〈과제 #5〉

Ex) 4.3

1) a) $W = k_1(0,-2,2) + k_2(1,3,-1) = (k_2, -2k_1+3k_2, 2k_1-k_2)$

$$\begin{aligned} k_2 &= 2 \\ -2k_1+3k_2 &= 2 \\ 2k_1-k_2 &= 2 \end{aligned} \Rightarrow Ax=b \Rightarrow \begin{bmatrix} 0 & 1 \\ -2 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \quad k_1=2,\ k_2=2$$

(2,2,2)는 linear combinations 이다.

b)

$$\begin{aligned} k_2 &= 0 \\ -2k_1+3k_2 &= 4 \\ 2k_1-k_2 &= 5 \end{aligned} \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ -2 & 3 & 4 \\ 2 & -1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 3 & 4 \\ 0 & 1 & 0 \\ 2 & -1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & -2 \\ 0 & 1 & 0 \\ 2 & -1 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & -2 \\ 0 & 1 & 0 \\ 0 & 2 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ 이므로

(0,4,5)는 not linear combinations 이다.

c)

$$\begin{aligned} k_2 &= 0 \\ -2k_1+3k_2 &= 0 \\ 2k_1-k_2 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ -2 & 3 & 0 \\ 2 & -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 3 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad k_1 = 0,\ k_2 = 0$$

(0,0,0)은 linear combinations 이다.

Ex) 4.4

1-d) d가 linearly dependent 하다. 그 이유는 $k_2$가 $k_1$의 scalar 곱이기 때문이다. $k_1 \cdot x_1 + k_2 \cdot x_2 = 0$ 을 만족하는 $x_1, x_2$의 값이 무수히 많기 때문이다.

Ex) 4.5

13) a) $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} x_3 =$

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2 \\ 2x_2 + 3x_3 &= -1 \\ 3x_3 &= 3 \end{aligned}$$

$\Rightarrow$ $x_3 = 1$ 이므로, $x_2 = -2$, $x_1 = 2 - 2x_2 - 3x_3 = 2 + 4 - 3 = 3$

$x_1 = 3,\ x_2 = -2,\ x_3 = 1$ $\therefore (x_1, x_2, x_3) = (3, -2, 1)$

b) $\begin{bmatrix} 5 \\ -12 \\ 3 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + k_2 \begin{bmatrix} -4 \\ 5 \\ 6 \end{bmatrix} + k_3 \begin{bmatrix} 7 \\ -8 \\ 9 \end{bmatrix} = \begin{bmatrix} k_1 - 4k_2 + 7k_3 \\ 2k_1 + 5k_2 - 8k_3 \\ 3k_1 + 6k_2 + 9k_3 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & 5 \\ 2 & 5 & -8 & -12 \\ 3 & 6 & 9 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 7 & 5 \\ 0 & 13 & -22 & -22 \\ 0 & 18 & -12 & -12 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 7 & 5 \\ 0 & 1 & -\frac{22}{13} & -\frac{22}{13} \\ 0 & 3 & -2 & -2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 7 & 5 \\ 0 & 1 & -\frac{22}{13} & -\frac{22}{13} \\ 0 & 0 & -\frac{40}{13} & -\frac{40}{13} \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -4 & 7 & 5 \\ 0 & 1 & -\frac{22}{13} & -\frac{22}{13} \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore (x_1, x_2, x_3) = (-2, 0, 1)$