ISEN 617 Final Project

MSC Corporation Optimization Report

By Group 4

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1. Introduction

This project is about helping MCS Corporation to minimize their operating cost by optimizing their production and transportation process using linear programming with Gurobi. We provide three linear programming models to solve the following problems:

- Original Model:
  What is the optimal quantity for each factory producing the same quantity?
- 'What If' question 1 Model:
  What is the optimal quantity for each factory producing a different quantity?
- 'What If' question 2 Model:
  What is the reserve price of the old production line if the company replaces the new production line?

After applying Gurobi, we will get the results related to the model, including units produced at each factory (decision variables), the minimum cost (the objective function) based on all the necessary limitations (constraints).

2. Model

Parameters

We were given some information about the factories and customers. We define that information into parameters in our model:

I = {potential facility locations} J = {stores} L = {production lines in factory i, $\forall i \in I$} T: Time period, T=12 months N: Number of factory h: holding cost, h is constant J: the number of retail stores, J = 8	C_{ij}: Cost to transport 1 unit of demand factory i to store j D_{jm}: Demand of store j in month m E_{il}: Energy cost per unit at factory i for production line l M_i: Maximum capacity of factory i W_{il}: Production rate of production line l at factory i (Units per hour)

Decision Variables

u_{ijm}: the number of units transported from factory i to store j in month m

S_{ilm}: the number of units produced at factory i from production line l in month m

Objective function

$$\min \sum_{m \in T} [\, 1.2 \sum_{j \in J} \sum_{i \in I} C_{ij} u_{ijm} + \sum_{i \in I} \sum_{l \in L} E_{il} S_{ilm}]$$

Our objective is minimizing the annual total cost which includes cost of production and cost of transportation. The summation of transportation cost from each factory [i] to store [j] will be the total transportation cost. The cost of production at each factory[i] for production line [l] is equal to the unit production cost [c] multiply by the total unit produced [S] at the respective factory [i] for production line[l]. There is 20% of premium charged by LM Trucking, this is reflected by the multiplication of 1.2 which is 120% of the total cost. The period is 1 year, hence summing the total cost of 12 months.

Constraints

$$\sum_{i} u_{ijm} \geq D_{jm} \quad \forall j \in J$$

Demand Fulfilment on Transportation: this constraint ensures the number of units transported from all the factory to store [j] can fulfil the demand of each store [j].

$$S_{ilm} \leq M_{il} \quad \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{m} \in \mathrm{T}, \forall \mathrm{l} \in \mathrm{L}$$

Production Capacity: this constraint ensures the total production for all the production lines does not exceed the production capacity of each factory [i] every month

$$\sum_{i \in \mathrm{I}} \sum_{l \in \mathrm{L}} S_{ilm} \geq \sum_{j \in \mathrm{J}} D_{jm} \qquad \forall \mathrm{m} \in \mathrm{T}$$

Demand Fulfilment on Factory: this constraint ensures the total units produced in 3 factories able to fulfil DinoBall's monthly demand [m].

$$\sum_{j \in \mathrm{J}} u_{ijm} \leq \sum_{l \in \mathrm{L}} S_{ilm} \qquad \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{m} \in \mathrm{T}$$

Transportation units can't exceed production units: this constraint ensures the total units transported from facility [i] to store [j] less than or equal to the total units of production at facility [i] from all the production lines [l] in month [m].

$$\sum_{l \in \mathrm{L}} S_{1lm} = \sum_{l \in \mathrm{L}} S_{2lm} \qquad \forall \mathrm{m} \in \mathrm{T}$$

$$\sum_{l \in \mathrm{L}} S_{1lm} = \sum_{l \in \mathrm{L}} S_{3lm} \qquad \forall \mathrm{m} \in \mathrm{T}$$

Consistent production: these two constraints ensure every factory produces the same number of toys. The summation of units produced by all the production line [l] in each factory [i] must equal to other factories.

$$\frac{S_{i1m}}{W_{i1}} = S_{i2m}/W_{i2} = \frac{S_{i3m}}{W_{i3}} \qquad \forall \mathrm{i} \in \mathrm{I}$$

Equal wear and tear: these constraints ensure consistent wear and tear for each production line. The run time for each production line is equal to units produced by each production line divide by the rate of production of each production line.

Non-negativity

$$u_{ijm}, S_{ijm} \geq 0 \qquad \forall \mathrm{i} \in \mathrm{I}, \forall \mathrm{j} \in \mathrm{J}, \forall \mathrm{m} \in \mathrm{T}$$

<u>Assumptions</u>

1. Every truck carries specific demand, but the unit transportation cost was calculated based on full capacity.
2. Fixed holding cost - Assume the holding cost for each store is known and fixed.
3. Inventory Model wasn't considered since inventory capacity at the stores wasn't known.
4. Each truck can only transport to one store - Assume there is no detour to another store when transporting from factory i to store j.

3. Data

(1) Demand: The demand for products to ensure they can be delivered and satisfy customers. Table 1 describes the historical demand of 8 different retail stores. In this case, since the historical is not enough for make prediction, here we use these historical demand data as our future demand.

Table 1.
Restail Stores Monthly Demand

	Pittsburgh	Cleveland	Buffalo	Philadephia	Boston	Newtork	Providen	Hartford
January	200	100	125	250	225	400	50	75
February	150	125	100	300	200	425	75	100
March	225	150	75	250	225	375	100	125
April	250	200	100	200	200	350	125	150
May	250	175	75	200	250	300	75	125
June	180	175	100	300	175	400	50	100
July	180	200	125	250	200	250	100	125
August	200	150	200	300	150	200	150	150
September	150	100	150	350	200	225	25	100
October	150	100	100	350	250	400	50	75
November	200	75	150	400	300	500	150	100
December	300	200	175	450	400	475	150	150

* we use historical demand as our future demand.

(2) Cost: In the cost part, we use two types of cost into our objective function in order to minimize our total cost.

(a) Transportation Cost: we use per unit cost as our shipping cost shown in Table 2.

(b) Production Cost: production cost here we use energy cost per unit (see Table 3).

Table 2

Shipping Costs Per Unit ($)

	Pittsburgh	Cleveland	Buffalo	Philadelphia	Boston	Newtork	Providen	Hartford
Troy,NY	1.073	1.018	0.625	0.501	0.371	0.335	0.358	0.250
Newark,NJ	0.768	1.065	0.610	0.188	0.480	0.026	0.405	0.271
Harrisburg,PA	0.433	0.698	0.625	0.226	0.821	0.361	0.745	0.610

Table 3.

Engerny cost for each facility

Troy, NY	Units per hour	Energy Cost per unit
Prod Line 1	6	5
Prod Line 2	2	19.125
Prod Line 3	1	41.25

Newark, NJ	Units per hour	Energy Cost per unit
Prod Line 1	5	7.56
Prod Line 2	2	22.95
Prod Line 3	1	49.5

Harrisburg,PA	Units per hour	Energy Cost per unit
Prod Line 1	4	4.95
Prod Line 2	3	7.05
Prod Line 3	0.5	54

Table 4.

Capacility for ecah facility

Troy, NY	Capacity
Prod Line 1	960
Prod Line 2	320
Prod Line 3	160

Newark, NJ	Capacity
Prod Line 1	800
Prod Line 2	320
Prod Line 3	160

Harrisburg,PA	Capacity
Prod Line 1	640
Prod Line 2	480
Prod Line 3	80

* assume consistant 160 workings hour per month

(4) New Production line, “FastProd”: Table 5 and 6 showed the production cost and capacity for each facility. The life cycle of each new production line will be 3 years.

Table 5

Engery cost in FastProb production line

	Engery cost per unit
Troy, NY	5.625
Newark, NJ	6.75
Harrisburg,PA	3.375

Table 6

Capacity in FastProb production line

	Capacity
FasProb	1280

* The capacity are same in 3 facilties.

4. Result

4.1 The original model: In Figure 4.1, we can see that the output of original model has fixed quantity for each factory. In the restriction formula of the original Model, we specifically set that all factories need to comply with the "even wear and tear" rule, so that the algorithm will take this setting into account after the calculation.

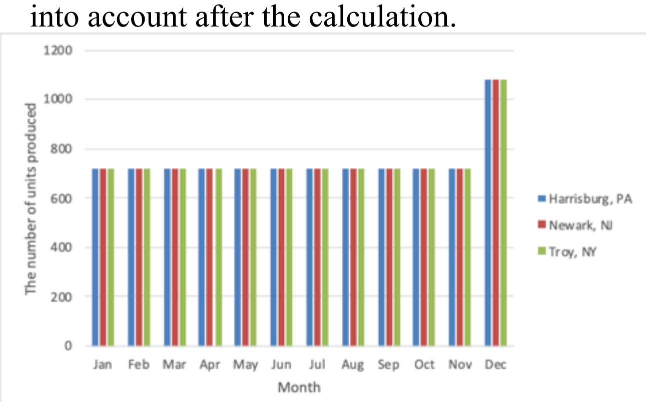

Figure 4.1

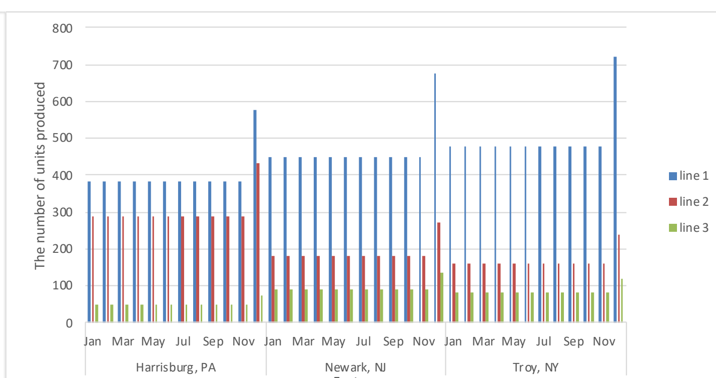

Figure 4.2

We can take a closer look at the production status of each factory production line with Figure 4.2 above. Since we know that the production line 1 of each factory are the production lines with the highest production efficiency, the answers we obtained through the optimal algorithm is consistent with the data. The principle is to produce through the production line with the best production efficiency, which can reduce the cost to minimum.

4.2 Model 1: Factories produce different quantity (what-if question 1)

Model 1 do not have the constraint that each factory must produce the same quantity. Therefore, from Figure 4.3 that not all factories are producing. According to our model 1, if the Newark factory is not operated, it can most effectively reduce costs and meet the needs of retailers. Harrisburg produces a fixed quantity regardless of the month; however, Tory's production varies from month to month.

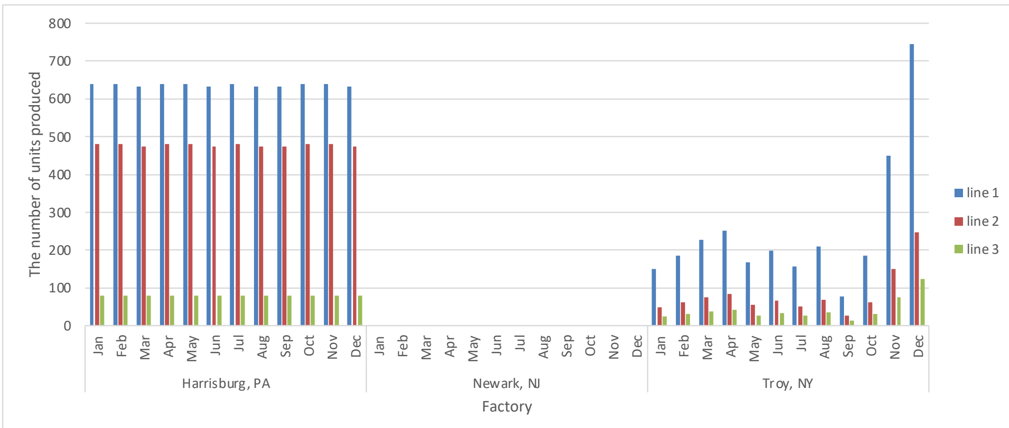

Figure 4.3

According to the calculation of Model 1, we find that the optimal production cost is **$1,505,200** under the condition that the production quantity can be different. Compared with the same output from original Model, the total saving is <u>**$1,242,638**</u>. Shown in Table 7.

Table 7 *The optimal cost of original model and model 1*

Original model	Model 1	Total saving
$2,747,837.50	$1,501,099	$1,246,738.50

4.3 Model 2: Update all production lines with 'FastProd' (what-if question 2)

The big difference between the model 2 and the first two is that the production cost of the model 2 is calculated by the new FastProd equipment, and the production quantities of the three production lines are the same because every production line has the same production efficiency. As shown in Figure 4.4.

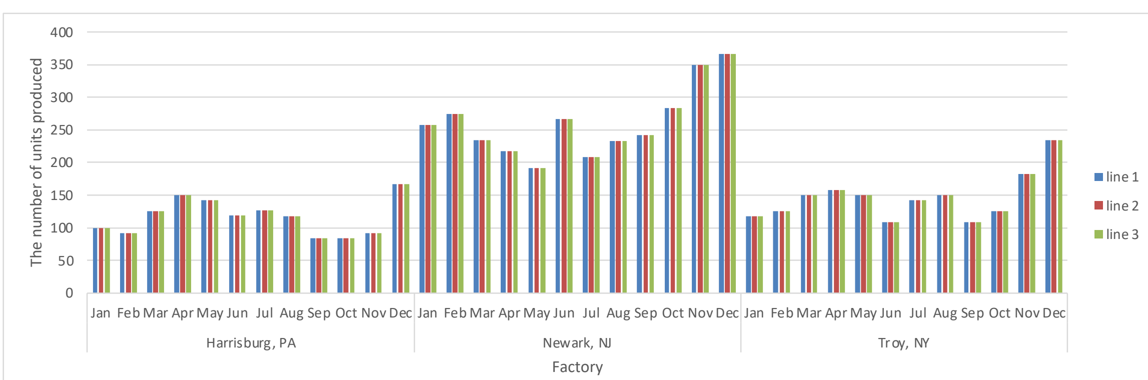

Figure 4.4

In addition, after considering factors such as transportation cost and production cost, Newark is the factory with the largest number of productions in model 3, which is different from the previous two, as shown in Figure 4.5.

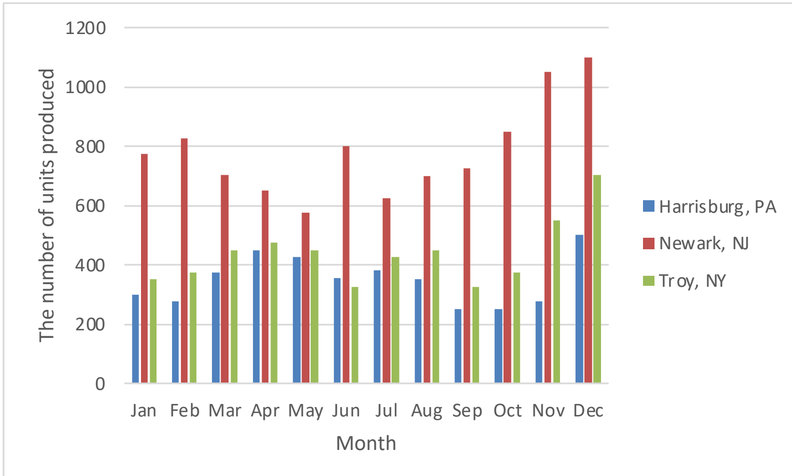

Figure 4.5

With Model 2, the total production cost of the production line in 3 years is $2,433,444. We adjusted the production cost of Model 1 for three years and found that the cost of continuous use of "even wear and tear" in the factory after three years was $8,243,513. Therefore, we set the reserve price to $8,243,513-$2,433,444, which is **$5,810,068**.

5. Conclusion

Based on our analysis, the total cost of the original model is **$2,747,837.5** and the total saving for removing constant unit production in every factory is **$1,242,638**. This model is recommended if the cost of restructuring the company is lower than the total saving amount from model 1. According to the analysis, "Fast Prod" production lines are capable to save $5,810,068 over three years compared to the old production lines. "Fast Prod" is highly recommended if the cost of upgrading the production lines is lower than **$5,810,068**. Besides, this saving has not included the salvage value of old production line. If we assume the old production lines have a total salvage value which is equal to 10% of the new production lines. The total saving would be **$5,810,068 – 0.9 * Cost of Installing "Fast Prod".**

The inventory model would also be recommended for lowering the cost of the model. Inventory could be potentially stored at stores where the holding cost per unit is less than the transportation cost of the unit. Also, the trucks could carry up to a full 250 units with normal optimization problem becoming a Routing and Scheduling problem.

6. Reference

1. Gurobi Jupyter notebook modeling examples. Gurobi. (2022, April 4). Retrieved May 4, 2022, from https://www.gurobi.com/resource/modeling-examples-using-the-gurobi-python-api-in-jupyter-notebook/