

Graph

- Introduction
- Graph API
- Elementary Graph Operations
 - **DFS: Depth first search**
 - BFS: Breadth first search
 - CC: Connected Components

Major references:

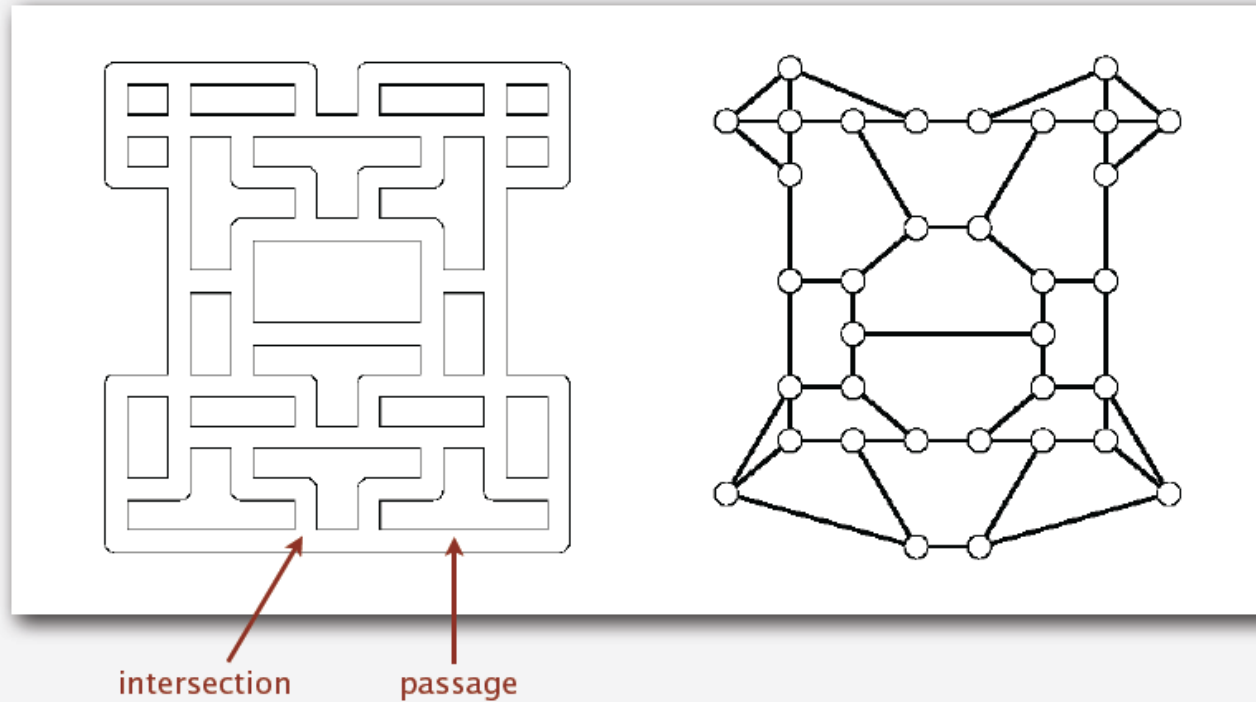
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

DFS: Depth-First Search

Algorithm:

- Vertex = intersection
- Edge = passage

pacman

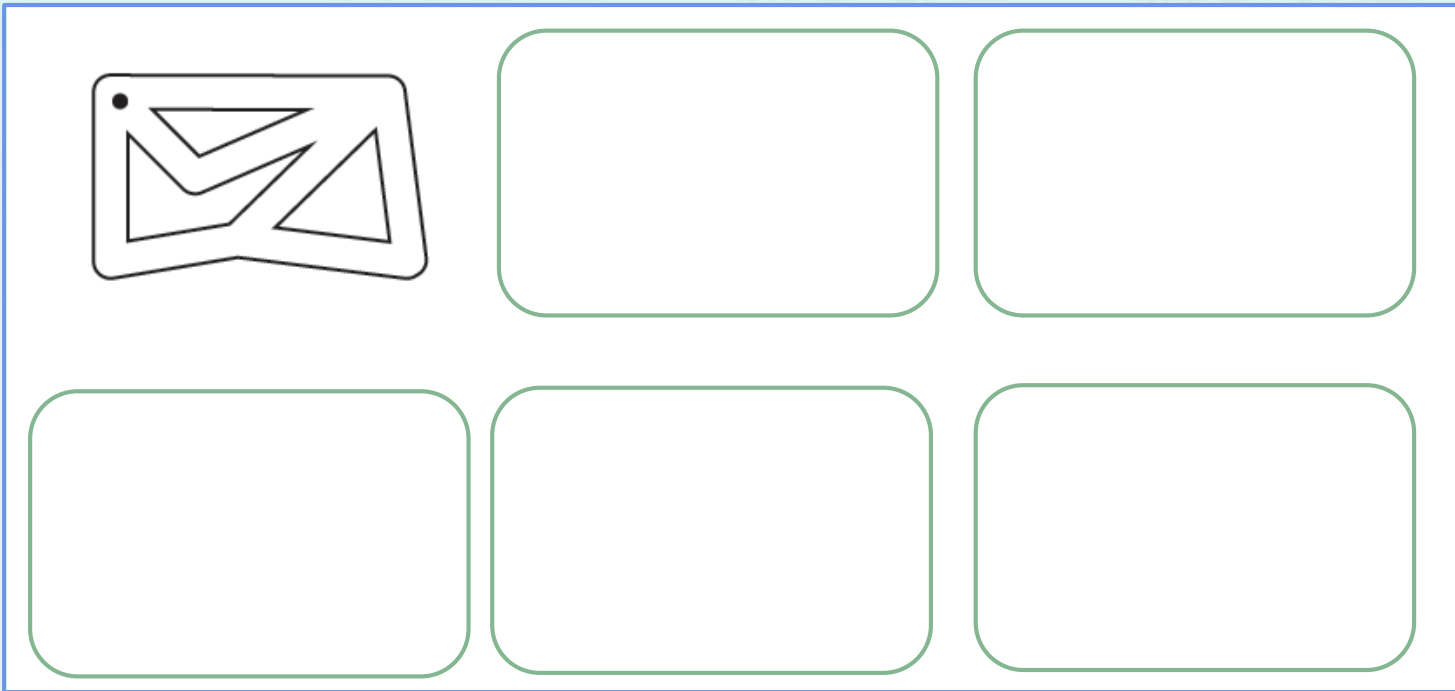


Maze Goal: Explore every intersection in the maze.

DFS: Depth-First Search

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options



Maze Goal: Explore every intersection in the maze.

Good Visualization: <https://www.cs.usfca.edu/~galles/visualization/DFS.html>

DFS: Depth-First Search

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
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Theseus, a hero of Greek mythology, is best known for slaying a monster called the Minotaur. When Theseus entered the Labyrinth where the Minotaur lived, he took a ball of yarn to unwind and mark his route. Once he found the Minotaur and killed it, Theseus used the string to find his way out of the maze.

Read more: <http://www.mythencyclopedia.com/Sp-Tl/Theseus.html#ixzz30wFO3ofe>

Maze Goal: Explore every intersection in the maze.

DFS: Depth-First Search

Maze graph:

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
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Shannon and his famous electromechanical mouse *Theseus* (named after Theseus from Greek mythology) which he tried to have solve the maze in one of the first experiments in artificial intelligence.

The Las Vegas connection: Shannon and his wife Betty also used to go on weekends to Las Vegas with MIT mathematician Ed Thorp, and made very successful forays in blackjack using game theory.

Maze Goal: Explore every intersection in the maze.

DFS: Depth-First Search

Design pattern: Decouple graph data type

Idea: Mimic maze exploration

DFS (to visit a vertex v)

- **Mark v as visited.**
- **Recursively visit all unmarked vertices w adjacent to v .**

Typical applications:

- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

Challenge:

- How to implement?

DFS: Depth-First Search

Goal: Systematically search through a graph from graph processing

- **Create a graph object**
- **Pass the graph to a graph processing routine**
- **Query the graph-processing routine**

DFS: Depth-First Search

Goal: Systematically search through a graph from graph processing

- **Create a graph object**
- **Pass the graph to a graph processing routine**
- **Query the graph-processing routine**

```
public class Paths
```

```
    Paths(Graph G, int s)           find paths in G from source s
```

```
    boolean hasPathTo(int v)       is there a path from s to v?
```

```
    Iterable<Integer> pathTo(int v) path from s to v; null if no such path
```

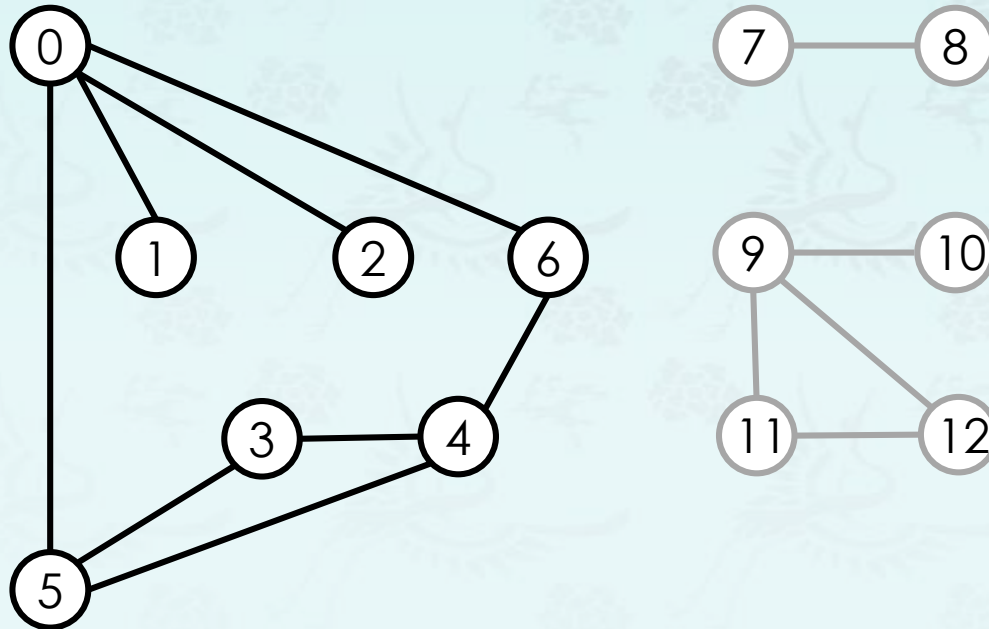
```
Paths paths = new Paths(G, s);  
for (int v = 0; v < G.V(); v++)  
    if (paths.hasPathTo(v))  
        stdout.println(v);
```

← print all vertices
connected to s

Graph – Coding

For each edge(v, w) in the list

- Insert front each vertex both $(\text{adj}[v], w)$ and $(\text{adj}[w], v)$
 `addEdgeFromTo(g, v, w);` `// add an edge from v to w.`



V-E lists →

graph3.txt
13 ← V
13 ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3

Graph g:

Challenge: build adjacency lists?

Graph Coding – graph.cpp

```
struct Graph {
    int V;           // number of vertices in the graph
    int E;           // number of edges in the graph
    gnode adj;       // an array of adjacency lists (or gnode pointers)

    Graph(int v = 0) { // constructs a graph with v vertices
        V = v;
        E = 0;

        // initialize each adjacency list as an empty list;
        for (int i = 0; i < V; i++) {
            g->adj[i].next = nullptr;
            g->adj[i].item = i;
        }
    }
    ~Graph() {}
};

using graph = Graph *;
```

← set each adj list nullptr
unused; but may store the size of degree.

Graph Coding

// add an edge to an **undirected** graph

```
void addEdgeFromTo(graph g, int v, int w) {  
    // add an edge from v to w.  
    // A new vertex is added to the adjacency list of v.  
    // The vertex is added at the beginning
```

```
    gnode node = new Gnode(w, g->adj[v].next);  
    g->E++;  
}
```

← instantiate a node w insert it
at **the front of** adjacency list[v]

// add an edge to an undirected graph

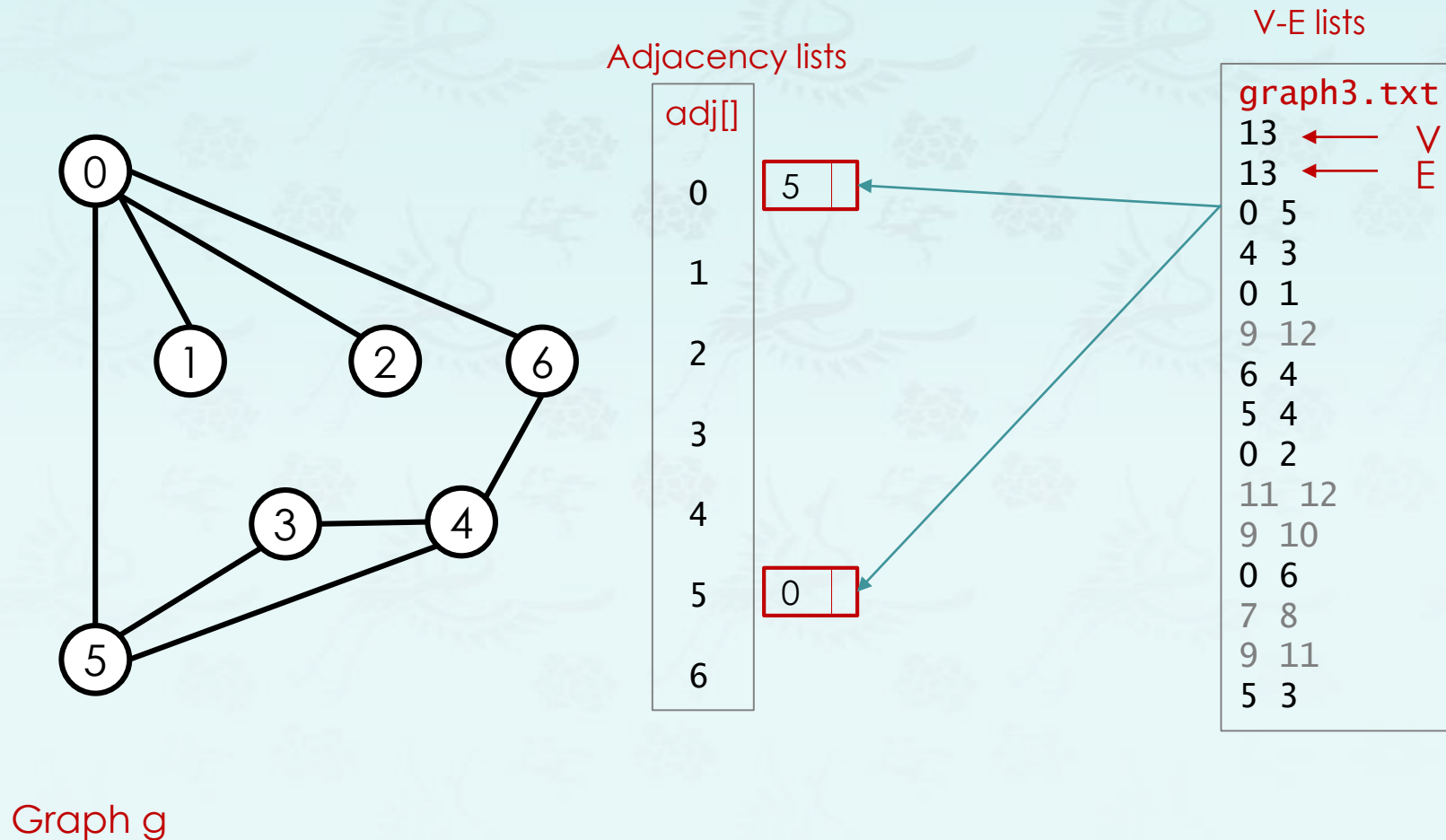
```
void addEdge(graph g, int v, int w) {  
    addEdgeFromTo(g, v, w); // add an edge from v to w.  
    addEdgeFromTo(g, w, v); // if graph is undirected, add both  
}
```

← add an edge for undirected graph

Graph – Build Adjacency list

For each edge(v, w) in the list

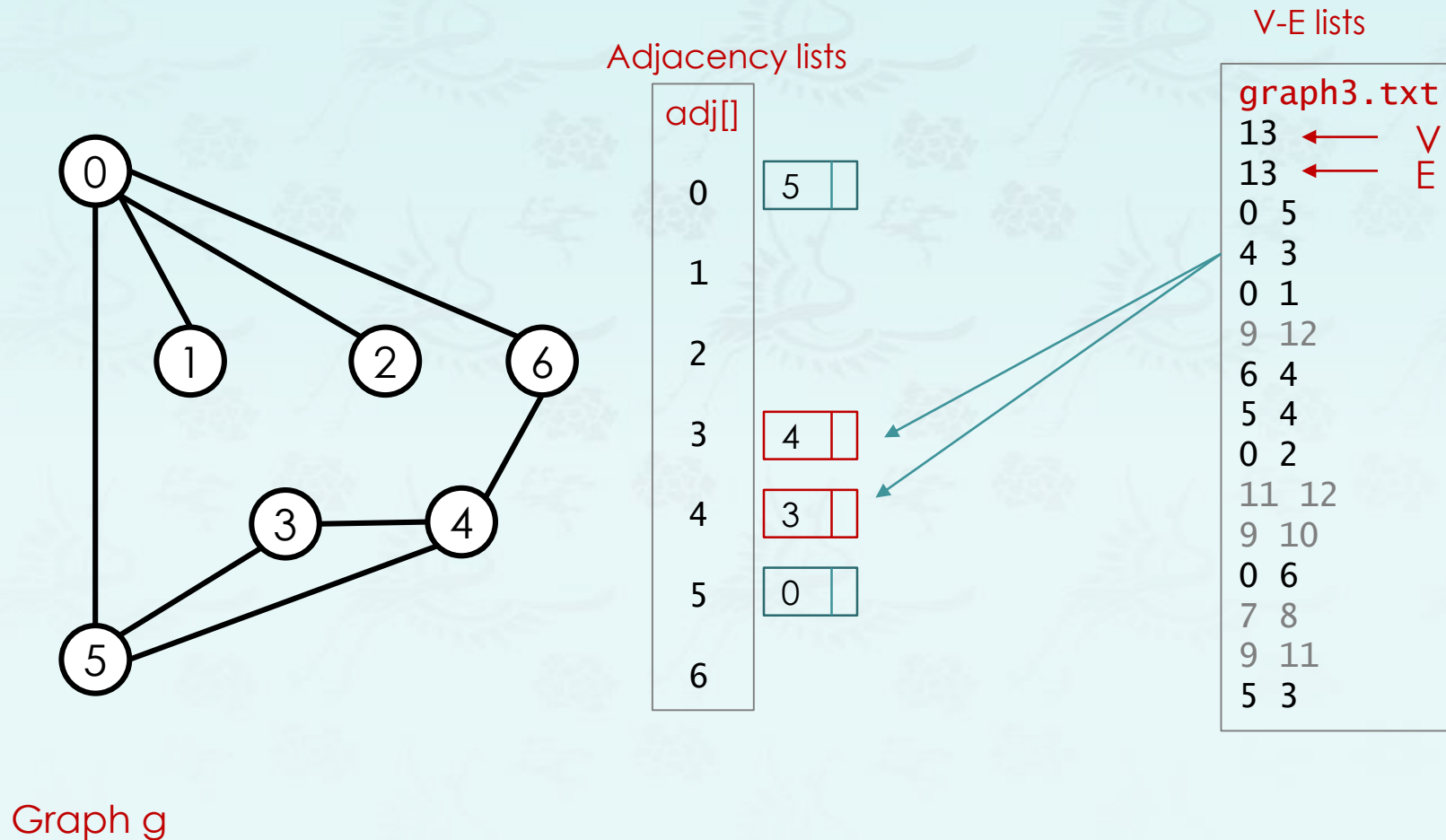
- Insert front each vertex both (adj[v] , w) and (adj[w], v)
addEdgeFromTo(g, v, w); // add an edge from v to w.



Graph – Build Adjacency list

For each edge(v, w) in the list

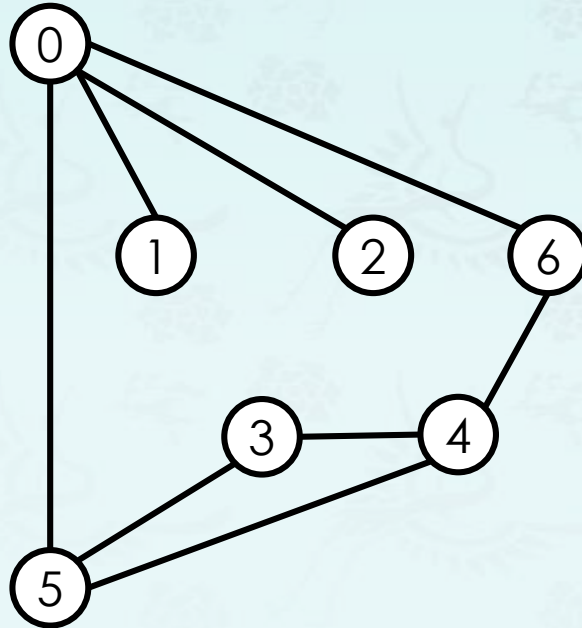
- Insert front each vertex both (adj[v] , w) and (adj[w], v)
addEdgeFromTo(g, v, w); // add an edge from v to w.



Graph – Build Adjacency list

For each edge(v, w) in the list

- Insert front each vertex both (adj[v] , w) and (adj[w], v)
addEdgeFromTo(g, v, w); // add an edge from v to w.



Graph g

Adjacency lists

adj[]	
0	5
1	
2	
3	4
4	3
5	0
6	

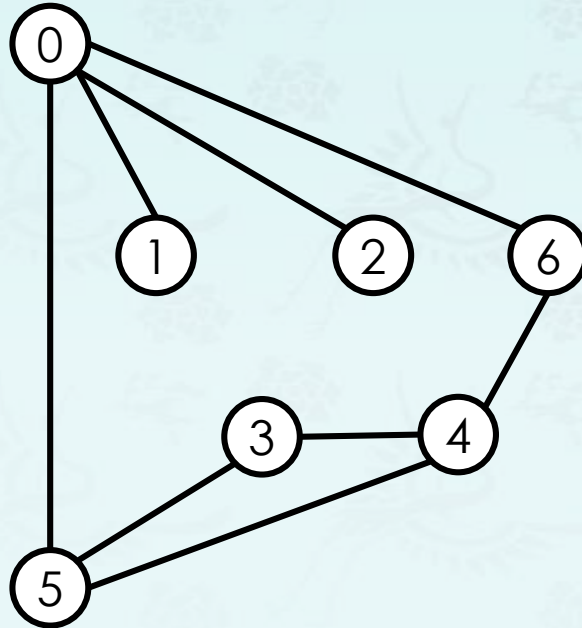
V-E lists

graph3.txt		
13	←	V
13	←	E
0	5	
4	3	
0	1	
9	12	
6	4	
5	4	
0	2	
11	12	
9	10	
0	6	
7	8	
9	11	
5	3	

Graph – Build Adjacency list

For each edge(v, w) in the list

- Insert front each vertex both ($\text{adj}[v], w$) and ($\text{adj}[w], v$)
`addEdgeFromTo(g, v, w);` `// add an edge from v to w.`



Graph g

Adjacency lists

adj[]	
0	1 5
1	0
2	
3	4
4	3
5	0
6	

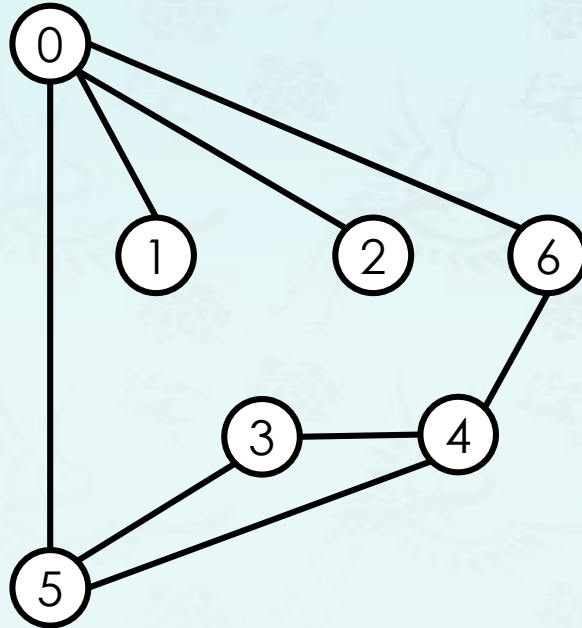
V-E lists

graph3.txt		
13	←	V
13	←	E
0 5		
4 3		
0 1		
9 12		
6 4		
5 4		
0 2		
11 12		
9 10		
0 6		
7 8		
9 11		
5 3		

Graph – Build Adjacency list

For each edge(v, w) in the list

- Insert front each vertex both ($\text{adj}[v], w$) and ($\text{adj}[w], v$)
`addEdgeFromTo(g, v, w);` `// add an edge from v to w.`



Graph g

Adjacency lists

adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

V-E lists

graph3.txt

13	← V
13	← E
0 5	
4 3	
0 1	
9 12	
6 4	
5 4	
0 2	
11 12	
9 10	
0 6	
7 8	
9 11	
5 3	

DFS: Depth-First Search Demo

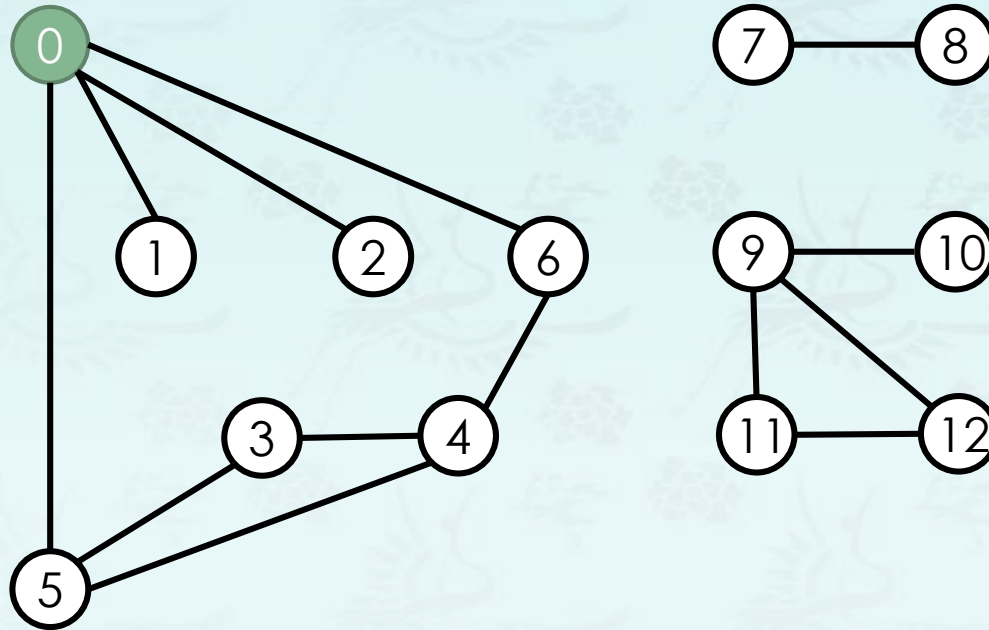
To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v .

DFS: Depth-First Search Demo

To visit a vertex v :

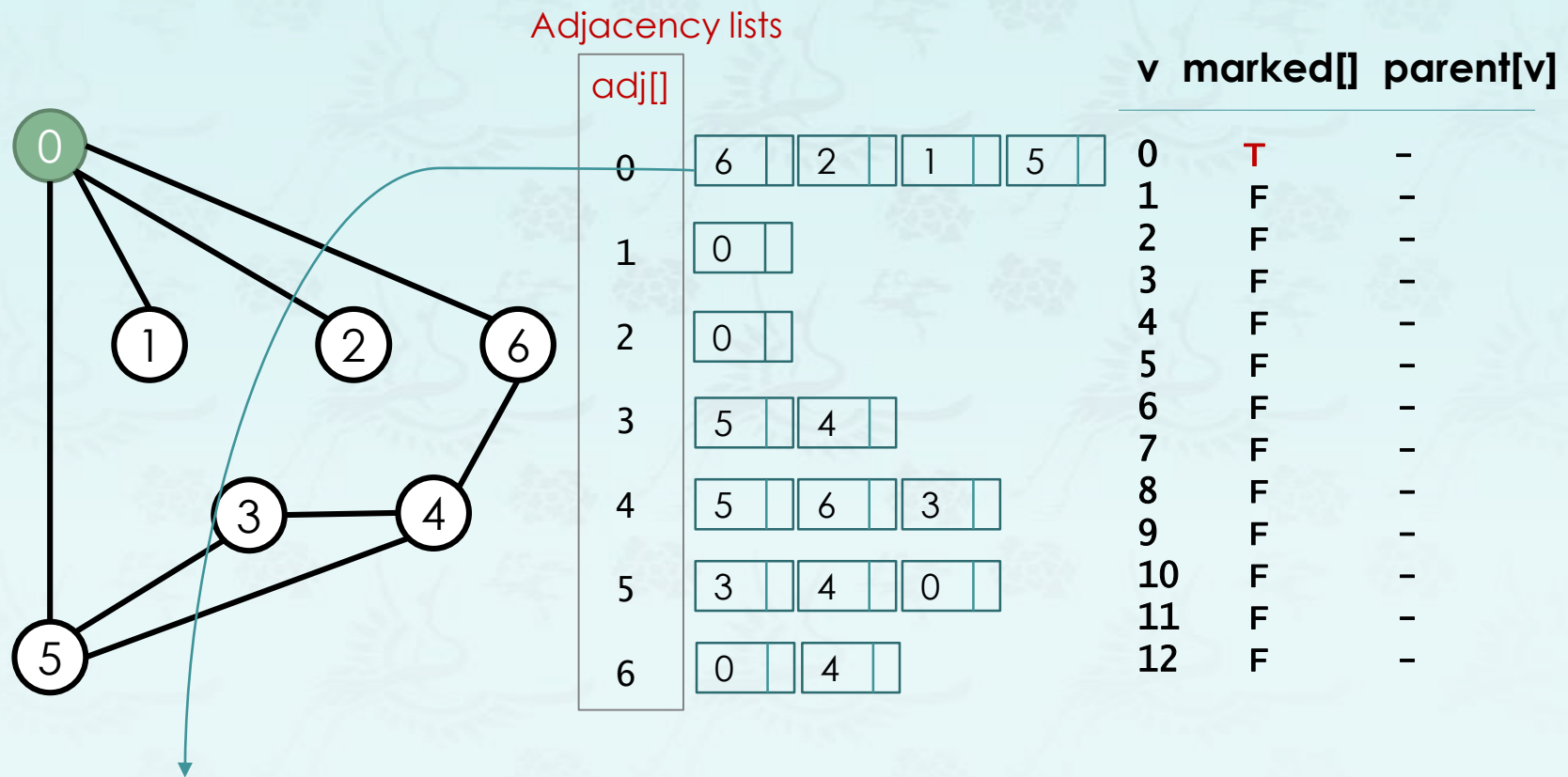
- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v .



v	marked[]	parent[v]
-----	----------	-----------

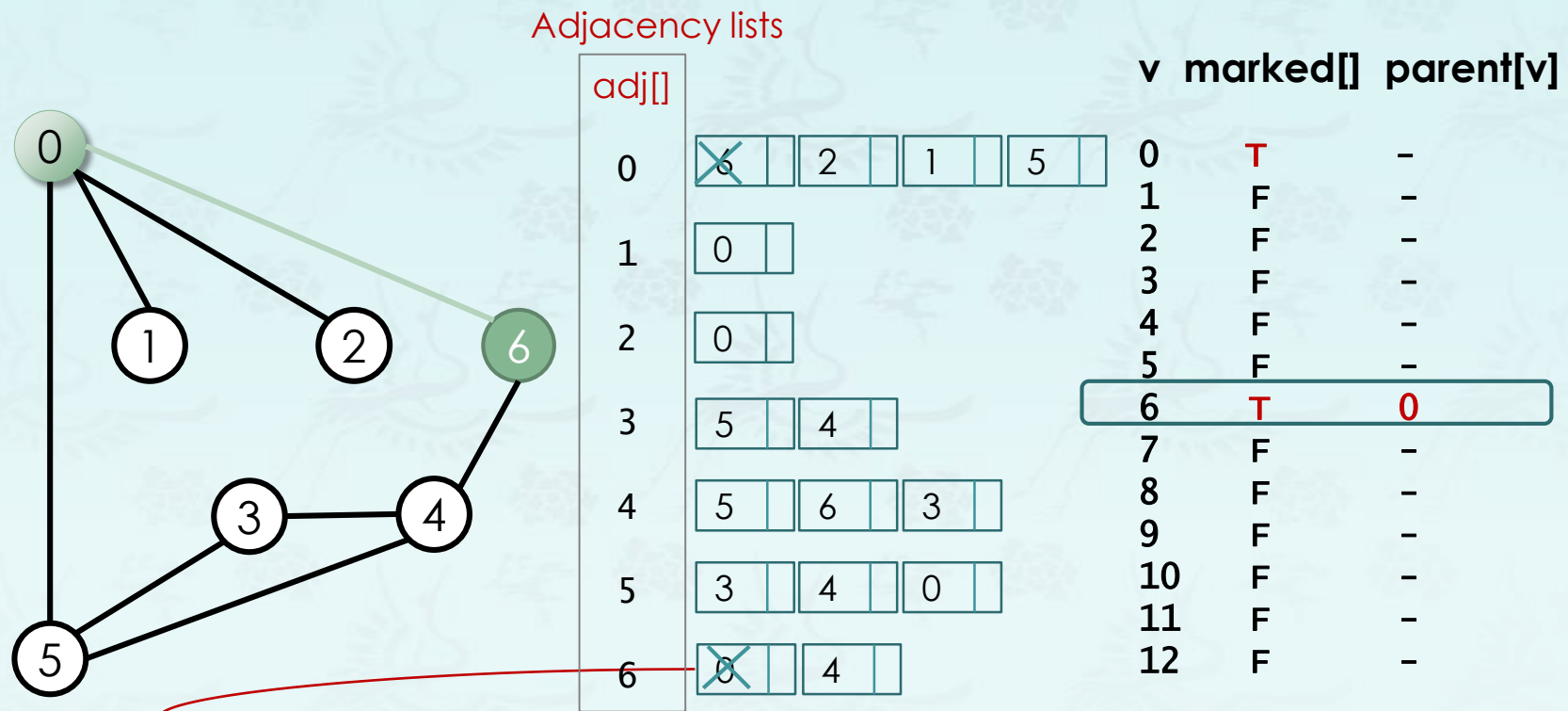
0	T	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 0: **Which one first?**



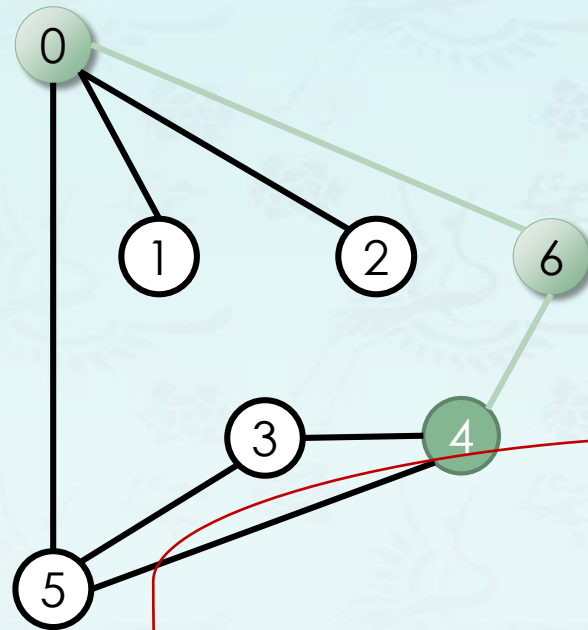
visit 0: **check 6**, check 2, check 1, and check 5

DFS 0



visit 6: **check 0**, check 4

DFS 0 6

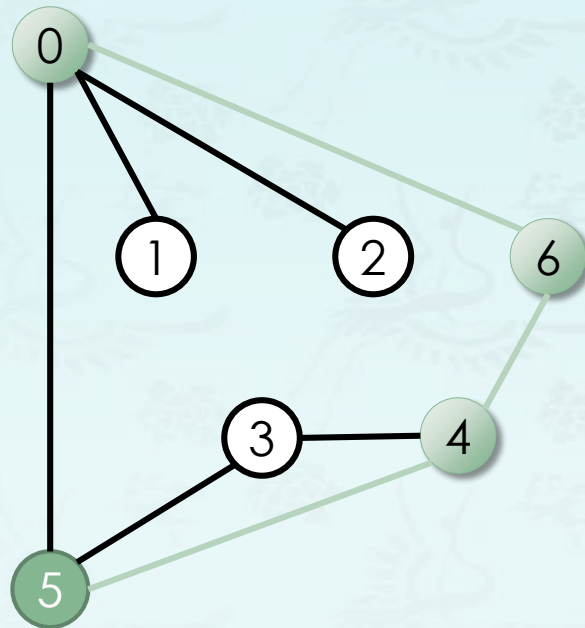


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	3	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	T	6
5	F	-
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 4: **check 5**, check 6, check 3

DFS 0 6 4

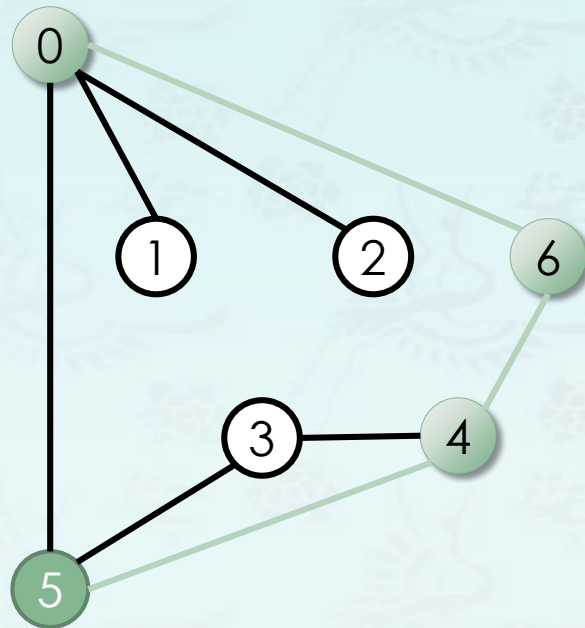


adj[]				
0	0	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	3	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: **check 3**, check 4, check 0

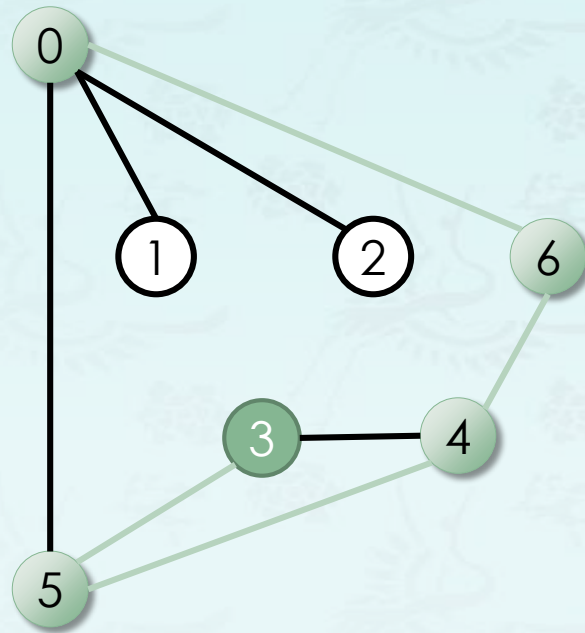
DFS 0 6 4 5



adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	F	-
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

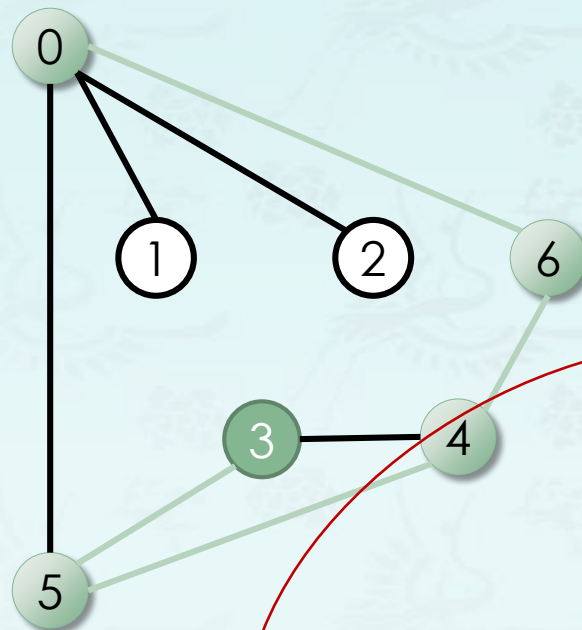
visit 5: **check 3**, check 4, check 0



adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: **check 3**, check 4, check 0



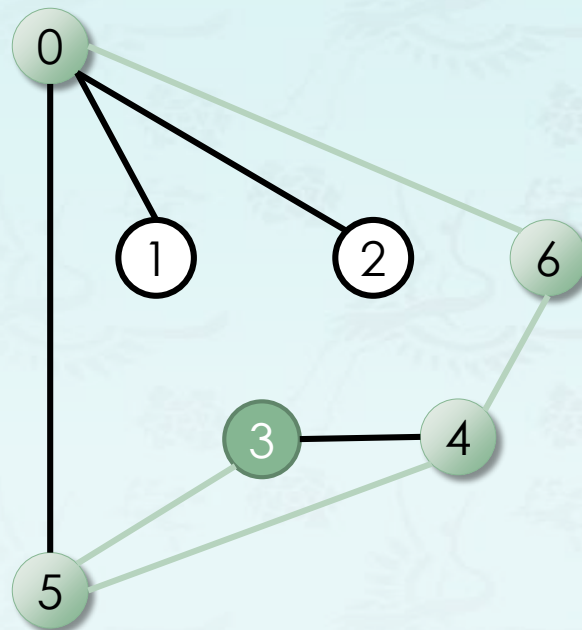
Adjacency lists

adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 3: **check 5**, **check 4**

DFS 0 6 4 5 3

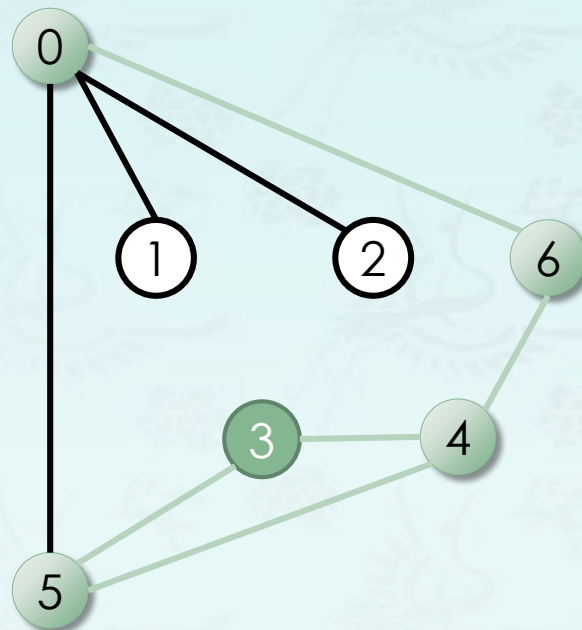


adj[]

0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 3: ~~check 5~~, check 4

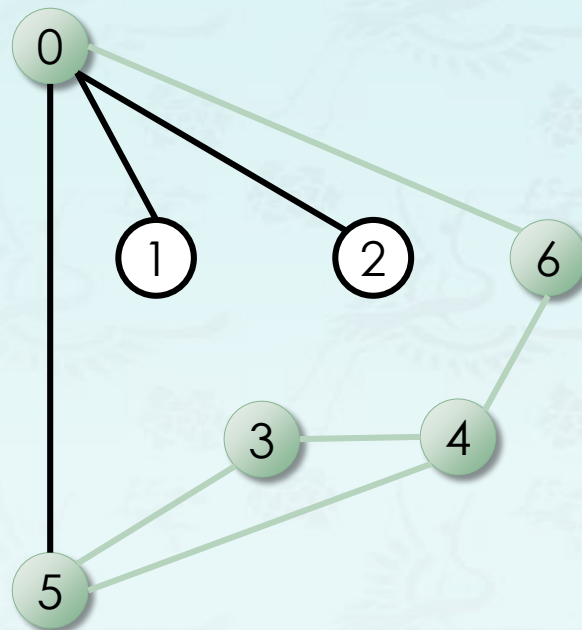


adj[]

0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

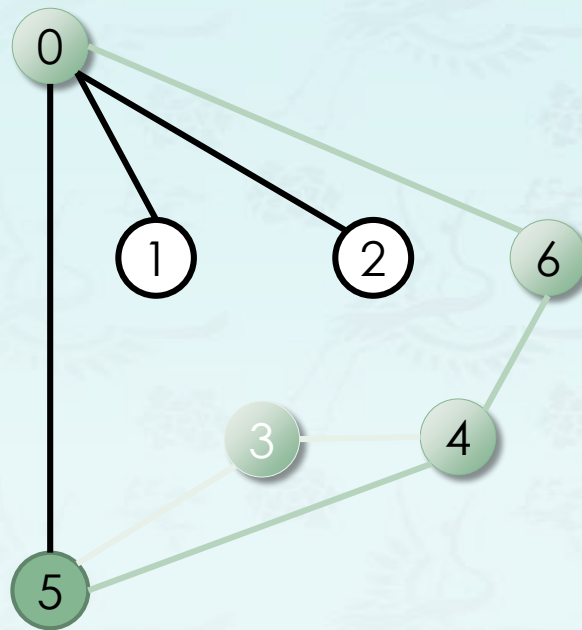
visit 3: ~~check 5, check 4~~



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

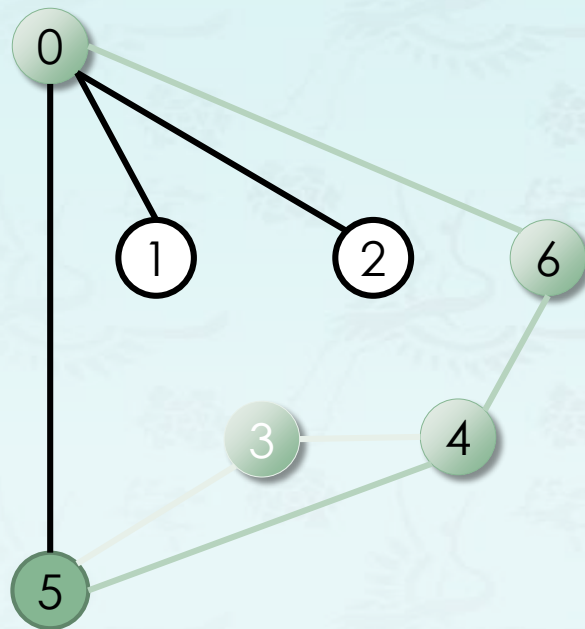
visit 3: ~~check 5, check 4~~



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

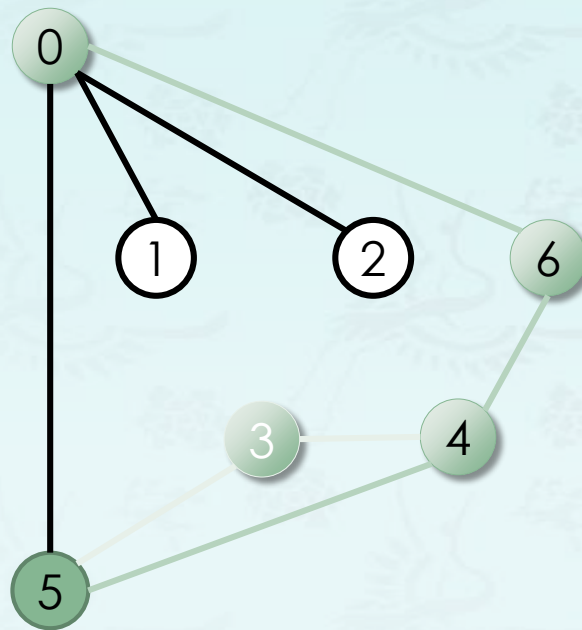
3 done: What's the next?



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

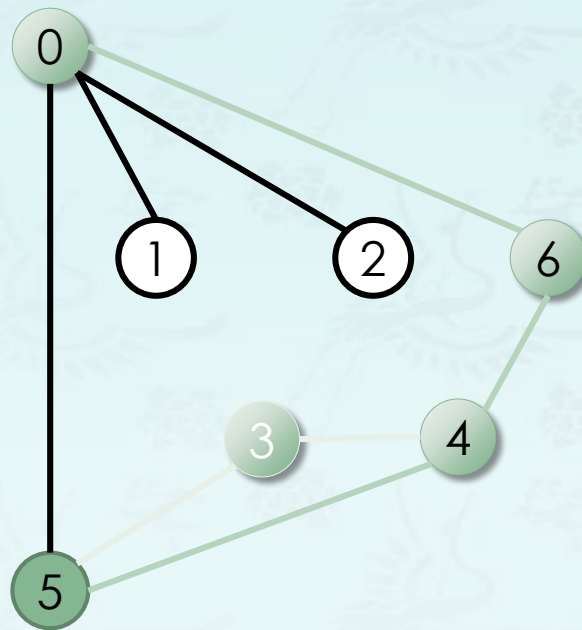
3 done: What's the next? **Backtrack!**



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

3 done: What's the next? **Backtrack!**
 How to?



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

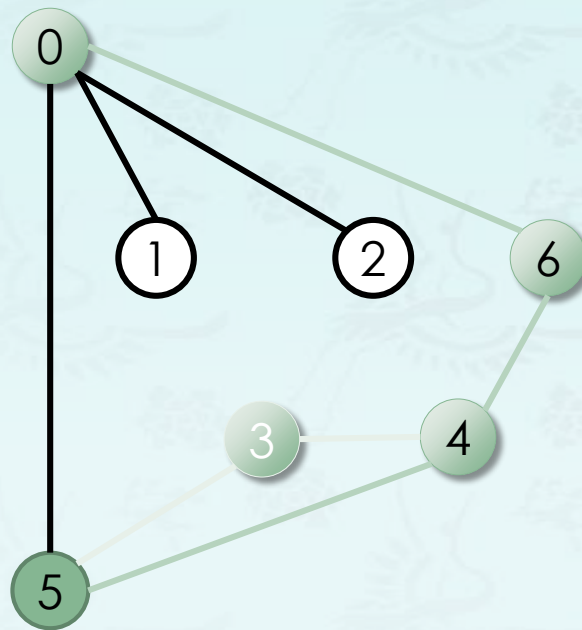
v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

3 done:

What's the next?
How to?

Backtrack!
Use parent[v]

parent[3] = 5

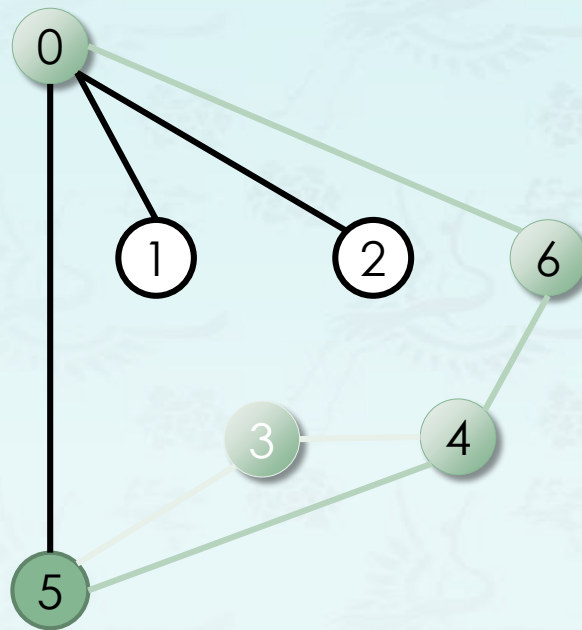


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: ~~check 3~~, **check 4**, check 0

done

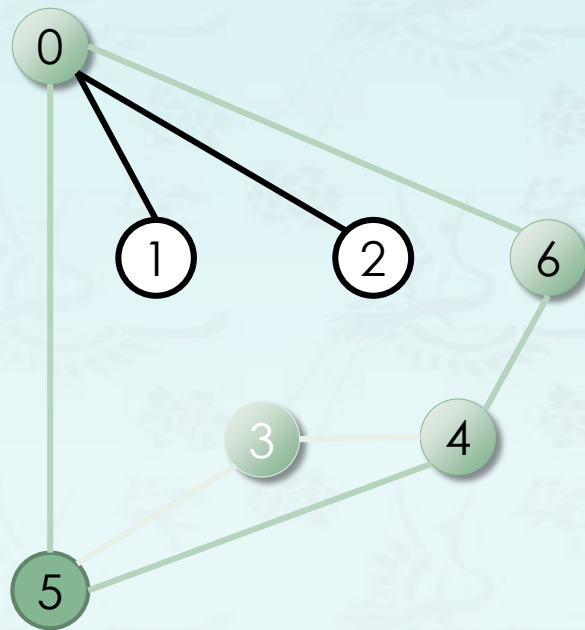


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	6	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: ~~check 3~~, **check 4**, check 0

done

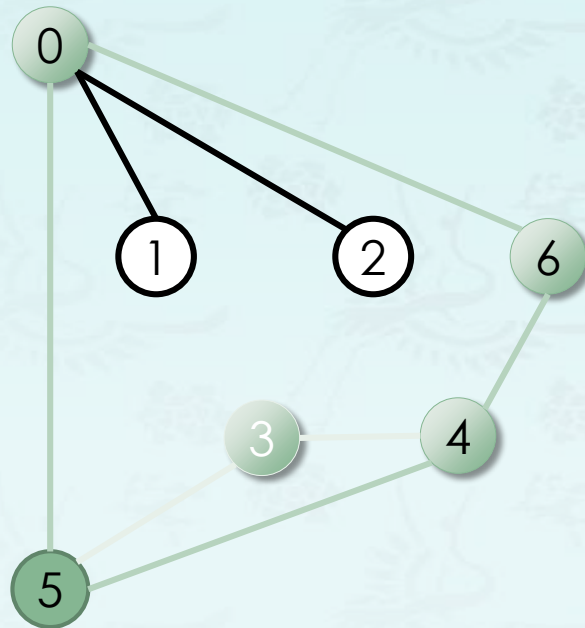


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	6	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: ~~check 3~~, ~~check 4~~, check 0

done

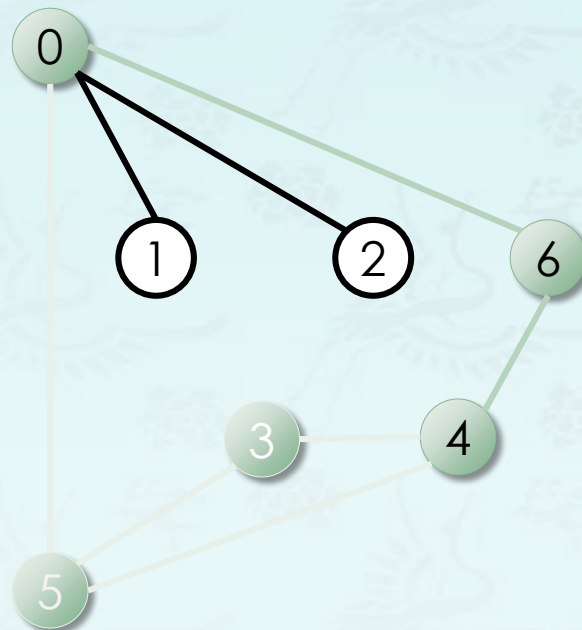


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	3	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 5: **check 3**, **check 4**, **check 0**

done

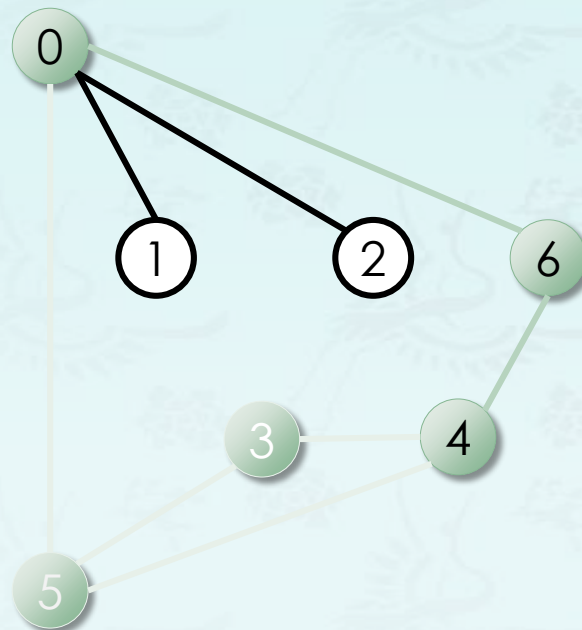


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	6	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

5 done

What's the next? **Backtrack!**
How to?



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	6	4	0	
6	0	4		

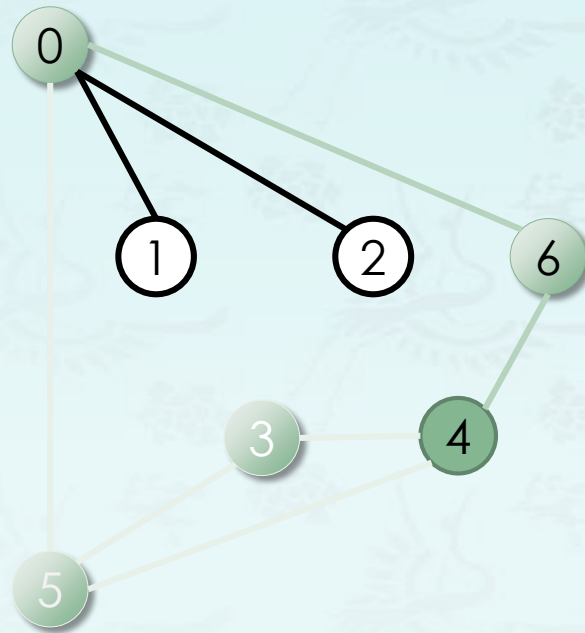
v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

5 done

What's the next?
How to?

Backtrack!
Use parent[v]

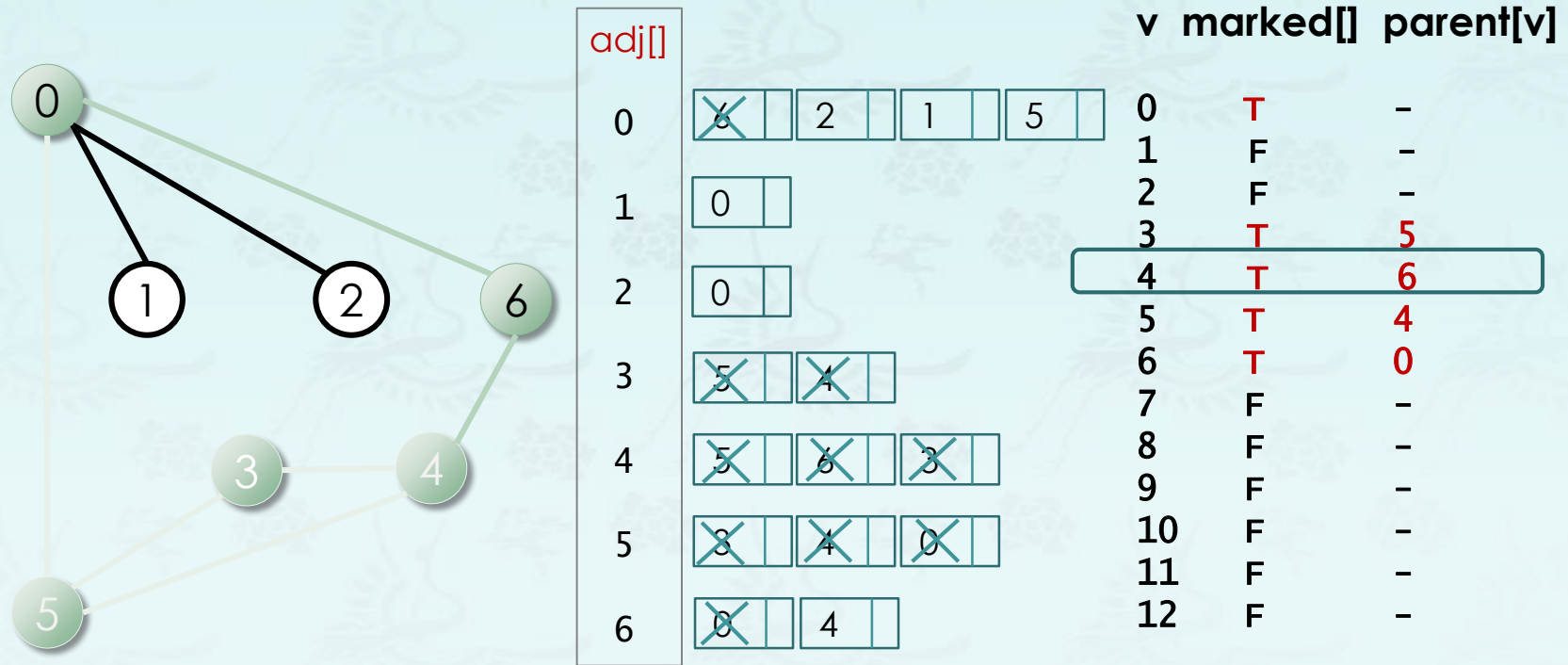
parent[5] = 4



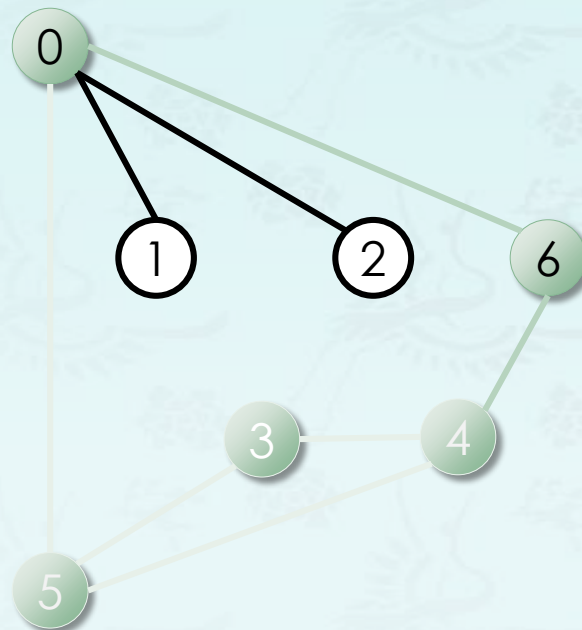
adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 4: check 5, **check 6**, check 3



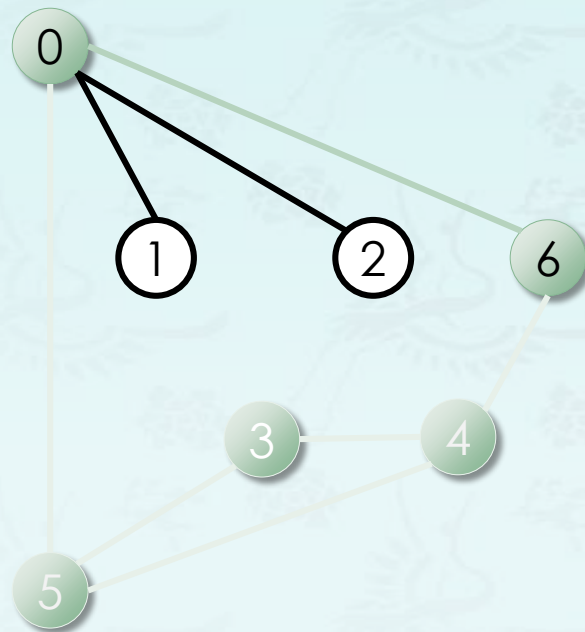
visit 4: check 5, check 6, **check 3** 4 done



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

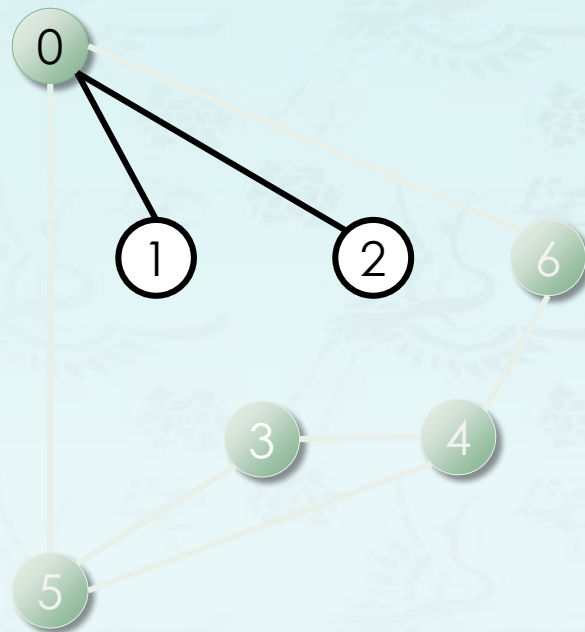
visit 4: check 5, check 6, **check 3** 4 done **Backtrack!** parent[4] = 6



adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	8 4 0
6	0 4

v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 6: check 0, check 4



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

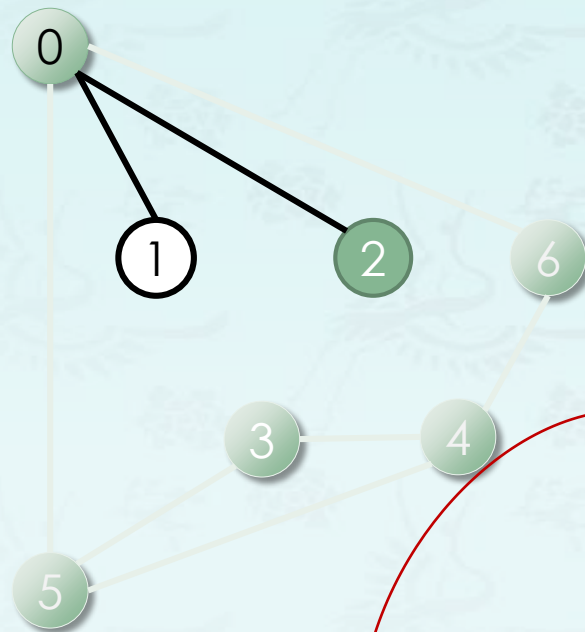
v	marked[]	parent[v]
0	T	-
1	F	-
2	F	-
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 6: check 0, check 4

done 6

Backtrack!

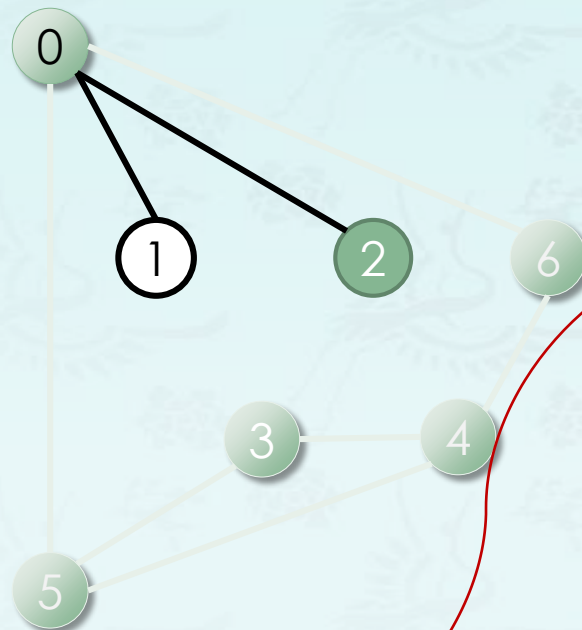
parent[6] = 0



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 0: check 6, **check 2**, check 1, and check 5

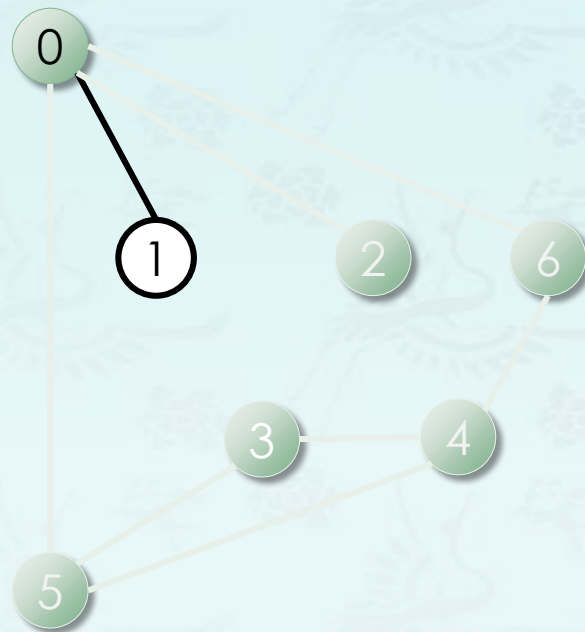


adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	8	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 2: check 0

DFS: 0 6 4 5 3 2



adj[]
0
1
2
3
4
5
6

6	2	1	5
0			
0			
5	4		
5	6	3	
8	4	0	
0	4		

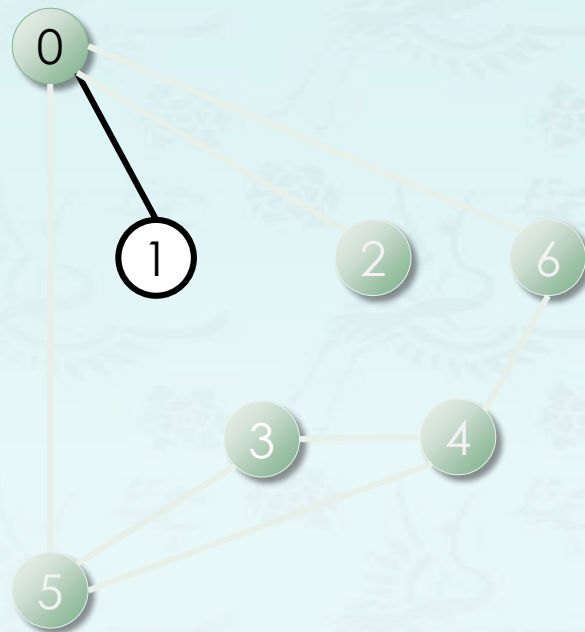
v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 2: check 0

2 done

Backtrack!

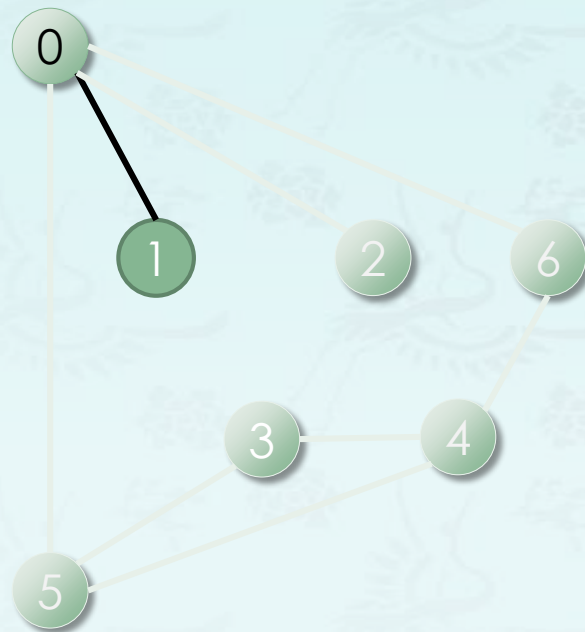
parent[2] = 0



adj[]				
0	6	2	1	5
1	0			
2	0			
3	5	4		
4	5	6	3	
5	0	4	0	
6	0	4		

v	marked[]	parent[v]
0	T	-
1	F	-
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

visit 0: check 6, check 2, **check 1**, and check 5



adj[]				v marked[] parent[v]		
0	<div><div>6</div><div>2</div><div>1</div><div>5</div></div>			0	T	-
1	<div><div>0</div><div></div></div>			1	T	0
2	<div><div>8</div><div></div></div>			2	T	0
3	<div><div>5</div><div>4</div></div>			3	T	5
4	<div><div>5</div><div>6</div><div>3</div></div>			4	T	6
5	<div><div>8</div><div>4</div><div>0</div></div>			5	T	4
6	<div><div>0</div><div>4</div></div>			6	T	0
				7	F	-
				8	F	-
				9	F	-
				10	F	-
				11	F	-
				12	F	-

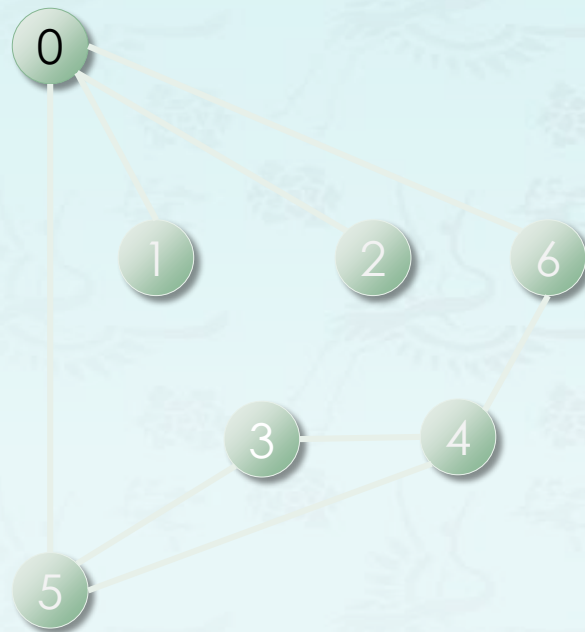
visit 1: check 0

1 done

Backtrack!

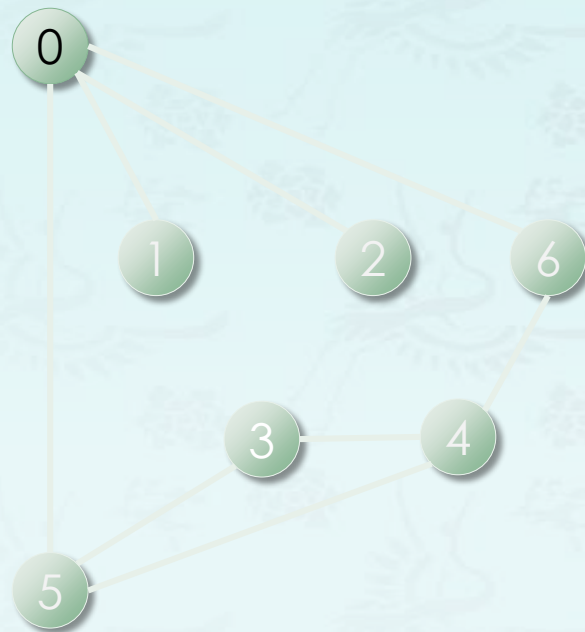
parent[1] = 0

DFS: 0 6 4 5 3 2 1



adj[]				v marked[] parent[v]		
0	<div><div>0</div><div>2</div><div>1</div><div>5</div></div>			0	T	-
1	<div><div>0</div></div>			1	T	0
2	<div><div>0</div></div>			2	T	0
3	<div><div>5</div><div>4</div></div>			3	T	5
4	<div><div>5</div><div>6</div><div>3</div></div>			4	T	6
5	<div><div>3</div><div>4</div><div>0</div></div>			5	T	4
6	<div><div>0</div><div>4</div></div>			6	T	0
				7	F	-
				8	F	-
				9	F	-
				10	F	-
				11	F	-
				12	F	-

visit 0: check 6, check 2, check 1, and **check 5**



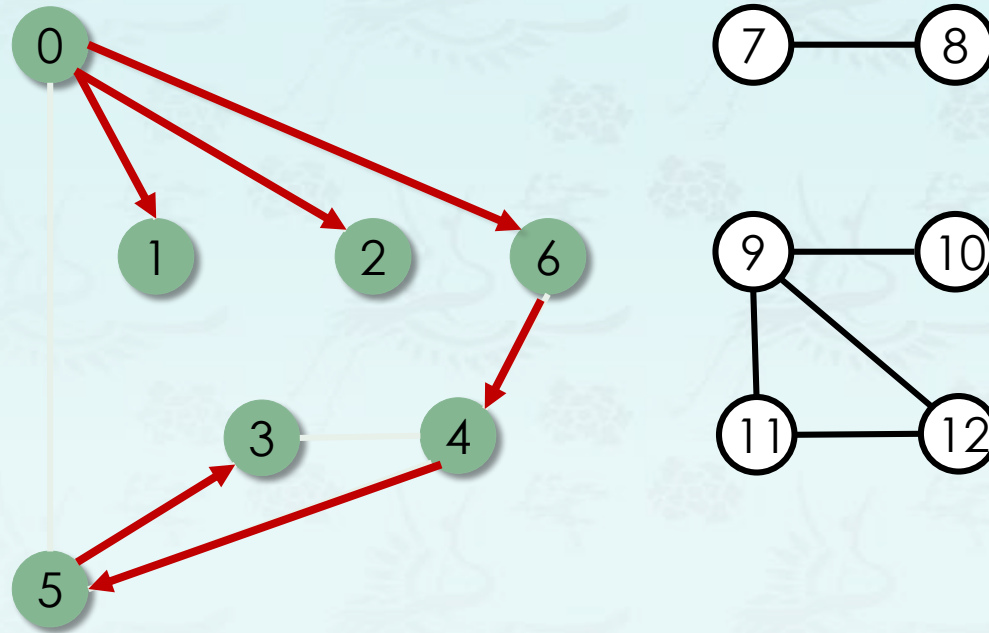
adj[]				v marked[] parent[v]		
0	<div><div>6</div><div>2</div><div>1</div><div>5</div></div>			0	T	-
1	<div><div>0</div></div>			1	T	0
2	<div><div>0</div></div>			2	T	0
3	<div><div>5</div><div>4</div></div>			3	T	5
4	<div><div>5</div><div>6</div><div>3</div></div>			4	T	6
5	<div><div>0</div><div>4</div><div>0</div></div>			5	T	4
6	<div><div>0</div><div>4</div></div>			6	T	0
				7	F	-
				8	F	-
				9	F	-
				10	F	-
				11	F	-
				12	F	-

visit 0: check 6, check 2, check 1, and check 5
0 done

DFS: Depth-First Search Demo

To visit a vertex v :

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v .



v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

DFS Output: DFS: 0 6 4 5 3 2 1

- found vertices reachable from 0
- build a data structure **parent[v]**

DFS: Depth-First Search Demo

Goal: Find all vertices connected to s (and a corresponding path).

Idea: Mimic maze exploration

Algorithm:

- Use recursion (ball of string).
- Mark each visited vertex (and keep track of edge taken to visit it).
- Return (retrace steps) when no unvisited options.

Data Structures:

- **Boolean[] marked** to mark visited vertices.
- **int[] parent** to keep tree of paths.
($\text{parent}[w] == v$) means that edge v - w taken to visit w for first time

DFS: Depth-First Search Coding

```
// DFS - find vertices connected to v
void DFS(graph g, int v, queue<int>& que) {
    if (empty(g)) return;
    for (int i = 0; i < V(g); i++)
        g->marked[i] = false;
    _DFS(g, v, que);    // recursive _DFS at v
    g->DFSv = que;      // save result at DFSv at v
}
```


DFS: Depth-First Search Coding

```
// DFS - find vertices connected to v
void DFS(graph g, int v, queue<int>& que) {
    if (empty(g)) return;
    for (int i = 0; i < V(g); i++)
        g->marked[i] = false;
    _DFS(g, v, que);    // recursive _DFS at v
    g->DFSv = que;      // save result at DFS
}
```

```
// Recursive _DFS does the work
void _DFS(graph g, int v, queue<int>& que) {
    g->marked[v] = true;    'v' current visiting vertex
    que.push(v);           // save the path

    for (gnode w = g->adj[v].next; w; w = w->next) {
        if (!g->marked[w->item]) {
            _DFS(g, w->item, que);
            g->parentDFS[w->item] = v;
        }
    }
}
```

keep where it reached from

DFS: Depth-First Search Properties

Proposition: After DFS, can find vertices connected to **s** in constant time and can find a path to **s** (if one exists) in time proportional to its length.


Proof: `parent[]` is parent-link representation of a tree rooted at **s**.

```
// returns a path from v to w using the result of DFS's parent[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
    if (empty(g)) return;

    // DFS at v, starting vertex
    queue<int> q;
    DFS(g, v, q);

    path = {};
    for (int x = w; x != v; x = g->parentDFS[x])
        path.push(x);

    path.push(v);
}
```



v	marked[]	parent[v]
0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

DFS: Depth-First Search Coding

Proposition: After DFS, can find vertices connected to **s** in constant time and can find a path to **s** (if one exists) in time proportional to its length.

Proof: `parent[]` is parent-link representation of a tree rooted at **s**.

What is the path from vertex 0 to vertex 3?

In this case, what is in the stack when `parent()` returns?

```
// returns a path from v to w using the result of DFS's parent[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
    if (empty(g)) return;

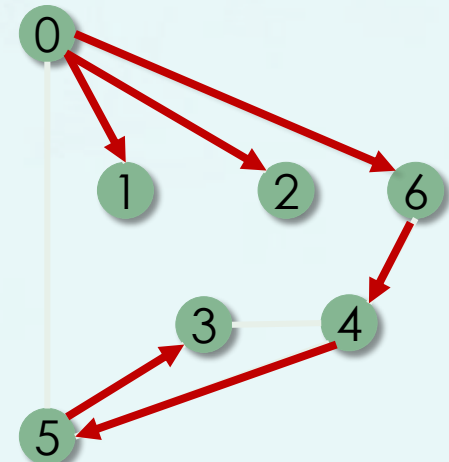
    // DFS at v, starting vertex
    queue<int> q;
    DFS(g, v, q);

    path = {};
    for (int x = w; x != v; x = g->parentDFS[x])
        path.push(x);

    path.push(v);
}
```

v marked[] parent[v]

0	T	-
1	T	0
2	T	0
3	T	5
4	T	6
5	T	4
6	T	0
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-



DFS: Depth-First Search Coding

Proposition: After DFS, can find vertices connected to s in constant time and can find a path to s (if one exists) in time proportional to its length.

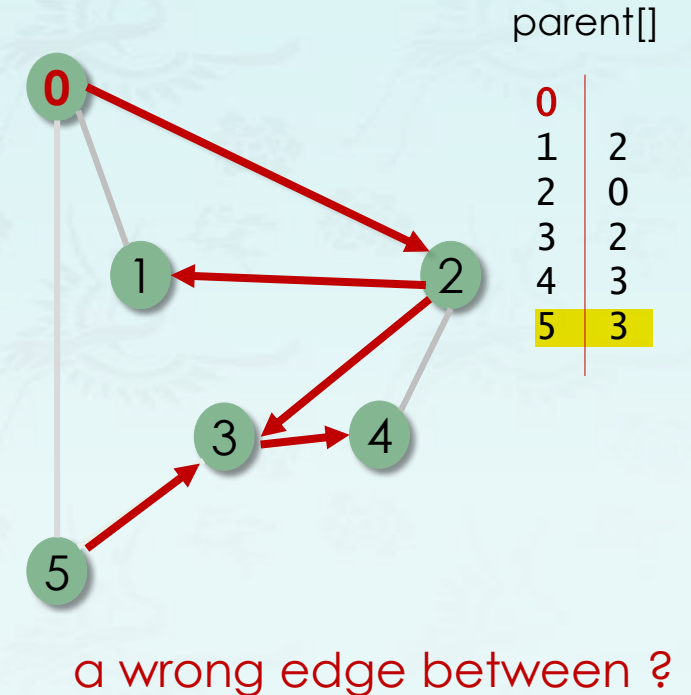
Proof: `parent[]` is parent-link representation of a tree rooted at s .

```
// returns a path from v to w using the result of DFS's parent[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
    if (empty(g)) return;

    // DFS at v, starting vertex
    queue<int> q;
    DFS(g, v, q);

    path = {};
    for (int x = w; x != v; x = g->parentDFS[x])
        path.push(x);

    path.push(v);
}
```



Graph

- Introduction
- Graph API
- Elementary Graph Operations
 - **DFS: Depth first search**
 - BFS: Breadth first search
 - CC: Connected Components

Major references:

1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

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Graph

- Introduction
- Graph API
- Elementary Graph Operations
 - DFS: Depth first search
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Connectivity Queries

Def.: Vertices v and w are connected if there is a path between them.

Goal: Preprocess graph to answer queries of the form “*is v connected to w ?*” in constant time.

```
public class CC
```

```
    CC(Graph G)
```

find connected components in G

```
    boolean connected(int v, int w)
```

are v and w connected?

```
    int count()
```

number of connected components

```
    int id(int v)
```

component identifier for v

Depth-first search? Yes ...

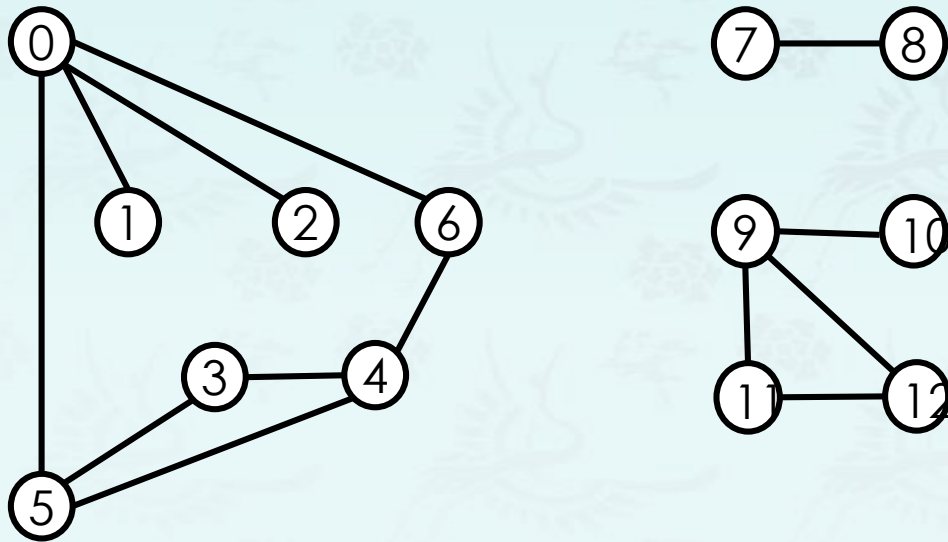
Connected Components

The relation “is connected to” is **equivalence relation**:

Reflexive: v is connected to v .

Symmetric: if v is connected to w , then w is connected to v .

Transitive: if v connected to w and w connected to x , then v connected to x



Connected Components

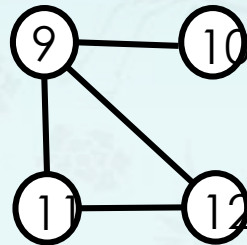
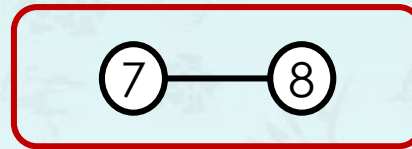
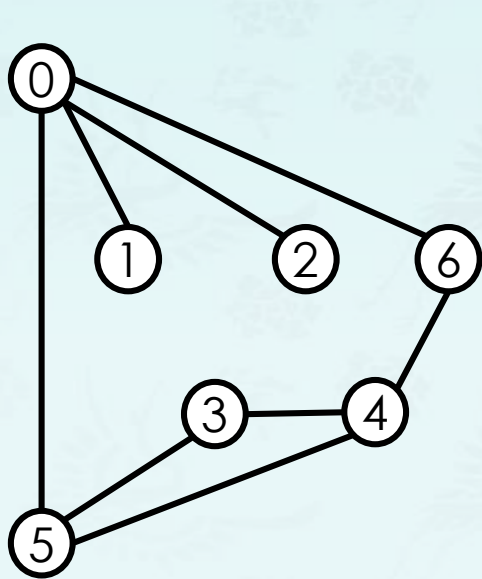
The relation “is connected to” is **equivalence relation**:

Reflexive: v is connected to v .

Symmetric: if v is connected to w , then w is connected to v .

Transitive: if v connected to w and w connected to x , then v connected to x

Def.: A connected component is a maximal set of connected vertices.



3 connected components

v	$id[v]$
0	1
1	1
2	1
3	1
4	1
5	1
6	1
7	2
8	2
9	3
10	3
11	3
12	3

Connected Components

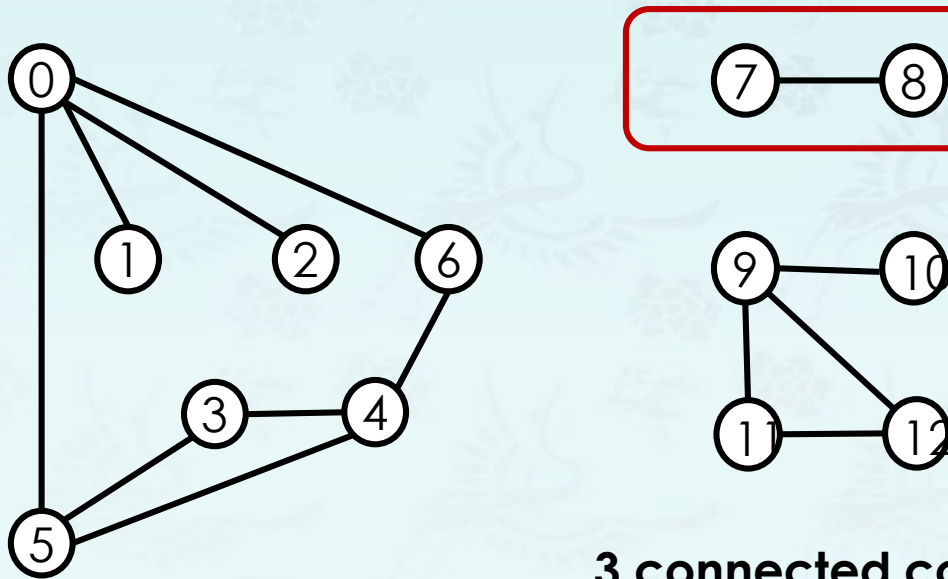
The relation “is connected to” is **equivalence relation**:

Reflexive: v is connected to v .

Symmetric: if v is connected to w , then w is connected to v .

Transitive: if v connected to w and w connected to x , then v connected to x

Def.: A connected component is a maximal set of connected vertices.



3 connected components

Remark: Given connected components,
can answer queries in constant time.

v	$\text{id}[v]$
0	1
1	1
2	1
3	1
4	1
5	1
6	1
7	2
8	2
9	3
10	3
11	3
12	3

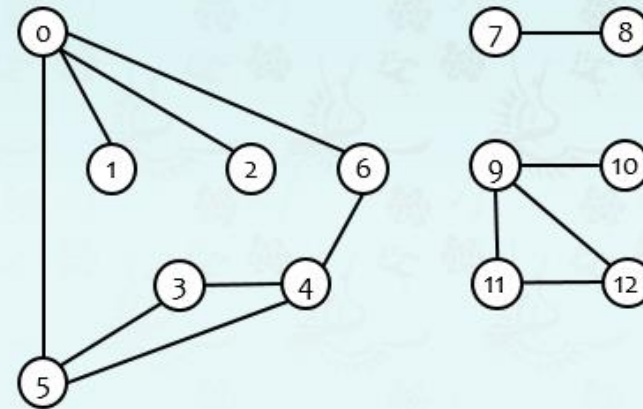
Connected Components

Goal: Partition vertices into connected components.

Connected components

Initialize all vertices v as unmarked.

For each unmarked vertex v , run DFS to identify all vertices discovered as part of the same component.



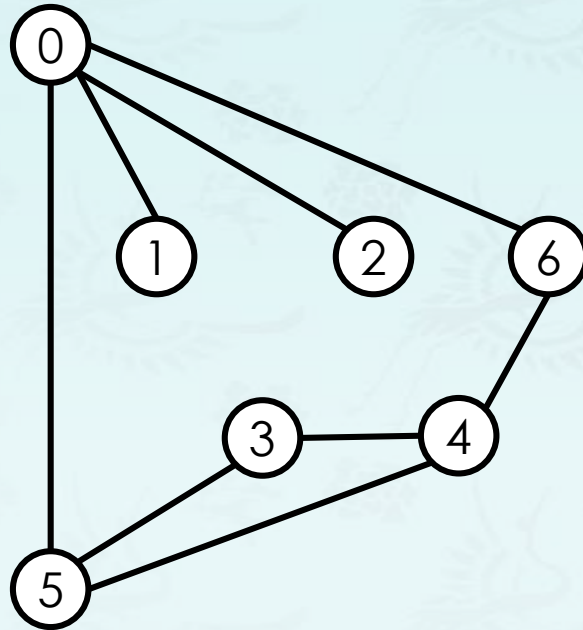
graph3.txt

```
13
13
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

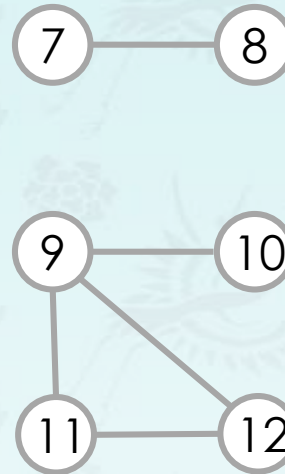
Connected Components

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



Graph g:



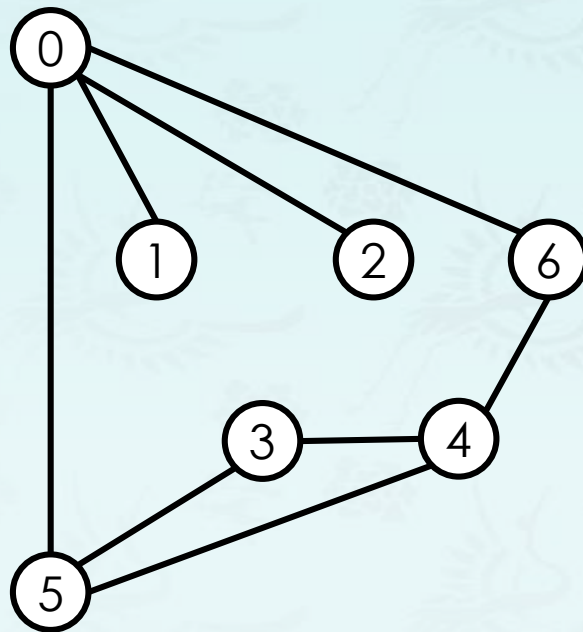
V-E lists →

graph3.txt

```
13 ← V
13 ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

Challenge: build adjacency lists?

Connected Components



Graph g

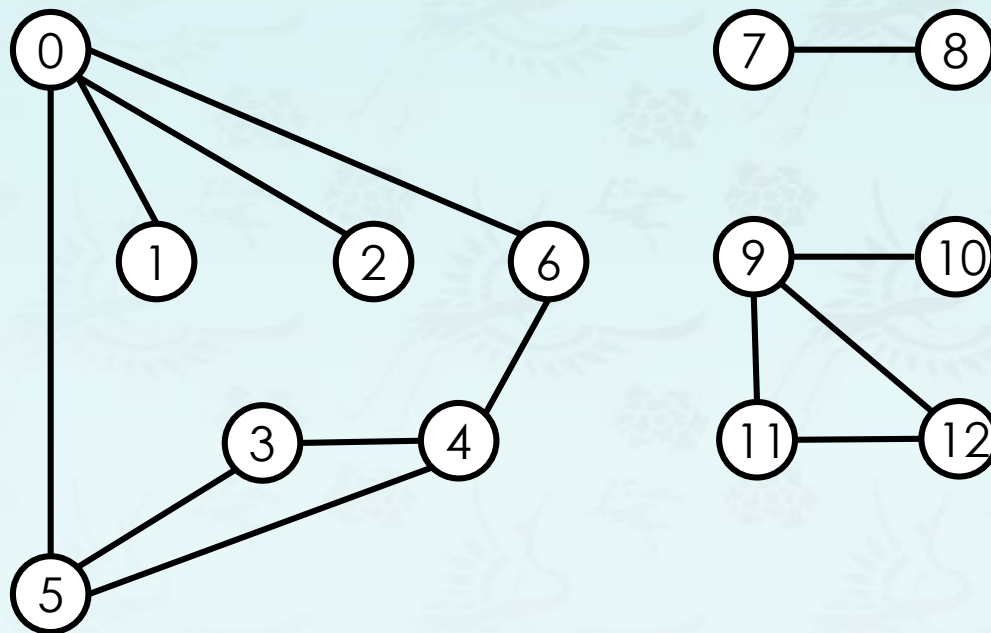
Adjacency lists

adj[]	
0	6 2 1 5
1	0
2	0
3	5 4
4	5 6 3
5	3 4 0
6	0 4

V-E lists

graph3.txt

```
13 ← V
13 ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

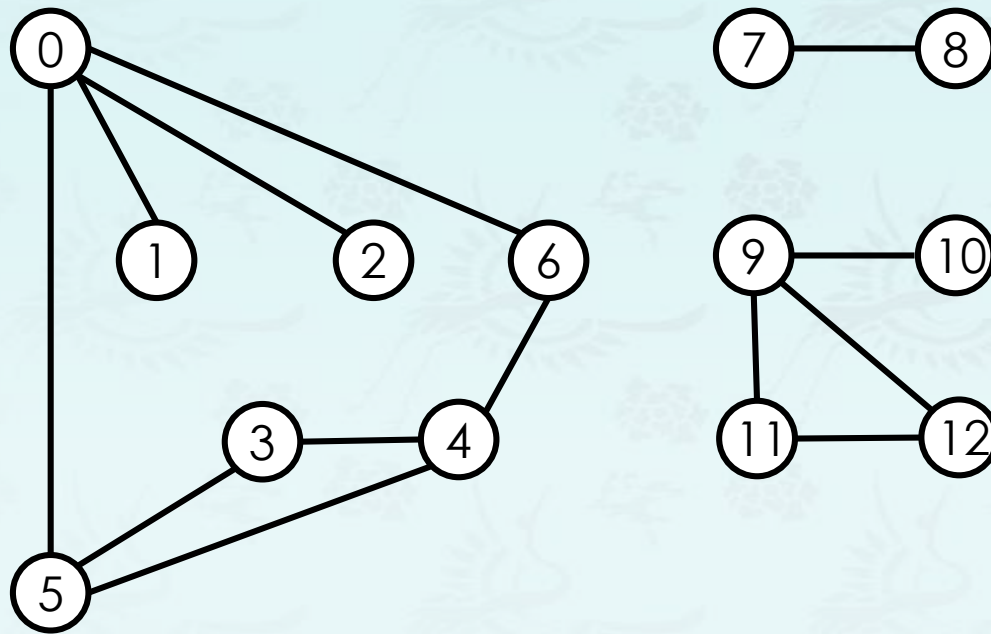


V-E lists →

graph3.txt
 13 ← V
 13 ← E
 0 5
 4 3
 0 1
 9 12
 6 4
 5 4
 0 2
 11 12
 9 10
 0 6
 7 8
 9 11
 5 3

Graph g:

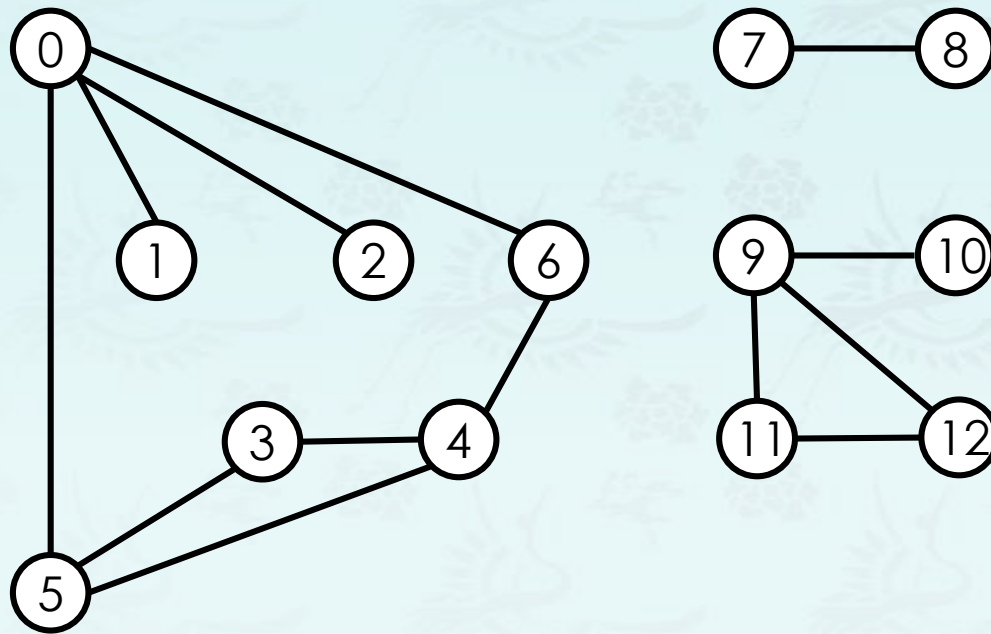
Connected Components



Graph g:

v	marked[]	id[]
0	F	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-
10	F	-
11	F	-
12	F	-

Connected Components



Done:

v	marked[]	id[]
0	T	0
1	T	0
2	T	0
3	T	0
4	T	0
5	T	0
6	T	0
7	T	1
8	T	1
9	T	2
10	T	2
11	T	2
12	T	2

Connected Components – Coding

```
// returns true if v and w are connected.
bool connected(graph g, int v, int w) {
    if (empty(g)) return true;

    queue<int> q;
    DFS(g, v, q);

    return g->CCID[v] == g->CCID[w];
}
```

```
// returns number of connected components.
int nCCs(graph g) {
    int id = g->CCID[0];
    int count = 1;
    for (int i = 0; i < V(g); i++)
        if (id != g->CCID[i]) {
            id = g->CCID[i];
            count++;
        }
    return id == 0 ? 0 : count;
}
```

Graph

- Introduction
- Graph API
- Elementary Graph Operations
 - DFS: Depth first search
 - BFS: Breadth first search
 - **CC: Connected Components**
- pset – graph.cpp
 - **implement DFS, BFS, CC and others**

Major references:

1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

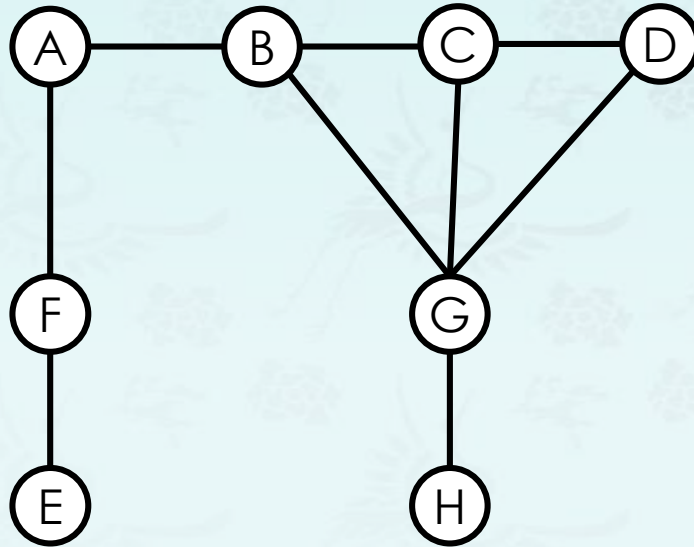
Prof. Youngsup Kim, idebtor@handong.edu, 2014 Data Structures, CSEE Dept., Handong Global University

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DFS - Exercise

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



adjacent list

A: B F

B: G C A

C: D G B

D: C G

E: F

F: E A

G: H B C D

H: G

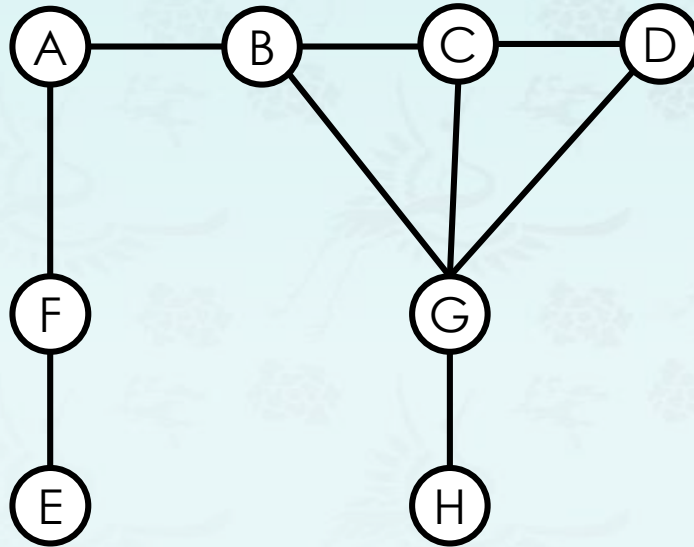
Graph g:

Hint: A B...?...F E

DFS - Exercise

To visit a vertex v:

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



Graph g:

adjacent list

A: B F
B: G C A
C: D G B
D: C G
E: F
F: E A
G: H B C D
H: G

Hint: A B G H C D F E

```
dfs(A)
  dfs(B)
    dfs(G)
      dfs(H)
        check G
        H done
      check B
      dfs(C)
        dfs(D)
          check C
          check G
          D done
        check G
        check B
        C done
      check D
      G done
    check C
    check A
    B done
  dfs(F)
    dfs(E)
      check F
      E done
    check A
    F done
  A done
  check B
  check C
  check D
  check E
  check F
  check G
  check H
```