CSE-321 Assignment - *Inductive Proofs*

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Due in class, Wednesday, March 19, 2pm

In this assignment, you will practice inductive proofs.

- Please write your proofs clearly and legibly.
- Begin your proof by stating how the proof proceeds, *e.g.*, "By rule induction on $\cdots$".
- In each proof step, write the conclusion on the left side and the justification on the right side. You receive no points if the teaching assistant fails to understand your proof.

1 Matched parentheses

Consider the inference rules for the judgment s mparen and s lparen for recognizing strings of matched parentheses:

$$\frac{}{\epsilon\ \mathsf{mparen}}\ \mathit{Meps} \qquad \frac{s\ \mathsf{mparen}}{(s)\ \mathsf{mparen}}\ \mathit{Mpar} \qquad \frac{s_1\ \mathsf{mparen} \quad s_2\ \mathsf{mparen}}{s_1\ s_2\ \mathsf{mparen}}\ \mathit{Mseq}$$

$$\frac{}{\epsilon\ \mathsf{lparen}}\ \mathit{Leps} \qquad \frac{s_1\ \mathsf{lparen} \quad s_2\ \mathsf{lparen}}{(s_1)\ s_2\ \mathsf{lparen}}\ \mathit{Lseq}$$

We use ϵ to denote an empty string (*i.e.*, $\epsilon s = s = s\epsilon$).

Theorem 1.1. *If* s lparen*, then* s mparen*.*

Question 1. (40 Points) Give a proof of Theorem 1.1.

Consider another inductive definition of strings of matched parentheses where we use a new judgment s tparen:

$$\frac{}{\epsilon\ \mathsf{tparen}}\ \mathit{Teps} \qquad \frac{s_1\ \mathsf{tparen} \quad s_2\ \mathsf{tparen}}{s_1\ (s_2)\ \mathsf{tparen}}\ \mathit{Tseq}$$

Lemma 1.2. *If* s tparen *and* s' tparen*, then* $s\ s'$ tparen*.*

Question 2. (40 Points) Give a proof of Lemma 1.2.

Theorem 1.3. *If* s mparen*, then* s tparen*.*

Question 3. (20 Points) Use Lemma 1.2 to prove Theorem 1.3.