

The SCLP solution, which covers 100% of all demands, required 149 new stations. This exceeded the IMM's budget for opening new stations, which allowed for the equivalent of 64 new stations. Therefore, the researchers imposed this budget constraint in the MCLP and found a solution that covers 93.9% of the demand, including 71.1% of demands from heritage subdistricts. It also double-covers 35.6% of the subdistricts, more than twice the number that are double-covered in the status quo solution. The problems were solved in the modeling language GAMS using the MIP solver CPLEX, with run times of less than 1 second.

As of their 2013 paper, Aktaş, et al. report that IMM had opened 25 new fire stations in subdistricts proposed by the model, with a subsequent slowdown due to economic conditions. Their solution provides a roadmap for future expansion of the fire station network that can be implemented as budgets allow.

PROBLEMS

8.1 (Locating DCs for Toy Stores) A toy store chain operates 100 retail stores throughout the United States. The company currently ships all products from a central distribution center (DC) to the stores, but it is considering closing the central DC and instead operating multiple regional DCs that serve the retail stores. It will use the UFLP to determine where to locate DCs. Planners at the company have identified 24 potential cities in which regional DCs may be located. The file `toy-stores.xlsx` lists the longitude and latitude for all of the locations (stores and DCs), as well as the annual demand (measured in pallets) at each store and the fixed annual location cost at each potential DC location. Using optimization software of your choice, implement the UFLP model from Section 8.2.2 and solve it using the data provided. Assume that transportation from DCs to stores costs \$1 per mile, as measured by the great circle distance between the two locations. Report the optimal cities to locate DCs in and the optimal total annual cost.

8.2 (10-Node UFLP Instance: Exact) The file `10node.xlsx` contains data for a 10-node instance of the UFLP, with nodes located on the unit square and $I = J$, pictured in Figure 8.22. The file lists the x - and y -coordinates, demands h_i , and fixed costs f_j for each node, as well as the transportation cost c_{ij} between each pair of nodes i and j . Transportation costs equal 10 times the Euclidean distance between the nodes. All fixed costs equal 200.

Solve this instance of the UFLP exactly by implementing the UFLP in the modeling language of your choice and solving it with a MIP solver. Report the optimal locations, optimal assignments, and optimal cost.

8.3 (10-Node UFLP Instance: Greedy-Add) Use the greedy-add heuristic to solve the 10-node UFLP instance described in Problem 8.2. Report the facility that is opened at each iteration, as well as the final locations, assignments, and cost.

8.4 (10-Node UFLP Instance: Swap) Suppose we have a solution to the 10-node UFLP instance described in Problem 8.2 in which $x_2 = x_3 = 1$ and $x_j = 0$ for all other j . Use the swap heuristic to improve this solution. Use a best-improving strategy (that is, search through the facilities in order of index, and at each iteration, implement the first swap

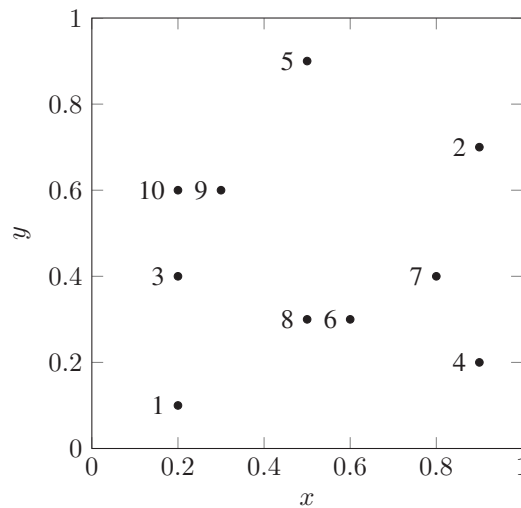


Figure 8.22 10-node facility location instance for Problems 8.2–8.11.

found that improves the cost.) Report the swaps made at each iteration, as well as the final locations, assignments, and cost.

8.5 (10-node p MP Instance: Exact) Using the file `10node.xlsx` (see Problem 8.2), solve the p MP exactly by implementing it in the modeling language of your choice and solving it with a MIP solver. Ignore the fixed costs in the data set and use $p = 4$. Report the optimal locations, optimal assignments, and optimal cost.

8.6 (10-Node p MP Instance: Swap) Suppose we have a solution to the 10-node p MP instance described in Problem 8.5 in which $x_2 = x_3 = x_5 = x_8 = 1$ and $x_j = 0$ for all other j . Use the swap heuristic to improve this solution. Use a best-improving strategy (that is, search through the facilities in order of index, and at each iteration implement the first swap found that improves the cost.) Report the swaps made at each iteration, as well as the final locations, assignments, and cost.

8.7 (10-Node p MP Instance: Neighborhood Search) Suppose we have a solution to the 10-node p MP instance described in Problem 8.5 in which $x_4 = x_5 = x_6 = x_{10} = 1$ and $x_j = 0$ for all other j . Use the neighborhood search heuristic to improve this solution. Report the swaps made at each iteration, as well as the final locations, assignments, and cost.

8.8 (10-node SCLP Instance) Using the file `10node.xlsx` (see Problem 8.2), solve the SCLP exactly by implementing it in the modeling language of your choice and solving it with a MIP solver. Set the fixed cost of every facility equal to 1. Assume that facility j covers customer i if $c_{ij} \leq 2.5$. Report the optimal locations.

8.9 (10-node MCLP Instance) Using the file `10node.xlsx` (see Problem 8.2), solve the MCLP exactly by implementing it in the modeling language of your choice and solving it with a MIP solver. Set $p = 4$. Assume that facility j covers customer i if $c_{ij} \leq 2.5$. Report the optimal locations and the total number of demands covered.

8.10 (10-node MCLP Instance: Coverage vs. p) Using the file `10node.xlsx` (see Problem 8.2), solve the MCLP exactly for $p = 1, 2, \dots, 10$ using the modeling language and solver of your choice. Assume that facility j covers customer i if $c_{ij} \leq 2.5$. Construct a plot similar to Figure 8.14.

8.11 (10-node p CP Instance) Use Algorithm 8.9 to solve the 10-node instance of the p CP specified in the file `10node.xlsx` (see Problem 8.2). Set $p = 3$. Use $r^L = 0$, $r^U = \max_{i \in I, j \in J} \{c_{ij}\}$, and $\epsilon = 0.1$. Report the value of r at each iteration, as well as the optimal locations, assignments, and objective function value.

8.12 (Locating Homework Centers for Chicago Schools) Suppose the City of Chicago wishes to establish homework-help centers at 12 of its public libraries. It wants the homework center locations to be as close as possible to Chicago public schools. In particular, it wants the homework centers to cover as many schools as possible, where a school is “covered” if there is a homework center located within 2 miles of it.

- Using the files `chicago-schools.csv` and `chicago-libraries.csv` and determining coverage using great circle distances, find the 12 libraries at which homework centers should be established. Report the indices of the libraries selected, as well as the total number of schools covered. (Chicago school and library data are adapted from Chicago Data Portal (2017a,b).)
- Suppose now that the city wishes to ensure that *all* schools are covered. What is the minimum number of homework centers that must be established to accomplish this?

8.13 (Easy or Hard Modifications?) Which of the following costs can be implemented in the UFLP by modifying the parameters only, without requiring structural changes to the model; that is, without requiring modifications to the variables, objective function, or constraints? Explain your answers briefly.

- A per-unit cost to ship items from a supplier to facility j . (The cost may be different for each j .)
- A per-unit processing cost at facility j . (The cost may be different for each j .)
- A fixed cost to ship items from facility j to customer i . (The cost is independent of the quantity shipped but may be different for each i and j .)
- A transportation cost from facility j to customer i that is a nonlinear function of the quantity shipped (for example, one of the quantity discount structures discussed in Section 3.4).
- A fixed capacity-expansion cost that is incurred if the demand served by facility j exceeds a certain threshold.
- Some facilities are already open; an open facility j can be closed at a cost of \hat{f}_j . (In addition, we can open new facilities, as in the UFLP.)

8.14 (LP Relaxation of UFLP) Develop a simple instance of the UFLP for which the optimal solution to the LP relaxation has fractional values of the x_j variables. This solution must be strictly optimal—that is, you can’t submit an instance for which the LP relaxation has an optimal solution with all integer values, even if there’s another optimal solution, that ties the integer one, with fractional values. Your instance must have $I = J$, that is, all customer nodes are also potential facility sites. Your instance must have at most four nodes.

Include the following in your report:

- A diagram of the nodes and edges.
- The values of h_i , f_j , and c_{ij} for all i, j .
- The optimal solution (x_{LP} and y_{LP}) and optimal objective value (z_{LP}) for the LP relaxation.
- The optimal solution (x^* and y^*) and optimal objective value (z^*) for the IP.

8.15 (LP Relaxation of p MP) Repeat Problem 8.14 but for the p MP instead of the UFLP.

8.16 (Ignoring Some Customers in the UFLP) The UFLP includes a constraint that requires every customer to be assigned to some facility. It is often the case that a small handful of customers in remote regions of the geographical area are difficult to serve and can influence the solution disproportionately. In this problem, you will formulate a version of the UFLP in which a certain percentage of the demands may be ignored when calculating the objective function.

Let α be the minimum fraction of demands to be assigned; that is, a set of customers whose cumulative demand is no more than $100(1 - \alpha)\%$ of the total demand may be ignored. The parameter α is fixed, but the model decides endogenously which customers to ignore. Customers must be either assigned or not—they cannot be assigned fractionally.

- Using the notation introduced in Section 8.2.2, formulate this problem—we'll call it the "partial assignment UFLP" (PAUFLP)—as a linear integer programming problem. Explain each of your constraints in words.
- Now consider adding a dummy facility, call it u , to the original UFLP. Facility u has a fixed capacity, so we are really dealing with the capacitated fixed-charge location problem (CFLP), not the UFLP. (See Section 8.3.1 for more on the CFLP.) Assigning customers to this dummy facility in the CFLP represents choosing not to assign them in the PAUFLP. Explain how to set the dummy facility's parameters—its fixed cost, capacity, and transportation cost to each customer—so that solving the CFLP with the dummy facility is equivalent to solving the PAUFLP. Formulate the resulting integer programming problem.
- Using Lagrangian relaxation, relax the assignment constraints in your model from part (b). Formulate the Lagrangian subproblem, using λ_i as the Lagrange multiplier for the assignment constraint for customer i .
- Explain how to solve the Lagrangian subproblem you wrote in part (c) for fixed values of λ .
- Once you have a solution to the Lagrangian subproblem for fixed values of λ , how can you convert it to a feasible solution to the CFLP?

8.17 (UFLP with Multiple Assignments) Suppose that, in the UFLP, customers do not receive 100% of their demand from their nearest open facility. For example, a given customer might receive 80% of its demand from the closest facility, 15% from the second-closest, and 5% from the third-closest. This situation might arise, for example, when locating ambulances, repair centers, or other services for which the primary facility may sometimes be busy.

Let m be the maximum number of facilities that serve each customer, and let b_{ik} be the fraction of demand that customer $i \in I$ receives from the k th-closest open facility, for $k = 1, \dots, m$. (In the example above, $m = 3$, $b_{i1} = 0.8$, $b_{i2} = 0.15$, and $b_{i3} = 0.05$.) The

b_{ik} are inputs to the model; that is, the assignment fractions are known in advance. Assume that, for a given i , the b_{ik} are nonincreasing in k .

- a) Formulate this problem as an integer linear optimization problem. Use the notation introduced in Section 8.2.2, with the following modification: y_{ijk} equals 1 if facility j serves customer i as the k th closest, and 0 otherwise. If you introduce any new notation, define it clearly. Explain the objective function and each constraint in words.
- b) If customer i is assigned to j_1 at level k_1 and j_2 at level k_2 for $k_1 < k_2$, then we must have $c_{ij_1} \leq c_{ij_2}$. Explain why the model does *not* need a constraint enforcing this condition.
- c) If we require $y_{ijk} \geq 0$ rather than $y_{ijk} \in \{0, 1\}$, as we did in the UFLP, does there always exist an optimal solution in which these variables are binary, as there is in the UFLP?
- d) In your model from part (a), you should have a constraint that requires each customer i to be assigned to exactly one facility j at each proximity level k . Relax this constraint via Lagrangian relaxation. Write the Lagrangian subproblem that results. Explain how to solve this problem efficiently for fixed values of the Lagrange multipliers. Your method must be exact (i.e., it must be guaranteed to find the optimal solution) and self-contained (i.e., it may not rely on CPLEX or another solver).
- e) *Bonus:* Suppose the b_{ik} are *not* nonincreasing in k . Then the distance-ordering property in part (c) may not hold unless we enforce it using constraints. Write constraints to enforce this condition.

8.18 (Relaxing x Variables in UFLP) Prove or disprove the following claim: If we constrain the y variables to be binary in the UFLP but allow the x variables to be continuous, then there always exists an optimal solution to the resulting problem in which the x variables are binary.

8.19 (Locating Paper Factories) A paper company needs to decide where to locate paper factories in order to supply its five regional branches, which are located in Akron, OH, Albany, NY, Nashua, NH, Scranton, PA, and Utica, NY. The Assistant to the Regional Manager of the Scranton office has selected four potential locations for factories: Bethlehem, PA, Pittsburgh, PA, Rochester, NY, and Springfield, MA. Table 8.3 lists the annual fixed costs and capacities at the four potential plant locations; the annual demand at each of the regional branches; and the cost to produce and ship one case of paper from each plant to each branch. Plant capacities and branch demands are expressed in cases per year.

Where should the company build its plants? Which plant(s) should each branch receive paper from? What is the total cost of your solution? Solve the problem using the modeling environment and solver of your choice.

8.20 (DUALOC #1) Figure 8.23 depicts an instance of the UFLP with three customers (marked as circles) and three potential facility sites (marked as squares). Fixed costs f_j are marked next to each facility. Each customer has a demand of $h_i = 1$, and transportation costs are equal to the Manhattan-metric distance between the facility and customer.

Apply DUALOC's dual-ascent procedure (Algorithm 8.4) to this instance. Report:

- The values of v_i for all $i \in I$ and s_j for all $j \in J$ at the end of the first complete iteration, i.e., after looping through all the customers once.

Table 8.3 Paper-company data for Problem 8.19.

	Production + Shipping Costs				Demand
	Bethlehem	Pittsburgh	Rochester	Springfield	
Akron	\$2.20	\$1.80	\$2.70	\$3.80	1,200,000
Albany	\$1.60	\$3.20	\$1.20	\$0.60	1,150,000
Nasuha	\$3.20	\$4.00	\$2.50	\$0.70	1,350,000
Scranton	\$0.80	\$2.10	\$1.40	\$1.30	1,800,000
Utica	\$1.60	\$2.40	\$0.70	\$1.50	900,000
Fixed Cost	\$4,000,000	\$7,500,000	\$4,500,000	\$5,200,000	
Capacity	3,300,000	4,800,000	4,200,000	3,750,000	

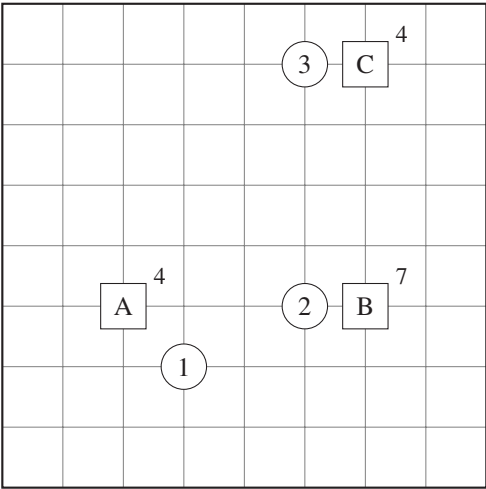


Figure 8.23 UFLP instance for Problem 8.20. Distances use Manhattan metric.

- The final values of v^+ , J^+ , x^+ , y^+ , and the dual and primal objective function values.
- Whether the solution to this instance of the UFLP is (a) definitely optimal, (b) definitely sub-optimal, or (c) you can't tell.

8.21 (DUALOC #2) Repeat Problem 8.20 for the instance depicted in Figure 8.24.

8.22 (Warehouses for Quikflix) Quikflix is a mail-order DVD-rental company. You choose which DVDs to rent on Quikflix's web site, and the company mails the DVDs to you. When you've finished watching the movies, you mail them back to Quikflix. Quikflix's business plan depends on fast shipping times (otherwise, customers will get impatient). But overnight delivery services like FedEx are prohibitively expensive. Instead, Quikflix has decided to open enough DCs so that roughly 90% of their customers enjoy 1-day delivery times.

In this problem, you will formulate and solve a model to determine where Quikflix should locate DCs to ensure that a desired percentage of the US population is within a

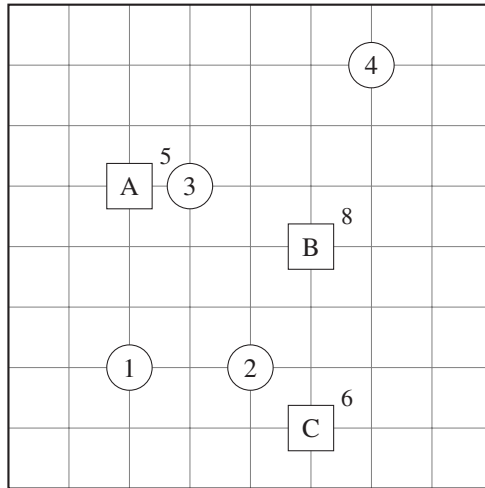


Figure 8.24 UFLP instance for Problem 8.21. Distances use Manhattan metric.

1-day mailing range while minimizing the fixed cost to open the DCs. (You may assume that the per-unit cost of processing and shipping DVDs is the same at every DC.)

- a) Formulate the following problem as an integer programming problem: We are given a set of cities, as well as the population of each city and the fixed cost to open a DC in that city. The objective is to decide in which cities to locate DCs in order to minimize the total fixed cost while also ensuring that at least α fraction of the population is within a 1-day mailing range.

Define your notation clearly and indicate which items are parameters (inputs) and which are decision variables. Explain each of your constraints in words.

- b) Implement your model using a modeling language of your choice. Solve the problem using the data set provided in `quikflix.xlsx`, which gives the locations and populations of the 250 largest cities in the United States (according to the 2000 US Census), as well as the average annual fixed costs to open a DC in the cities (which are fictitious). The file also contains the distance between each pair of cities in the data set, in miles. Assume that two cities are within a 1-day mailing radius if they are no more than 150 miles apart.

Using these data and a coverage percentage of $\alpha = 0.9$, find the optimal solution to the Quikflix DC location problem. Include a printout of your model file (data not necessary) in your report. Report the total cost of your solution and the total number of DCs open.

8.23 (Solving the Quikflix Problem) In Problem 8.22, you formulated an IP model to solve Quikflix's problem of locating DCs to ensure that a given fraction (α) of the population is within a 1-day mailing range of its nearest DC. In this problem, you will develop a method for solving this IP using Lagrangian relaxation.

The IP formulation for Problem 8.22 contains two sets of decision variables. We'll assume that the x variables represent location decisions, while the z variables indicate whether or not a city is covered (i.e., is within a 1-day mailing radius of an open facility). If you defined z as a continuous variable, make sure you have added a constraint requiring

it to be less than or equal to 1. (This constraint is not strictly necessary since it is implied by other constraints, but it strengthens the Lagrangian relaxation formulation.)

The IP formulation also has a set of constraints that allow city i to be covered only if there is an open facility that is less than 150 miles away. If necessary, rewrite your model so that those constraints are written as \leq constraints. Then relax those constraints, and let λ_i be the Lagrange multiplier for the constraint corresponding to node $i \in J$, where J is the set of cities.

- Write out the Lagrangian subproblem that results from this relaxation.
- The subproblem should decompose into two separate problems, one containing only the x variables and one containing only the z variables. Write out these two separate problems.
- Explain how to solve each of the two subproblems, the x -subproblem and the z -subproblem. Your solution method may *not* rely on using the simplex method or any other general-purpose LP or IP algorithm.
- Suppose that the problem parameters and Lagrange multipliers are given by the following values:

i	f_i	h_i	λ_i
1	100	80	-50
2	100	120	-50
3	100	40	-40
4	100	90	-200

Suppose also that $\alpha = 0.7$ and that node 1 covers nodes 1, 2, 3; node 2 covers nodes 1, 2, and 4; node 3 covers nodes 1 and 3; and node 4 covers nodes 2 and 4.

Determine the optimal values of x and z , as well as the optimal objective value, for this iteration of the Lagrangian subproblem.

8.24 (UFLP with Enemy Customers) Suppose that, in the UFLP model, some pairs of customers are “enemies” and cannot be served by the same facility. Let $a_{ik} = 1$ if customers $i, k \in I$ ($i \neq k$) are enemies of each other, 0 otherwise. (a_{ik} is a parameter.) Assume that the enemy pairs don’t overlap: If i and k are enemies of each other, then i and k aren’t enemies of any other customers.

- Write one or more linear constraints that can be added to the UFLP to enforce the condition that two customers may not be assigned to the same facility if they are enemies of each other. If you introduce any new notation, define it clearly.
- Suppose we add your constraints from part (a) to the UFLP and then relax constraints (8.4) using Lagrangian relaxation, with Lagrange multipliers λ_i . Write the resulting Lagrangian subproblem.
- Explain how to solve the Lagrangian subproblem you formulated in part (b) for fixed values of λ . Your solution method may not rely on using the simplex method or any other general-purpose LP or IP algorithm.
- Choose one option and briefly explain your reasoning: For every instance of the UFLP with enemy constraints, the optimal objective function value will be [\leq , $<$, $=$, $>$, \geq] the optimal objective function value of the corresponding instance of the classical UFLP.

- e) Choose one option and briefly explain your reasoning: For every instance of the UFLP with enemy constraints, the optimal number of open facilities will be $[\leq, <, =, >, \geq]$ the optimal number of open facilities in the corresponding instance of the classical UFLP.

8.25 (Locating Warehouses for Vandelay Industries) Vandelay Industries manufactures latex products at several plants (whose locations must be chosen from among a set of potential locations) and ships products to customers (whose locations and demands are known). There is a fixed cost to open each plant, and each has a fixed production capacity.

For each unit of demand shipped to a given customer, Vandelay Industries earns a certain amount of revenue. However, the company may choose to satisfy only a part of a given customer's demand, or not to satisfy its demand at all (for example, if it is too expensive to ship to that customer). The only penalty for failing to serve a customer is the lost revenue.

In order to ensure adequate service to customers spread throughout the country, Vandelay Industries also wishes to ensure that no two plants are located less than a certain distance apart.

The company's objective is to maximize the total profit, accounting for the revenue from serving customers and the costs of opening facilities and shipping goods to customers.

Formulate this problem as a linear mixed-integer optimization problem (MIP). In addition to the notation in Sections 8.2.2 and 8.3.1, please use the following notation. If you use any additional notation, define it clearly.

c_{jk} = distance (miles) between plant $j \in J$ and plant $k \in J$

c_{\min} = minimum allowable distance (miles) between two open plants

8.26 (Locating Snack Bars) You have been hired as a consultant for a new theme park to help choose locations for the park's snack bars (restaurants). The park has been divided into sectors, each representing a small area of land. The management team has forecast the number of people that are expected to be in each sector at any point in time.

Let I be the set of sectors and let J be the set of possible locations for the snack bars. The set J is a subset of I because each possible snack bar location is also a sector. Let h_i be the number of people located in sector i , for $i \in I$. (Of course, h_i is just an estimate, because this number will constantly be changing, but we'll treat it as though the number of people in sector i is static and deterministic.) Let t_{ij} be the number of minutes it takes to walk from sector i to sector j .

The management team has decided there will be four snack bars in the theme park. The snack bars are to be located so as to maximize the number of people that are within a 5-minute walk of a snack bar. Let a_{ij} equal 1 if sector j is within a 5-minute walk of sector i ; that is,

$$a_{ij} = \begin{cases} 1, & \text{if } t_{ij} \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Let x_j equal 1 if we locate a snack bar in sector j and 0 otherwise ($j \in J$). Let z_i equal 1 if sector i is within a 5-minute walk of a snack bar ($i \in I$).

- Formulate this problem as a linear mixed-integer optimization problem (MIP). If you use any new notation, define it clearly. Explain your constraints in words.
- Suppose that the management team wants instead to maximize the number of customers covered by at least *two* snack bars. We can redefine z_i to equal 1 if

- sector i is covered by at least two open snack bars. Explain how to modify your model from part (a) to enforce this new requirement. Clearly define any new notation you introduce and explain your new constraint(s) in words.
- c) Return to the original formulation—assume again that a customer is “covered” if there is one open snack bar within 5 minutes. Suppose now the management team also wants to ensure that the average distance traveled by a customer to his or her closest snack bar is no more than 6 minutes. (The average is taken across all customers.) That is, we want to maximize the number of customers within 5 minutes of a snack bar, but we also want to ensure that the average time for *all* customers is no more than 6 minutes. Revise the model to include this requirement. Clearly define any new notation you introduce and explain any new constraints in words.
- d) Continuing with the model in part (c), suppose that the management wants to require that the average distance traveled by a customer to his or her *second-closest* snack bar is no more than 6 minutes. Explain how to modify your model from part (c) to include this requirement. Clearly define any new notation you introduce and explain any new constraints in words.

8.27 (Locating RFID Readers) The theme park from Problem 8.26 issues bands to all of the visitors to the park. The bands are worn on the wrist, and they contain RFID chips that allow the park to identify visitors, without paper tickets, barcodes, etc. The RFID chips are “read” by RFID readers that are located throughout the park—at the park entrance, near the entrances to rides, and so on. RFID is wireless, and each RFID reader can detect RFID chips that are within a certain radius. In fact, there are two types of RFID readers—short-range and long-range—and the wrist bands contain *both* types of RFID chips. Some locations within the park must be covered by a short-range reader, some by a long-range reader, and some by both.

Two technical constraints restrict the locations of the readers:

1. Short- and long-range readers cannot be placed at the same location.
2. No location can be covered by more than four readers, total (including both types).

Park planners want to locate RFID readers throughout the park to cover all of the necessary sites with the reader types required, at minimum possible cost, while satisfying the technical constraints.

- a) Let I be a set of nodes representing locations in the park that must be covered by an RFID reader. (We’ll call these “demand nodes.”) Let J be a set of nodes representing potential sites for the readers. Let $k = 1, 2$ be the two types of readers (1 = short-range, 2 = long-range). Let r_{ik} be a parameter (an input) that equals 1 if demand node $i \in I$ must be covered by a reader of type k . Let f_{jk} be the fixed cost to locate a type- k reader at location $j \in J$. Let x_{jk} be a decision variable that equals 1 if we locate a reader of type k at location $j \in J$.

Using this notation, formulate the problem as a linear integer optimization model. Explain the objective function and the constraints in words. If you introduce any new notation, define it clearly.

- b) Now suppose the theme park’s engineers have found a way to locate a short-range and a long-range RFID reader in the same location $j \in J$, but due to the expense

involved in doing so, planners wish to have at most two locations that have both types of readers. Write one or more linear constraints to enforce this restriction.

8.28 (Locating Compost Sites) The city of Greentown is planning to open several composting facilities, which will convert organic matter (kitchen waste, leaves, yard waste, shredded paper, etc.) into fertilizer instead of sending it to landfills. While the population of Greentown agrees that this is a good idea, nobody wants a new compost site too close to their homes, due to the noise, smell, and truck traffic to and from the site. The city's mayor has hired you to develop a model to choose locations for the new compost facilities.

The population of the city has been aggregated into a set I of neighborhoods, each with population h_i . City planners have identified a set J of potential sites for the compost facilities. The distance between neighborhood i and site j is given by c_{ij} miles. The city wishes to locate p compost sites in order to *maximize the minimum distance between a neighborhood and its nearest open compost facility*.

Define the following decision variables:

$$x_j = \begin{cases} 1, & \text{if we locate a compost facility at site } j, \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if site } j \text{ is the nearest open compost facility to neighborhood } i, \\ 0, & \text{otherwise} \end{cases}$$

- a) Formulate this problem as a linear integer optimization model. Explain the objective function and constraints in words as well as formulating them in mathematical notation. If you introduce any new notation (sets, parameters, decision variables), define it clearly.
- b) Now suppose that, instead of maximizing the minimum distance between a neighborhood and its nearest open facility, the mayor wants to *maximize the shortest distance between any two open compost facilities*. Note that this objective function focuses only on the distances among compost facilities and ignores distances between facilities and neighborhoods.

Formulate this modified problem as a linear integer optimization model. Explain the objective function and new constraints in words. If you introduce any new notation, define it clearly.

8.29 (Convex Hulls are Nonoverlapping) Consider a facility location instance with nodes in \mathbb{R}^2 and Euclidean distances. Suppose we open a set $J' \subseteq J$ of facilities and assign each customer in I to the nearest open facility. Recall that the *neighborhood* of an open facility j is $N_j \equiv \{i \in I | y_{ij}=1\}$. Prove that the convex hulls of the neighborhoods of the open facilities do not overlap.

8.30 (LR Iteration for UFLP) The file LR-UFLP.xlsx contains data for a 50-node instance of the UFLP, as well as the Lagrange multipliers for a single iteration of the Lagrangian relaxation algorithm described in Section 8.2.3. For each facility $j \in J$, column B lists the fixed cost f_j . For each customer $i \in I$, row 2 lists the demand h_i and row 3 lists the Lagrange multiplier λ_i . Finally, the cells in the range C6:AZ55 contain the matrix of transportation costs c_{ij} .

- a) For each $j \in J$, calculate the benefit β_j , the optimal value of x_j , and the optimal objective value of (UFLP-LR $_{\lambda}$). The worksheet labeled “solution” contains spaces to list β_j (column B), x_j (column C), and the objective value (cell C5).

Hint: To double-check your calculations, we’ll tell you that if $i = 6$ and $j = 3$, then $h_i c_{ij} - \lambda_i = 12422.34$.

- b) Using the method described in Section 8.2.3.4, generate a feasible solution to the UFLP. In row 2 of the “solution” worksheet, list the index of the facility that each customer is assigned to in your solution. In cell C6, list the objective value of your solution.

8.31 (Maxisum Location Problem) Consider the following problem: We must locate exactly p facilities, for fixed p . The objective is to *maximize* the sum of the demand-weighted distances between each customer and its nearest facility. Formulate this problem as an IP. Define any new notation clearly. Explain the objective function and each of the constraints in words.

8.32 (Supplier–Facility Capacities) Consider the following extension of the UFLP: We are given a set K of suppliers whose locations are fixed. Each supplier $k \in K$ can ship at most b_{jk} units to facility $j \in J$. This is like a capacity constraint, but it is (supplier, facility)-specific rather than the facility-specific capacities discussed in Section 8.3.1. Such constraints might arise from, say, the capacity of the truck transporting goods from k to j . Let d_{jk} be the cost to transport one unit of demand from supplier $k \in K$ to facility $j \in J$, and let z_{jk} be a decision variable representing the number of units transported from k to j . Note that z_{jk} is a flow-type variable ($z_{jk} \geq 0$), whereas y_{ij} is a fractional variable ($0 \leq y_{ij} \leq 1$). Multiple-sourcing is allowed; that is, facility j may receive shipments from more than one supplier k . In addition to the notation just defined, use the notation in Section 8.2.2. If you need to define any additional notation, define it clearly.

- Formulate this extension of the UFLP as a linear mixed-integer optimization problem. Explain the objective function and each constraint clearly in words.
- In Section 8.2.3, we solved the UFLP by relaxing the “assignment” constraints that require each customer to be assigned to exactly 1 facility. Write the objective function of the Lagrangian subproblem that results from relaxing the analogous constraint in your model from part (a).
- Consider the special case in which $h_i = h$ for all $i \in I$, i.e., all of the customers have the same demand, and b_{jk} is an integer multiple of h for all j, k . Explain how to solve the Lagrangian subproblem from part (b) for this special case. Your method must be exact (i.e., it must be guaranteed to find the optimal solution) and self-contained (i.e., it may not rely on a general-purpose optimization solver).
- Describe a method that, given a feasible solution to the Lagrangian subproblem, produces a feasible solution for the original problem.

8.33 (Salt Stockpiles) You are the director of your local Department of Transportation. You have decided to build silos to stockpile the salt the department uses on roadways during winter weather. A stockpile is considered to cover a town if they are within r miles of each other. Your job is to determine where to locate up to p stockpiles to maximize the total population of the towns that are *double-covered*, i.e., covered by at least *two* stockpiles. Local planners have provided you with the population of each town that you would like to be covered.

- a) Formulate this problem as an integer programming problem. Define any new notation clearly.
- b) Now suppose that the two stockpiles that double-cover a given town must be *at least* s miles from each other. (Two stockpiles may be *located* less than s miles from each other, but a given town doesn't count as double-covered unless there are two stockpiles that cover it and that are at least s miles apart.) Formulate the new model, and define any new notation clearly.

8.34 (Pre-Positioning Disaster Relief Shelters) A disaster relief agency plans to establish shelters in preparation for a hurricane that has been forecast for the coming days. The agency wishes to choose shelters from a set J of potential locations in order to cover every population center in the set I . A shelter covers a population center if it is within r miles of it. As in the set covering and maximal covering models, we define the parameter a_{ij} to equal 1 if a shelter at site $j \in J$ covers population center $i \in I$.

If we locate a shelter at site j , we incur a fixed cost of f_j , as well as an “assignment cost” of w_j for each population center assigned to the shelter at j (regardless of the size of these population centers). For example, if shelter j serves 12 population centers, then we pay an assignment cost of $12w_j$.

Define the following decision variables:

$$x_j = \begin{cases} 1, & \text{if we locate a shelter at site } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if a shelter at site } j \text{ serves population center } i \\ 0, & \text{otherwise} \end{cases}$$

- a) Formulate this problem as a linear integer optimization problem. If you introduce any new notation, define it clearly. Briefly explain your objective function and constraints.
- b) In part (a), the assignment cost is a linear function of the number of population centers assigned to each shelter: It equals $w_j n$, where n is the number of population centers assigned to j . Suppose instead that the assignment cost is a *nonlinear* function $g_j(n)$, where n is the number of population centers assigned to j . Define the following decision variables:

$$z_{jn} = \begin{cases} 1, & \text{if exactly } n \text{ population centers are assigned to a shelter at } j \\ 0, & \text{otherwise} \end{cases}$$

Formulate this problem as a linear integer programming problem. Define any new notation clearly, and explain the objective function and any new constraints.

8.35 (Stochastic Pre-Positioning) A humanitarian relief agency wishes to pre-position stockpiles of emergency supplies (food, water, blankets, medicine, etc.) for use in the aftermath of disasters. Its objective is to locate the smallest possible number of stockpiles while ensuring a low probability that, for each population center, a disaster strikes and the population center cannot be served by any stockpile. Whether a given stockpile can serve a given population center depends on their physical distance as well as on the disaster that strikes.

Disasters are represented by scenarios. A scenario can be thought of as a disaster type, magnitude, and location (e.g., magnitude 7.5 earthquake in city A, influenza pandemic in city B, etc.). However, mathematically each scenario simply specifies whether a given population center can be served by a given stockpile during a given disaster.

Let I be the set of population centers, and let J be the set of potential stockpile locations. Let S be the set of scenarios (including the scenario in which no disaster occurs), and let q_s be the probability that scenario s occurs. Stockpile j is said to “cover” population center i in scenario s if *either* stockpile j can serve population center i in scenario s *or* population center i does not need disaster relief in scenario s . Let a_{ijs} be a parameter that equals 1 if stockpile j covers population center i in scenario s , and 0 otherwise. Assume each stockpile is sufficiently large to serve the needs of the entire population it covers.

Formulate a linear integer programming problem that chooses where to locate stockpiles in order to minimize the total number of stockpiles located while ensuring that, for each $i \in I$, the probability that i is not covered by any open stockpile is less than or equal to α , for given $0 \leq \alpha \leq 1$. Clearly define any new notation you introduce. Explain the objective function and all constraints in words.

8.36 (Error Bias) Suppose the transportation costs are estimated badly in the UFLP. It is natural to expect that the true cost of the solution found under the erroneous data has an equal probability of being larger or smaller than the cost calculated when solving the problem. Test this hypothesis by solving the instance given in `random-errors.xlsx` 100 times, each time perturbing the transportation costs by multiplying them by $U[0.75, 1.25]$ random variates. For each instance generated this way, record the objective function value, as well as the objective function of the same solution when the correct costs are used. If the hypothesis is correct, the objective function should be less than the true cost for roughly half of the instances and greater for the other half. Do your results confirm the hypothesis? In a few sentences, explain your results, and why they occurred. Also comment on the implications your results have for the importance of having accurate data when choosing facility locations.

8.37 (1-Center on a Tree) Consider the 1-center problem on a tree network in which all of the demands are 1. Prove that the Algorithm 8.10 finds the optimal solution to both the absolute and the vertex 1-center problem. (Recall from Section 8.4.3 that the *absolute p -center problem* allows facilities to be located on either the edges or the nodes of the network, whereas the *vertex p -center problem* restricts facilities to the nodes.)

Algorithm 8.10 1-Center on a tree

- 1: $v_1 \leftarrow$ any point on the tree
 - 2: $v_2 \leftarrow$ node that is farthest from v_1
 - 3: $v_3 \leftarrow$ node that is farthest from v_2
 - 4: absolute 1-center is at the midpoint of the (unique) path from v_2 to v_3 ; vertex 1-center is at the vertex of the tree that is closest to the absolute 1-center
-

8.38 (2-Center on a Tree) Prove that Algorithm 8.11 finds the optimal solution to the absolute 2-center problem.

8.39 (N -Echelon Location Problem) By extending the approach used in Section 8.7.1, formulate a facility location model with N echelons, for general $N \geq 3$. Echelon N ships

Algorithm 8.11 2-Center on a tree

- 1: using Algorithm 8.10, find the absolute 1-center of the tree
- 2: delete from the tree the link containing the absolute 1-center. (If the absolute 1-center is on a vertex, delete one of the links incident to the center on the path from v_1 to v_2 .) This divides the tree into two disconnected subtrees
- 3: use Algorithm 8.10 to find the absolute 1-center of each of the subtrees; these constitute a solution to the absolute 2-center problem

products to echelon $N - 1$, which ships products to echelon $N - 2$, and so on; echelon 1 serves the end customer. The locations of the facilities in echelons $2, \dots, N$ are to be decided by the model, and there are fixed costs for each. Define any new notation clearly. Explain the objective function and each of the constraints in words. *Note:* No decision variables should have more than 3 indices.

8.40 (UFLP Duality Gap) Prove Lemma 8.4.

8.41 (Another Relaxation for the p MP) Suppose that we use Lagrangian relaxation to relax constraint (8.72) in the p MP. Write the resulting Lagrangian subproblem. This problem is structurally identical to another problem discussed in this chapter; what is it? Briefly summarize the advantages and disadvantages of this relaxation compared to the relaxation discussed in Section 8.3.2.2: Which subproblem is harder to solve? Which approach will give a tighter bound? For which approach will the subgradient optimization procedure converge more quickly?

8.42 (Tightening the CFLP Relaxation) Suppose we add the following constraint to the CFLP:

$$\sum_{j \in J} v_j x_j \geq \sum_{i \in I} h_i. \quad (8.152)$$

Explain in words what this constraint says. Explain why this constraint is redundant for the CFLP (adding it does not change the optimal solution for the CFLP) and why adding it tightens the Lagrangian relaxation discussed in Section 8.3.1. Finally, explain how to solve the Lagrangian subproblem when constraint (8.152) is included in the model.

8.43 (Variable-Splitting Method for CFLP) In this problem, you will develop a *variable-splitting* method for the CFLP. Variable splitting (also known as *Lagrangian decomposition*) is a method that involves duplicating one or more sets of variables, adding a constraint that requires those variables to be equal to their duplicates, and then relaxing that constraint using Lagrangian relaxation. (See Guignard and Kim (1987).)

- a) Introduce new decision variables w_{ij} for $i \in I, j \in J$. Rewrite the objective function as

$$\text{minimize} \quad \sum_{j \in J} f_j x_j + \beta \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij} + (1 - \beta) \sum_{i \in I} \sum_{j \in J} h_i c_{ij} w_{ij},$$

where $0 \leq \beta \leq 1$ is a constant. Rewrite constraints (8.55) using w instead of y . Add the following new constraints, which require w and y to be equal, and require w to be nonnegative:

$$w_{ij} = y_{ij} \quad \forall i \in I, \forall j \in J \quad (8.153)$$

$$w_{ij} \geq 0 \quad \forall i \in I, \forall j \in J \quad (8.154)$$

Write the resulting problem. This problem is equivalent to (CFLP).

- b) Relax constraints (8.153) using Lagrangian relaxation. Write the resulting subproblem.
- c) Explain how to solve the subproblem from part (b).
- d) Based on your intuition, will this relaxation provide a tighter, weaker, or equivalent bound to the relaxation discussed in Section 8.3.1?

8.44 (Accuracy of Spherical Law of Cosines Formula) Calculate the distances between every pair of nodes in the 88-node data set (88node.xlsx) using both the great circle distance formula (8.1) and the spherical law of cosines formula (8.2). Compare the results. Are there any cases for which the two formulas produce distances that differ by more than a mile or so? If so, what characterizes those cases?

8.45 (Swap vs. Neighborhood Search for p -Median) Implement the swap and neighborhood search heuristics for the p MP (Algorithms 8.7 and 8.8). Conduct a numerical experiment to compare the effectiveness (as measured by objective function value) and efficiency (as measured by CPU time) of these two heuristics. Your experiment should use randomly generated p -median instances with at least 100 nodes.

8.46 (Hakimi Property for SCLP) Does the Hakimi property hold for the set covering problem? Explain your answer.

8.47 (Proof of Lemma 8.8) Prove Lemma 8.8.

8.48 (Solving the p CP using the MCLP) Algorithm 8.9 relies on the relationship between the p CP and the SCLP stated in Lemma 8.8. A similar relationship exists between the p CP and the MCLP.

- a) State a lemma similar to Lemma 8.8 that describes this relationship.
- b) Write pseudocode similar to Algorithm 8.9 for an exact algorithm that solves the p CP by iteratively solving MCLPs.

8.49 (The MCLP with Mandatory Closeness Constraints) The *MCLP with mandatory closeness constraints* is identical to the MCLP except that it also requires every customer to be covered within a distance of s , with $s \geq r$. That is, we wish to maximize the number of demands that are covered within r , but every customer must be covered within s . Write an integer programming formulation for this problem. If you introduce any new notation, define it clearly. Explain the objective function and each constraint in words.

8.50 (MCLP is a Special Case of p MP) Show that the MCLP is a special case of the p MP by showing how to set the parameters of the p MP so that solving it is equivalent to solving the MCLP.

8.51 (A Dynamic Location Problem) Consider a dynamic facility location problem in which the demands over a finite time horizon are known but change in each time period: h_{it} is the demand at node $i \in I$ in period t , where $t = 1, \dots, T$. We can open and close as many facilities as we like in each time period. Facility $j \in J$ incurs a fixed cost of f_{jt}^+ if it is opened in period t (but was closed in period $t - 1$), a fixed cost of f_{jt}^- if it is closed in period t (but was open in period $t - 1$), and a fixed cost of f_{jt} if it remains open in period t (that is, if it was also open in period $t - 1$). Assume that no facilities are open at the

start of the horizon, that is, in period 0. The transportation cost from facility j to customer i in period t is given by c_{ijt} . Formulate an integer programming model to optimize the locations of facilities over the time horizon to minimize the total fixed and transportation costs. If you introduce any new notation, define it clearly. Explain your objective function and constraints in words.

8.52 (MCLP Modifications) Modify the MCLP to accommodate each of the changes described below (one at a time). For each modification, change *either* the objective function *or* exactly one constraint to reflect the modification. Indicate the number of the equation (objective function or constraint) you are changing.

- We wish to maximize the total number of nodes covered, not the total population covered.
- Each facility j has a fixed construction cost of f_j . Rather than restricting the number of facilities to equal p , restrict the total amount spent to construct facilities to a budget of b .
- A demand node only counts as covered if there are *two* facilities within the coverage radius.

8.53 (Subproblem Assignments) Prove that, if customer i is assigned to at least one facility in the optimal solution to (UFLP-LR $_{\lambda}$), then one of the facilities it is assigned to is the nearest open facility. (This implies that in step 4 of Algorithm 8.2, it suffices to check only those j such that $y_{ij} = 1$ in the optimal solution to (UFLP-LR $_{\lambda}$).)

8.54 (Location of Power Generators) Consider the problem of locating generators within an electricity network.

- First consider a single generator. Suppose the generator's load (i.e., the total demand for electricity from the generator) is given by $D \sim N(\mu, \sigma^2)$, where D is measured in kilowatt-hours (kWh). The cost to generate enough electricity to meet a load of d kWh is given by $\frac{1}{2}\gamma d^2$, where $\gamma > 0$ is a constant. Prove that the expected generation cost is given by $\frac{1}{2}\gamma(\mu^2 + \sigma^2)$.
- Now consider an electricity network consisting of multiple generators, whose locations we need to choose. Let I be the set of loads (demand nodes), with load i having a daily demand distributed $N(\mu_i, \sigma_i^2)$. Let J be the set of potential generators. The daily fixed cost if generator j is open is f_j , and the generation cost coefficient for j is γ_j . Formulate the problem of choosing generator locations and assigning loads to generators in order to minimize the expected daily cost of the system. Assume that, once location and assignment decisions are made, the power network for a given generator and its loads is disconnected from the remaining generators and loads (so that the physics of power flows can be ignored). Also assume that the cost to transmit power is negligible.

8.55 (Stochastic Location for Toy Stores) Return to Problem 8.1, and suppose now that the demands are stochastic. The file `toy-stores-stochastic.xlsx` gives the demands for five scenarios, as well as the probability that each scenario occurs.

- Implement the stochastic fixed-charge location problem in a modeling language of your choice. Find the optimal solution for the instance given in the data set. Report the optimal set of facilities and the corresponding cost.

- b) Now implement and solve the minimax fixed-charge location problem. Report the optimal set of facilities and the corresponding cost.

8.56 (Minimax Cost \neq Minimax Regret) Construct a small example of the minimax fixed-charge location problem (MFLP) in which minimizing the maximum cost results in an optimal solution that is different from the solution that minimizes the maximum regret. (You may choose either relative or absolute regret.) Your instance may have at most five nodes.

8.57 (Side Constraints for Arc Design) Formulate each side constraint listed below for the arc design model in Section 8.7.2.2. Your constraints must be linear. If you introduce any new notation, define it clearly.

- a) We have a set $P \subseteq E \times E$ of ordered pairs of arcs such that, for $(e_1, e_2) \in P$, if arc e_1 is opened, then arc e_2 must be opened.
- b) We have a set of $E' \subseteq E$ of arcs such that at most r arcs in E' may be opened.
- c) We have a set of $E' \subseteq E$ of arcs such that at least r arcs in E' must be opened.
- d) We have an upper bound B on the transportation cost that may be spent shipping on a subset $E' \subseteq E$ of the arcs.

8.58 (Modified Hungary Network) Consider the Hungary instance of the arc design problem shown in Figure 8.20. The file `hungary2.xlsx` contains a modification of the instance described in Example 8.11. It lists the latitude and longitude of each node, the available units for each node and product, and the fixed cost and capacity for each arc. The variable cost is 1 for every arc and product. Formulate the arc design model in a modeling language of your choice, and solve this instance. Report the optimal arcs to open, the optimal flows, and the optimal total cost.

8.59 (Campaign Offices) A candidate for a national political position wishes to establish campaign offices and decide how much money to spend on campaign activities at those offices. The candidate's staff has identified a set J of potential locations for campaign offices (facilities) and a set I of neighborhoods (demand nodes) that they wish to "cover" using these offices. Let a_{ij} be a parameter that equals 1 if office location $j \in J$ covers neighborhood $i \in I$, and 0 otherwise. Neighborhood $i \in I$ has h_i registered voters living in it. Opening an office at location $j \in J$ incurs a fixed cost of f_j .

In addition to choosing *where* to locate offices, the candidate's staff needs to determine how much money to spend on campaign activities (get-out-the-vote, marketing, etc.) at each office. They can only perform campaign activities at offices that they have chosen to open. Staffers have estimated that each \$1 spent on these activities will earn the candidate exactly one extra vote.

For example: Suppose the candidate opens an office at location $j \in J$, and location j covers 1000 registered voters. If the campaign spends \$1000 on campaign activities (*not* including the fixed cost f_j), the candidate will earn all of their votes; if it spends \$500, the candidate will earn half of their votes; and if it spends \$0, the candidate will earn none of their votes. Note that there is no advantage to spending more than \$1000 on campaign activities in this example. There is also no advantage to opening an office at j if we spend \$0 since the candidate will not earn any votes.

If a neighborhood is covered by more than one open campaign office, its votes can only be earned once. Therefore, only one office should direct its campaign activities at that

neighborhood. Your model should choose which of the open offices should “serve” each customer.

The candidate’s objective is to maximize the number of votes earned. The campaign has a total budget of $\$B$ to spend on *both* fixed costs *and* campaign activities.

Define the following decision variables:

$x_j = 1$, if we open a campaign office at location $j \in J$, 0 otherwise

w_j = the number of dollars we spend on campaign activities at office $j \in J$

Formulate this problem as an integer linear optimization problem. If you introduce any new notation, define it clearly. Explain your objective function and each constraint in words.

8.60 (Exchange Rate Hedging) An automobile manufacturer wishes to decide where to locate factories around the world in order to account for random fluctuations in currency exchange rates. The company will change the production levels at the various factories to take advantage of changes in the exchange rates. Exchange rates are expressed as α $\$/\mathfrak{A}$, where $\$$ stands for US dollars (USD) and \mathfrak{A} stands for the local currency in the other country. For example, if the exchange rate between the United States and Thailand is $\alpha = 0.028$ $\$/\text{B}$, then 1 Thai baht is worth US\\$0.028.

The manufacturer is considering a set J of potential locations for the factories, which will ship automobiles directly to the customers in a set I . Customer $i \in I$ has a demand of h_i units per year. We have the following costs:

- Building a factory at site $j \in J$ incurs a fixed annual cost of $\$f_j$, which is deterministic and expressed in USD.
- The cost to produce one automobile at factory $j \in J$ is $\mathfrak{A}b_j$, which is deterministic and expressed in the local currency of the country in which factory j is located.
- The cost to ship one automobile from factory $j \in J$ to customer $i \in I$ is $\$c_{ij}$, which is deterministic and expressed in USD.

The factories have effectively unlimited capacity.

Once the factories are built, the random exchange rates are realized, and the company then decides how much to produce at each factory, as well as how much to ship from each factory to each customer. The exchange rates are described by a set S of scenarios, such that α_{js} is the exchange rate (in $\$/\mathfrak{A}$) in scenario s for the country in which facility $j \in J$ is located. Let q_s be the probability that scenario s occurs.

Let x_j equal 1 if we open a factory at site $j \in J$, 0 otherwise. Let y_{ijs} equal the number of automobiles to be shipped from a factory at site $j \in J$ to customer $i \in I$ in scenario $s \in S$. These are our decision variables. You may treat y_{ijs} as a continuous variable.

- a) Formulate a stochastic optimization problem that minimizes the total expected annual cost of locating facilities and producing and transporting automobiles. If you introduce any new notation, define it clearly. Explain your objective function and each constraint in words.
- b) Suppose we allow y_{ijs} to be continuous and nonnegative. If the demands h_i are expressed as integers, will there necessarily exist an optimal solution in which the y_{ijs} are integers? Why or why not?
- c) Suppose that, instead of minimizing the total expected cost, the company wishes to minimize the maximum absolute regret that can occur, across all exchange rate scenarios. Formulate this new problem. If you introduce any new notation, define it clearly.

