Because $p(x|y) \propto ye^{-yx}$ , $0 < x < B < \infty$ , we can write $p(x|y) = a \times ye^{-yx}$ .

We know $0 < x < B$ , we get $a = \frac{1}{\int_0^B f(x)\,\mathrm{d}x} = \frac{1}{1-e^{-By}}$ .

Now we can write $p(x|y) = \frac{ye^{-yx}}{1-e^{-By}}$ ; similarly, $p(y|x) = \frac{xe^{-yx}}{1-e^{-Bx}}$

To use the inversion sampling, we need to first drawn a random variable $u\ U(0,1)$ and then inverting this draw using $F^{-1}$ .

Therefore, we calculate $F(x|y) = \int_0^x f(x|y)dx = \frac{1-e^{-yx}}{1-e^{-By}}$ .

Let $u = F(z)$

$$z = F^{-1}(u) = \frac{\log\left(1-u\times(1-e^{-yB})\right)}{-y}$$

Given a draw of u and y, z is a draw from $p(x|y)$ . The same can be done for $p(y|x)$ and we obtain that the function is $\frac{\log\left(1-u\times(1-e^{-xB})\right)}{-x}$ for drawing from the $p(y|x)$ .

We first choose an initial value for x and y as x0, y0 respectively within the range of 0 to B. Then, we generate a $u$ from the Uniform distribution [0,1], plug the value of u and y0 into the inversed function and get an updated x. Similarly, we will update y accordingly. Repeating the steps will generate a series of x and y's which forms our new distribution.

See the code below: