

$$P_i = (x_i, y_i), (i = 1, 2, \dots, n)$$

$$f(x_i) = ax_i + b$$

$$L(a,b)=\sum_{i=1}^n (y_i-f(x_i))^2$$

$$f(x)=\begin{pmatrix}x_i&1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}\begin{cases}f(x_1)=ax_1+b\\f(x_2)=ax_2+b\\f(x_3)=ax_3+b\end{cases}$$

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad \hat{\mathbf{Y}} = \begin{pmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$min|\mathbf{Y}-\hat{\mathbf{Y}}|^2$$

$$\mathbf{A} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$$

$$\mathbf{X} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{pmatrix} = (\mathbf{X}_1 \quad \mathbf{X}_2)$$

$$\hat{\mathbf{Y}} = \mathbf{X}\mathbf{A}$$

$$\begin{aligned}\hat{\mathbf{Y}} &= \mathbf{X}\mathbf{A} = (\mathbf{X}_1 \quad \mathbf{X}_2) \begin{pmatrix} a \\ b \end{pmatrix} \\ &= a\mathbf{X}_1 + b\mathbf{X}_2\end{aligned}$$

$$min|\mathbf{Y}-\hat{\mathbf{Y}}|^2 \rightarrow (\mathbf{Y}-\hat{\mathbf{Y}}) \perp \mathbf{X}_1, \mathbf{X}_2$$

$$\mathbf{X_1} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \mathbf{X_2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{X_1}^T(\mathbf{Y}-\hat{\mathbf{Y}}) = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}(\mathbf{Y}-\hat{\mathbf{Y}}) = 0$$

$$\mathbf{X_2}^T(\mathbf{Y}-\hat{\mathbf{Y}}) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}(\mathbf{Y}-\hat{\mathbf{Y}}) = 0$$

$$\mathbf{X}^T(\mathbf{Y}-\hat{\mathbf{Y}}) = 0$$

$$\rightarrow \begin{pmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \end{pmatrix}(\mathbf{Y}-\hat{\mathbf{Y}}) = 0$$

$$\stackrel{2}{\rightarrow} \mathbf{X}^T(\mathbf{Y}-\mathbf{X}\mathbf{A}) = 0$$

$$\rightarrow \mathbf{X}^T\mathbf{Y}-\mathbf{X}^T\mathbf{X}\mathbf{A} = 0$$

$$\rightarrow \mathbf{X}^T\mathbf{Y} = \mathbf{X}^T\mathbf{X}\mathbf{A}$$

$$U_0(x)=V_{k,1}^1(x)=1$$

$$U_1(x)=V_{k,1}^2(x)=\sqrt{3}(1-2x), 0\leq x\leq 1$$

$$U_2(x)=V_{k,1}^3(x)=\sqrt{5}(6x^2-6x+1)$$

$$U_3(x)=V_{k,1}^4(x)=\sqrt{7}(-20x^3+30x^2-12x+1)$$

$$\mathbf{X_2}^T(\mathbf{Y}-\hat{\mathbf{Y}}) = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}(\mathbf{Y}-\hat{\mathbf{Y}}) = 0$$

$$\mathbf{X}^T(\mathbf{Y}-\hat{\mathbf{Y}}) = 0$$

(1)