Ex 6.3)

47) $A = QR$,

$$\begin{array}{c} \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{bmatrix} \end{array} \begin{bmatrix} \alpha_1 \cdot q_1 & \alpha_2 q_1 & \alpha_3 q_1 \\ \cdot & \alpha_2 q_2 & \alpha_3 q_2 \\ \cdot & \cdot & \alpha_3 q_3 \end{bmatrix}$$

$$A \quad = \quad Q \quad R$$

$$R = \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} \\ \cdot & \frac{3}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \cdot & \cdot & \frac{4}{\sqrt{6}} \end{bmatrix} \Rightarrow \underset{R}{\begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} & -\frac{\sqrt{3}}{3} \\ 0 & 0 & \frac{2\sqrt{6}}{3} \end{bmatrix}}$$

$$\therefore A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} & -\frac{\sqrt{3}}{3} \\ 0 & 0 & \frac{2\sqrt{6}}{3} \end{bmatrix}$$

P. 388 Example 1 (Least squares straight line fit)

points $(0,1), (1,3), (2,4), (3,4)$, $Y = a^* + b^* X$

$$M = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad M^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}, \quad M^T M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix}$$

$$(M^T M)^{-1} = \frac{1}{10} \begin{bmatrix} 7 & -3 \\ -3 & 2 \end{bmatrix}, \quad v^* = (M^T M)^{-1} \cdot M^T y = \frac{1}{10} \begin{bmatrix} 7 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

$= \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$, $v^* = \begin{bmatrix} a^* \\ b^* \end{bmatrix}$ 이므로, $Y = 1.5 + X$ 이다.

Exercise 6.5)

3) points : (2, 0) (3, -10) (5, -48), (6, -76)

$Y = \alpha_0 + \alpha_1 X + \alpha_2 X^2$,

$$\begin{matrix} Y_1 \\ Y_2 \\ \vdots \\ \vdots \\ Y_n \end{matrix} = \begin{matrix} \alpha_0 + \alpha_1 X + \alpha_2 X^2 \\ \alpha_0 + \alpha_1 X + \alpha_2 X^2 \\ \alpha_0 \quad \vdots \quad \vdots \\ \vdots \quad \vdots \quad \vdots \\ \alpha_n + \alpha_n X \quad \alpha_n X^2 \end{matrix} = \underset{Y}{\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ \vdots \\ Y_n \end{bmatrix}} = \underset{M}{\begin{bmatrix} 1 & X_1 & X_1^2 \\ 1 & X_2 & X_2^2 \\ \vdots & \vdots & \vdots \\ 1 & X_n & X_n^2 \end{bmatrix}} \underset{V^*}{\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix}}$$

$$\rightarrow M = \begin{array}{c} \begin{matrix} x^0 & x^1 & x^2 \end{matrix} \\ \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \end{bmatrix} \end{array}, \quad Y = \begin{bmatrix} 0 \\ -10 \\ -48 \\ -76 \end{bmatrix}, \quad Y = MV^*, \quad \underline{M^T Y = M^T M V^*}$$

$M^T Y = M^T M V$, $V = (M^T M)^{-1} \cdot M^T Y$ 이므로

$$V = \left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 5 & 6 \\ 4 & 9 & 25 & 36 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 5 & 6 \\ 4 & 9 & 25 & 36 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -10 \\ -48 \\ -76 \end{bmatrix} \text{ 이다}$$

$$\therefore \quad V = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix} \begin{matrix} \rightarrow x^0 \\ \rightarrow x^1 \\ \rightarrow x^2 \end{matrix} \qquad \underline{Y = -3X^2 + 5X + 2}$$