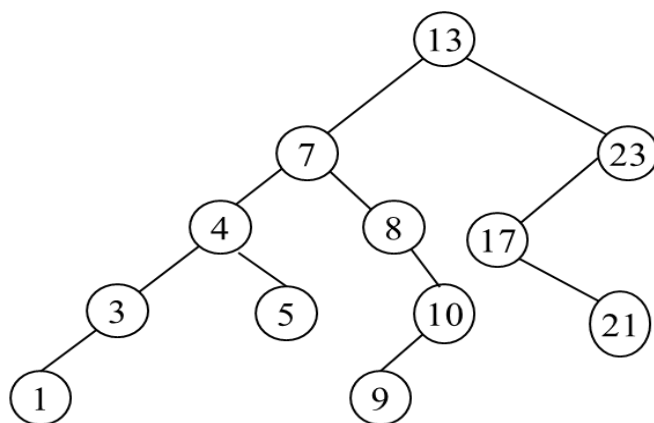


## Algorithm Final Exam.

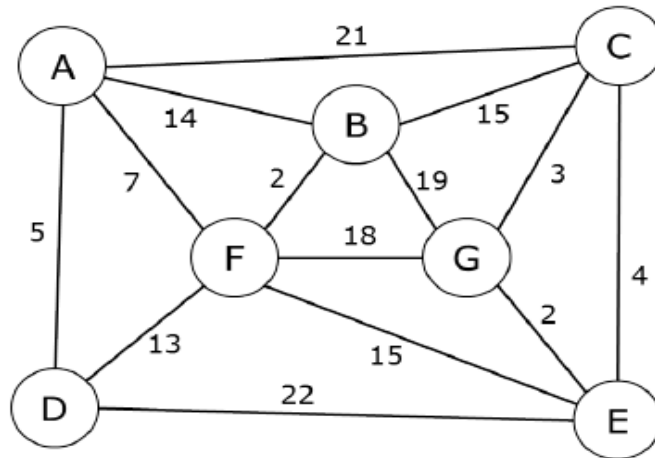
1. (10 pts) True/False and justify your answer briefly
  - a) Any problem that can be solved with a greedy algorithm can also be solved with dynamic programming.
  - b) Prim's algorithm is based on greedy algorithm but Kruskal's algorithm is not.
  - c) Linear probing is equivalent to double hashing if a secondary hash function of  $h_2(k) = 1$ .
  - d) NP stands for Not Polynomial time.
  - e) maximum spanning tree(i.e spanning tree that maximize the sum of weights) can be computed in  $O(E \log V)$  time
2. (10 pts) Draw the hash table(size  $m=11$ ) that results from using the hash function,  $h(i)=(2i+5) \bmod 11$ , to hash the keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, and 5,
  - a) when collision resolution by chaining
  - b) when collision resolution by linear probing
3. (10 pts) Given the following binary search tree, draw binary search tree after deleting 7.



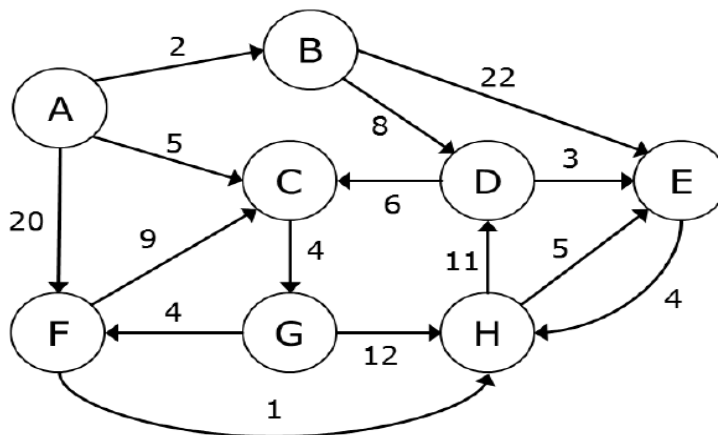
4. (10 pts) Describe an optimal dynamic programming algorithm to find the maximum product of a contiguous sequence of positive numbers  $A[1..n]$ . For example, if  $A = (0.1, 17, 1, 5, 0.05, 2, 4, 1, 0.7, 0.02, 12, 0.3)$ , then the answer

would be 85 because of the subsequence 17,1, 5.

5. (20 pts) The following graph is given.



- Find the minimum spanning tree by Prim's algorithm.
  - Find the minimum spanning tree by Kruskal's algorithm.
6. (10 pts) Consider the following directed, weighted graph



Use Dijkstra's algorithm to calculate the single-source shortest paths from vertex A to every other vertex.

7. (10 pts) a) For the character frequency array shown below, construct the Huffman coding tree and produce the Huffman code for each character in the frequency array.

<i>character</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
<i>Frequency</i>	8	10	12	15	20	35

b) How many bits can Huffman code be saved when compared to the fixed length code (three bits/letter)?

8. (5 pts) A set of  $n$  customers at a bank is given. Every customer  $i$  needs  $t_i$  time for being served. Write an algorithm that determines an order of the customers that minimizes the total amount of time spent at the bank. (We assume all customers leave once being served.)
9. (5 pts) A table with  $N \times M$  cells is given, and each cell has number of jewels. You start from the upper-left corner. At each step you can go down or right one cell. Write an algorithm to find the maximum number of jewels you can collect. To understand the problem, refer to the below example.

2	0	5	3
1	3	4	2
1	0	3	8
8	5	3	1
1	4	5	5

Max sum = 31

10. (10 pts) Consider a positive weighted directed graph. Write an algorithm for finding a minimum weight simple cycle in the graph. (including the description of data structure for efficiency)