$$P_i = (x_i, y_i), (i = 1, 2, \ldots, n)$$

$$f(x_i) = ax_i + b$$

$$L(a, b) = \sum_{i=1}^{n} (y_i - f(x_i))^2$$

$$f(x) = \begin{pmatrix} x_i & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \begin{cases} f(x_1) = ax_1 + b \\ f(x_2) = ax_2 + b \\ f(x_3) = ax_3 + b \end{cases}$$

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \hat{\mathbf{Y}} = \begin{pmatrix} f(x_1) \\ f(x_2) \\ f(x_3) \end{pmatrix} \mathbf{X} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{pmatrix} \mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$min|\mathbf{Y} - \hat{\mathbf{Y}}|^2$$

$$\mathbf{A} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}$$

$$\mathbf{X} = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 \end{pmatrix} \tag{1}$$

$$\hat{\mathbf{Y}} = \mathbf{X}\mathbf{A}$$

$$\begin{aligned} \hat{\mathbf{Y}} = \mathbf{X}\mathbf{A} &= \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \\ &= a\mathbf{X}_1 + b\mathbf{X}_2 \\ min|\mathbf{Y} - \hat{\mathbf{Y}}|^2 &\to (\mathbf{Y} - \hat{\mathbf{Y}}) \perp \mathbf{X_1}, \mathbf{X_2} \end{aligned}$$

$$\begin{aligned} \mathbf{X_1} &= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \mathbf{X_2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ \mathbf{X_1}^{\mathrm{T}}(\mathbf{Y} - \hat{\mathbf{Y}}) &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} (\mathbf{Y} - \hat{\mathbf{Y}}) = 0 \\ \mathbf{X_2}^{\mathrm{T}}(\mathbf{Y} - \hat{\mathbf{Y}}) &= \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} (\mathbf{Y} - \hat{\mathbf{Y}}) = 0 \\ \mathbf{X}^{\mathrm{T}}(\mathbf{Y} - \hat{\mathbf{Y}}) &= 0 \\ &\to \begin{pmatrix} x_1 & x_2 & x_3 \\ 1 & 1 & 1 \end{pmatrix} (\mathbf{Y} - \hat{\mathbf{Y}}) = 0 \\ 2 \quad &\to \mathbf{X}^{\mathrm{T}}(\mathbf{Y} - \mathbf{X}\mathbf{A}) = 0 \\ &\to \mathbf{X}^{\mathrm{T}}\mathbf{Y} - \mathbf{X}^{\mathrm{T}}\mathbf{X}\mathbf{A} = 0 \\ &\to \mathbf{X}^{\mathrm{T}}\mathbf{Y} = \mathbf{X}^{\mathrm{T}}\mathbf{X}\mathbf{A} \end{aligned}$$