A Synthetic Aperture Focusing Method for Ultrasonic Imaging

Let $a(\boldsymbol{x}, t; \boldsymbol{x}_s)$ be an A-scan waveform of time t measured at $\boldsymbol{x}$ when the incident wave is excited by a source located at $\boldsymbol{x}_s$. We consider synthesizing an ultrasonic image of scattering object for given set of waveforms $\mathcal{D}$ such that

$$\mathcal{D} = \{a(\boldsymbol{x}, t; \boldsymbol{x}_s) \,|\boldsymbol{x} \in \mathcal{R}, t > 0\} \tag{1}$$

where $\mathcal{R}$ denotes the measurement aperture. It is assumed that the waveform $a(\boldsymbol{x}, t; \boldsymbol{x}_s)$ may be decomposed into the incident and scattered components as

$$a(\boldsymbol{x}, t; \boldsymbol{x}_s) = a^{in}(\boldsymbol{x}, t; \boldsymbol{x}_s) + a^{sc}(\boldsymbol{x}, t; \boldsymbol{x}_s). \tag{2}$$

In what follows, quantities associated with the incident and scattered wave components are denoted by the superscripts "in" and "sc", respectively. To explain the delay-and-sum operation we introduce the time-of-flight function:

$$t = T_f(\boldsymbol{x}, \boldsymbol{y}) \tag{3}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the points of observation and transmission ,respectively. The time-of-flight function T_f is a fuction that returns the travel time required for a wave excited at $\boldsymbol{y}$ to arrive at $\boldsymbol{x}$. The incident wave

$$t^{in}(\boldsymbol{x}) = T_f(\boldsymbol{x}, \boldsymbol{x}_s) \tag{4}$$

$$t^{sc}_P(\boldsymbol{x}) = T_f(\boldsymbol{x}_P, \boldsymbol{x}_s) + T_f(\boldsymbol{x}, \boldsymbol{x}_P) \tag{5}$$

$$\bar{a}^{in}(t) = \int_{\mathcal{R}} a^{in}(\boldsymbol{x}, t - t^{in}(\boldsymbol{x}) + t^{in}(\boldsymbol{x}_R))d\boldsymbol{x} \tag{6}$$

$$\bar{a}^{sc}_P(t) = \int_{\mathcal{R}} a^{sc}(\boldsymbol{x}, t - t^{sc}_P(\boldsymbol{x}) + t^{sc}_P(\boldsymbol{x}_R))d\boldsymbol{x} \tag{7}$$

$$I(\boldsymbol{x}_P) = \mathcal{S}\left[\bar{a}^{sc}_P(t)\right] \tag{8}$$

$$\mathcal{S}\left[\bar{a}^{sc}_P(t)\right] = \bar{a}^{sc}_P(t^{sc}_P)\,(= \langle\delta(t - t^{sc}_P),\, \bar{a}^{sc}_P\rangle) \tag{9}$$

$$\langle f, g\rangle = \int f(t)g(t)dt \tag{10}$$

$$\mathcal{S}\left[\bar{a}^{sc}_P(t)\right] = \left\langle \bar{a}^{in}(t + t^{in} - t^{sc}_P), \bar{a}^{sc}_P(t)\right\rangle \tag{11}$$

In the followings, the incident wave component is used as a rerference waveform, while the scattered wave component as a signal to synthesize an image of scattering object. The imaging alogorithm is consisted of two steps. The first step is the delay-and-sum operation on the A-scans to amplifiy the scattered wave packet of interest. The second step is a sampling operation on the resulting A-scan to assign an image intensity to a particular pixel. We go through those steps as follows.

If the phase velocity c of ultasonic wave is given, the time-of-flight required for the wave to travel from a given source $\boldsymbol{x}_s$ to an observation point $\boldsymbol{x}$ can be caluculated theoretically. Let the theoretically obtained time-of-flight be To