta_m = 0.2$, $\beta_n = 0.04$.

In the experiments, the iteration period was set as $t_f = 50$ s. The results of the iterative control terms m , n are shown in Fig. 4, which are bounded and converge to cancel out disturbances as the iterations continue. The position errors and the impedance errors after different iterations ($k = 3$, $k = 15$, $k = 30$) are shown in Fig. 5 and Fig. 6 respectively. When $k = 3$, the position error is around 0.003 rad, and the impedance error is around 0.02 N m. When $k = 30$, the position error reduces to 0.001 rad (no interaction forces exerted) and the impedance error reduces to 0.01 N m. That is, both the position error and the impedance error converge to zero as the iteration goes on. During the experiments, the subject exerted forces on the robot by holding the robot link at a certain position (e.g. $t \approx 14$ –20 s in Fig. 6(a) and $t \approx 5$ –14 s in Fig. 6(c)). While the robot joint angle is drifted away from the sinusoid trajectory because of the interaction force, the convergence of the impedance error to zero is guaranteed, which shows the realization of the impedance control task. The control inputs after different iterations are shown in Fig. 7. Therefore, the experimental results validate the performance of the proposed controller.

From Fig. 4 and Fig. 7, it is seen that the ratio of m over θ_d after different iterations (i.e. 3, 15, 30) is 0.0006, 0.0043, 0.008 respectively, while the ratio of n over τ after different iterations is 0.0064,

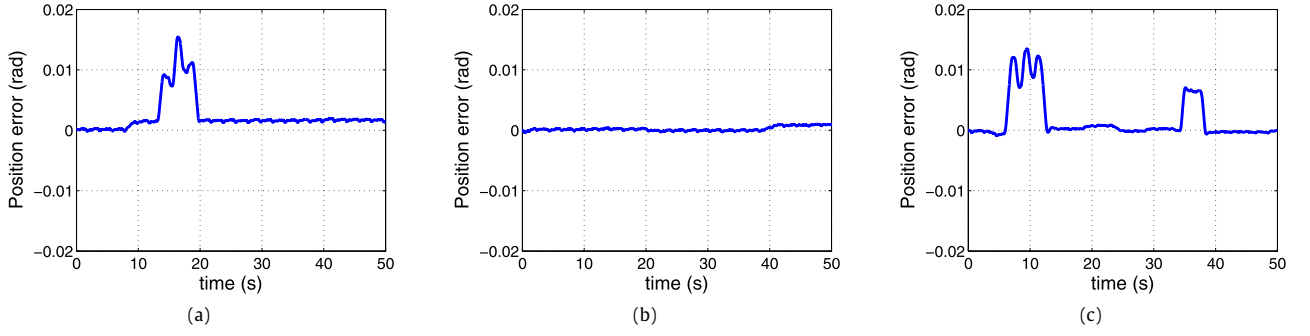


Fig. 5. Position error Δq : (a) 3 iterations; (b) 15 iterations; (c) 30 iterations.

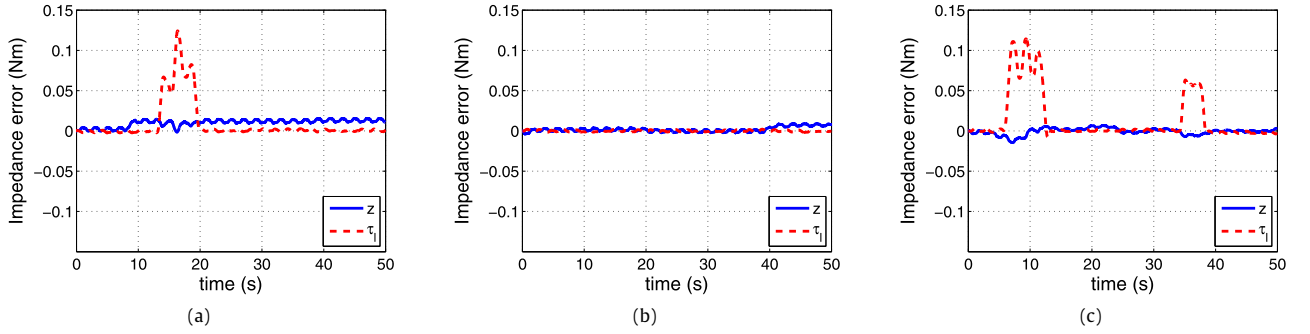


Fig. 6. Impedance vector z : (a) 3 iterations; (b) 15 iterations; (c) 30 iterations.

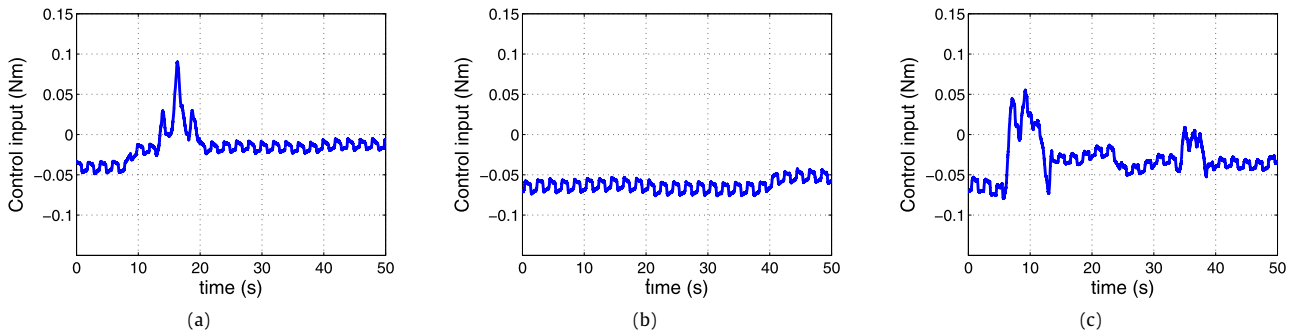


Fig. 7. Control input τ : (a) 3 iterations; (b) 15 iterations; (c) 30 iterations.

0.1616, 0.2803 respectively. That is, both ratios increase as the iteration continues, indicating the iterative feedforward control is playing a more dominant role in the proposed controller.

5. Conclusion

An iterative learning impedance controller has proposed for rehabilitation robots driven by SEAs in this paper. The convergence of the desired impedance to zero is realized by learning from repetitive tasks through therapeutic process. The proposed iterative method guarantees the transient performance of robot. The stability of the overall system, consisting both the rigid-link subsystem and the actuator subsystem, is rigorously proved with Lyapunov methods. Experimental results are presented to validate the effectiveness of the proposed learning controller, i.e. smaller impedance errors after more iterations.

Appendix

First, an index function $V_k(t)$ is defined as:

$$V_k(t) = V_k(0) + \int_0^t \left(\frac{1}{\beta_m^2} \mathbf{m}_k^T(\varsigma) \mathbf{m}_k(\varsigma) + \frac{1}{\beta_n^2} \mathbf{n}_k^T(\varsigma) \mathbf{n}_k(\varsigma) \right) d\varsigma, \quad (21)$$

for all $t \in [0, t_f]$, where $V_k(0)$ represents the initial value at $t = 0$.

Define $\Delta V_k = V_{k+1} - V_k$. From (13), (18), and (21), it is obtained that:

$$\begin{aligned} \Delta V_k &= V_{k+1}(0) - V_k(0) + \\ &\int_0^t \left(\frac{1}{\beta_m^2} \mathbf{m}_{k+1}^T(\varsigma) \mathbf{m}_{k+1}(\varsigma) + \frac{1}{\beta_n^2} \mathbf{n}_{k+1}^T(\varsigma) \mathbf{n}_{k+1}(\varsigma) \right) d\varsigma \\ &- \int_0^t \left(\frac{1}{\beta_m^2} \mathbf{m}_k^T(\varsigma) \mathbf{m}_k(\varsigma) + \frac{1}{\beta_n^2} \mathbf{n}_k^T(\varsigma) \mathbf{n}_k(\varsigma) \right) d\varsigma \end{aligned}$$

$$\begin{aligned}
&= V_{k+1}(0) - V_k(0) + \int_0^t \mathbf{z}_k^T(\varsigma) \mathbf{z}_k(\varsigma) d\varsigma + \int_0^t \mathbf{s}_k^T(\varsigma) \mathbf{s}_k(\varsigma) d\varsigma \\
&\quad - \frac{2}{\beta_m} \int_0^t \mathbf{z}_k^T(\varsigma) \mathbf{m}_k(\varsigma) d\varsigma - \frac{2}{\beta_n} \int_0^t \mathbf{s}_k^T(\varsigma) \mathbf{n}_k(\varsigma) d\varsigma. \quad (22)
\end{aligned}$$

From (14) and (19), the two learning inputs, i.e. \mathbf{m} and \mathbf{n} , can be specified as:

$$\begin{aligned}
\mathbf{m}_k(t) &= \mathbf{M}(\mathbf{q}_k(t)) \dot{\mathbf{z}}_k(t) + (\mathbf{C}(\mathbf{q}_k(t), \dot{\mathbf{q}}_k(t)) + \mathbf{K}_z) \mathbf{z}_k(t) \\
&\quad - \mathbf{d}_r(t) + k_r \mathbf{sgn}(\mathbf{z}_k(t)) - \mathbf{K} \Delta \theta_k(t), \quad (23)
\end{aligned}$$

$$\begin{aligned}
\mathbf{n}_k(t) &= \mathbf{B} \dot{\mathbf{s}}_k(t) + \mathbf{K}_s \mathbf{s}_k(t) + \mathbf{K}_\theta \Delta \theta_k(t) \\
&\quad - \mathbf{d}_a(t) + k_a \mathbf{sgn}(\mathbf{s}_k(t)). \quad (24)
\end{aligned}$$

Substituting (23) and (24) into (22) yields:

$$\begin{aligned}
\Delta V_k &= V_{k+1}(0) - V_k(0) - \frac{2}{\beta_m} \int_0^t \mathbf{z}_k^T(\varsigma) [\mathbf{M}(\mathbf{q}_k(\varsigma)) \dot{\mathbf{z}}_k(\varsigma) \\
&\quad + (\mathbf{C}(\mathbf{q}_k(\varsigma), \dot{\mathbf{q}}_k(\varsigma)) + \mathbf{K}_z) \mathbf{z}_k(\varsigma) - \mathbf{K} \Delta \theta_k(\varsigma) \\
&\quad - \mathbf{d}_r(\varsigma) + k_r \mathbf{sgn}(\mathbf{z}_k(\varsigma))] d\varsigma \\
&\quad + \int_0^t \mathbf{z}_k^T(\varsigma) \mathbf{z}_k(\varsigma) d\varsigma + \int_0^t \mathbf{s}_k^T(\varsigma) \mathbf{s}_k(\varsigma) d\varsigma \\
&\quad - \frac{2}{\beta_n} \int_0^t \mathbf{s}_k^T(\varsigma) [\mathbf{B} \dot{\mathbf{s}}_k(\varsigma) + \mathbf{K}_s \mathbf{s}_k(\varsigma) + \mathbf{K}_\theta \Delta \theta_k(\varsigma) \\
&\quad - \mathbf{d}_a(\varsigma) + k_a \mathbf{sgn}(\mathbf{s}_k(\varsigma))] d\varsigma. \quad (25)
\end{aligned}$$

Since $\mathbf{z}^T(k_r \mathbf{sgn}(\mathbf{z}) - \mathbf{d}_r) \geq (k_r - B_r) \|\mathbf{z}\|$, $\mathbf{s}^T(k_a \mathbf{sgn}(\mathbf{s}) - \mathbf{d}_a) \geq (k_a - B_a) \|\mathbf{s}\|$, and

$$\begin{aligned}
&\int_0^t \mathbf{z}_k^T(\varsigma) \mathbf{M}(\mathbf{q}_k(\varsigma)) \dot{\mathbf{z}}_k(\varsigma) d\varsigma \\
&= \frac{1}{2} \mathbf{z}_k^T(t) \mathbf{M}(\mathbf{q}_k(t)) \mathbf{z}_k(t) - \frac{1}{2} \mathbf{z}_k^T(0) \mathbf{M}(\mathbf{q}_k(0)) \mathbf{z}_k(0) \\
&\quad - \frac{1}{2} \int_0^t \mathbf{z}_k^T(\varsigma) \dot{\mathbf{M}}(\mathbf{q}_k(\varsigma)) \mathbf{z}_k(\varsigma) d\varsigma, \quad (26)
\end{aligned}$$

by setting k_r and k_a such that (C4) and (C5) are satisfied and using initial conditions (Arimoto et al., 1984; Bristow et al., 2006) and Property (iv), we have:

$$\begin{aligned}
\Delta V_k &\leq \int_0^t \mathbf{z}_k^T(\varsigma) \mathbf{z}_k(\varsigma) d\varsigma - \frac{1}{\beta_m} \mathbf{z}_k^T(t) \mathbf{M}(\mathbf{q}_k(t)) \mathbf{z}_k(t) \\
&\quad - \frac{2}{\beta_m} \int_0^t (\mathbf{z}_k^T(\varsigma) \mathbf{K}_z \mathbf{z}_k(\varsigma) - \mathbf{z}_k^T(\varsigma) \mathbf{K} \Delta \theta_k(\varsigma)) d\varsigma + \\
&\quad \int_0^t \mathbf{s}_k^T(\varsigma) \mathbf{s}_k(\varsigma) d\varsigma - \frac{1}{\beta_n} \mathbf{s}_k^T(t) \mathbf{B} \mathbf{s}_k(t) - \frac{1}{\beta_n} \Delta \theta_k^T(t) \mathbf{K}_\theta \Delta \theta_k(t) \\
&\quad - \frac{2}{\beta_n} \int_0^t (\mathbf{s}_k^T(\varsigma) \mathbf{K}_s \mathbf{s}_k(\varsigma) + \alpha_\theta \Delta \theta_k^T(\varsigma) \mathbf{K}_\theta \Delta \theta_k(\varsigma)) d\varsigma, \quad (27)
\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}
\Delta V_k &\leq -\frac{1}{\beta_m} \mathbf{z}_k^T(t) \mathbf{M}(\mathbf{q}_k(t)) \mathbf{z}_k(t) - \frac{1}{\beta_n} \mathbf{s}_k^T(t) \mathbf{B} \mathbf{s}_k(t) \\
&\quad - \frac{1}{\beta_n} \Delta \theta_k^T(t) \mathbf{K}_\theta \Delta \theta_k(t) - \int_0^t [\mathbf{s}_k^T(\varsigma) (\frac{2}{\beta_n} \mathbf{K}_s - \mathbf{I}_n) \mathbf{s}_k(\varsigma)] d\varsigma \\
&\quad - \int_0^t \{[\mathbf{z}_k^T(\varsigma), \Delta \theta_k^T(\varsigma)] \mathbf{P} [\mathbf{z}_k^T(\varsigma), \Delta \theta_k^T(\varsigma)]^T\} d\varsigma, \quad (28)
\end{aligned}$$

where

$$\mathbf{P} = \begin{bmatrix} \frac{2}{\beta_m} \mathbf{K}_z - \mathbf{I}_n & -\frac{1}{\beta_m} \mathbf{K} \\ -\frac{1}{\beta_m} \mathbf{K} & \frac{2\alpha_\theta}{\beta_n} \mathbf{K}_\theta \end{bmatrix}. \quad (29)$$

If the control parameters are chosen such that conditions (C1)–(C3) are satisfied, \mathbf{P} is non-negative definite, and $\Delta V_k \leq 0$. This implies that $V_k(t)$ converges to a nonnegative constant because $V_0(t)$ is bounded. Therefore, $\Delta V_k(t) \rightarrow 0$ as $k \rightarrow \infty$. From (28), we have:

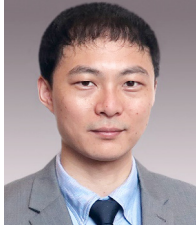
$$\mathbf{z}_k(t) \rightarrow \mathbf{0}, \quad (30)$$

for all $t \in [0, t_f]$ as $k \rightarrow \infty$. That is, the impedance control objective is achieved.

References

- Agarwal, P., Fox, J., Yun, Y., O'Malley, M., & Deshpande, A. (2015). An index finger exoskeleton with series elastic actuation for rehabilitation: Design, control and performance characterization. *International Journal of Robotics Research*, 1–26.
- Albu-Schaffer, A., Ott, C., & Hirzinger, G. (2007). A unified passivity-based control framework for position, torque and impedance control of flexible joint robots. *International Journal of Robotics Research*, 26(1), 23–39.
- Arimoto, S. (1996). *Control theory of non-linear mechanical systems*. Oxford University Press.
- Arimoto, S., Kawamura, S., & Miyazaki, F. (1984). Better operation of robots by learning. *Journal of Robotic Systems*, 1, 123–140.
- Bristow, D. A., Tharayil, M., & Alleyne, A. G. (2006). A survey of iterative learning control. *IEEE Control Systems Magazine*, 26(3).
- Cheah, C. C., & Li, X. (2015). *Task-space sensory feedback control of robot manipulators*. Springer.
- Cheah, C. C., & Wang, D. (1998). Learning impedance control for robotic manipulators. *IEEE Transactions on Robotics and Automation*, 14(3), 452–465.
- Colombo, G., Joerg, M., Schreier, R., & Dietz, V. (2000). Treadmill training of paraplegic patients using a robotic orthosis. *Journal of Rehabilitation Research and Development*, 37, 693–700.
- Duschau-Wicke, A., Zitzewitz, J. Von., Caprez, A., Lunenburger, L., & Riener, R. (2000). Path control: A method for patient-cooperative robot-aided gait rehabilitation. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 18(1), 38–48.
- Freeman, C. T., Rogers, E., Hughes, A.-M., Burrage, J. H., & Meadmore, K. L. (2012). Iterative learning control in health care. *IEEE Control Systems Magazine*, 32(1), 18–43.
- Hallett, M. (1999). Plasticity in the human motor system. *Neuroscientist*, 5, 324–332.
- Hogan, N. (1985). Impedance control - An approach to manipulation. *Journal of Dynamic Systems, Measurement and Control*, 107(1), 1–24.
- Jenkins, W. M., & Merzenich, M. M. (1987). Reorganization of neurocortical representations after brain injury: A neurophysiological model of the bases of recovery from stroke. *Progress Brain Research*, 71, 249–266.
- Kong, K. C., & Jeon, D. (2006). Design and control of an exoskeleton for the elderly and patients. *IEEE/ASME Transactions on Mechatronics*, 11, 428–432.
- Krebs, H. I., Hogan, N., Aisen, M. L., & Volpe, B. T. (1998). Robot-aided neurorehabilitation. *IEEE Transactions on Rehabilitation Engineering*, 6, 75–87.
- Li, X., & Cheah, C. C. (2013). Adaptive neural network control of robot based on a unified objective bound. *IEEE Transactions on Control Systems Technology*, 22(3), 1032–1043.
- Li, X., Pan, Y., Chen, G., & Yu, H. (2017a). Adaptive human–robot interaction control for robots driven by series elastic actuators. *IEEE Transactions on Robotics*, 33(1), 169–182.
- Li, X., Pan, Y., Chen, G., & Yu, H. (2017b). Multi-modal control scheme for rehabilitation robotic exoskeletons. *International Journal of Robotics Research* online.
- Paine, N., Oh, S., & Sentis, L. (2014). Design and control considerations for high-performance series elastic actuators. *IEEE/ASME Transactions on Mechatronics*, 19(3), 1080–1091.
- Petit, F., Dietrich, A., & Albu-Schaffer, A. (2015). Generalizing torque control concepts: Using well-established torque control methods on variable stiffness robots. *IEEE Robotics & Automation Magazine*, 22(4), 37–51.

- Pratt, G. A., & Williamson, M. M. (1995). Series elastic actuators. In: *IEEE/RSJ Int. Conf. Intelligent Robots Syst.* (pp. 399–406).
- Veneman, J. F., Kruidhof, R., Hekman, E. E. G., Ekkelenkamp, R., Van Asseldonk, E. H. F., & Van Der Kooij, H. (2007). Design and evaluation of the LOPES exoskeleton robot for interactive gait rehabilitation. *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, 15(3), 379–386.
- Wang, D., & Cheah, C. C. (1998). An iterative learning control scheme for impedance control of robotic manipulators. *International Journal of Robotics Research*, 17(10), 1091–1104.
- Zhang, J., & Cheah, C. C. (2015). Passivity and stability of human–robot interaction control for upper-limb rehabilitation robots. *IEEE Transactions on Robotics*, 31(2), 233–245.



Xiang Li received the Bachelor's and Master's degrees from Beijing Institute of Technology, Beijing, China, in 2006 and 2008 respectively, and the Ph.D. degree from Nanyang Technological University, Singapore, in 2013. He is currently a Research Assistant Professor with the Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Hong Kong. His current research interests include robot control, visual servoing, human–robot interaction, and cell manipulation.



Yun-Hui Liu is Choh-Ming Li Professor of Mechanical and Automation Engineering at the Faculty of Engineering, The Chinese University of Hong Kong (CUHK). He received the Bachelor of Engineering degree at Beijing Institute of Technology in 1985, the Master of Engineering from Osaka University in 1989, and the Ph.D. degree in Mathematical Engineering from the University of Tokyo in 1992. After working at the Electrotechnical Laboratory of Japan as a Research Scientist, he joined The Chinese University of Hong Kong in 1995 and was promoted to a Professor at the Department of Mechanical and Automation Engineering in 2002. He is currently the Director of the CUHK T Stone Robotics Institute.



Haoyong Yu received the B.S. and M.S. degrees in Mechanical Engineering from Shanghai Jiao Tong University, Shanghai, China, in 1988 and 1991 respectively. He received the Ph.D. degree in Mechanical Engineering from Massachusetts Institute of Technology, Cambridge, Massachusetts, USA, in 2002. He is currently an Associate Professor with the Department of Biomedical Engineering. His areas of research include medical robotics, rehabilitation engineering and assistive technologies, system dynamics, and intelligent control.