Geometric Approach to Robot Deceleration Control

Nonlinear Distance-Based Control Method Using a Parabolic Cylinder

Youngook Kim

Independent Researcher, Daejeon, Republic of Korea

Email: a01058228126@gmail.com

Abstract

Accurate and smooth deceleration is crucial for the safety and efficiency of autonomous robotic systems. Traditional methods, such as time-based linear and S-curve profiles, suffer from limited real-time adaptability and can cause mechanical stress due to jerk. This paper introduces the Parabolic Cylinder Velocity – Nonlinear Approach (PCV-NA), a novel, distance-based deceleration control method derived from a three-dimensional geometric model. The method directly links remaining distance to velocity, enabling instant adaptation to sensor feedback. It also accounts for non-zero initial velocities, path curvature, and controller response delays. While the current implementation does not explicitly minimize jerk, this approach provides a robust and computationally efficient solution for real-time deceleration control, with potential for widespread deployment in diverse robotic platforms.

Keywords: robot control; deceleration; distance-based control; parabolic cylinder; autonomous systems

---

1. Introduction

In autonomous robotic systems, accurate and smooth deceleration is essential for ensuring safety and operational efficiency. Time-based linear profiles and S-curve profiles are widely used but have inherent limitations: Jerk generation: Rapid changes in acceleration can cause mechanical stress and destabilize payloads. Limited real-time adaptability: Preplanned time-based velocity profiles cannot instantly respond to environmental changes. This paper proposes a nonlinear, distance-based deceleration control method derived from a three-dimensional geometric model, hereafter referred to as the Parabolic Cylinder Velocity – Nonlinear Approach (PCV-NA). The method directly links remaining distance to velocity, enabling instantaneous adaptation to sensor feedback, and incorporates: Non-zero initial velocity $v_0 \neq 0$ Path curvature compensation Consideration of controller response delays The current implementation does not perform explicit jerk minimization; jerk control is left as a topic for future work.

---

2. Related Work

S-curve deceleration: Smooths acceleration changes but is time-based, requiring re-planning when conditions change.

Spline/Bezier path planning: Produces smooth paths but does not directly control the speed–distance relationship.

The proposed method defines a nonlinear speed–distance relationship using a parabolic cylinder and achieves real-time adaptability through distance-based control. Curvature compensation can be added without altering the core structure.

---

3. Proposed Method

3.1 3D Geometric Model

The robot's state is represented as a point on a parabolic cylinder: $[z^2 = kx]$ where:

(x): remaining distance to the target (m)

(z): robot speed (m/s)

(y): path coordinate (m)

(k): acceleration gain (m/s²), (k < 0) for deceleration

This model produces a nonlinear deceleration curve that naturally converges to zero velocity as $(x \to 0)$.

3.2 Initial Speed Consideration

For a non-zero initial speed ($v_0 \neq 0$), the velocity–distance relationship becomes:

$\left[v(d) = \sqrt{v_0^2 + 2k(d_0 - d)}\right]$ here:

(d_0): initial distance (m)

(v_0): initial speed (m/s)

(k): control gain (m/s²)

This formulation preserves the parabolic relationship while accommodating arbitrary starting speeds.

3.3 Discrete Real-Time Implementation

In practice, the control loop operates at a discrete sampling interval (Δt):

$$\left[d_{n+1} = d_n - v_n \Delta t\right]\left[v_{n+1} = \sqrt{\max\left(0, v_n^2 + 2k(d_n - d_{n+1})\right)}\right]$$

Safety: The $(\max(0,\cdot))$ term prevents negative arguments inside the square root.

Feature: Being distance-driven, the method can instantly respond to changes in (d_n) from sensor input.

3.4 Path Curvature Compensation

For straight or gently curved paths, (y)-axis independence holds.

For high-curvature paths, arc length (s) is used: $\left[s = \int_0^y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}, dy, \quad v(s) = \sqrt{v_0^2 + 2k(d_0 - s)}\right]$

Optional curvature compensation coefficients can be applied to enhance stability.

3.5 Jerk Characteristics and Limitations

The current method does not perform jerk minimization or limitation. Cause: Sudden changes in sensor input, discontinuous updates to target/obstacle data, and sampling or actuator saturation effects can produce large acceleration change rates. Observation: In certain scenarios (e.g., emergency stops, abrupt target changes, noise bursts), peak jerk spikes may occur. Note: Jerk control is beyond the scope of this study and is planned as an optional module in future work. Possibility: In future implementations, jerk can be optimized through an appropriate control function without altering the core structure of the method.

---

4. Experiments and Results

4.1 Simulation Setup

Paths: Straight and high-curvature

Comparisons: Linear deceleration, S-curve deceleration

Scenarios: Target changes, sensor noise, controller delays

4.2 Performance Summary

Responsiveness: Adapts instantly to distance changes without re-planning.

Stopping Precision: Reaches zero velocity exactly at the target.

Curvature Robustness: Maintains stable speed control on high-curvature paths with compensation enabled.

Jerk: No jerk control in this version; peak jerk increases observed in certain cases.

4.X Large-Scale Deployment Potential

The proposed nonlinear distance-based deceleration control method combines structural simplicity with real-time adaptability, making it well-suited for large-scale deployment across diverse hardware platforms and environments. Key attributes include:

Computational Efficiency

Operates using a simple distance–velocity relationship, enabling implementation on low-spec MCUs or embedded boards.

Eliminates the need for high-performance processors, reducing hardware costs.

Platform Versatility

The same control logic can be ported to various robots, AGVs, drones, and autonomous platforms.

Only the control gain (k) and initial conditions need adjustment for each product.

Ease of Production and Maintenance

Minimal parameters allow rapid, intuitive tuning during production.

Low risk during firmware updates or feature expansions, reducing maintenance costs.

Environmental Adaptability

Responds instantly to sensor input changes, ensuring stable operation in varied environments.

Handles target changes and obstacle avoidance without re-planning.

These characteristics suggest that the method can deliver both cost efficiency and reliability when applied to mass-produced commercial robots and autonomous systems.

---

5. Conclusion

This paper presented a nonlinear, distance-based deceleration control method using a parabolic cylinder model.

Advantages: Real-time adaptability through direct distance–velocity linkage, consideration of initial speed, curvature, and controller delays, and precise stopping performance.

Limitations: No jerk minimization in the current version.

Future work will focus on integrating jerk-limiting mechanisms and predictive compensation to further improve motion smoothness.

Acknowledgements

The authors would like to acknowledge the assistance of the Google Gemini large language model for its help in refining the language, grammar, and overall clarity of this manuscript. The final content, ideas, and conclusions of this paper remain the sole responsibility of the authors.