Study on the Solution Set of Knot Colorings

MATH010T

Yang Sihyeong | Kim Dongin | Seokyoung Yoon, Korea Science Academy of KAIST, Busan, Republic of Korea

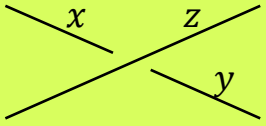
Q1. Problem or Question

Well-Known Fact

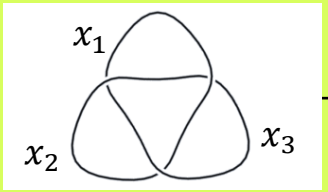
1. p-colorable

1) $2z - x - y \equiv 0 (mod\ p)$

2) More than 2 colors



2. p-colorability : invariant



3_1 Knot is 3-colorable

"Let's Color the Knot!"

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Find the properties of knot coloring solution sets by solving linear congruences.

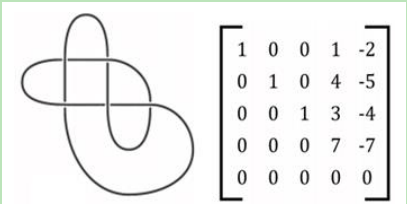
➔ Find out new invariant

Q2. Framework

Knot ➔ Matrix

Colorability ⇔ ∃ Solutions
Coloring ⇔ Solving Congruences

So, we map Knot to a matrix
Null space of matrix = Solution set



$$\begin{bmatrix} 1 & 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 4 & -5 \\ 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Coloring Matrix of 5_2 Knot

Reduced Knot Diagram Matrix

p-coloring : Operation on $\mathbb{Z}_p$
m-coloring : Operation on $\mathbb{Z}$

Algorithm Using Matrix

Knot Coloring Algorithm

DT notation, n

↓ Using Coloring Matrix

n-Coloring Solution Set

: Print all possible solution of prime knot under crossing 12

Find & prove the common properties of the solution set

Q3. Findings

Algorithm to analyze Knots

p-coloring

Ex) Solution set of 5_2 knot with 7-coloring

$\{a(6,3,4,1,0) + t(1,1,1,1,1)\ mod 7 |$
$0 \le a, t \le 6.\ a \neq 0\}$

# of free variables = mod p rank +1

→ Number of solution is invariant

Define a new concept :
'Coloring matrix'
to analyze m-coloring

m-coloring

Essentiality Solution set size

Theorems to determine these features

Solution set analysis

➔ Essentiality & Solution set size are INVARIENT

Q4. Interpretation & Conclusions

Study about properties of knot coloring solution sets

p-coloring

- p-colorability
  ↓
  solution set
- Free variable

m-coloring

- Devised an d-coloring solution formula
- Relationship between d, m – coloring
- Proved that essentiality, solution set size are invariants – **New invarients**

Link coloring

- Proved that it shares most of the properties of knots coloring

New perspective on classifying knots – solution set of a coloring