Instant Continuation Marks–Just Add $\boldsymbol{\lambda}$ Technical Report

Kimball Germane and Jay McCarthy

Brigham Young University, Provo, Utah

This report contains a formal, partially-mechanical proof of the correctness of $\mathcal{C}_{\text{cps}}$.

1 Proof

The evaluation of a λ_{cm} program proceeds with the evolution of an evaluation context and possibly reducible expression. When evaluation begins, the evaluation context is merely $\bullet$, a placeholder for the eventual result, and the reducible expression, or redex, is the program itself. The evaluation of arguments–both in application and continuation mark forms–defers evaluation of the expression at hand by storing its evaluation context and evaluating subterms. As these results are applied and evaluation continues, the size of the context fluctuates until finally, if the program terminates, we are left with a single value to plug in $\bullet$. This value is the value of the program.

The state of evaluation at any given point can be encapsulated by a pair of an evaluation context E and an expression e which we write in unorthodox style as $E[e]$. In order to prove that evaluation in the transformation corresponds to native evaluation, we must relate this state with its corresponding transformation.

We do this by overloading $\mathcal{C}_{\text{cps}}$ to accommodate evaluation contexts which allows us to formally relate $E[e]$ and $\mathcal{C}_{\text{cps}}[E[e]]$. We first define

Definition 1.

$$\xi(E) = \begin{cases} \textbf{true} & \textit{if } E = E'[(\mathrm{wcm}\, v'\, \bullet)] \textit{ for some } E' \textit{ and } v' \\ \textbf{false} & \textit{otherwise} \end{cases}$$

to denote the *flags* argument and assume that the *marks* argument is $\mathcal{C}'_{\text{cps}}[\chi(E)]$. We can now define $\mathcal{C}_{\text{cps}}$ over contexts $E \in \lambda_{cm}$:

Definition 2. $\mathcal{C}_{\text{cps}}[\bullet]$

(λ (value)
value)

Definition 3. $\mathcal{C}_{\text{cps}}[E[(\bullet \textit{ rand-value})]]$

(λ (rator-value)
((($\mathcal{C}_{\text{cps}}$*[rand-expr]*

$(\lambda$ *(rand-value)*
$((((\textit{rator-value rand-value})\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])))$
false)
$\mathcal{C}'_{\text{cps}}[\chi(E)]))$

Definition 4. $\mathcal{C}_{\text{cps}}[E[(v_0\ \bullet)]]$

$(\lambda$ *(rand-value)*
$((((v_0\ \textit{rand-value})\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)]))$

Definition 5. $\mathcal{C}_{\text{cps}}[E[(\textbf{\textit{wcm}}\ \bullet\ \textit{body-expr})]]$

$(\lambda$ *(mark-value)*
$((\lambda$ *(rest-marks)*
$(((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \hat{\mathcal{C}}_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))$
$((\xi(E)\ \hat{\mathcal{C}}_{\text{cps}}[(\textbf{snd}\ \chi(E))])\ \mathcal{C}'_{\text{cps}}[\chi(E)])))$

Definition 6. $\mathcal{C}_{\text{cps}}[E[(\textbf{\textit{wcm}}\ v_0\ \bullet)]]$

$\mathcal{C}_{\text{cps}}[E]$

This allows us to define $\mathcal{C}_{\text{cps}}$ over a context-expression pair.

Definition 7. $\mathcal{C}_{\text{cps}}[E[e]]$

$(((\mathcal{C}_{\text{cps}}[e]\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])$

From this definition, it is apparent that $\hat{\mathcal{C}}_{\text{cps}}[p]{=}(((\mathcal{C}_{\text{cps}}[p]\ \mathcal{C}_{\text{cps}}[\bullet])\ \xi(\bullet))\ \mathcal{C}'_{\text{cps}}[\bullet]){=}\mathcal{C}_{\text{cps}}[\bullet[p]]$. Now we show that substitution is preserved by the transformation.

Lemma 1 (Substitution). *For all* $e, x, v \in \lambda_{cm}$, $\mathcal{C}[e[x \leftarrow v]] = \mathcal{C}[e][x \leftarrow \mathcal{C}'[v]]$.

See section 2 for proof.

Finally, we define “filling the hole”, the insertion of a value in the context from which it came.

Definition 8. $\mathcal{C}_{\text{cps}}[E[v]]$

$(\mathcal{C}_{\text{cps}}[E]\ \mathcal{C}'_{\text{cps}}[v])$

With each significant step of native evaluation formally related with the transformation, we can express a simulation lemma.

Lemma 2 (Simulation). *For all contexts* $E \in \lambda_{cm}$ *and expressions* $e \in \lambda_{cm}$, $E[e] \rightarrow_{cm} E'[e'] \implies \mathcal{C}[E[e]] \rightarrow_v^* \mathcal{C}[E'[e']]$

We will reason by structural induction on both contexts E and terms e. Instead of nesting the induction, which requires the consideration of $|E|\cdot|e|$ cases, we will take first E and then e in isolation, in each assuming the correctness of the other, which requires the consideration of only $|E| + |e|$ cases.

First, we prove it holds for terms e. In each case, let E be an arbitrary context.

Proof. Case $e = (e_0\, e_1)$ By steps app1-app3, $\mathcal{C}_{\mathrm{cps}}[E[(e_0\, e_1)]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E[(\bullet\, e_1)][e_0]]$.

Proof. Case $e = (\mathrm{wcm}\, e_0\, e_1)$ By steps wcm1-wcm3, $\mathcal{C}_{\mathrm{cps}}[E[(\mathrm{wcm}\, e_0\, e_1)]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E[(\mathrm{wcm}\ \bullet\ e_1)][e_0]]$.

Proof. Case $e = (\mathrm{ccm})$ By steps ccm1-ccm3, $\mathcal{C}_{\mathrm{cps}}[E[(\mathrm{ccm})]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E[\chi(E)]]$.

Proof. Case $e = v_0$ By steps value1-value3, $\mathcal{C}_{\mathrm{cps}}[E[v_0]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E[v_0]]$.

Proof. Case $e = x$ By steps x1-x4, $\mathcal{C}_{\mathrm{cps}}[E[x]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E[\mathrm{error}]]$.

Now, we prove it holds for contexts E. In each case, let v_0 be an arbitrary value.

Proof. Case $E = \bullet$ This is identical to the case that $e = v_0$.

Proof. Case $E = E'[(\bullet\, e_1)]$ By step app4, $\mathcal{C}_{\mathrm{cps}}[E[v_0]] \rightarrow_v \mathcal{C}_{\mathrm{cps}}[E'[(v_0\, \bullet)][e_1]]$.

Proof. Case $E = E'[(v_0\, \bullet)]$ By step app5, $\mathcal{C}_{\mathrm{cps}}[E[v_0]] \rightarrow_v \mathcal{C}_{\mathrm{cps}}[E'[(v_0\, v_1)]]$.
By step app6 and lemma 1, $\mathcal{C}_{\mathrm{cps}}[E'[(v_0\, v_1)]] \rightarrow_v \mathcal{C}_{\mathrm{cps}}[E'[e']]$.

Proof. Case $E = E'[(\mathrm{wcm}\ \bullet\ e_1)]$ If $E' = E''[(\mathrm{wcm}\, v'\, \bullet)$ for some E'' and v', then $\mathcal{C}_{\mathrm{cps}}[E'[(\mathrm{wcm}\ \bullet\ e_1)][v_0]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E''[(\mathrm{wcm}\, v_0\, \bullet)][e_1]]$ by steps wcm4tail-wcm6tail.
Otherwise, $\mathcal{C}_{\mathrm{cps}}[E'[(\mathrm{wcm}\, \bullet\, e_1)][v_0]] \rightarrow_v^* \mathcal{C}_{\mathrm{cps}}[E'[(\mathrm{wcm}\, v_0\, \bullet)][e_1]]$ by steps wcm4nontail-wcm6nontail.

Proof. Case $E = E'[(\mathrm{wcm}\, v_0\, \bullet)]$ By definition, $\mathcal{C}_{\mathrm{cps}}[E[v_1]] = \mathcal{C}_{\mathrm{cps}}[E'[v_1]]$.

Corollary 1. *For all contexts E and terms $e \in \lambda_{cm}$, if $E[e] \rightarrow_{cm}^* v$, then $\mathcal{C}[E[e]] \rightarrow_v^* \mathcal{C}'[v]$.*

Proof. By simulation.

Theorem 1. *For all programs $p \in \lambda_{cm}$, $\hat{\mathcal{C}}_{\mathrm{cps}}[\mathrm{eval}_{cm}(p)] = \mathrm{eval}_v(\hat{\mathcal{C}}_{\mathrm{cps}}[p])$.*

Proof.

$$\begin{aligned}
p \rightarrow_{cm}^* v &\implies \mathrm{eval}_{cm}(p) = v && \text{by definition of } \mathrm{eval}_{cm}\\
&\implies \hat{\mathcal{C}}[\mathrm{eval}_{cm}(p)] \rightarrow_v^* \mathcal{C}'[v] && \text{by definition of } \hat{\mathcal{C}}\\
&\implies \hat{\mathcal{C}}[\mathrm{eval}_{cm}(p)] \equiv \mathcal{C}'[v] && \text{by definition of equivalency}
\end{aligned}$$

$$\begin{aligned}
p \rightarrow_{cm}^* v &\implies \bullet[p] \rightarrow_{cm}^* v && (\text{since } p \rightarrow_{cm} \bullet[p])\\
&\implies \mathcal{C}[\bullet[p]] \rightarrow_v^* \mathcal{C}'[v] && (\text{by corollary 1})\\
&\implies (((\mathcal{C}[p]\, \mathcal{C}[\bullet])\, \xi(\bullet))\, \mathcal{C}'[\chi(\bullet)]) \rightarrow_v^* \mathcal{C}[v] && \text{by definition of } \mathcal{C}\\
&\implies \hat{\mathcal{C}}[p] \rightarrow_v^* \mathcal{C}[v] && (\text{by definition of } \hat{\mathcal{C}})\\
&\implies \mathrm{eval}_v(\hat{\mathcal{C}}[p]) = \mathcal{C}'[v] && (\text{by definition of } \mathrm{eval}_v)
\end{aligned}$$

Therefore, $\mathcal{C}[\mathrm{eval}_{cm}(p)] \equiv \mathrm{eval}_v(\mathcal{C}[p])$.

2 Proof of Lemma 1

Proof. Case $e = (\mathit{rator\text{-}expr}\ \mathit{rand\text{-}expr})$
$\mathcal{C}_{\text{cps}}[(\mathit{rator\text{-}expr}\ \mathit{rand\text{-}expr})[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\mathit{rator\text{-}expr}[x \leftarrow v]\ \mathit{rand\text{-}expr}[x \leftarrow v])]$
$\mathcal{C}_{\text{cps}}[(\mathit{rator\text{-}expr}\ \mathit{rand\text{-}expr})][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$$\begin{array}{l}
(\lambda\ (\mathit{kont}) \\
\quad (\lambda\ (\mathit{flag}) \\
\qquad (\lambda\ (\mathit{marks}) \\
\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rator\text{-}expr}] \\
\qquad\qquad (\lambda\ (\mathit{rator\text{-}value}) \\
\qquad\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rand\text{-}expr}] \\
\qquad\qquad\qquad (\lambda\ (\mathit{rand\text{-}value}) \\
\qquad\qquad\qquad\ \ ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks}))) \\
\qquad\qquad\qquad \mathbf{false})\ \mathit{marks}))) \\
\qquad\qquad \mathbf{false})\ \mathit{marks}))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\mathit{kont}) \\
\quad (\lambda\ (\mathit{flag}) \\
\qquad (\lambda\ (\mathit{marks}) \\
\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rator\text{-}expr}] \\
\qquad\qquad (\lambda\ (\mathit{rator\text{-}value}) \\
\qquad\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rand\text{-}expr}] \\
\qquad\qquad\qquad (\lambda\ (\mathit{rand\text{-}value}) \\
\qquad\qquad\qquad\ \ ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks}))) \\
\qquad\qquad\qquad \mathbf{false})\ \mathit{marks}))) \\
\qquad\qquad \mathbf{false})\ \mathit{marks}))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\mathit{kont}) \\
\quad (\lambda\ (\mathit{flag}) \\
\qquad (\lambda\ (\mathit{marks}) \\
\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rator\text{-}expr}] \\
\qquad\qquad (\lambda\ (\mathit{rator\text{-}value}) \\
\qquad\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rand\text{-}expr}] \\
\qquad\qquad\qquad (\lambda\ (\mathit{rand\text{-}value}) \\
\qquad\qquad\qquad\ \ ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks}))) \\
\qquad\qquad\qquad \mathbf{false})\ \mathit{marks}))) \\
\qquad\qquad \mathbf{false})\ \mathit{marks})[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])))
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\mathit{kont}) \\
\quad (\lambda\ (\mathit{flag}) \\
\qquad (\lambda\ (\mathit{marks}) \\
\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rator\text{-}expr}] \\
\qquad\qquad (\lambda\ (\mathit{rator\text{-}value}) \\
\qquad\qquad\ \ (((\mathcal{C}_{\text{cps}}[\mathit{rand\text{-}expr}]
\end{array}$$

$$
\begin{array}{l}
\hspace{6em}(\lambda\ (\textit{rand-value})\\
\hspace{7.5em}((((\textit{rator-value}\ \ \textit{rand-value})\ \ \textit{kont})\ \ \textit{flag})\ \ \textit{marks})))\\
\hspace{6em}\mathbf{false})\ \ \textit{marks})))\\
\hspace{2.5em}\mathbf{false})[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ \ \textit{marks}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\hspace{1.5em}(\lambda\ (\textit{flag})\\
\hspace{3em}(\lambda\ (\textit{marks})\\
\hspace{4.5em}(((\mathcal{C}_{\text{cps}}[\textit{rator-expr}]\\
\hspace{6.5em}(\lambda\ (\textit{rator-value})\\
\hspace{8em}(((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\hspace{10em}(\lambda\ (\textit{rand-value})\\
\hspace{11.5em}((((\textit{rator-value}\ \ \textit{rand-value})\ \ \textit{kont})\ \ \textit{flag})\ \ \textit{marks})))\\
\hspace{10em}\mathbf{false})\ \ \textit{marks})))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\\
\hspace{6.5em}\mathbf{false}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])\ \ \textit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\hspace{1.5em}(\lambda\ (\textit{flag})\\
\hspace{3em}(\lambda\ (\textit{marks})\\
\hspace{4.5em}(((\mathcal{C}_{\text{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\\
\hspace{6.5em}(\lambda\ (\textit{rator-value})\\
\hspace{8em}(((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\hspace{10em}(\lambda\ (\textit{rand-value})\\
\hspace{11.5em}((((\textit{rator-value}\ \ \textit{rand-value})\ \ \textit{kont})\ \ \textit{flag})\ \ \textit{marks})))\\
\hspace{10em}\mathbf{false})\ \ \textit{marks}))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])\\
\hspace{6.5em}\mathbf{false})\ \ \textit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\hspace{1.5em}(\lambda\ (\textit{flag})\\
\hspace{3em}(\lambda\ (\textit{marks})\\
\hspace{4.5em}(((\mathcal{C}_{\text{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\\
\hspace{6.5em}(\lambda\ (\textit{rator-value})\\
\hspace{8em}(((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\hspace{10em}(\lambda\ (\textit{rand-value})\\
\hspace{11.5em}((((\textit{rator-value}\ \ \textit{rand-value})\ \ \textit{kont})\ \ \textit{flag})\ \ \textit{marks})))\\
\hspace{10em}\mathbf{false})\ \ \textit{marks})[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))\\
\hspace{6.5em}\mathbf{false})\ \ \textit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\hspace{1.5em}(\lambda\ (\textit{flag})\\
\hspace{3em}(\lambda\ (\textit{marks})\\
\hspace{4.5em}(((\mathcal{C}_{\text{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\\
\hspace{6.5em}(\lambda\ (\textit{rator-value})\\
\hspace{8em}(((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\hspace{10em}(\lambda\ (\textit{rand-value})
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\quad ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks})))\\
\qquad\qquad\qquad \mathbf{false})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \mathit{marks}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])))\\
\qquad\qquad \mathbf{false})\ \mathit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\mathit{kont})\\
\quad (\lambda\ (\mathit{flag})\\
\quad\ \ (\lambda\ (\mathit{marks})\\
\qquad\ (((\mathcal{C}_{\mathrm{cps}}[\mathit{rator\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\quad (\lambda\ (\mathit{rator\text{-}value})\\
\qquad\qquad (((\mathcal{C}_{\mathrm{cps}}[\mathit{rand\text{-}expr}]\\
\qquad\qquad\quad (\lambda\ (\mathit{rand\text{-}value})\\
\qquad\qquad\qquad ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks})))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\quad \mathbf{false}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\ \mathit{marks})))\\
\qquad\quad \mathbf{false})\ \mathit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\mathit{kont})\\
\quad (\lambda\ (\mathit{flag})\\
\quad\ \ (\lambda\ (\mathit{marks})\\
\qquad\ (((\mathcal{C}_{\mathrm{cps}}[\mathit{rator\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\quad (\lambda\ (\mathit{rator\text{-}value})\\
\qquad\qquad (((\mathcal{C}_{\mathrm{cps}}[\mathit{rand\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\quad (\lambda\ (\mathit{rand\text{-}value})\\
\qquad\qquad\qquad ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks}))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\\
\qquad\qquad\quad \mathbf{false})\ \mathit{marks})))\\
\qquad\quad \mathbf{false})\ \mathit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\mathit{kont})\\
\quad (\lambda\ (\mathit{flag})\\
\quad\ \ (\lambda\ (\mathit{marks})\\
\qquad\ (((\mathcal{C}_{\mathrm{cps}}[\mathit{rator\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\quad (\lambda\ (\mathit{rator\text{-}value})\\
\qquad\qquad (((\mathcal{C}_{\mathrm{cps}}[\mathit{rand\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\quad (\lambda\ (\mathit{rand\text{-}value})\\
\qquad\qquad\qquad ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))\\
\qquad\qquad\quad \mathbf{false})\ \mathit{marks})))\\
\qquad\quad \mathbf{false})\ \mathit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\mathit{kont})\\
\quad (\lambda\ (\mathit{flag})\\
\quad\ \ (\lambda\ (\mathit{marks})\\
\qquad\ (((\mathcal{C}_{\mathrm{cps}}[\mathit{rator\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\quad (\lambda\ (\mathit{rator\text{-}value})\\
\qquad\qquad (((\mathcal{C}_{\mathrm{cps}}[\mathit{rand\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\quad (\lambda\ (\mathit{rand\text{-}value})\\
\qquad\qquad\qquad ((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \mathit{marks}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])))
\end{array}
$$

$$\begin{array}{l}
\qquad\qquad\qquad\quad \textbf{false})\ \textit{marks})))\\
\qquad\qquad\quad \textbf{false})\ \textit{marks}))))
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad (\lambda\ (\textit{rator-value})\\
\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rand-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\qquad (\lambda\ (\textit{rand-value})\\
\qquad\qquad\qquad\quad ((((\textit{rator-value}\ \textit{rand-value})\ \textit{kont})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \textit{flag}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\ \textit{marks})))\\
\qquad\qquad\qquad \textbf{false})\ \textit{marks})))\\
\qquad\qquad \textbf{false})\ \textit{marks}))))
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad (\lambda\ (\textit{rator-value})\\
\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rand-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\qquad (\lambda\ (\textit{rand-value})\\
\qquad\qquad\qquad\quad ((((\textit{rator-value}\ \textit{rand-value})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \textit{kont}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\ \textit{flag})\ \textit{marks})))\\
\qquad\qquad\qquad \textbf{false})\ \textit{marks})))\\
\qquad\qquad \textbf{false})\ \textit{marks}))))
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad (\lambda\ (\textit{rator-value})\\
\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rand-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\qquad (\lambda\ (\textit{rand-value})\\
\qquad\qquad\qquad\quad ((((\textit{rator-value}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \textit{rand-value}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\ \textit{kont})\ \textit{flag})\ \textit{marks})))\\
\qquad\qquad\qquad \textbf{false})\ \textit{marks})))\\
\qquad\qquad \textbf{false})\ \textit{marks}))))
\end{array}$$

$$\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rator-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad (\lambda\ (\textit{rator-value})\\
\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{rand-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\qquad (\lambda\ (\textit{rand-value})\\
\qquad\qquad\qquad\quad ((((\textit{rator-value}\ \textit{rand-value})\ \textit{kont})\ \textit{flag})\ \textit{marks})))\\
\qquad\qquad\qquad \textbf{false})\ \textit{marks})))\\
\qquad\qquad \textbf{false})\ \textit{marks}))))
\end{array}$$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{rator\text{-}expr}[x \leftarrow v]]$
$\quad\quad\quad\quad(\lambda\ (\mathit{rator\text{-}value})$
$\quad\quad\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{rand\text{-}expr}[x \leftarrow v]]$
$\quad\quad\quad\quad\quad\quad(\lambda\ (\mathit{rand\text{-}value})$
$\quad\quad\quad\quad\quad\quad\quad((((\mathit{rator\text{-}value}\ \mathit{rand\text{-}value})\ \mathit{kont})\ \mathit{flag})\ \mathit{marks})))$
$\quad\quad\quad\quad\quad\quad\mathbf{false})\ \mathit{marks})))$
$\quad\quad\quad\quad\mathbf{false})\ \mathit{marks}))))$

$\mathcal{C}_{\text{cps}}[(\mathit{rator\text{-}expr}[x \leftarrow v]\ \mathit{rand\text{-}expr}[x \leftarrow v])]$
Therefore, $\mathcal{C}_{\text{cps}}[(\mathit{rator\text{-}expr}\ \mathit{rand\text{-}expr})[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\mathit{rator\text{-}expr}\ \mathit{rand\text{-}expr})][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$.

Proof. Case $e = (\mathbf{wcm}\ \mathit{mark\text{-}expr}\ \mathit{body\text{-}expr})$
$\mathcal{C}_{\text{cps}}[(\mathbf{wcm}\ \mathit{mark\text{-}expr}\ \mathit{body\text{-}expr})[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\mathbf{wcm}\ \mathit{mark\text{-}expr}[x \leftarrow v]\ \mathit{body\text{-}expr}[x \leftarrow v])]$
$\mathcal{C}_{\text{cps}}[(\mathbf{wcm}\ \mathit{mark\text{-}expr}\ \mathit{body\text{-}expr})][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{mark\text{-}expr}]$
$\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$
$\quad\quad\quad\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\text{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$\quad\quad\quad\quad\quad((\mathit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))))$
$\quad\quad\quad\mathbf{false})\ \mathit{marks}))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{mark\text{-}expr}]$
$\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$
$\quad\quad\quad\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\text{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$\quad\quad\quad\quad\quad((\mathit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))))$
$\quad\quad\quad\mathbf{false})\ \mathit{marks})))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\text{cps}}[\mathit{mark\text{-}expr}]$
$\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$

$$
\begin{array}{l}
\qquad\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{body-expr}]\ \textit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\qquad\qquad ((\textit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks}))))\\
\qquad\quad \mathbf{false})\ \textit{marks}))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{mark-expr}]\\
\qquad\qquad\quad (\lambda\ (\textit{mark-value})\\
\qquad\qquad\qquad ((\lambda\ (\textit{rest-marks})\\
\qquad\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{body-expr}]\ \textit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\qquad\qquad ((\textit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks}))))\\
\qquad\quad \mathbf{false})\ \textit{marks})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{mark-expr}]\\
\qquad\qquad\quad (\lambda\ (\textit{mark-value})\\
\qquad\qquad\qquad ((\lambda\ (\textit{rest-marks})\\
\qquad\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{body-expr}]\ \textit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\qquad\qquad ((\textit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks}))))\\
\qquad\quad \mathbf{false})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \textit{marks}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{mark-expr}]\\
\qquad\qquad\quad (\lambda\ (\textit{mark-value})\\
\qquad\qquad\qquad ((\lambda\ (\textit{rest-marks})\\
\qquad\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{body-expr}]\ \textit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\qquad\qquad ((\textit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks}))))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\quad \mathbf{false}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\ \textit{marks}))))
\end{array}
$$

$$
\begin{array}{l}
(\lambda\ (\textit{kont})\\
\quad (\lambda\ (\textit{flag})\\
\qquad (\lambda\ (\textit{marks})\\
\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{mark-expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\\
\qquad\qquad\quad (\lambda\ (\textit{mark-value})\\
\qquad\qquad\qquad ((\lambda\ (\textit{rest-marks})\\
\qquad\qquad\qquad\quad (((\mathcal{C}_{\mathrm{cps}}[\textit{body-expr}]\ \textit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\qquad\qquad ((\textit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks})))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\\
\qquad\quad \mathbf{false})\ \textit{marks}))))
\end{array}
$$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$
$\quad\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$
$\quad\quad\quad\quad\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$\quad\quad\quad\quad\quad\quad\ ((\mathit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))$
$\quad\quad\quad\quad\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$
$\quad\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$
$\quad\quad\quad\quad\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$
$\quad\quad\quad\quad\quad\quad\ ((\mathit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])))$
$\quad\quad\quad\quad\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$
$\quad\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$
$\quad\quad\quad\quad\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})])[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])$
$\quad\quad\quad\quad\quad\quad\ ((\mathit{flag}\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \mathit{marks})])[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \mathit{marks}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))))$
$\quad\quad\quad\quad\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$
$\quad\quad\quad\quad\quad(\lambda\ (\mathit{mark\text{-}value})$
$\quad\quad\quad\quad\quad\quad((\lambda\ (\mathit{rest\text{-}marks})$
$\quad\quad\quad\quad\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})\ \mathbf{true})[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \hat{\mathcal{C}}_{\mathrm{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])$
$\quad\quad\quad\quad\quad\quad\ ((\mathit{flag}[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ \hat{\mathcal{C}}_{\mathrm{cps}}[(\mathbf{snd}\ \mathit{marks})][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])\ \mathit{marks}))))$
$\quad\quad\quad\quad\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(((\mathcal{C}_{\mathrm{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$

$(\lambda\ (\mathit{mark\text{-}value})$
$((\lambda\ (\mathit{rest\text{-}marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{body\text{-}expr}]\ \mathit{kont})[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ \mathbf{true}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])\ \hat{\mathcal{C}}_{\text{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$((\mathit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))))$
$\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$
$(\lambda\ (\mathit{mark\text{-}value})$
$((\lambda\ (\mathit{rest\text{-}marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{body\text{-}expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ \mathit{kont}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])\ \mathbf{true})\ \hat{\mathcal{C}}_{\text{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$((\mathit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))))$
$\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{mark\text{-}expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$
$(\lambda\ (\mathit{mark\text{-}value})$
$((\lambda\ (\mathit{rest\text{-}marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{body\text{-}expr}][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\text{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$((\mathit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))))$
$\mathbf{false})\ \mathit{marks}))))$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{mark\text{-}expr}[x \leftarrow v]]$
$(\lambda\ (\mathit{mark\text{-}value})$
$((\lambda\ (\mathit{rest\text{-}marks})$
$(((\mathcal{C}_{\text{cps}}[\mathit{body\text{-}expr}[x \leftarrow v]]\ \mathit{kont})\ \mathbf{true})\ \hat{\mathcal{C}}_{\text{cps}}[((\mathbf{cons}\ \mathit{mark\text{-}value})\ \mathit{rest\text{-}marks})]))$
$((\mathit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \mathit{marks})])\ \mathit{marks}))))$
$\mathbf{false})\ \mathit{marks}))))$

$\mathcal{C}_{\text{cps}}[(\mathbf{wcm}\ \mathit{mark\text{-}expr}[x \leftarrow v]\ \mathit{body\text{-}expr}[x \leftarrow v])]$
Therefore, $\mathcal{C}_{\text{cps}}[(\mathbf{wcm}\ \mathit{mark\text{-}expr}\ \mathit{body\text{-}expr})[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\mathbf{wcm}\ \mathit{mark\text{-}expr}\ \mathit{body\text{-}expr})][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$.

Proof. Case e=$(\mathbf{ccm})$
$\mathcal{C}_{\text{cps}}[(\mathbf{ccm})[x \leftarrow v]]$=$\mathcal{C}_{\text{cps}}[(\mathbf{ccm})]$
$\mathcal{C}_{\text{cps}}[(\mathbf{ccm})][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$

$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ \mathit{marks}))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ \mathit{marks})))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ \mathit{marks}))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ \mathit{marks})[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])))$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ \mathit{marks}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))))$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ \mathit{marks}))))$

$\mathcal{C}_{\text{cps}}[(\mathbf{ccm})]$
Therefore $\mathcal{C}_{\text{cps}}[(\mathbf{ccm})[x \leftarrow v]]{=}Ccps[(\mathbf{ccm})][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$.

Proof. Case $e = (\lambda\ (x)\ e')$
$\mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e')[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e')]$
$\mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e')][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e']))))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$(\lambda\ (\mathit{flag})$
$(\lambda\ (\mathit{marks})$
$(\mathit{kont}\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e'])))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e'])))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e']))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e'])[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e'])))))$

$\mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e')]$

Therefore, $\mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e')[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e')][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$.

Proof. Case $e = (\lambda\ (x')\ e')$ where $x' \neq x$

$\mathcal{C}_{\text{cps}}[(\lambda\ (x')\ e')[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(\lambda\ (x)\ e'[x \leftarrow v])]$

$\mathcal{C}_{\text{cps}}[(\lambda\ (x')\ e')][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e'])))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e']))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e'])))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))$

$(\lambda\ (\mathit{kont})$
$\quad(\lambda\ (\mathit{flag})$
$\quad\quad(\lambda\ (\mathit{marks})$
$\quad\quad\quad(\mathit{kont}\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e']))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])))$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e'])[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))))$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e'][x \leftarrow \mathcal{C}'_{\text{cps}}[v]])))))$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont\ (\lambda\ (x')\ \mathcal{C}_{\text{cps}}[e'[x \leftarrow v]])))))$

$\mathcal{C}_{\text{cps}}[(lamba\ (x')\ e'[x \leftarrow v])]$

Therefore, $\mathcal{C}_{\text{cps}}[(lamba\ (x')\ e')[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[(lamba\ (x')\ e')][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$.

Proof. Case $e = x$

$\mathcal{C}_{\text{cps}}[x[x \leftarrow v]] = \mathcal{C}_{\text{cps}}[v]$
$\mathcal{C}_{\text{cps}}[x][x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont\ x))))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont\ x)))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont\ x))[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont\ x)[x \leftarrow \mathcal{C}'_{\text{cps}}[v]])))$

$(\lambda\ (kont)$
$\quad(\lambda\ (\mathit{flag})$
$\qquad(\lambda\ (marks)$
$\qquad\quad(kont[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]\ x[x \leftarrow \mathcal{C}'_{\text{cps}}[v]]))))$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont\ \mathcal{C}'_{\mathrm{cps}}[v]))))$

$\mathcal{C}_{\mathrm{cps}}[v]$

Therefore, $\mathcal{C}_{\mathrm{cps}}[x[x \leftarrow v]] = \mathcal{C}_{\mathrm{cps}}[x][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$.

Proof. Case $e = x'$ where $x' \neq x$

$\mathcal{C}_{\mathrm{cps}}[x'[x \leftarrow v]]$=$\mathcal{C}_{\mathrm{cps}}[x']$

$\mathcal{C}_{\mathrm{cps}}[x'][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont\ x'))))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont\ x')))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont\ x'))[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont\ x')[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]])))$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]\ x'[x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]]))))$

$(\lambda\ (kont)$
$\quad(\lambda\ (flag)$
$\quad\quad(\lambda\ (marks)$
$\quad\quad\quad(kont\ x'))))$

$\mathcal{C}_{\mathrm{cps}}[x']$

Therefore, $\mathcal{C}_{\mathrm{cps}}[x'[x \leftarrow v]] = \mathcal{C}_{\mathrm{cps}}[x'][x \leftarrow \mathcal{C}'_{\mathrm{cps}}[v]$.

3 Reduction

3.1 application form

$\mathcal{C}_{\text{cps}}[E[(\textit{rator-expr}\ \ \textit{rand-expr})]]$

$$
\begin{array}{l}
((((\lambda\ (\textit{kont})\\
\quad\quad (\lambda\ (\textit{flag})\\
\quad\quad\quad (\lambda\ (\textit{marks})\\
\quad\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rator-expr}]\\
\quad\quad\quad\quad\quad\quad\quad (\lambda\ (\textit{rator-value})\\
\quad\quad\quad\quad\quad\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (\lambda\ (\textit{rand-value})\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (((((\textit{rator-value}\ \ \textit{rand-value})\ \textit{kont})\ \textit{flag})\ \textit{marks})))\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\ \mathbf{false})\ \textit{marks})))\\
\quad\quad\quad\quad\quad\ \mathbf{false})\ \textit{marks}))))\\
\quad \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

app1

$$
\begin{array}{l}
(((\lambda\ (\textit{flag})\\
\quad\quad (\lambda\ (\textit{marks})\\
\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rator-expr}]\\
\quad\quad\quad\quad\quad\quad (\lambda\ (\textit{rator-value})\\
\quad\quad\quad\quad\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (\lambda\ (\textit{rand-value})\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (((((\textit{rator-value}\ \ \textit{rand-value})\ \mathcal{C}_{\text{cps}}[E])\ \textit{flag})\ \textit{marks})))\\
\quad\quad\quad\quad\quad\quad\quad\quad \mathbf{false})\ \textit{marks})))\\
\quad\quad\quad\quad \mathbf{false})\ \textit{marks})))\\
\quad \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

app2

$$
\begin{array}{l}
((\lambda\ (\textit{marks})\\
\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rator-expr}]\\
\quad\quad\quad\quad\quad (\lambda\ (\textit{rator-value})\\
\quad\quad\quad\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad (\lambda\ (\textit{rand-value})\\
\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (((((\textit{rator-value}\ \ \textit{rand-value})\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \textit{marks})))\\
\quad\quad\quad\quad\quad\quad\quad \mathbf{false})\ \textit{marks})))\\
\quad\quad\quad \mathbf{false})\ \textit{marks}))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

app3

$$
\begin{array}{l}
(((\mathcal{C}_{\text{cps}}[\textit{rator-expr}]\\
\quad\quad\quad (\lambda\ (\textit{rator-value})\\
\quad\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{rand-expr}]\\
\quad\quad\quad\quad\quad\quad\quad (\lambda\ (\textit{rand-value})\\
\quad\quad\quad\quad\quad\quad\quad\quad (((((\textit{rator-value}\ \ \textit{rand-value})\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)]))))\\
\quad\quad\quad\quad\quad \mathbf{false})\ \mathcal{C}'_{\text{cps}}[\chi(E)])))
\end{array}
$$

false) $\mathcal{C}'_{\text{cps}}[\chi(E)]$)

app4

$$
\begin{array}{l}
(((\mathcal{C}_{\text{cps}}[\textit{rand-expr}] \\
\qquad (\lambda\ (\textit{rand-value}) \\
\qquad\quad ((((\mathcal{C}'_{\text{cps}}[v_0]\ \textit{rand-value})\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)]))) \\
\quad \mathbf{false})\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

app5

$$((((\mathcal{C}'_{\text{cps}}[v_0]\ \mathcal{C}'_{\text{cps}}[\textit{v\_1}])\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])$$

$$((((\mathcal{C}'_{\text{cps}}[(\lambda\ (x)\ \textit{e\_0})]\ \mathcal{C}'_{\text{cps}}[\textit{v\_1}])\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])$$

app6

$$(((\mathcal{C}_{\text{cps}}[\textit{e\_0}][x \leftarrow \mathcal{C}'_{\text{cps}}[\textit{v\_1}]]\ \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])$$

3.2 wcm form

$$\mathcal{C}_{\text{cps}}[E[(\mathbf{wcm}\ \textit{mark-expr}\ \textit{body-expr})]]$$

$$
\begin{array}{l}
((((\lambda\ (\textit{kont}) \\
\quad (\lambda\ (\textit{flag}) \\
\quad\quad (\lambda\ (\textit{marks}) \\
\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{mark-expr}] \\
\qquad\qquad\quad (\lambda\ (\textit{mark-value}) \\
\qquad\qquad\quad\quad ((\lambda\ (\textit{rest-marks}) \\
\qquad\qquad\quad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \textit{kont})\ \mathbf{true})\ \mathcal{C}'_{\text{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})])) \\
\qquad\qquad\quad\quad ((\textit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks})))) \\
\qquad\quad \mathbf{false})\ \textit{marks})))) \\
\quad \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

wcm1

$$
\begin{array}{l}
(((\lambda\ (\textit{flag}) \\
\quad (\lambda\ (\textit{marks}) \\
\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{mark-expr}] \\
\qquad\qquad (\lambda\ (\textit{mark-value}) \\
\qquad\qquad\quad ((\lambda\ (\textit{rest-marks}) \\
\qquad\qquad\quad\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \mathbf{true})\ \mathcal{C}'_{\text{cps}}[((\mathbf{cons}\ \textit{mark-value})\ \textit{rest-marks})])) \\
\qquad\qquad\quad ((\textit{flag}\ \hat{\mathcal{C}}_{\text{cps}}[(\mathbf{snd}\ \textit{marks})])\ \textit{marks})))) \\
\qquad \mathbf{false})\ \textit{marks}))) \\
\quad \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

wcm2

$$
\begin{array}{l}
((\lambda\ (\textit{marks}) \\
\quad (((\mathcal{C}_{\text{cps}}[\textit{mark-expr}] \\
\qquad\quad (\lambda\ (\textit{mark-value})
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad ((\lambda\ (\textit{rest-marks})\\
\qquad\qquad\qquad\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\qquad\qquad ((\xi(E)\ \hat{\mathcal{C}}_{\text{cps}}[(\textbf{snd}\ \textit{marks})])\ \textit{marks}))))\\
\qquad \textbf{false})\ \textit{marks}))\\
\ \mathcal{C}'_{\text{cps}}[\chi(E)]])
\end{array}
$$

wcm3

$$
\begin{array}{l}
(((\mathcal{C}_{\text{cps}}[\textit{mark-expr}]\\
\qquad (\lambda\ (\textit{mark-value})\\
\qquad\quad ((\lambda\ (\textit{rest-marks})\\
\qquad\qquad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\qquad\quad ((\xi(E)\ \hat{\mathcal{C}}_{\text{cps}}[(\textbf{snd}\ \chi(E))])\ \mathcal{C}'_{\text{cps}}[\chi(E)]))))\\
\quad \textbf{false})\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

wcm4tail

$$
\begin{array}{l}
((\lambda\ (\textit{rest-marks})\\
\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\ ((\textbf{true}\ \hat{\mathcal{C}}_{\text{cps}}[(\textbf{snd}\ \chi(E))])\ \mathcal{C}'_{\text{cps}}[\chi(E)]))
\end{array}
$$

wcm5tail

$$
\begin{array}{l}
((\lambda\ (\textit{rest-marks})\\
\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\ \mathcal{C}'_{\text{cps}}[(\textbf{snd}\ \chi(E))])
\end{array}
$$

wcm6tail

$$
(((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ (\textbf{snd}\ \chi(E)))])
$$

wcm4nontail

$$
\begin{array}{l}
((\lambda\ (\textit{rest-marks})\\
\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\ ((\textbf{false}\ \hat{\mathcal{C}}_{\text{cps}}[(\textbf{snd}\ \chi(E))])\ \mathcal{C}'_{\text{cps}}[\chi(E)]))
\end{array}
$$

wcm5nontail

$$
\begin{array}{l}
((\lambda\ (\textit{rest-marks})\\
\quad (((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \textit{rest-marks})]))\\
\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

wcm6nontail

$$
(((\mathcal{C}_{\text{cps}}[\textit{body-expr}]\ \mathcal{C}_{\text{cps}}[E])\ \textbf{true})\ \mathcal{C}'_{\text{cps}}[((\textbf{cons}\ \textit{mark-value})\ \chi(E))])
$$

3.3 ccm form

$$
\mathcal{C}_{\text{cps}}[E[(\textbf{ccm})]]
$$

$$
\begin{array}{l}
((((\lambda\ (\textit{kont})\\
\qquad (\lambda\ (\textit{flag})
\end{array}
$$

$$
\begin{array}{l}
\quad\quad (\lambda\ (marks) \\
\quad\quad\quad (kont\ marks)))) \\
\quad \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

ccm1

$$
\begin{array}{l}
(((\lambda\ (flag) \\
\quad\quad (\lambda\ (marks) \\
\quad\quad\quad (\mathcal{C}_{\text{cps}}[E]\ marks))) \\
\quad \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

ccm2

$$
\begin{array}{l}
((\lambda\ (marks) \\
\quad (\mathcal{C}_{\text{cps}}[E]\ marks)) \\
\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

ccm3

$$(\mathcal{C}_{\text{cps}}[E]\ \mathcal{C}'_{\text{cps}}[\chi(E)])$$

3.4 value form

$\mathcal{C}_{\text{cps}}[E[v]]{=}\mathcal{C}_{\text{cps}}[E[(\lambda\ (x)\ e)]]$

$$
\begin{array}{l}
((((\lambda\ (kont) \\
\quad\quad (\lambda\ (flag) \\
\quad\quad\quad (\lambda\ (marks) \\
\quad\quad\quad\quad (kont\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e]))))) \\
\quad \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

value1

$$
\begin{array}{l}
(((\lambda\ (flag) \\
\quad\quad (\lambda\ (marks) \\
\quad\quad\quad (\mathcal{C}_{\text{cps}}[E]\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e])))) \\
\quad \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

value2

$$
\begin{array}{l}
((\lambda\ (marks) \\
\quad (\mathcal{C}_{\text{cps}}[E]\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e]))) \\
\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

value3

$$(\mathcal{C}_{\text{cps}}[E]\ (\lambda\ (x)\ \mathcal{C}_{\text{cps}}[e]))$$

3.5 variable form

$\mathcal{C}_{\text{cps}}[E[x]]$

$$
\begin{array}{l}
((((\lambda\ (\mathit{kont}) \\
\qquad (\lambda\ (\mathit{flag}) \\
\qquad\quad (\lambda\ (\mathit{marks}) \\
\qquad\qquad (\mathit{kont}\ x)))) \\
\quad \mathcal{C}_{\text{cps}}[E])\ \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

x1

$$
\begin{array}{l}
(((\lambda\ (\mathit{flag}) \\
\qquad (\lambda\ (\mathit{marks}) \\
\qquad\quad (\mathcal{C}_{\text{cps}}[E]\ x))) \\
\quad \xi(E))\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

x2

$$
\begin{array}{l}
((\lambda\ (\mathit{marks}) \\
\quad (\mathcal{C}_{\text{cps}}[E]\ x)) \\
\ \mathcal{C}'_{\text{cps}}[\chi(E)])
\end{array}
$$

x3

$(\mathcal{C}_{\text{cps}}[E]\ x)$

x4

$(\mathcal{C}_{\text{cps}}[E]\ \mathsf{error})$

x5

error