

# Optimization Assignment 3

## Linear Programs and Applications

Kim Paolo Laberinto (7771083)

Winter 2021

# Contents

|          |   |          |
|----------|---|----------|
| <b>1</b> | <b>Q1. Linear Programming Applied to Diet Optimizaton</b>                     | <b>3</b> |
| 1.1      | Problem Set-up . . . . .  | 3        |
| 1.1.1    | Variables . . . . .   | 3        |
| 1.1.2    | Objective Function . . . . .  | 3        |
| 1.1.3    | Constraints . . . . .   | 3        |
| 1.2      | Solution using Julia . . . . .  | 4        |
| 1.3      | Solution using Matlab . . . . .   | 5        |
| 1.4      | Observations . . . . .  | 5        |
| 1.5      | Follow-up Question and Solution - Doubling Calories in Bran Muffins . . . . . | 5        |
| <b>2</b> | <b>Q2. Example of Conversion to Standard Form</b>                             | <b>7</b> |
| <b>3</b> | <b>Q3. Example of Graphically Solving a Linear Program</b>                    | <b>7</b> |
| <b>4</b> | <b>Q4. Textbook Questions</b>   | <b>7</b> |
| 4.1      | 12.9 . . . . .  | 7        |
| 4.2      | 12.15 . . . . .   | 7        |
| 4.3      | 12.21 . . . . .   | 7        |
| 4.4      | 12.22 . . . . .   | 7        |
| <b>A</b> | <b>Source Code</b>  | <b>8</b> |

# 1 Q1. Linear Programming Applied to Diet Optimizatoin

## 1.1 Problem Set-up

In the assignment, a word problem was given in which a diet had to be constructed such that it minimized the total calorie content, while also still meeting the minimum daily nutritional requirements given.

In this section the word problem will be formulated into linear program consisting of the following elements.

- Variables
- Objective Function
- Constraints

### 1.1.1 Variables

The variables are the number of servings of each food item as denoted in Tab. 1.

| Variable | Description                             |
|----------|---|
| $x_1$    | Number of servings of provolone         |
| $x_2$    | Number of servings of mozzarella        |
| $x_3$    | Number of servings of 2% milk           |
| $x_4$    | Number of servings of salami            |
| $x_5$    | Number of servings of ham               |
| $x_6$    | Number of servings of brussel sprouts   |
| $x_7$    | Number of servings of lettuce           |
| $x_8$    | Number of servings of french fries      |
| $x_9$    | Number of servings of orange            |
| $x_{10}$ | Number of servings of whole wheat bread |
| $x_{11}$ | Number of servings of bran muffin       |

Table 1: Variables for Linear Program for Diet Optimization

### 1.1.2 Objective Function

Let  $E(x_i)$  denote the caloric (energy) content of 1 serving of food  $x_i$  in units of kcal (equivalent to in units of food calories (i.e 1 kcal = 1 food calorie)).

The objective of this linear program is the following. Shown without any constraints.

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_i E(x_i)x_i \quad (1)$$

### 1.1.3 Constraints

In this linear program, the constraints are to meet all the daily nutritional values. There are also constraints to ensure that serving portions are non-negative.

Let the following functions denote the nutritional content of 1 serving of the food  $x_i$

- Protein( $x_i$ ) - protein in grams
- Carb( $x_i$ ) - carbohydrates in grams
- Fat( $x_i$ ) - fat in grams
- VitA( $x_i$ ) - Vitamin A in REs
- VitB1( $x_i$ ) - Vitamin B1 Thiamin in milligrams

- $\text{VitB2}(x_i)$  - Vitamin B2 Riboflavin in milligrams
- $\text{VitC}(x_i)$  - Vitamin C in milligrams
- $\text{Fibre}(x_i)$  - Fibre in grams

Then the linear program can be summarized as the following.

$$\begin{aligned}
& \underset{\mathbf{x}}{\text{minimize}} && \sum_i E(x_i)x_i \\
& \text{subject to} && x_i \geq 0, \ i = 1, \dots, 11 \\
& && \sum_i \text{Protein}(x_i)x_i \geq 60 \\
& && \sum_i \text{Carb}(x_i)x_i \geq 300 \\
& && \sum_i \text{Fat}(x_i)x_i \geq 40 \\
& && \sum_i \text{VitA}(x_i)x_i \geq 800 \\
& && \sum_i \text{VitB1}(x_i)x_i \geq 1.0 \\
& && \sum_i \text{VitB2}(x_i)x_i \geq 1.2 \\
& && \sum_i \text{VitC}(x_i)x_i \geq 60 \\
& && \sum_i \text{Fibre}(x_i)x_i \geq 10
\end{aligned} \tag{2}$$

## 1.2 Solution using Julia

The linear program was solved using JuMP.jl from the Julia ecosystem, yielding the following solution.

| Variable | Food              | Diet Serving Count | Diet Quantity |
|----------|-------------------|--------------------|---------------|
| $x_1$    | Provolone         | 0.0                | -             |
| $x_2$    | Mozzarella        | 0.0                | -             |
| $x_3$    | 2% Milk           | 0.0                | -             |
| $x_4$    | Salami            | 0.0                | -             |
| $x_5$    | Ham               | 0.0                | -             |
| $x_6$    | Brussel Sprouts   | 6.96               | 1740.0 mL     |
| $x_7$    | Lettuce           | 0.0                | -             |
| $x_8$    | French Fries      | 0.0                | -             |
| $x_9$    | Orange            | 2.17               | 2.17 medium   |
| $x_{10}$ | Whole Wheat Bread | 0.0                | -             |
| $x_{11}$ | Bran Muffin       | 10.0               | 10.0 medium   |

Table 2: Optimized diet solution using Julia. Serving counts, and quantity shown.

|                         | Servings | Energy<br>[kcal] | Protein<br>[g] | Carbs<br>[g] | Fat<br>[g] | Vitamin A<br>[RE] | Vitamin B1<br>[mg] | Vitamin B2<br>[mg] | Vitamin C<br>[mg] | Fibre<br>[g] |
|-------------------------|----------|------------------|----------------|--------------|------------|-------------------|--------------------|--------------------|-------------------|--------------|
| $x_6$ - Brussel Sprouts | 6.96     | 445.44           | 27.84          | 97.44        | 0.0        | 828.24            | 1.25               | 0.90               | 709.92            | 34.8         |
| $x_8$ - French Fries    | 0.0      | 0.0              | 0.0            | 0.0          | 0.0        | 0.0               | 0.0                | 0.0                | 0.0               | 0.0          |
| $x_9$ - Oranges         | 2.17     | 134.54           | 2.17           | 32.55        | 0.0        | 60.76             | 0.24               | 0.11               | 151.9             | 5.64         |
| $x_{11}$ - Bran Muffin  | 10.0     | 1040.0           | 30             | 170          | 40         | 180               | 0.5                | 0.8                | 0.0               | 18           |
| Diet Summary            |          | 1620.0           | 60             | 300          | 40         | 1069              | 2.0                | 1.8                | 861.8             | 58.4         |
| Daily Requirement       |          |                  | 60             | 300          | 40         | 800               | 1.0                | 1.2                | 60                | 10           |

Table 3: Nutrition Summary of Optimized Diet using Julia showing all nutritional daily requirements are met. Rounded values for clarity.

### 1.3 Solution using Matlab

The linear program was solved using Matlab, yielding the following solution.

### 1.4 Observations

There are several observations to make from these solutions.

- 

### 1.5 Follow-up Question and Solution - Doubling Calories in Bran Muffins

A follow up question in the assignment was to determine what happens to the diet if the calories in the muffins are increased by a factor of 2.

Using this new calorie content for the bran muffins yields the following solution.

| Variable | Food              | Diet Serving Count | Diet Quantity |
|----------|-------------------|--------------------|---------------|
| $x_1$    | Provolone         | 0.0                | -             |
| $x_2$    | Mozzarella        | 0.0                | -             |
| $x_3$    | 2% Milk           | 0.0                | -             |
| $x_4$    | Salami            | 0.0                | -             |
| $x_5$    | Ham               | 0.0                | -             |
| $x_6$    | Brussel Sprouts   | 11.96              | 2990 mL       |
| $x_7$    | Lettuce           | 0.0                | -             |
| $x_8$    | French Fries      | 5.0                | 50 strips     |
| $x_9$    | Orange            | 2.17               | 2.17 medium   |
| $x_{10}$ | Whole Wheat Bread | 0.0                | -             |
| $x_{11}$ | Bran Muffin       | 0.0                | -             |

Table 4: Optimized diet solution, where bran muffins had 2x calories. Serving counts, and quantity shown. (Solution found using Julia).

|                         | Servings | Energy<br>[kcal] | Protein<br>[g] | Carbs<br>[g] | Fat<br>[g] | Vitamin A<br>[RE] | Vitamin B1<br>[mg] | Vitamin B2<br>[mg] | Vitamin C<br>[mg] | Fibre<br>[g] |
|-------------------------|----------|------------------|----------------|--------------|------------|-------------------|--------------------|--------------------|-------------------|--------------|
| $x_6$ - Brussel Sprouts | 11.96    | 765.44           | 47.84          | 167.44       | 0.0        | 1423.24           | 2.1582             | 1.55               | 1219.92           | 59.80        |
| $x_8$ - French Fries    | 5        | 790              | 10.0           | 100.0        | 40.0       | 0                 | 0.45               | 0.05               | 25.0              | 0            |
| $x_9$ - Oranges         | 2.17     | 134.54           | 2.17           | 32.55        | 0.0        | 60.76             | 0.24               | 0.11               | 151.9             | 5.64         |
| $x_{11}$ - Bran Muffin  | 0.0      | 0.0              | 0.0            | 0.0          | 0.0        | 0.0               | 0.0                | 0.0                | 0.0               | 0.0          |
| Diet Summary            |          | 1690             | 60             | 300          | 40         | 1484              | 2.8                | 1.7                | 1396.8            | 65.4         |
| Daily Requirement       |          |                  | 60             | 300          | 40         | 800               | 1.0                | 1.2                | 60                | 10           |

Table 5: Nutritional Summary of Diet (double calorie muffins).

There are several observations to make from this new solution:

-

## 2 Q2. Example of Conversion to Standard Form

## 3 Q3. Example of Graphically Solving a Linear Program

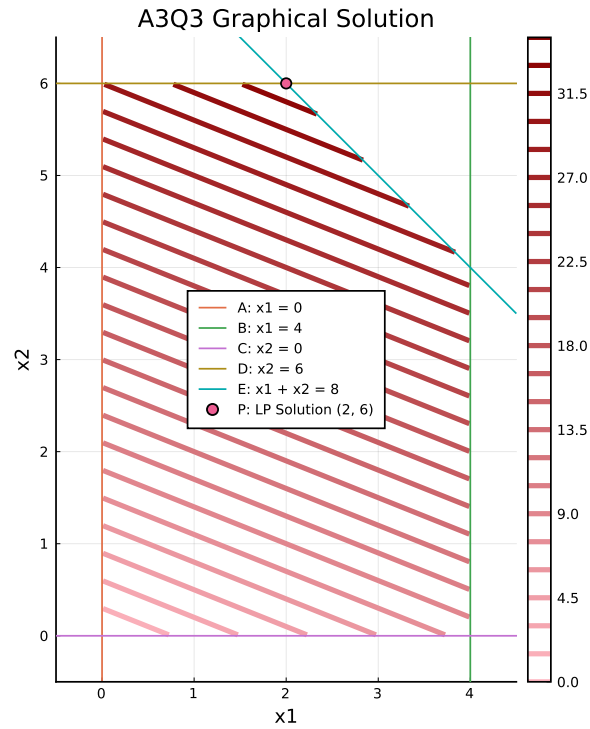


Figure 1: Graphical Solution to A3Q3 question, showing feasible region, objective contours, and solution.

## 4 Q4. Textbook Questions

4.1 12.9

4.2 12.15

4.3 12.21

4.4 12.22

## A Source Code