선형대수 과제 #4

<1.8>

2) $T = [T(e_1) | T(e_2) | T(e_3)] = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 0 & -3 \\ 0 & 1 & -1 \end{bmatrix}$, $T(x) = \begin{bmatrix} 1 & 0 & 4 \\ 3 & 0 & -3 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$

4) $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = C_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$, $\begin{cases} C_1 + 2C_2 = 1 \\ C_1 + 3C_2 = 0 \end{cases}$, $C_1 = 3$, $C_2 = -1$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = K_1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + K_2\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$, $\begin{cases} K_1 + 2K_2 = 0 \\ K_1 + 3K_2 = 1 \end{cases}$, $K_1 = -2$, $K_2 = 1$

$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = 3 \times T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) + (-1) \times T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3+2 \\ -6-5 \end{bmatrix} = \begin{bmatrix} 5 \\ -11 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = -2 \times T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) + 1 \times T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -2-2 \\ 4+5 \end{bmatrix} = \begin{bmatrix} -4 \\ 9 \end{bmatrix}$

$T = T\left(T(e_1) | T(e_2)\right) = \begin{bmatrix} 5 & -4 \\ -11 & 9 \end{bmatrix}$

<1.10>

3) a)

- $300 + x_2 = 400 + x_3 \Rightarrow x_2 - x_3 = 100$
- $x_3 + 750 = 250 + x_4 \Rightarrow -x_3 + x_4 = 500$
- $x_4 + 200 = 300 + x_1 \Rightarrow -x_1 + x_4 = 100$
- $x_1 + 100 = 400 + x_2 \Rightarrow x_1 - x_2 = 300$

b)
$$\begin{aligned} x_2 - x_3 &= 100 \\ -x_3 + x_4 &= 500 \\ -x_1 + x_4 &= 100 \\ x_1 - x_2 &= 300 \end{aligned} \Rightarrow \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 100 \\ 500 \\ 100 \\ 300 \end{bmatrix}$$

$A \quad x = b$

$x_4 = t$ 일때, $x_1 = -100 + t$, $x_3 = -500 + t$, $x_2 = 100 + x_3$

$100 - 500 + t$

$= -400 + t$

따라서, $x_1 = t - 100$, $x_2 = t - 400$, $x_3 = t - 500$, $x_4 = t$

$t \geq 500$, $x_1 \geq 400$, $x_2 \geq 100$, $x_3 \geq 0$, $x_4 \geq 500$

c) b에서 구한식 $x_1 \geq 400$, $x_2 \geq 100$, $x_3 \geq 0$, $x_4 \geq 500$ 을 이용해 minimum 값을 구할 수 있다. $x_1 = 400$, $x_2 = 100$, $x_3 = 0$, $x_4 = 500$ 일때 minimum flow가 된다.

〈Chapter 3〉

20) Q1) k along d $\Rightarrow$ $\text{proj}_d k = \frac{v \cdot d}{\|d\|^2} \times d$

$$\Rightarrow \frac{(5,0,-3,7) \cdot (2,1,-1,-1)}{(2,1,-1,-1) \cdot (2,1,-1,-1)} \cdot (2,1,-1,-1) = \frac{6}{7}(2,1,-1,-1)$$

$$= \left(\frac{12}{7}, \frac{6}{7}, -\frac{6}{7}, -\frac{6}{7}\right)$$

Q2) k orthogonal to d

$$\rightarrow k - \text{proj}_d k = (5,0,-3,7) - \left(\frac{12}{7}, \frac{6}{7}, -\frac{6}{7}, -\frac{6}{7}\right) = \left(\frac{23}{7}, -\frac{6}{7}, -\frac{15}{7}, \frac{55}{7}\right)$$