

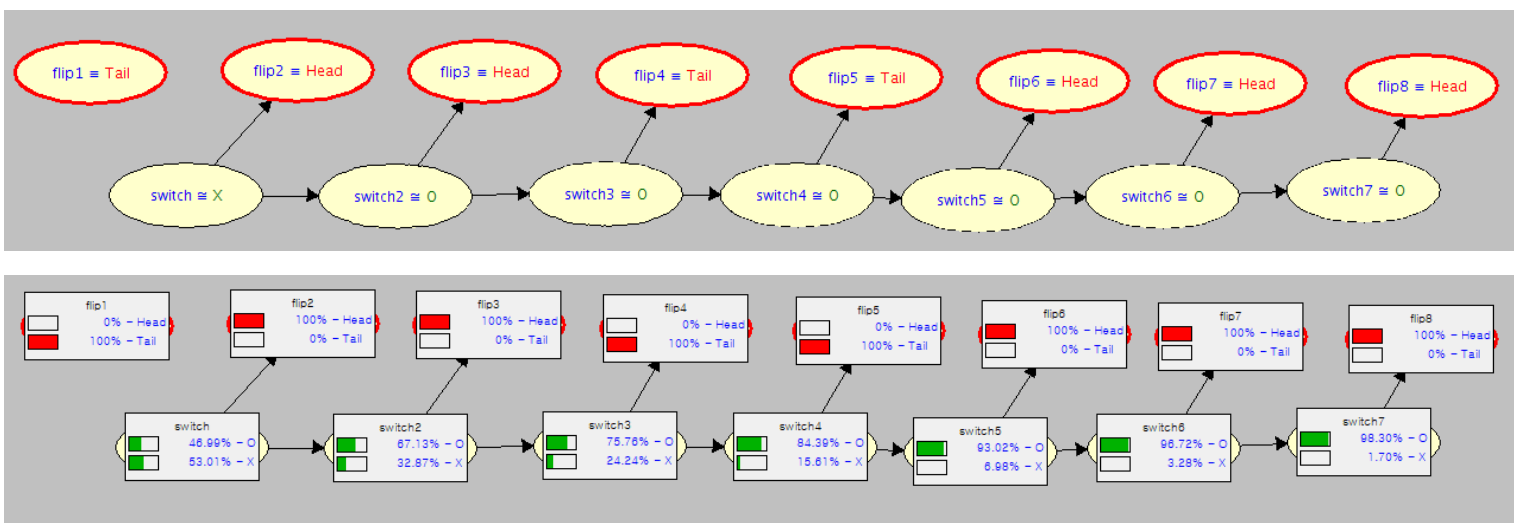
1 The Problem

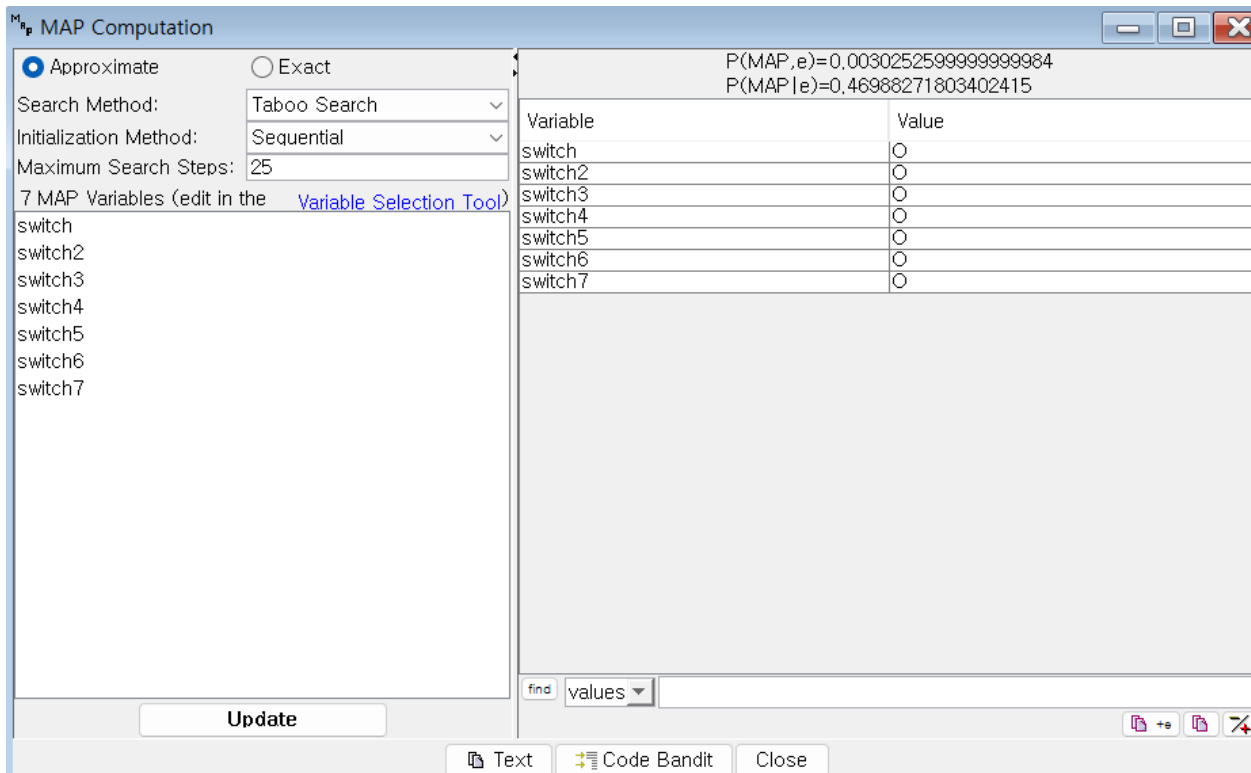
Lisa is given a fair coin C1 and asked to flip it eight times in a row. Lisa also has a biased coin C2 with a probability .7 of landing heads. All we know is that Lisa flipped the fair coin initially (the first flip), then she intends to switch to the biased coin, and that she tends to be 40% successful in performing the switch (per attempt). Lisa will keep using the biased coin if switched successfully. Suppose that we observe the outcomes of the eight coin flips. We want to find out whether Lisa managed to perform a coin switch and when.

Suppose that the outcome of the eight coin flips are: tail, head, head, tail, tail, head, head, head. Has Lisa managed to perform a coin switch? When?

2 Tasks

1. Construct a Bayesian network for solving the above problem using a Bayesian network software tool (e.g. SamIam). Save the Bayesian network file (e.g. as Coin.net in SamIam).





2. Describe a probabilistic query for solving the problem, and answer the queries using the Bayesian network software tool. (Note: probabilistic queries refer to queries such as $P(X)$, $P(X = \text{True} | Y = \text{False})$, specific MAP, MPE queries, etc.)

Figures below are the Bayesian network performed using Samiam program. By using the nodes and edges in Samiam program, it is possible to show A probability graph model that represents the causal relationship between probability variables in graph form and then learns the distribution of probability variables for a given data.

From my model, since flip 1 is independent; does not relate to any other process. Therefore, it is there independently. After the first flip, Lisa now attempts to switch to a fraud coin. Which, flip 2 ~ 8 will be dependent on the result of the switch.

For Flip 2 to 8, if the switch is a success, then the probability will be head: 0.7 and tail 0.3. If not, it will be 50:50. For switch nodes, switches 2 to 7 are set if the previous switch had been succeeded, then it is 100% chance to say it is switched. Because in the problem condition, it is given that if Lisa manages to switch her coin, then it is possible to keep that coin. O represents the success of switch, and X represents the failure. If previous switch was failure, Lisa gets another chance to change her coin, which has 4:6 probability.

As a result, I got $P(\text{MAP}, e) = 0.003025\dots$ and $P(\text{MAP} | e) = 0.469\dots$

Where I have mapped switch, and set flip value to tail, head, head, tail, tail, head, head, head.

From the second figure, it is possible to say that Lisa managed to perform a coin switch during her second switch, which also means that starting with a third coin, it is the biased coin. On switch 1, it was 53.01% whereas on switch2, it was 67.13%. After the coin is switched, the switched coin will be maintained.