Optimization Assignment 3
Linear Programs and Applications

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Winter 2021

Contents

1 Q1. Linear Programming Applied to Diet Optimizaton

1.1 Problem Set-up

In the assignment, a word problem was given in which a diet had to be constructed such that it minimized the total calorie content, while also still meeting the minimum daily nutriential requirements given.

In this section the word problem will be formulated into linear program consisting of the following elements.

- Variables
- Objective Function
- Constraints

1.1.1 Variables

The variables are the number of servings of each food item as denoted in Tab. 1.

Variable	Description
x_1	Number of servings of provolone
x_2	Number of servings of mozzarella
x_3	Number of servings of 2% milk
x_4	Number of servings of salami
x_5	Number of servings of ham
x_6	Number of servings of brussel sprouts
x_7	Number of servings of lettuce
x_8	Number of servings of french fries
x_9	Number of servings of orange
x_{10}	Number of servings of whole wheat bread
x_{11}	Number of servings of bran muffin

Table 1: Variables for Linear Program for Diet Optimization

1.1.2 Objective Function

Let $E(x_i)$ denote the caloric (energy) content of 1 serving of food x_i in units of kcal (equivalent to in units of food calories (i.e 1 kcal = 1 food calorie)).

The objective of this linear program is the following. Shown without any constraints.

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_i E(x_i)x_i \tag{1}$$

1.1.3 Constraints

In this linear program, the constraints are to meet all the daily nutriential values. There are also constraints to ensure that serving portions are non-negative.

Let the following functions denote the nutritional content of 1 serving of the food x_i

- Protein(x_i) - protein in grams
- Carb(x_i) - carbohydrates in grams
- Fat(x_i) - fat in grams
- VitA(x_i) - Vitamin A in REs
- VitB1(x_i) - Vitamin B1 Thiamin in milligrams

- $\text{VitB2}(x_i)$ - Vitamin B2 Riboflavin in milligrams
- $\text{VitC}(x_i)$ - Vitamin C in milligrams
- $\text{Fibre}(x_i)$ - Fibre in grams

Then the linear program can be summarized as the following.

$$\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \sum_i E(x_i) x_i \\
\text{subject to} \quad & x_i \geq 0,\ i = 1, \ldots, 11 \\
& \sum_i \text{Protein}(x_i) x_i \geq 60 \\
& \sum_i \text{Carb}(x_i) x_i \geq 300 \\
& \sum_i \text{Fat}(x_i) x_i \geq 40 \\
& \sum_i \text{VitA}(x_i) x_i \geq 800 \\
& \sum_i \text{VitB1}(x_i) x_i \geq 1.0 \\
& \sum_i \text{VitB2}(x_i) x_i \geq 1.2 \\
& \sum_i \text{VitC}(x_i) x_i \geq 60 \\
& \sum_i \text{Fibre}(x_i) x_i \geq 10
\end{aligned} \tag{2}$$

1.2 Solution using Julia

The linear program was solved using JuMP.jl from the Julia ecosystem, yielding the following solution.

Variable	Food	Diet Serving Count	Diet Quantity
x_1	Provolone	0.0	-
x_2	Mozzarella	0.0	-
x_3	2% Milk	0.0	-
x_4	Salami	0.0	-
x_5	Ham	0.0	-
x_6	Brussel Sprouts	6.96	1740.0 mL
x_7	Lettuce	0.0	-
x_8	French Fries	0.0	-
x_9	Orange	2.17	2.17 medium
x_{10}	Whole Wheat Bread	0.0	-
x_{11}	Bran Muffin	10.0	10.0 medium

Table 2: Optimized diet solution using Julia. Serving counts, and quantity shown.

	Servings	Energy [kcal]	Protein [g]	Carbs [g]	Fat [g]	Vitamin A [RE]	Vitamin B1 [mg]	Vitamin B2 [mg]	Vitamin C [mg]	Fibre [g]
x_6 - Brussel Sprouts	6.96	445.44	27.84	97.44	0.0	828.24	1.25	0.90	709.92	34.8
x_8 - French Fries	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
x_9 - Oranges	2.17	134.54	2.17	32.55	0.0	60.76	0.24	0.11	151.9	5.64
x_{11} - Bran Muffin	10.0	1040.0	30	170	40	180	0.5	0.8	0.0	18
Diet Summary		1620.0	60	300	40	1069	2.0	1.8	861.8	58.4
Daily Requirement			60	300	40	800	1.0	1.2	60	10

Table 3: Nutrition Summary of Optimized Diet using Julia showing all nutritional daily requirements are met. Rounded values for clarity.

1.3 Solution using Matlab

The linear program was solved using Matlab, yielding the following solution.

1.4 Observations

There are several observations to make from these solutions.

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1.5 Follow-up Question and Solution - Doubling Calories in Bran Muffins

A follow up question in the assignment was to determine what happens to the diet if the calories in the muffins are increased by a factor of 2.

Using this new calorie content for the bran muffins yields the following solution.

Variable	Food	Diet Serving Count	Diet Quantity
x_1	Provolone	0.0	-
x_2	Mozzarella	0.0	-
x_3	2% Milk	0.0	-
x_4	Salami	0.0	-
x_5	Ham	0.0	-
x_6	Brussel Sprouts	11.96	2990 mL
x_7	Lettuce	0.0	-
x_8	French Fries	5.0	50 strips
x_9	Orange	2.17	2.17 medium
x_{10}	Whole Wheat Bread	0.0	-
x_{11}	Bran Muffin	0.0	-

Table 4: Optimized diet solution, where bran muffins had 2x calories. Serving counts, and quantity shown. (Solution found using Julia).

	Servings	Energy [kcal]	Protein [g]	Carbs [g]	Fat [g]	Vitamin A [RE]	Vitamin B1 [mg]	Vitamin B2 [mg]	Vitamin C [mg]	Fibre [g]
x_6 - Brussel Sprouts	11.96	765.44	47.84	167.44	0.0	1423.24	2.1582	1.55	1219.92	59.80
x_8 - French Fries	5	790	10.0	100.0	40.0	0	0.45	0.05	25.0	0
x_9 - Oranges	2.17	134.54	2.17	32.55	0.0	60.76	0.24	0.11	151.9	5.64
x_{11} - Bran Muffin	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Diet Summary		1690	60	300	40	1484	2.8	1.7	1396.8	65.4
Daily Requirement			60	300	40	800	1.0	1.2	60	10

Table 5: Nutritional Summary of Diet (double calorie muffins).

There are several observations to make from this new solution:

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2 Q2. Example of Conversion to Standard Form

3 Q3. Example of Graphically Solving a Linear Program

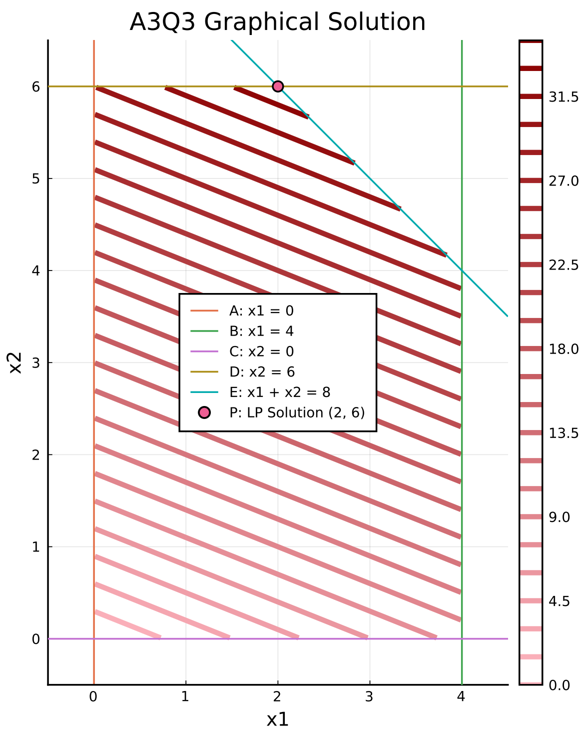

Figure 1: Graphical Solution to A3Q3 question, showing feasible region, objective contours, and solution.

Fig. 1 shows the graphical solution of the linear program in the assignment. Lines A, B, C, D, and E show the lines that bound the feasible region of the linear program. These lines are from the constraints given in the linear program.

The contours in the feasible region then show how the objective function increases within the feasible region. Clearly seen in the figure is that the maximum occurs at the insection of lines D and E.

Solving for the intersection of lines D and E yield the linear program solution (x1, x2) = (2, 6), as seen below.

$$x_2 = 6 \qquad \text{and} \qquad x_1 + x_2 = 8$$

$$x_1 + 6 = 8$$

$$x_1 = 2$$

$$(x_1, x_2) = (2, 6)$$

4 Q4. Textbook Questions

4.1 12.9

4.2 12.15

4.3 12.21

4.4 12.22

A Source Code