CSED433 Assignment - *Inductive Proofs*

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Theorem 1.1. If s lparen, then s mparen.

Question 1.

Proof. By rule induction on judgement s lparen.

Case $\dfrac{}{\epsilon \text{ lparen}}\ Leps$ where $s = \epsilon$:

ϵ mparen	by the rule $Meps$

Case $\dfrac{s_1 \text{ lparen} \qquad s_2 \text{ lparen}}{(s_1)s_2 \text{ lparen}}\ Lseq$ where $s = (s_1)s_2$:

s_1 mparen	by the induction hypothesis on s_1 lparen
s_2 mparen	by the induction hypothesis on s_2 lparen
(s_1) mparen	by the rule $Mpar$
$(s_1)s_2$ mparen	by the rule $Mseq$ with (s_1) mparen and s_2 mparen

□

Lemma 1.2. If s tparen and s' tparen, then ss' tparen.

Question 2.

Proof. We shall show that if s' tparen then s tparen implies ss' tparen. By rule induction on judgement s' tparen.

Case $\dfrac{}{\epsilon \text{ tparen}}\ Teps$ where $s' = \epsilon$:

s tparen	assumption
$ss' = s\epsilon = s$	
ss' tparen	from s tparen and $s = ss'$

Case $\dfrac{s_1 \text{ tparen} \qquad s_2 \text{ tparen}}{s_1(s_2) \text{ tparen}}\ Tseq$ where $s' = s_1(s_2)$:

s tparen	assumption
$ss' = ss_1(s_2)$	
"s tparen implies ss_1 tparen"	by the induction hypothesis on s_1 tparen
ss_1 tparen	from the assumption s tparen
$ss_1(s_2)$ tparen	by the rule $Tseq$ with ss_1 tparen and s_2 tparen
ss' tparen	from $ss_1(s_2)$tparen and $ss' = ss_1(s_2)$

□

Theorem 1.3. If s mparen, then s tparen.

Question 3.

Proof. By rule induction on judgement s mparen.

Case $\dfrac{}{\epsilon \text{ mparen}}\ Meps$ where $s = \epsilon$:

ϵ tparen	by the rule $Teps$

Case $\dfrac{s' \text{ mparen}}{(s') \text{ mparen}}\ Mpar$ where $s = (s')$:

ϵ tparen	by the rule $Teps$
s' tparen	by the induction hypothesis
$\epsilon(s') = (s')$	
$\epsilon(s')$ tparen	by the rule $Tseq$ with ϵ tparen and s' tparen
(s') tparen	$\epsilon(s') = (s')$

Case $\dfrac{s_1 \text{ mparen} \qquad s_2 \text{ mparen}}{s_1 s_2 \text{ mparen}}\ Mseq$ where $s = s_1 s_2$:

s_1 tparen	by the induction hypothesis on s_1 mparen
s_2 tparen	by the induction hypothesis on s_2 mparen
$s_1 s_2$ tparen	by Lemma 1.2

□

Theorem 1.4. If s lparen, then s tparen.

Question 4.

Proof. By rule induction on judgement s lparen.

Case $\dfrac{}{\epsilon \text{ lparen}}\ Leps$ where $s = \epsilon$:

ϵ tparen	by the rule $Teps$

Case $\dfrac{s_1 \text{ lparen} \qquad s_2 \text{ lparen}}{(s_1)s_2 \text{ lparen}}\ Lseq$ where $s = (s_1)s_2$:

s_1 tparen	by the induction hypothesis on s_1 lparen
s_2 tparen	by the induction hypothesis on s_2 lparen
ϵ tparen	by the rule $Teps$
$(s_1)\epsilon$ tparen	by the rule $Tseq$ with s_1 tparen and ϵ tparen
(s_1) tparen	$(s_1)\epsilon = (s_1)$
$(s_1)s_2$ tparen	by Lemma 1.2 with (s_1) tparen and s_2 tparen

□