EE386 Digital Signal Processing Lab **Jul-Dec 2021**

7: FIR Filters

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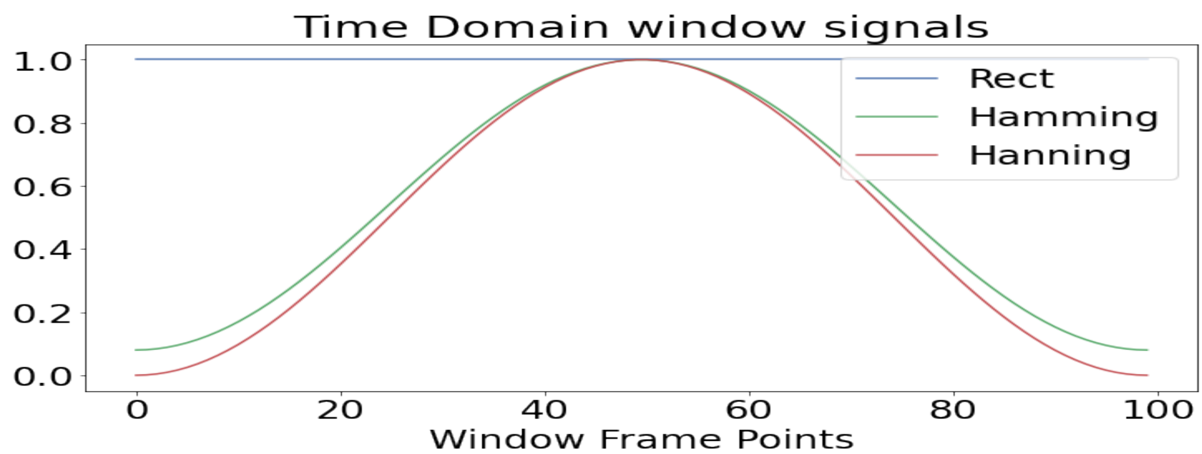
For all problems $\alpha = 3$

1 Problem 1

1.1

Windows are used as weights for a given point of the signal to which the window is applied. Windows in time domain are designed so as to give more weightage to the actual signal than the distortions brought about by various other factors namely to keep spectral leakage in check. Every window type has its own benefits and according to the need one must use a particular type of window.

The below image shows the window signals for Rectangular,Hanning and Hamming window :



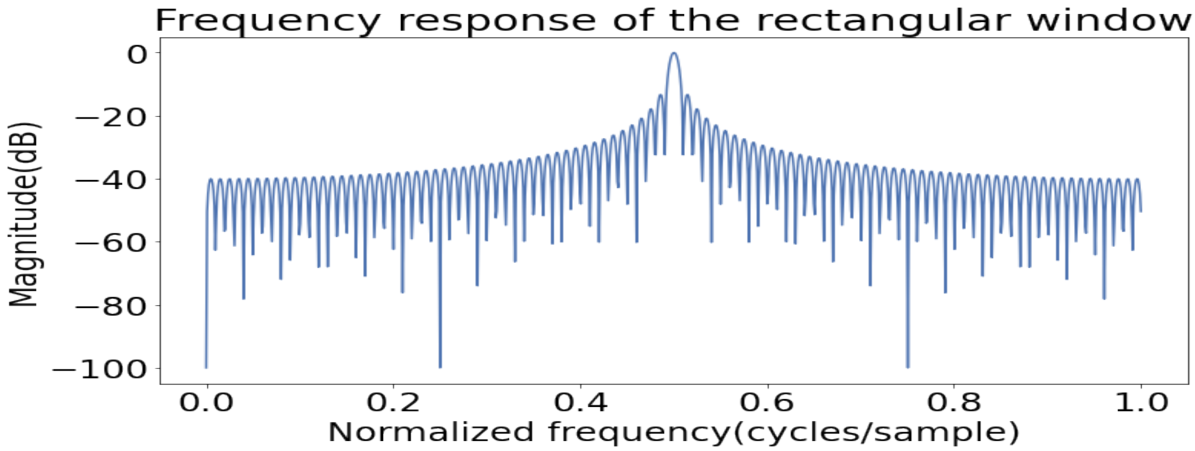
We will discuss the following windows :

(i) **Rectangular :**

Its basically a signal which gives equal weightage to each point and is horizontal in shape with unit weight.

$$\omega(n) = 1; N = 100 \tag{1.1}$$

Frequency Response of any window signal determines the affect on spectral leakage and resolution due to window :



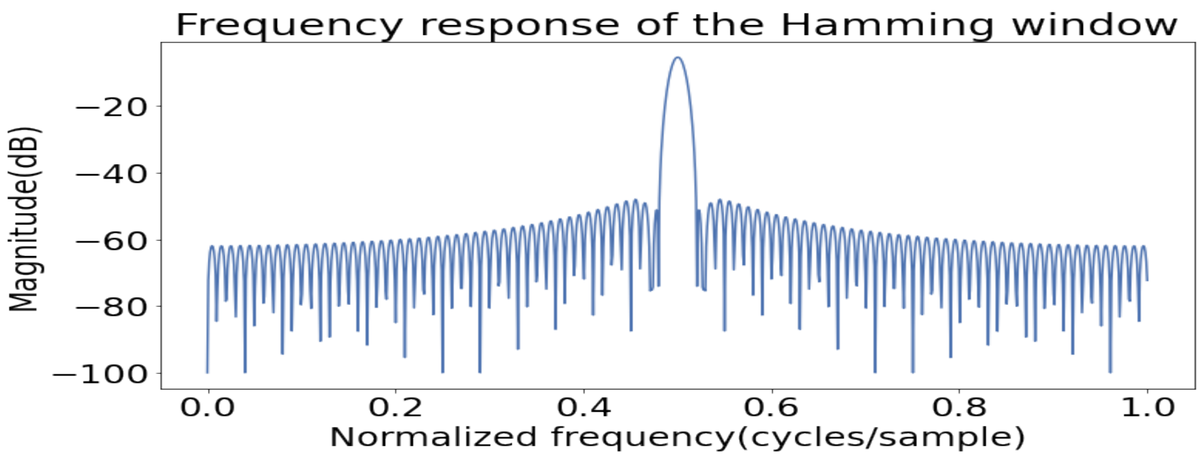
From the above image it is clear that each of the lobes end up at a comparatively higher weightage to each point. Also the lobe height does not drastically decrease as compared to other window signals.

Spectral Leakage is determined by the side lobes magnitude and behaviour while frequency resolution is determined by the thickness of the central lobe.

It is noted from the thickness of the central lobe that frequency resolution is lower for the rectangular window while spectral leakage is higher than the other two.Also the window does not reach zero at the ends. Number of DFT points can be increased to suppress the spectral leakage overall.

(ii) **Hamming :**

$$\omega(n) = 0.54 - 0.46(\cos(\frac{2\pi n}{N}); N = 100 \tag{1.2}$$

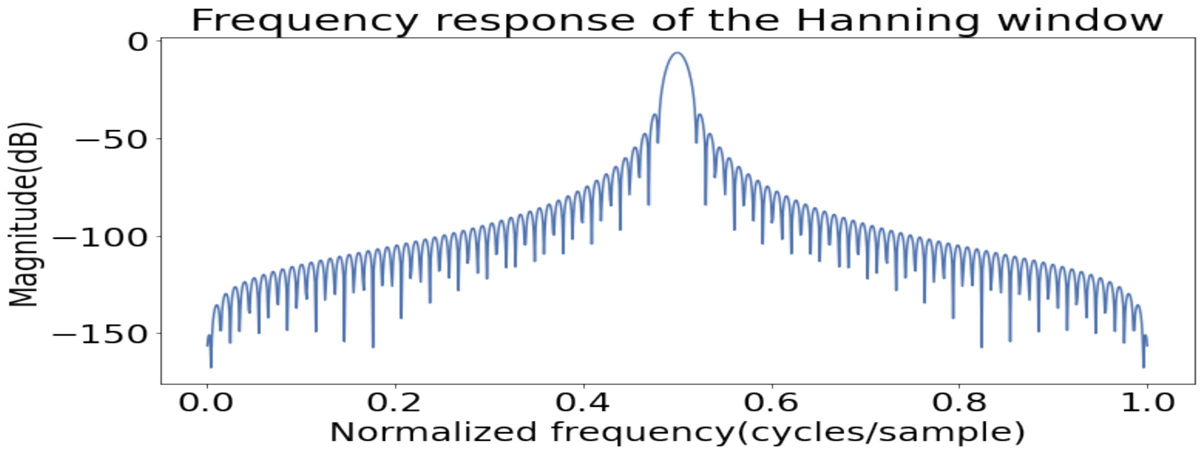


Window does not reach zero at the ends. Has frequency resolution lower than hanning but higher than rectangular while spectral leakage is better than rectangular but not as good as Hanning.

Number of DFT points can be increased to suppress the spectral leakage overall.

(iii) **Hanning :**

$$\omega(n) = (\sin^2(\frac{\pi n}{N}); N = 100 \tag{1.3}$$

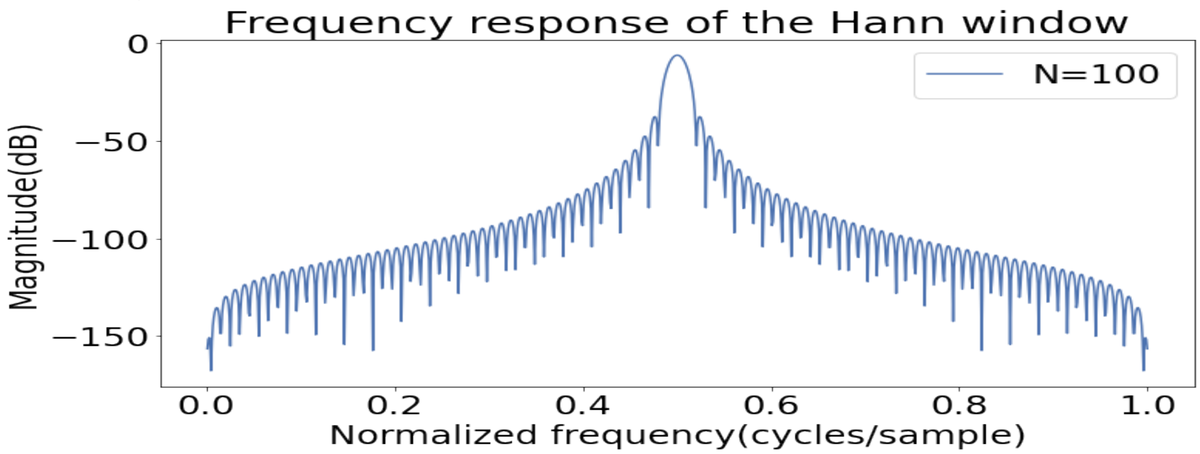


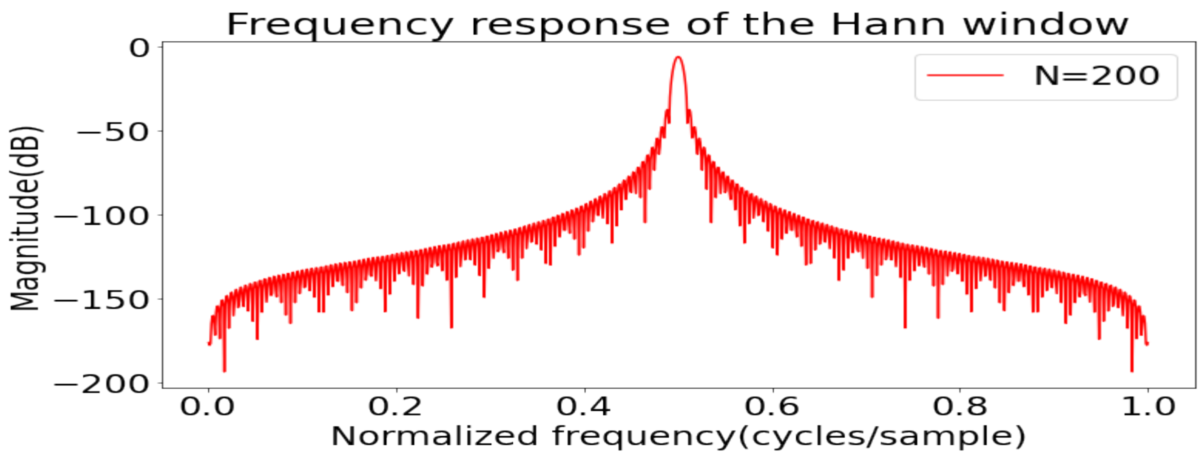
Only Window among the three to reach zero at the ends. Has bad frequency resolution compared to hanning and rectangular while spectral leakage is better than the other two.

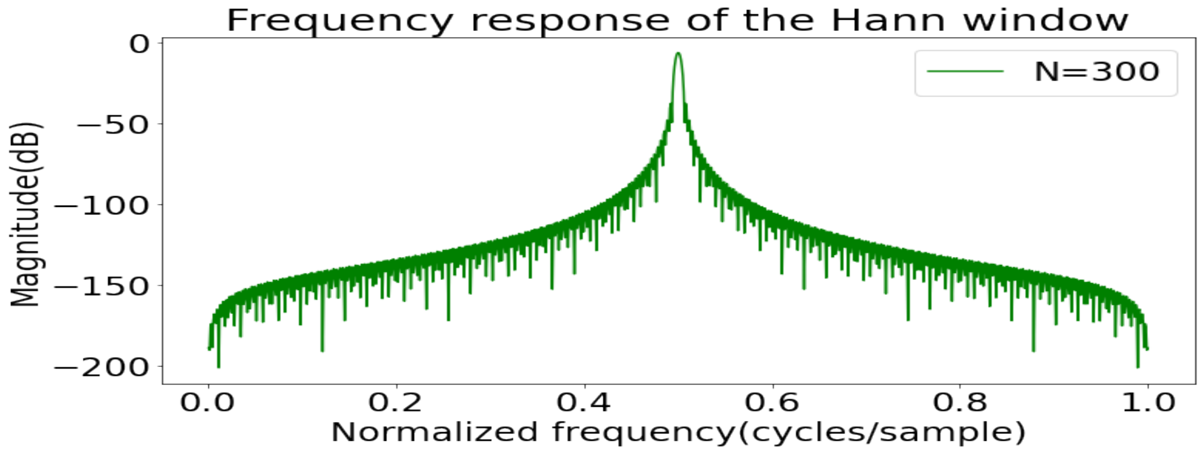
Number of DFT points can be increased to suppress the spectral leakage overall.

1.2

The spectrum/DFT for different lengths is as follows :



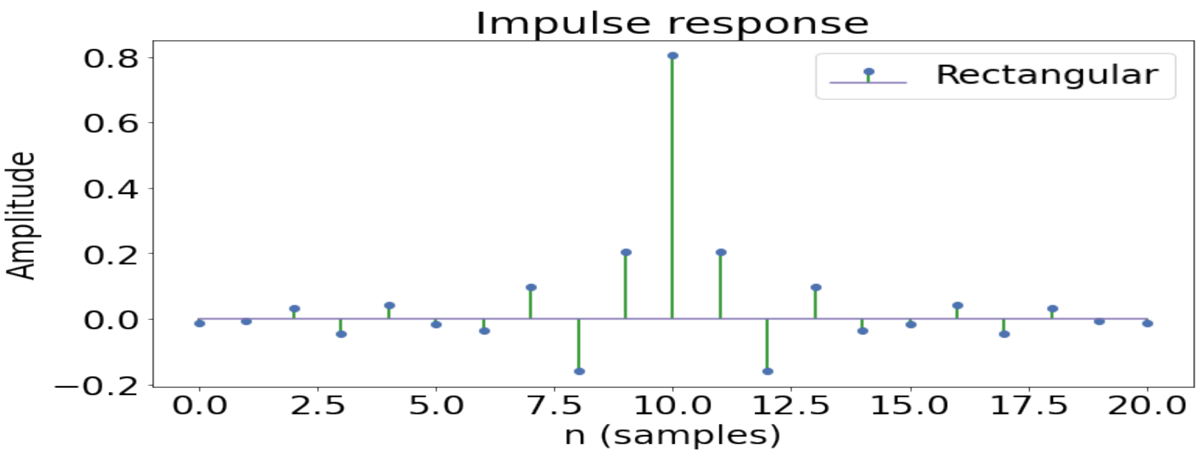


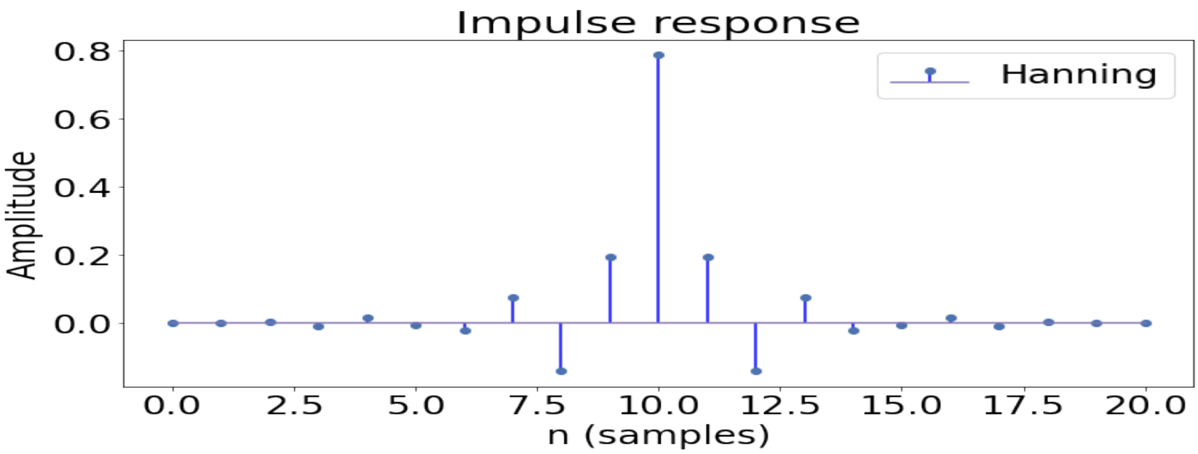


From the above figures it is noted that on increasing the number N i.e the window length the lobes tend to compress itself thus effectively should improve resolution although no real changes seem to be there in overall spectral leakage control. It is noted that there are some changes in height of the lobes.

2 Problem 2

2.1

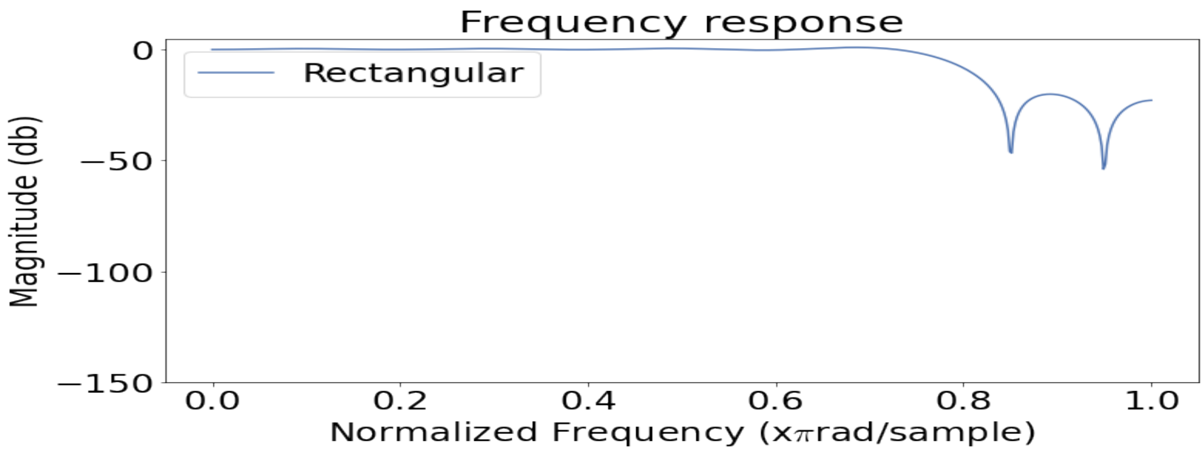


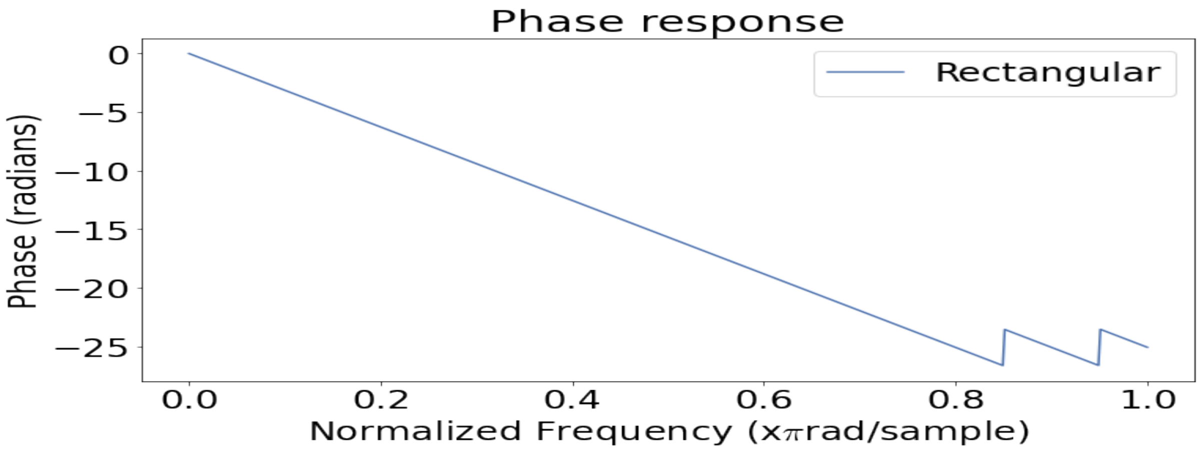


From the above impulse responses it is clearly the sinc function which is expected and also since the axis of plot remains positive the system is causal and therefore this FIR filter is stable as required.

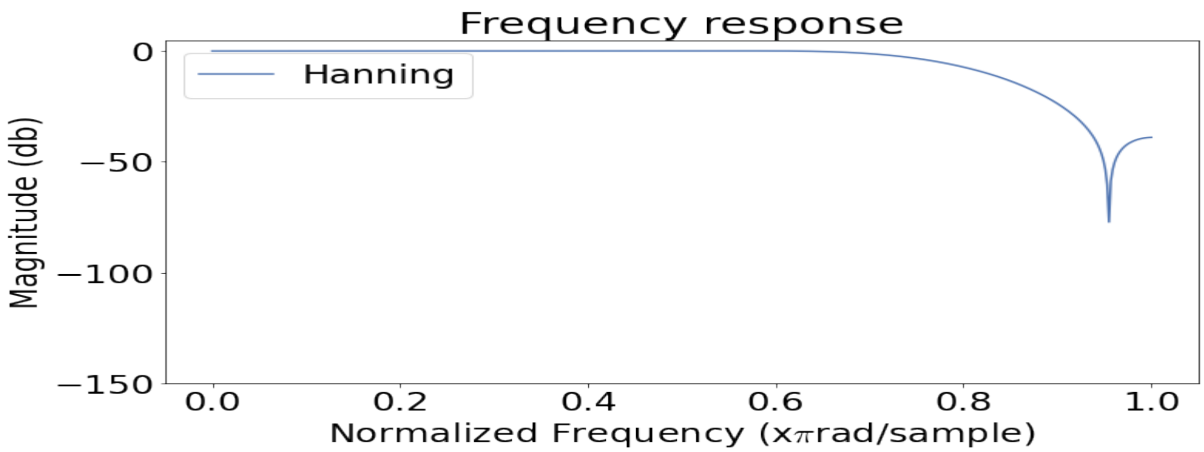
2.2

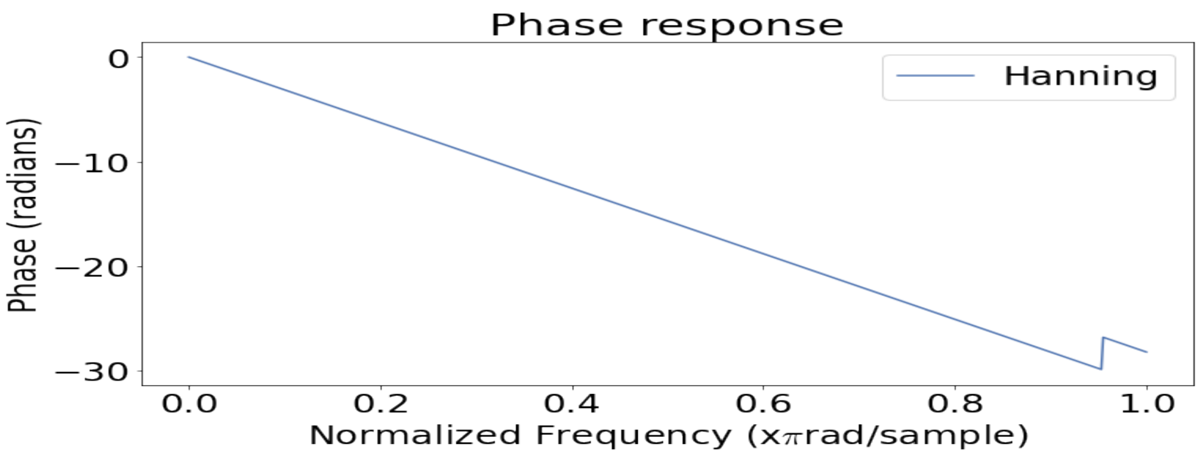
Bode Plot for Rectangular Window





Bode Plot for Hanning Window





Conclusions from above plots :

(i) Transition region for Hanning window is more than that of rectangular window.

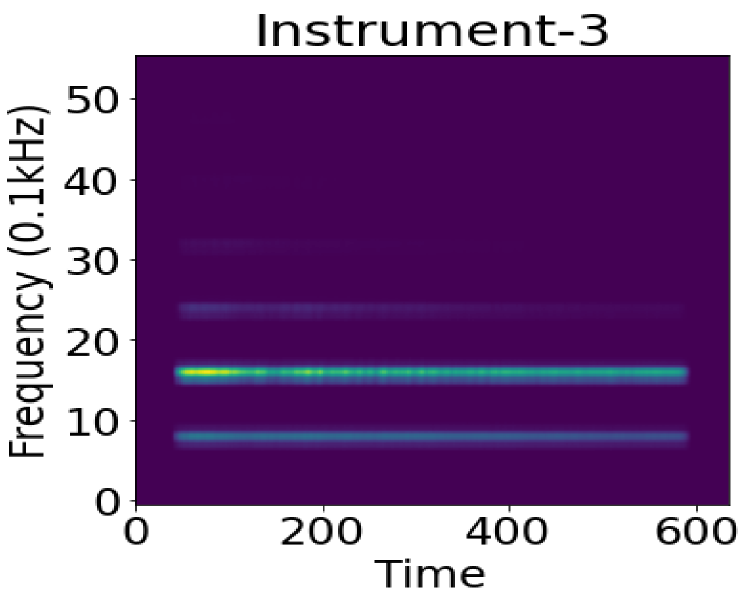
(ii) Passband ripple is evidently observed in rectangular window than the Hanning window.

(iii) There is a sharp decrease in magnitude in the following lobe indicating reduced weightage,however for rectangular the decrease in this magnitude is less.

(iv) Phase is more piecewise linear in Hanning than the rectangular since there are more spikes for the rectangular window.

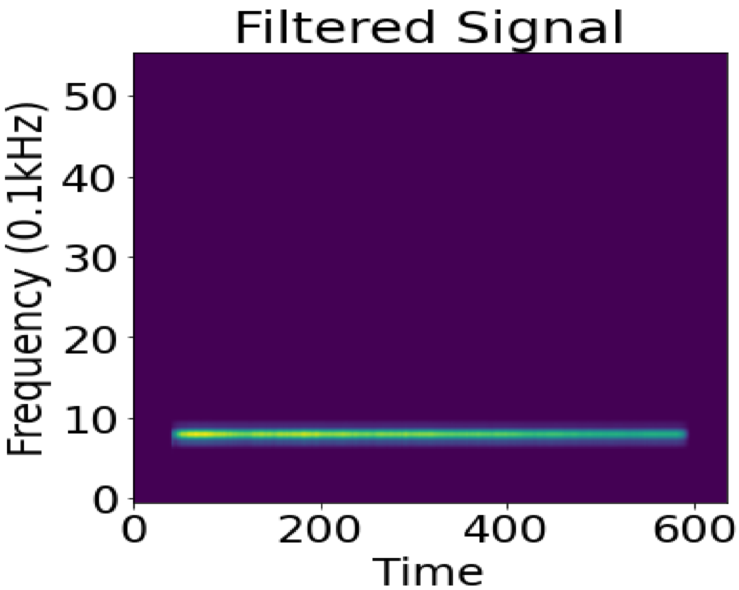
3 Problem 3

Spectrogram of the original signal generated by Instrument-3 :



From This spectrogram,with hamming window applied, and the previous analysis it is noted that the fundamental peak lies in the range of 750-800Hz and to be specific at 784 Hz.
To filter all other frequencies from the signal we need an FIR Bandpass Filter. We use the windowing technique to generate this filter and eventually filter the original signal for fundamental frequency.

The specification with which this filter is generated are as follows:
Taps/Coefficients = 111 ——— Sampling frequency = 11025Hz
Lower cut-off = 600Hz ——— Higher cut-off = 800Hz
Window = Hamming
The filter is generated using *firwin* command in signal module.
Spectrogram of the Filtered signal :



It is now clear from the spectrogram that only the fundamental frequency is allowed to pass through this filter.

Note : *Incase the filtered signal file does not play, please execute the jupyter script file to hear audio of filtered signal.*

4 Problem 4

Windowing in design of FIR filter design is basically multiplying an ideal FIR filter with a particular window in time domain and get a close to ideal filter. However in frequency domain this multiplication changes to convolution of ideal FIR transform and window transform, thus to create the perfect FIR filter possible for given conditions.It is basically used to improve effects on side lobes and its frequency response.

(i) For pass-band gain focus lies on main lobe.
(ii) For stop band gain focus is on the area under the side lobes.
(iii) Transition bandwidth is the bandwidth of the main lobe.
(iv) Gives smoother transition region compared to ideal filter.

On the other hand time-domain windowing is just the multiplication of weights of windows to corresponding weights of the signal. It is used to decrease spectral leakage. The length of the window is used as a frame and this frame is moved across the signal to get Windowed signal.

Window in FIR filter design focuses on the frequency response i.e lobes and its features while in time domain focus lies on spectral leakage and frequency resolution.

Time domain windowing pays no attention to the requirement of the user for specific frequencies. Its major task is to improve spectral leakage in frequency domain and some smoothening of the plot. The filtering function is the same as that of the actual window.

FIR window based filter design focuses on cutoff frequency requirements as well as other features including pass band ripple, stop band attenuation,transition width and other parameters. This method produces a transfer function in order to filter out the signal.Number of coeeficients decide the accuracy to which the filter works.