Algorithm Midterm Exam 24th, Oct

1. (20 pts)For each of the following statements, indicate whether TRUE or FALSE and justify your answer.
   a) Running merge sort on an array of size n which is already correctly sorted takes O(n) time.
   b) Building a heap from an array of n items requires Ω(nlogn) time
   c) In a heap of depth d, there must be at least 2^d elements. (Assume the depth of the first element (or root) is zero).
   d) The following array is a max heap: [10, 3, 5, 1, 4, 2].
2. (10 pts) Solve the following recurrence equations by master theorem.
   a) $T(n) = 3T(n/2) + n$
   b) $T(n) = 3T(n/2) + 3^n$
3. (10 pts) What happens when the array is already sorted (what is the running time for quick sort in the following cases)?
   a) last element in the array as a pivot.
   b) median-of-three as a pivot.
4. (20 pts) Consider the following four sorting algorithms:
   insertion sort, Merge sort, Quick Sort, Heap sort
   a) Which of above sorting algorithms are stable?
   b) Which of above sorting algorithms run in worst-case time O(nlogn)?
   c) Which of above sorting algorithms run in time O(n) in the best case?
   d) Which of above sorting algorithms are the Divide & Conquer strategy?
5. (10 pts) The following array is not a heap.

22	15	16	14	11	13	7	10	18	2	9	8	4

   After calling MAX-HEAPIFY to correct that violation, the resulting array will be a heap. Show it in array form:
6. (10 pts) Given the following array as input, illustrate how the Merge sort algorithm performs.

3	8	4	10	1	5	6	9

7. (20 pts) a) Write MAX-HEAPIFY procedure in pseudocode.
   b) Write BUILD-MAX-HEAP procedure in pseudocode