CSE-433 Assignment: Inductive Set

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Part 2

The simple language uses the following definition of terms:

$$t \quad ::= \quad \mathbf{true} \mid \mathbf{false} \mid \mathbf{if}\ t\ \mathbf{then}\ t\ \mathbf{else}\ t \mid \mathbf{0} \mid \mathbf{S}\ t \mid \mathbf{P}\ t \mid \mathbf{iszero}\ t$$

A judgment nvalue t means that t is a natural number value (which cannot be further reduced):

$$\frac{}{\text{nvalue}\ \mathbf{0}}\ \textsf{nzero} \qquad \frac{\text{nvalue}\ t}{\text{nvalue}\ \mathbf{S}\ t}\ \textsf{nsucc}$$

The small-step semantics uses a reduction judgment $t \mapsto t'$ which means that term t reduces to term t' in a single step. Reduction rules for the small-step semantics are given as follows:

$$\frac{}{\mathbf{if\ true\ then}\ t_2\ \mathbf{else}\ t_3 \mapsto t_2}\ \textsf{iftrue}$$

$$\frac{}{\mathbf{if\ false\ then}\ t_2\ \mathbf{else}\ t_3 \mapsto t_3}\ \textsf{iffalse}$$

$$\frac{t_1 \mapsto t_1'}{\mathbf{if}\ t_1\ \mathbf{then}\ t_2\ \mathbf{else}\ t_3 \mapsto \mathbf{if}\ t_1'\ \mathbf{then}\ t_2\ \mathbf{else}\ t_3}\ \textsf{if}$$

$$\frac{t \mapsto t'}{\mathbf{S}\ t \mapsto \mathbf{S}\ t'}\ \textsf{succ}$$

$$\frac{}{\mathbf{P}\ \mathbf{0} \mapsto \mathbf{0}}\ \textsf{predzero}$$

$$\frac{\text{nvalue}\ t}{\mathbf{P}\ (\mathbf{S}\ t) \mapsto t}\ \textsf{predsucc}$$

$$\frac{t \mapsto t'}{\mathbf{P}\ t \mapsto \mathbf{P}\ t'}\ \textsf{pred}$$

$$\frac{}{\mathbf{iszero}\ \mathbf{0} \mapsto \mathbf{true}}\ \textsf{iszerozero}$$

$$\frac{\text{nvalue}\ t}{\mathbf{iszero}\ (\mathbf{S}\ t) \mapsto \mathbf{false}}\ \textsf{iszerosucc}$$

$$\frac{t \mapsto t'}{\mathbf{iszero}\ t \mapsto \mathbf{iszero}\ t'}\ \textsf{iszero}$$

The specification for your function `interp` is:

- `interp` t returns t' if and only if $t \mapsto^* t'$ and there is no term t'' such that $t' \mapsto t''$.