无穷级数与函数逼近数学实验

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本次试验的目的：

- 探究级数 $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2}{n(2n+1)(2n+2)}$ 的部分和的变化趋势。
- 利用幂级数的部分和对函数 $y = x^x$ 进行逼近
- 利用傅立叶级数的部分和对函数 $y = \arctan x$ 进行逼近

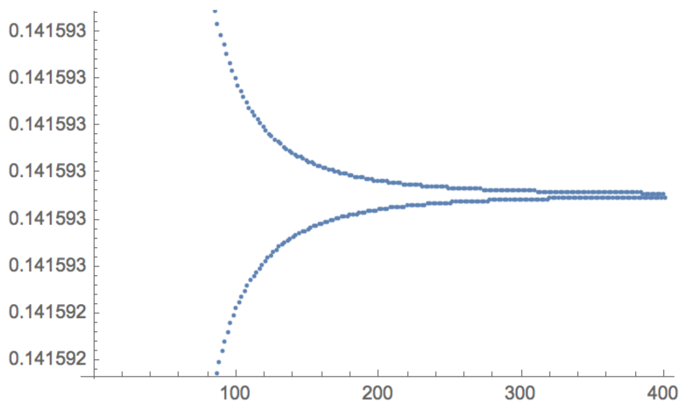
1 幂级数部分和的变化趋势

1.1 代码

1.1.1 用数据点集法观察

```
In[85]:= s[n_] := Sum[(-1) ^ (k - 1) * 2 / (k (2 k + 1) (2 k + 2)), {k, 1, n}];
                  求和
         data = Table[s[n], {n, 1, 400}];
                表格
         ListPlot[data]
         绘制点集
```

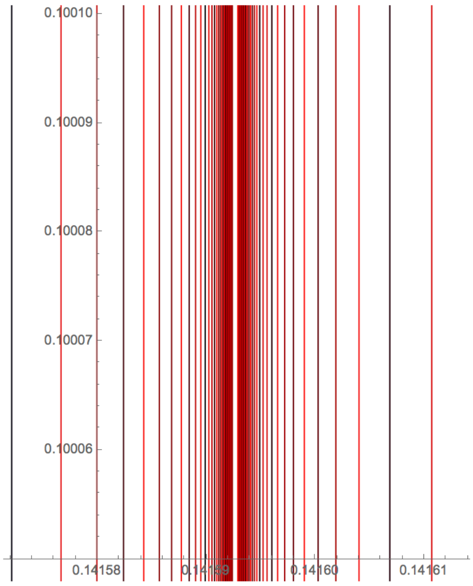
Out[87]=



1.1.2 用竖直线段法观察

```
In[91]:= sn = 0; n = 1; h = {}; m = 2;
While[1 / n > 10 ^ (-m), sn = sn + (-1) ^ (n - 1) * 2 / (n (2 n + 1) (2 n + 2));
|While循环
  h = Append[h, Graphics[{RGBColor[Abs[Sin[n]], 0, 1 / n], Line[{{sn, 0}, {sn, 1}}]}]];
      |追加       |图形      |RGB颜色  |…  |正弦                      |线段
  n++];
Show[h, PlotRange → {0.10005, 0.1001}, Axes → True]
|显示     |绘制范围                        |坐标轴   |真
```

Out[93]=



1.1.3 直接求和

```
In[95]:= Sum[(-1) ^ (n - 1) * 2 / (n (2 n + 1) (2 n + 2)), {n, 1, Infinity}]
         |求和                                                 |无穷大

Out[95]= -3 + π
```

1.2 结果

由图像可知，级数 $\sum_{n=1}^{\infty}(-1)^{n-1}\dfrac{2}{n(2n+1)(2n+2)}$ 的部分和非常快地趋向于 $\pi-3$.

2 幂级数部分和逼近函数

2.1 代码

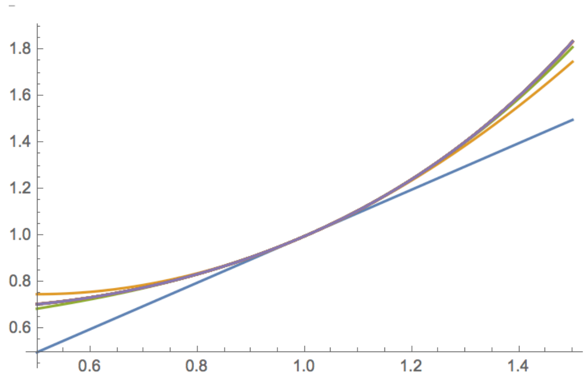
2.1.1 在 $x=1$ 处创建幂级数部分和

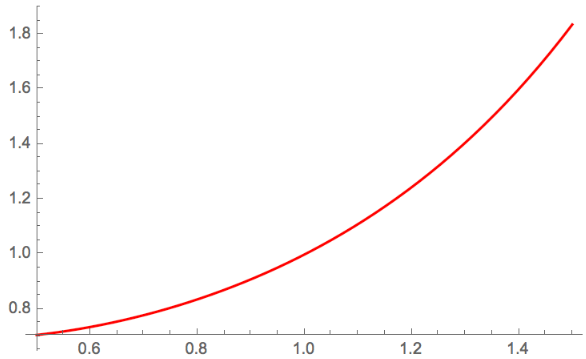
```
In[1]:= f[x_] := x ^ x; x0 = 1;
        g[n_, x0_] := D[f[x], {x, n}] /. x → x0;
                      |偏导
        s[n_, x_] := Sum[g[k, x0] / k ! * (x - x0) ^ k, {k, 0, n}];
                     |求和
```

2.1.2 用幂级数部分和逼近函数

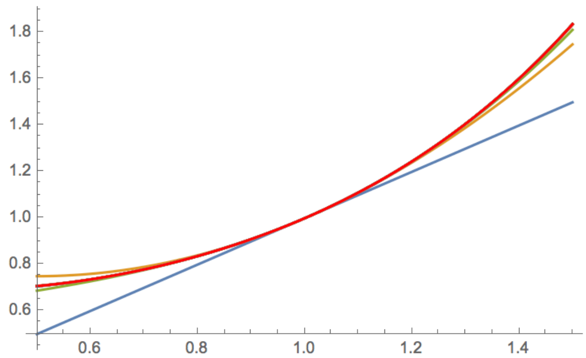
```
In[4]:= t = Table[s[n, x], {n, 20}];
            表格
        p1 = Plot[Evaluate[t], {x, 0.5, 1.5}];
             绘图   计算
        p2 = Plot[x ^ x, {x, 0.5, 1.5}, PlotStyle → RGBColor[1, 0, 0]];
             绘图                         绘图样式      RGB颜色
        Print[p1];
        打印
        Print[p2];
        打印
        Show[p1, p2]
        显示
```

图像为：





Out[9]=



2.2 结果

由图像可知，在 $x = 1$ 处，将 $y = x^x$ 展开至 20 阶 Taylor 展开式即可很好地逼近函数。

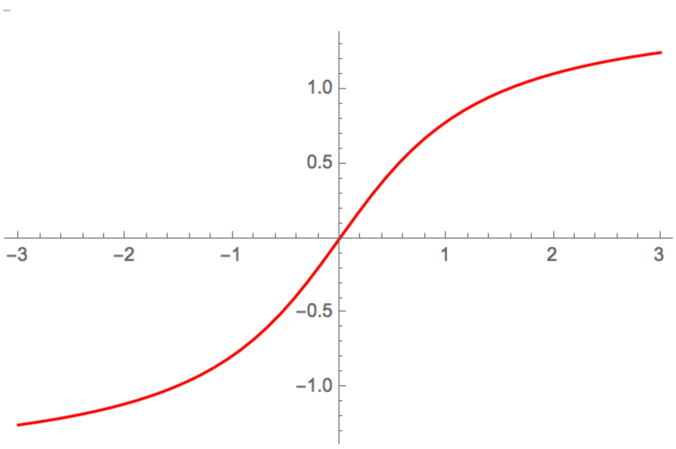
3 傅立叶级数部分和逼近函数

3.1 代码

3.1.1 创建周期为 3 的傅立叶级数并逼近

```
In[12]:= fourier[f_, T_, k_] := Module[{a, b, i, t, s, g1, g2},
                                 模块
    a[0] = Integrate[f, {x, -T, T}] / T; s = a[0] / 2;
           积分
    For[i = 1, i ≤ k, i++, t = i * Pi / T; a[i_] := Integrate[f * Cos[t * x], {x, -T, T}] / T;
    For循环                        圆周率          积分              余弦
     b[i_] := Integrate[f * Sin[t * x], {x, -T, T}] / T;
              积分           正弦
     s = s + a[i] Cos[t * x] + b[i] Sin[t * x]];
                  余弦              正弦
    Print[s];
    打印
    Plot[Evaluate[s], {x, -T, T}]];
    绘图 计算
f = ArcTan[x]; T = 3; n = 8; g = Plot[f, {x, -T, T}, PlotStyle → RGBColor[1, 0, 0]];
    反正切                          绘图                 绘图样式        RGB颜色
Print[g];
打印
For[j = 1, j ≤ n, j += 2, p = fourier[f, T, j]];
For循环
Show[g, p]
显示
```

3.1.2 函数 $y = \arctan x$ 的图像



3.1.3 傅立叶级数前 7 项和函数

$$\frac{1}{3\pi}\text{Sin}\left[\frac{\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)\right)$$

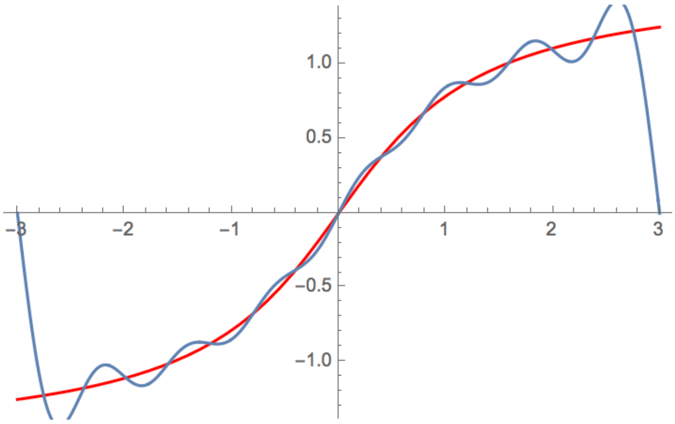
$$\frac{1}{3\pi}\text{Sin}\left[\frac{\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{6\pi}\text{Sin}\left[\frac{2\pi x}{3}\right]\left(-6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{2\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-2+\frac{2\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(2+\frac{2\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{2\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-2+\frac{2\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(2+\frac{2\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{3\pi}\text{Sin}[\pi x]\left(2\,\text{ArcTan}[3]-\text{i}\,\text{Cosh}[\pi]\left(\text{CosIntegral}[(-3+\text{i})\pi]-\text{CosIntegral}[(3+\text{i})\pi]\right)+\text{Sinh}[\pi]\left(\text{SinIntegral}[(-3+\text{i})\pi]-\text{SinIntegral}[(3+\text{i})\pi]\right)\right)$$

$$\frac{1}{3\pi}\text{Sin}\left[\frac{\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{6\pi}\text{Sin}\left[\frac{2\pi x}{3}\right]\left(-6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{2\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-2+\frac{2\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(2+\frac{2\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{2\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-2+\frac{2\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(2+\frac{2\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{3\pi}\text{Sin}[\pi x]\left(2\,\text{ArcTan}[3]-\text{i}\,\text{Cosh}[\pi]\left(\text{CosIntegral}[(-3+\text{i})\pi]-\text{CosIntegral}[(3+\text{i})\pi]\right)+\text{Sinh}[\pi]\left(\text{SinIntegral}[(-3+\text{i})\pi]-\text{SinIntegral}[(3+\text{i})\pi]\right)\right)+\frac{1}{12\pi}\text{Sin}\left[\frac{4\pi x}{3}\right]\left(-6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{4\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-4+\frac{4\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(4+\frac{4\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{4\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-4+\frac{4\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(4+\frac{4\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{15\pi}\text{Sin}\left[\frac{5\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{5\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-5+\frac{5\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(5+\frac{5\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{5\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-5+\frac{5\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(5+\frac{5\text{i}}{3}\right)\pi\right]\right)\right)$$

$$\frac{1}{3\pi}\text{Sin}\left[\frac{\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-1+\frac{\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(1+\frac{\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{6\pi}\text{Sin}\left[\frac{2\pi x}{3}\right]\left(-6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{2\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-2+\frac{2\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(2+\frac{2\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{2\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-2+\frac{2\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(2+\frac{2\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{3\pi}\text{Sin}[\pi x]\left(2\,\text{ArcTan}[3]-\text{i}\,\text{Cosh}[\pi]\left(\text{CosIntegral}[(-3+\text{i})\pi]-\text{CosIntegral}[(3+\text{i})\pi]\right)+\text{Sinh}[\pi]\left(\text{SinIntegral}[(-3+\text{i})\pi]-\text{SinIntegral}[(3+\text{i})\pi]\right)\right)+\frac{1}{12\pi}\text{Sin}\left[\frac{4\pi x}{3}\right]\left(-6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{4\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-4+\frac{4\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(4+\frac{4\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{4\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-4+\frac{4\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(4+\frac{4\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{15\pi}\text{Sin}\left[\frac{5\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{5\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-5+\frac{5\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(5+\frac{5\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{5\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-5+\frac{5\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(5+\frac{5\text{i}}{3}\right)\pi\right]\right)\right)+\frac{1}{6\pi}\text{Sin}[2\pi x]\left(-2\,\text{ArcTan}[3]-\text{i}\,\text{Cosh}[2\pi]\left(\text{CosIntegral}[(-6+2\text{i})\pi]-\text{CosIntegral}[(6+2\text{i})\pi]\right)+\text{Sinh}[2\pi]\left(\text{SinIntegral}[(-6+2\text{i})\pi]-\text{SinIntegral}[(6+2\text{i})\pi]\right)\right)+\frac{1}{21\pi}\text{Sin}\left[\frac{7\pi x}{3}\right]\left(6\,\text{ArcTan}[3]-3\,\text{i}\,\text{Cosh}\left[\frac{7\pi}{3}\right]\left(\text{CosIntegral}\left[\left(-7+\frac{7\text{i}}{3}\right)\pi\right]-\text{CosIntegral}\left[\left(7+\frac{7\text{i}}{3}\right)\pi\right]\right)+3\,\text{Sinh}\left[\frac{7\pi}{3}\right]\left(\text{SinIntegral}\left[\left(-7+\frac{7\text{i}}{3}\right)\pi\right]-\text{SinIntegral}\left[\left(7+\frac{7\text{i}}{3}\right)\pi\right]\right)\right)$$

3.1.4 傅立叶级数部分和逼近函数的图像

Out[16]=



3.2 结果

由图像可知，在 $x = 0$ 的一个小邻域内，$y = \arctan x$ 的前 7 项傅立叶级数部分和能较好地逼近函数。