

## 6: Filter Designing

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$\alpha = 3$  for all problems.

### 1 Problem-1

Specifications for Low Pass Butterworth filter is :

Sampling Frequency : 720Hz , Pass-band edge frequency = 10Hz , Stop-band edge Frequency = 20Hz , Pass-Band ripple = 3dB , Stop-Band Attenuation = 40dB

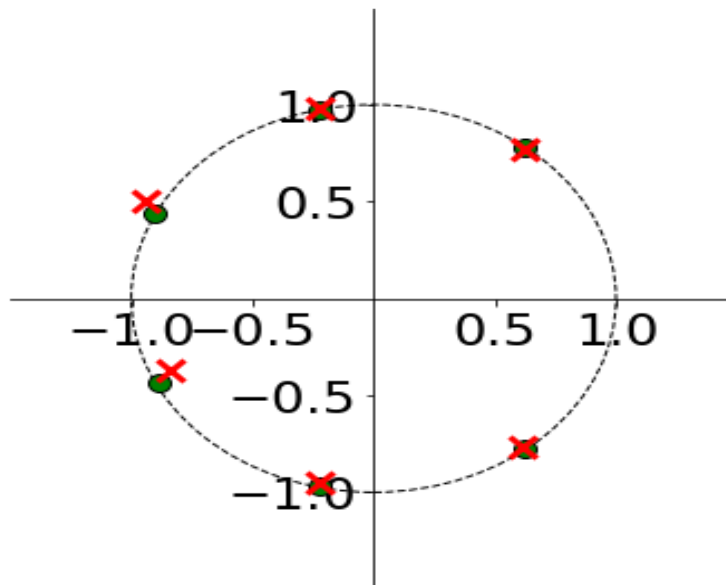
#### 1.1

We obtain order of the filter as 7 with Normalized cutoff frequency as 0.028.

$$Filter = 10^{-10} \left( \frac{2.49 + 17.44z^{-1} + 5.23z^{-2} + 8.72z^{-3} + 8.72z^{-4} + 5.23z^{-5} + 1.74z^{-6} + 2.49z^{-7}}{1 - 6.60z^{-1} + 18.72z^{-2} - 29.48z^{-3} + 27.88z^{-4} - 15.82z^{-5} + 4.99z^{-6} - 0.67z^{-7}} \right) \quad (1.1)$$

#### 1.2

The Below image is for a pole-zero plot of the above filter in normalized circumstances :



For a Filters transfer function to be stable in case of Digital butterworth filter these are the conditions :

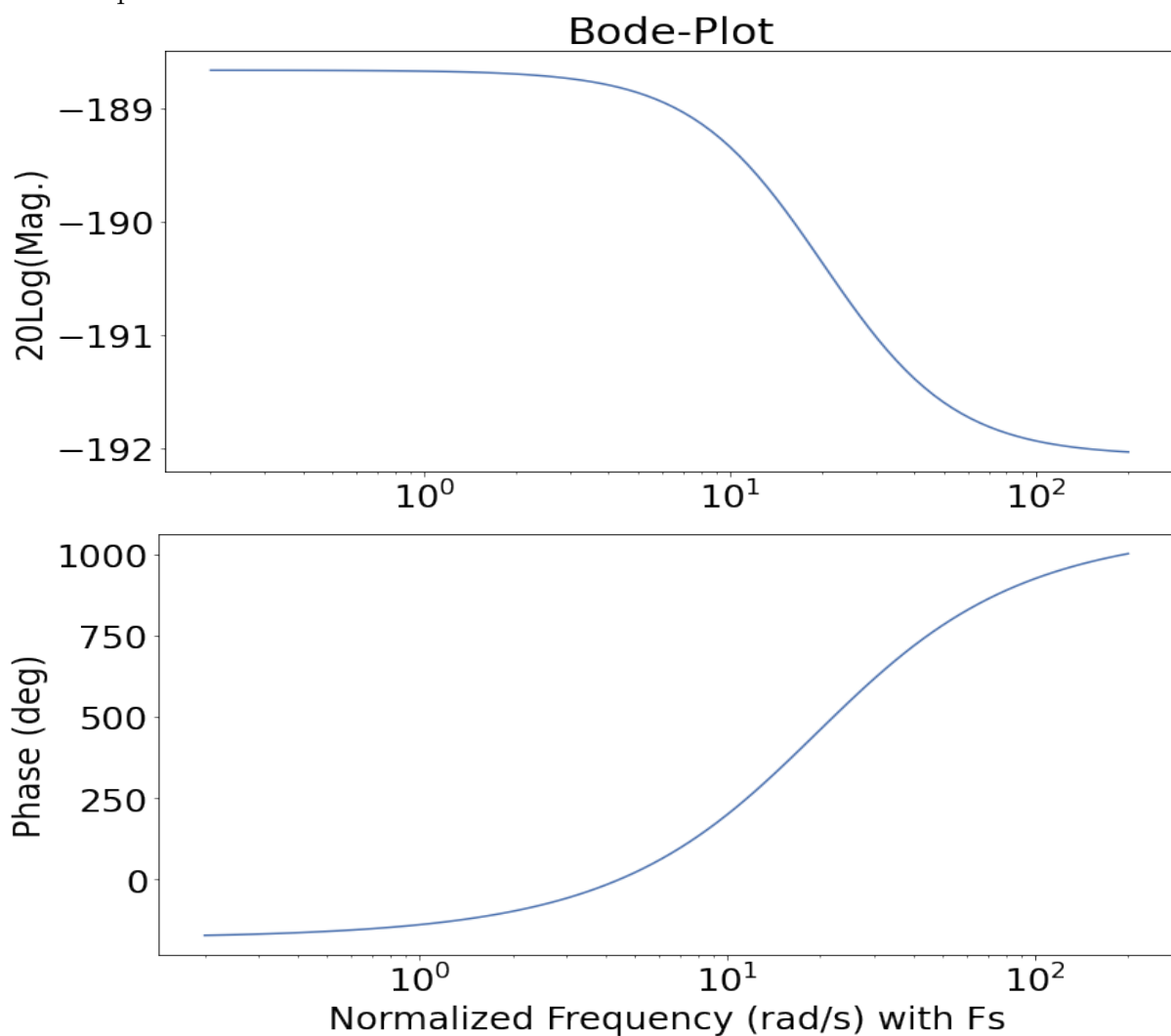
(i) Poles determine the stability. If they are within unit circle in Z-plane then system is said to be stable.

(ii) If poles go beyond unit circle then the system becomes unstable.

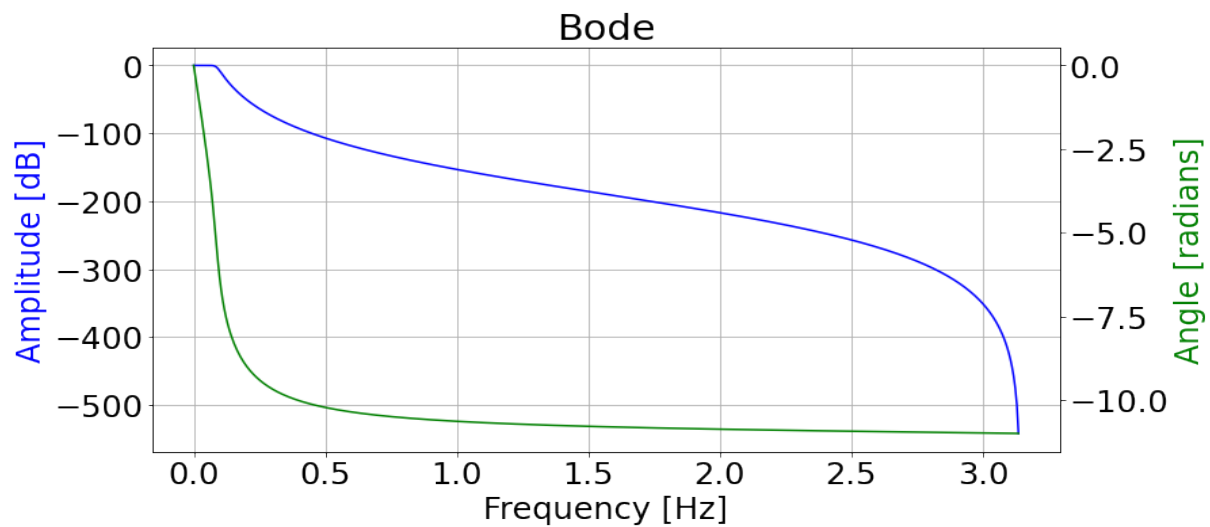
Since in the above plot one of the pole crosses the unit circle it is known to us that the filter transfer function is unstable.

### 1.3

Bode Plot using in-built bode() function. Not sure but uses numerator and denominator coefficients in s-plane.

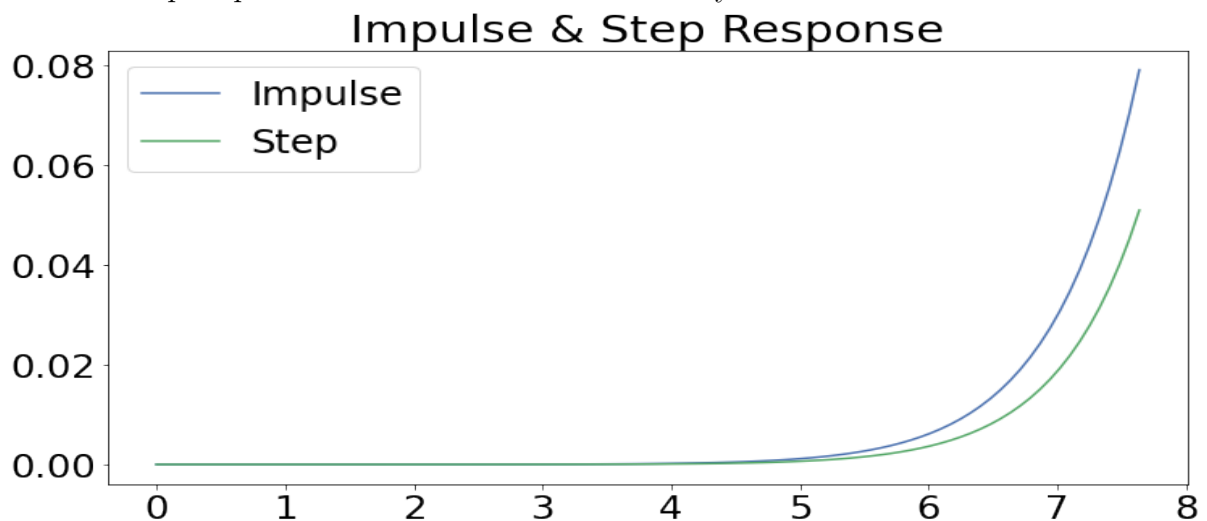


Manually plotting Bode in this case, the shape of output is not that of the desirable one.



## 1.4

Impulse and step response tell us the behaviour of the system

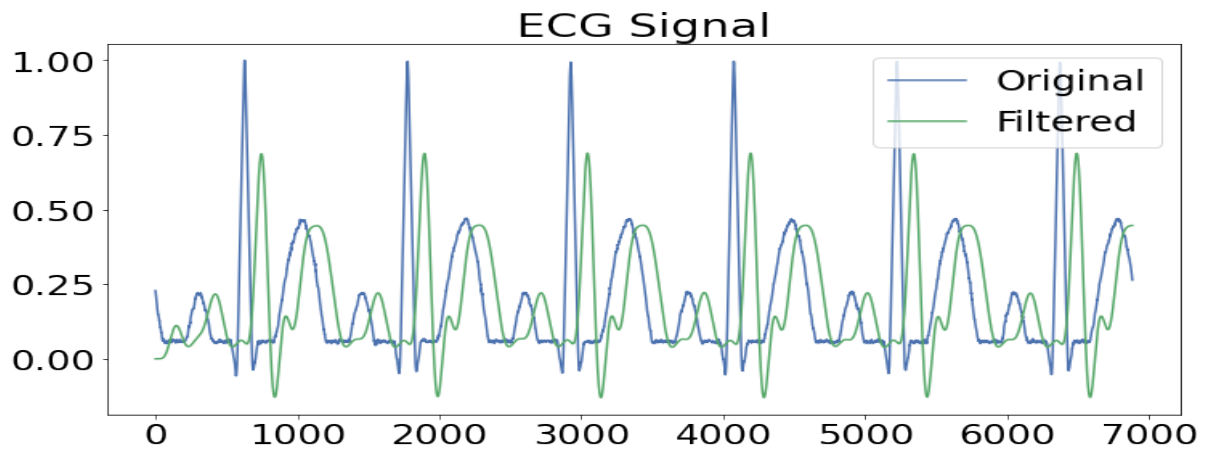


From the above graph it's clear that the system isn't bounded and thus leads to the conclusion that it's unstable as was determined in the previous sub-problem. Also the response is close to zero for better part of the plot.

## 2 Problem-2

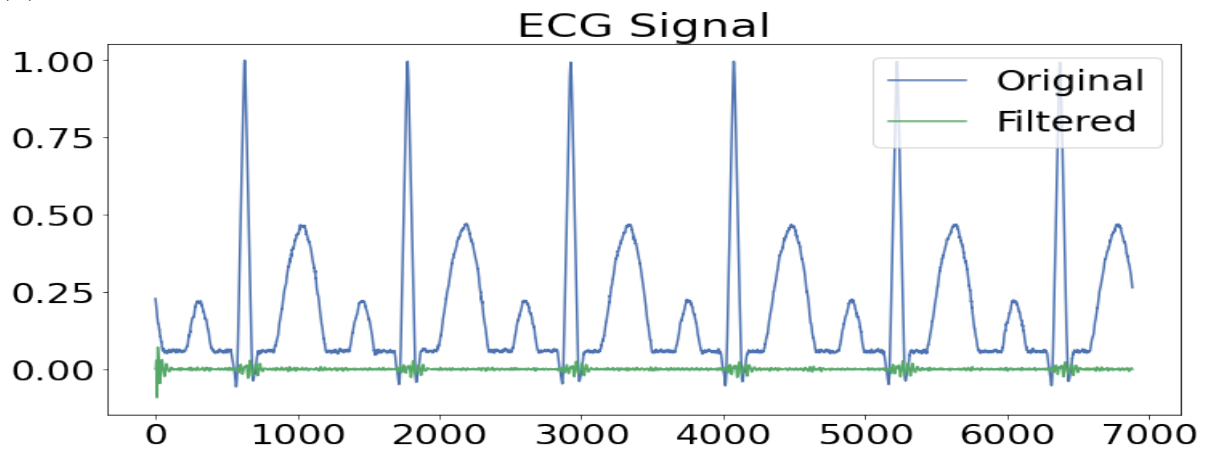
In the following images following specifications are used :

- (i) Butterworth Low pass Filter
- (ii) Order= 7 , Cut-off Frequency = 5Hz , Sampling Frequency = 720Hz



Specifications for below figure :

- (i) Butterworth Band Pass Filter
- (ii) Order=7 , Lower Cutoff =20Hz , Higher Cutoff = 100Hz , Sampling Frequency = 720Hz

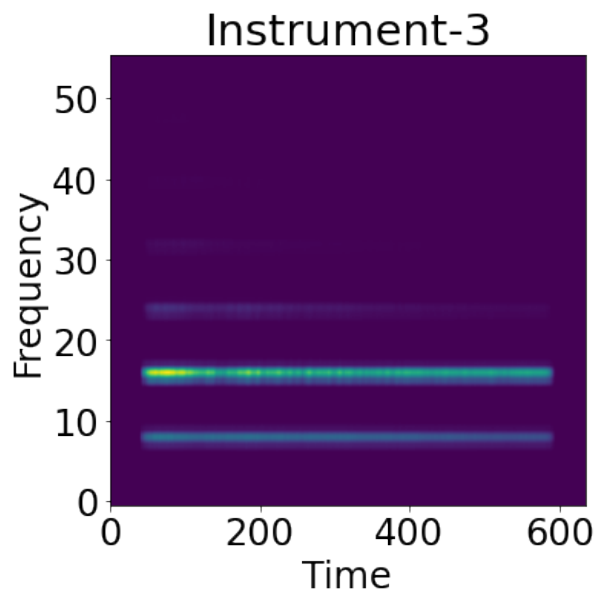


Observations and Conclusions :

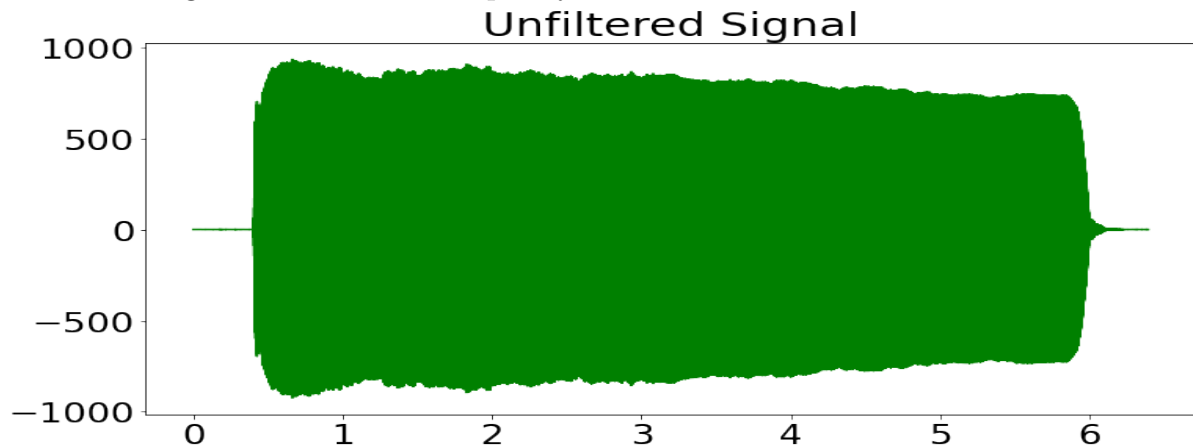
- (i) Image 1 provides more consistent signal with respect to original signal than that provided by image 2.
- (ii) ECG signal provided is contributing and consists of more frequency components from 0-5Hz band than the contribution from 20-100Hz band.

### 3 Problem-3

Spectrogram of original Instrument-3 signal is as shown below :



It can be observed from the above plot that the signal has one fundamental frequency and rest of them being harmonic of this frequency.



The above is the original signal plotted against samples.

It is required to apply a Digital Band Pass Butterworth Filter to obtain just the fundamental frequency from the signal.

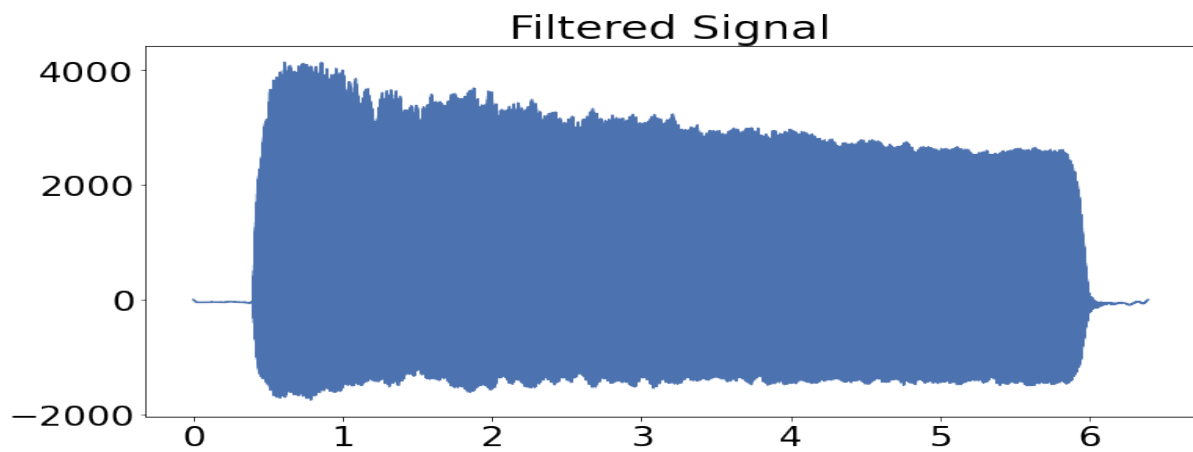
For that a Butterworth band pass filter with these specifications is designed :

(i) Order = 3

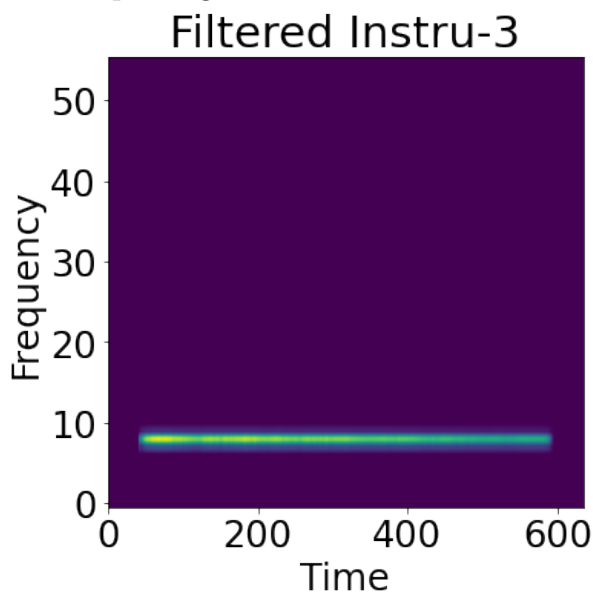
(ii) Lower cut-off = 600Hz , Higher cut-off = 800Hz , Sampling rate = 11025Hz as is that of the original signal.

This is designed keeping in mind the fact that fundamental frequency for this instrument lies in this range , to be specific is 784Hz.

Below is the signal obtained after Filtering :



To prove that the above signal contains only the fundamental frequency components we obtain the spectrogram for the same :



As it is inferred from the above spectrogram there is only one frequency available in this filtered signal.

**Note :** *Audio signal obtained is played properly in the Jupyter Python file , in case the file stored isn't played properly.*

## 4 Problem-4

Specifications for Low Pass Chebyshev Type 1 filter is :

**Sampling Frequency : 720Hz , Pass-band edge frequency = 10Hz , Stop-band edge Frequency = 20Hz , Pass-Band ripple = 3dB , Stop-Band Attenuation = 40dB**

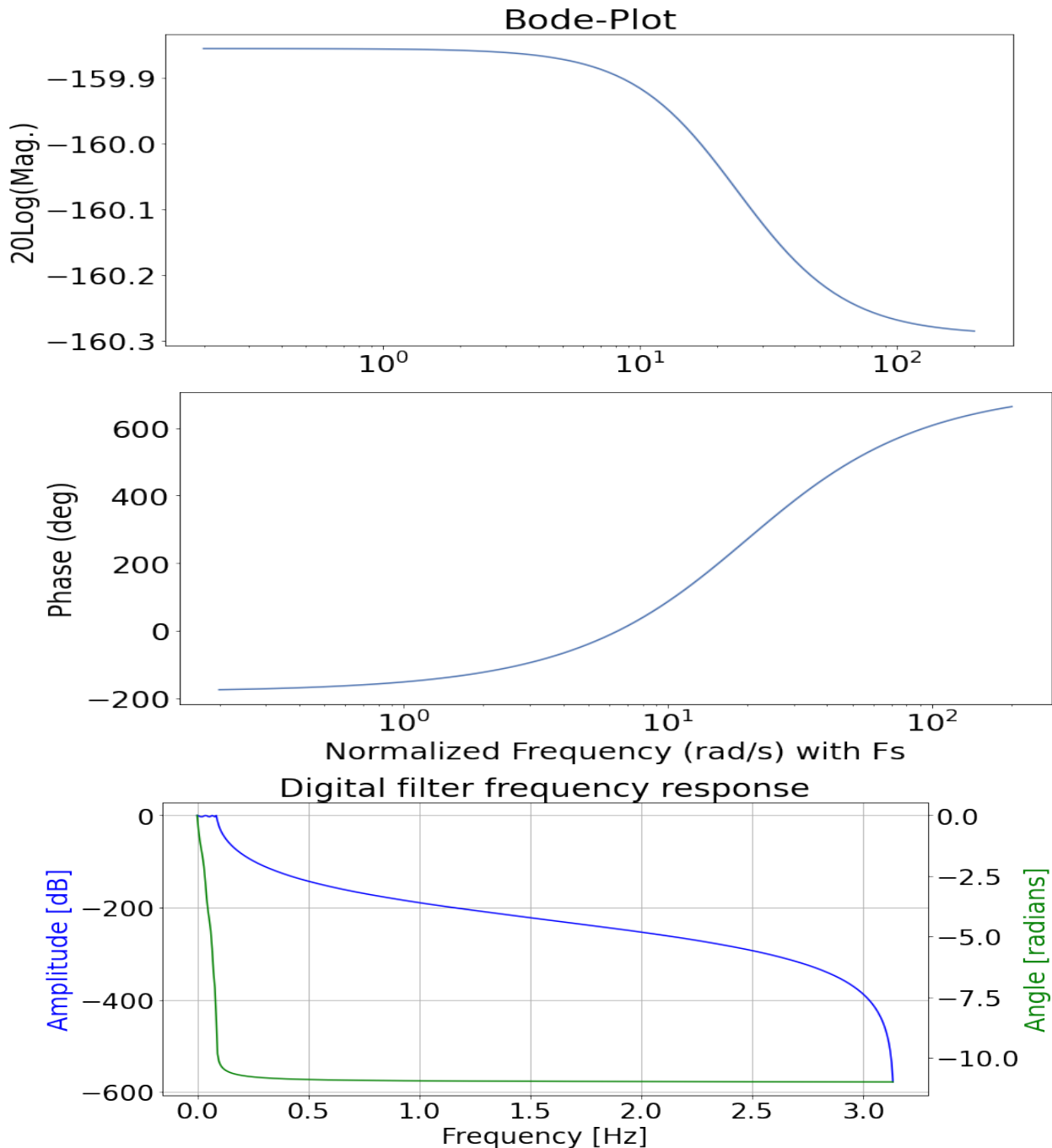
Using the given specifications we can obtain Chebyshev Type 1 order and cutoff frequency for Low pass type using respective python commands

Order Obtained = 5

Normalized Cut-off frequency = 0.028

It is noticed that Order of this Chebyshev Type 1 filter is lower than the order of butterworth filter for the same specifications. Therefore butterworth filter requires higher order to get the same specifications indicating Chebyshev Type 1 is better in this aspect.

The Bode Plot for the above filter is as follows :



The first two plots are using the function `bode()` while the last one is done in manual way.

Chebyshev Type 1 also has a parameter pass band ripple which differentiates it from butterworth filter.

Some general conclusions :

- (i) Filtering a signal requires knowledge of the type of frequencies that are desired in the first place.
- (ii) Digital and analog filters have to be dealt with differently.
- (iii) Chebyshev Type-1 and Butterworth filter differ in pass band ripple, order and slight difference in cutoff frequency too.