Lecture Note 1, Civil Engineering I - an introduction to structural mechanics

1 Review of Vector Algebra

1.1 Scalar and vector quantites

Scalar is a quantitiy that has only a magnitude. The magnitude can either be positive or negative. The examples of scalar quantities appearing in physics are mass, temperature, concentration, length, weight and so on. Vector is a quantity that has both a magnitude and direction. The examples are displacement, velocity, acceleration, force, heat flux, mass flux, electrip poloarization, etc. Graphically, a vector is represented by an arrow. When drawing the arrow, the magnitude and directions of the vector are indicated by the length and the orientation of the arrow head, respectively. For mathematical manipulations of vectors, symbolic representations are far more convenient. To denote vector quantities in typed manuscripts symbolically, boldface letters such as $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{v}, \dots$ are often used instead of writting simply as x, u and v. This is to distinguish vectors from scalar qunantities at sight. To denote the magnitude of a vector $\boldsymbol{a}$, we either place vertical bars on both sides of $\boldsymbol{a}$ and write as $|\boldsymbol{a}|$,or simply as a. If $\boldsymbol{a}$ has a unit magnitude, namely $|\boldsymbol{a}| = a = 1$, $\boldsymbol{a}$ is said to be a unit vector.

1.2 Vector Identity

Two vectors, say $\boldsymbol{a}$ and $\boldsymbol{b}$, are considered identical if their magnitude and direction are the same. In that case, we write $\boldsymbol{a} = \boldsymbol{b}$ regardless of their locations. If a pair of forces, say $\boldsymbol{f}$ and $\boldsymbol{g}$ of equal magnitude and direction ($|\boldsymbol{f}| = |\boldsymbol{g}|$) are acting on two different points on a body, $\boldsymbol{f}$ and $\boldsymbol{g}$ are physically different but mathematically identical.

1.3 Basic Algebraic Operation on Vectors

1.3.1 Scalar multiplication

A multiplication between a scalar s and a vector a are called "scalar multiplication" and is written as $s\boldsymbol{a}$. Scalar multiplication is an operation that gives us a new vector $s\boldsymbol{a}$ whose magnitude is as $|s|$ times greater ($|s| > 1$) or smaller ($|s| < 1$) than $|\boldsymbol{a}|$. When $s > 0$, $s\boldsymbol{a}$ points in the same direction as $\boldsymbol{a}$. When $s < 0$, $s\boldsymbol{a}$ points in the opposite direction to $\boldsymbol{a}$. If $s = 1$, then the multiplication does vertiually nothing to $\boldsymbol{a}$, thus $1\boldsymbol{a} = \boldsymbol{a}$. When $s = 0$, $s\boldsymbol{a}$ is equal to zero vector denoted by $\boldsymbol{0}$. Zero vector is a special vector whose length is zero. The direction of $\boldsymbol{0}$ is indeterminate.

1.3.2 Vector addition

Vector addition is an operation that produces a new vector by connecting a pair of vectors. When we have two vectors, say $\boldsymbol{a}$ and $\boldsymbol{b}$, we can form a directed broken line by connecting the tail of one vector with the head of the other. The sum $\boldsymbol{a} + \boldsymbol{b}$ is defined to be a vector extending from the tail to the head of the broken line. Note that vector addition is "commutative" since

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$$

alwyas hold true. This means the order of summation is immaterial. Clearly, we can add more than two variables by summing two vectors, repeatedly. For example, we can perform summation of three vectors either by

$$\boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c})$$

or by

$$(\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c}$$

We can prove graphically that the addition by result in a same vector.

1.4 Inner product

1.5 Exterior product

When a pair of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ make a right angle, they are said to be mutually orothogonal.