Optimization Assignment 3
Linear Programs and Applications

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Contents

1 Q1. Linear Programming Applied to Diet Optimizaton

1.1 Problem Set-up

In the assignment, a word problem was given in which a diet had to be constructed such that it minimized the total calorie content, while also still meeting the minimum daily nutriential requirements given.

In this section the word problem will be formulated into linear program consisting of the following elements.

- Variables
- Objective Function
- Constraints

1.1.1 Variables

The variables are the number of servings of each food item as denoted in Tab. 1.

Variable	Description
x_1	Number of servings of provolone
x_2	Number of servings of mozzarella
x_3	Number of servings of 2% milk
x_4	Number of servings of salami
x_5	Number of servings of ham
x_6	Number of servings of brussel sprouts
x_7	Number of servings of lettuce
x_8	Number of servings of french fries
x_9	Number of servings of orange
x_{10}	Number of servings of whole wheat bread
x_{11}	Number of servings of bran muffin

Table 1: Variables for Linear Program for Diet Optimization

1.1.2 Objective Function

Let $E(x_i)$ denote the caloric (energy) content of 1 serving of food x_i in units of kcal (equivalent to in units of food calories (i.e 1 kcal = 1 food calorie)).

The objective of this linear program is the following. Shown without any constraints.

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_i E(x_i)x_i \tag{1}$$

1.1.3 Constraints

In this linear program, the constraints are to meet all the daily nutriential values. There are also constraints to ensure that serving portions are non-negative.

Let the following functions denote the nutritional content of 1 serving of the food x_i

- $\text{Protein}(x_i)$ - protein in grams
- $\text{Carb}(x_i)$ - carbohydrates in grams
- $\text{Fat}(x_i)$ - fat in grams
- $\text{VitA}(x_i)$ - Vitamin A in REs
- $\text{VitB1}(x_i)$ - Vitamin B1 Thiamin in milligrams

3

- VitB2(x_i) - Vitamin B2 Riboflavin in milligrams

- VitC(x_i) - Vitamin C in milligrams

- Fibre(x_i) - Fibre in grams

Then the linear program can be summarized as the following.

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \sum_i E(x_i)x_i \\
\text{subject to} \quad & x_i \geq 0, \ i = 1, \ldots, 11 \\
& \sum_i \text{Protein}(x_i)x_i \geq 60 \\
& \sum_i \text{Carb}(x_i)x_i \geq 300 \\
& \sum_i \text{Fat}(x_i)x_i \geq 40 \\
& \sum_i \text{VitA}(x_i)x_i \geq 800 \\
& \sum_i \text{VitB1}(x_i)x_i \geq 1.0 \\
& \sum_i \text{VitB2}(x_i)x_i \geq 1.2 \\
& \sum_i \text{VitC}(x_i)x_i \geq 60 \\
& \sum_i \text{Fibre}(x_i)x_i \geq 10
\end{aligned}
\tag{2}
$$

1.2 Solution using Julia

The linear program was solved using JuMP.jl from the Julia ecosystem, yielding the following solution.

1.3 Solution using Matlab

The linear program was solved using Matlab, yielding the following solution.

1.4 Observations

There are several observations to make from these solutions.

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1.5 Follow-up Question and Solution - Doubling Calories in Bran Muffins

A follow up question in the assignment was to determine what happens to the diet if the calories in the muffins are increased by a factor of 2.

Using this new calorie content for the bran muffins yields the following solution.
There are several observations to make from this new solution:

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2   Q2. Example of Conversion to Standard Form

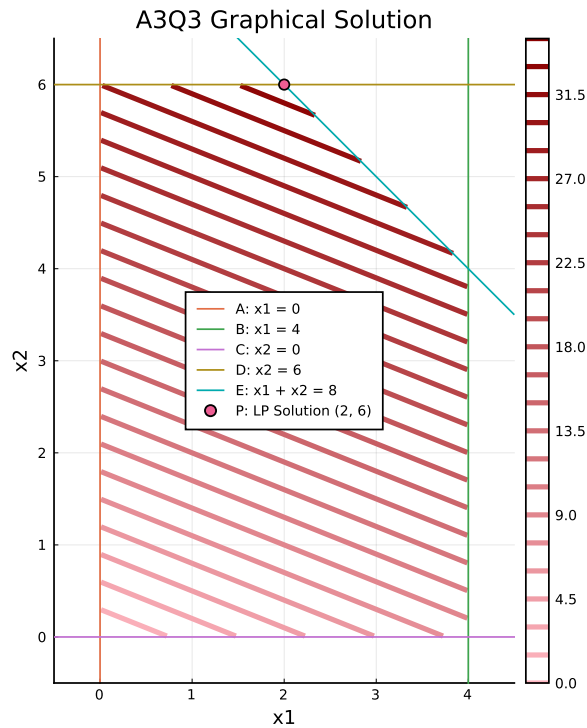# 3   Q3. Example of Graphically Solving a Linear Program

Figure 1: Graphical Solution to A3Q3 question, showing feasible region, objective contours, and solution.

4   Q4. Textbook Questions

4.1   12.9

4.2   12.15

4.3   12.21

4.4   12.22

A   Source Code