EE386 Digital Signal Processing Lab Jul-Dec 2021

8: Epidemic Modelling

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1 Problem-1

$$y[n] = \delta[n] + R_o y[n-1] \tag{1.1}$$

1.1

The transfer function for the above difference equation is :

$$H(z) = \frac{z}{z - R_o} \tag{1.2}$$

$$H(z) = \frac{1}{1 - R_o z^{-1}} \tag{1.3}$$

The pole-zero plot for the above transfer function is as follows:

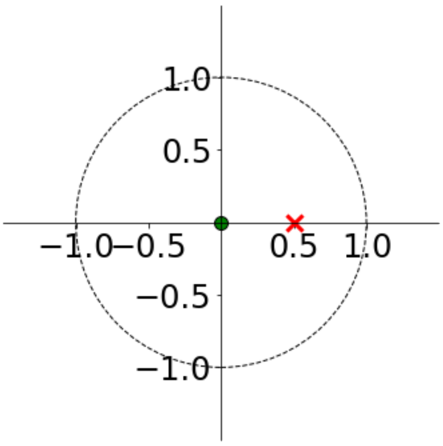

Figure 1: When $R_o <= 1$

The above pole-zero plot is for $R_o <= 1$ and since its pole lies within the unit circle it is stable and bounded that is it shall reach a point when the value becomes zero.

The below pole-zero plot is for $R_o > 1$ and since its pole lies outside the unit circle it is unstable and unbounded that is it shall never reach a point of zero and continues for infinite time.

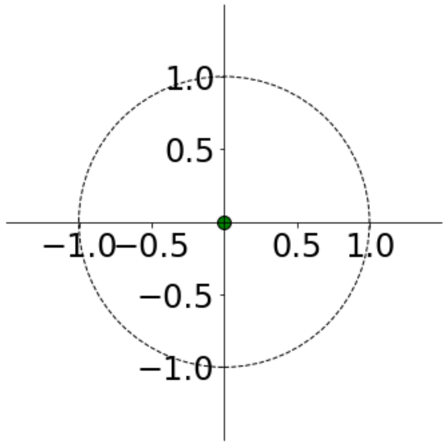

Figure 2: When $R_o > 1$

1.2

On solving the above difference equation using z-transform and inverse z-transform we get the following function which gives Number of daily infections for a given day :

$$y[n] = R_o^n \tag{1.4}$$

Since the above equation is a power equation we know that for R_o¿1 the above equation shall increase as a geometric function and is unbounded. While if R_o¡1 the above power function shall decrease and is bounded,therefore it shall eventually become zero.
It can be inferred from the pole-zero plot whether the system shall be bounded or unbounded ,nothing more can be inferred from that plot.

1.3

Solving the above time domain function for n with y[n]=1000000

$$1000000 = 2.5^n \tag{1.5}$$

We get n=15.078 days. That is by on 16th day daily infections shall pass 1 million mark.

1.4

From the data provided in the experiment it is noted that daily infections during the first wave lasting from May 2020 to September 2020 rises from 1 to 100000 in a matter of 153 days. Using some approximations we get the following equation :

$$100000 = R_o^{153} \tag{1.6}$$

Solving The above equation we get : $R_o = 1.078$

It is not a reliable technique to estimate R_o because it does not take into consideration the initial cases before first covid wave. It also assumes the function is geometrically increasing while the first covid wave had decreasing function too later on.

1.5

Plot for daily infections is as follows :

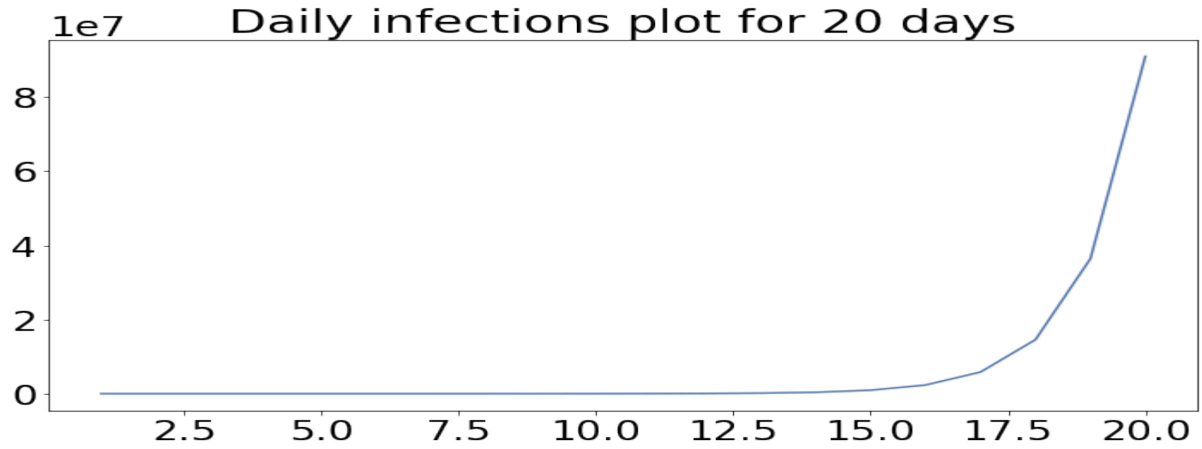

Figure 3: Daily infections plot for first 20 days using the geometric function

Equation for digital integration in z-domain is as follows :

$$I(z) = \frac{z}{z-1} \tag{1.7}$$

Applying this filter to the above plot and obtaining value for final day we get total infections upto that day :

Total infections for 20 days = 6,06,32,980

2 Problem-2

2.1

Using the filter given in the problem statement and passing Kronecker-delta as input we get the following plot for 100 days :

Using 1.7 and applying it to the given function we get the integration upto each day. So integration for 100 days is as follows :

Total infections upto 100 days = 1,69,95,618

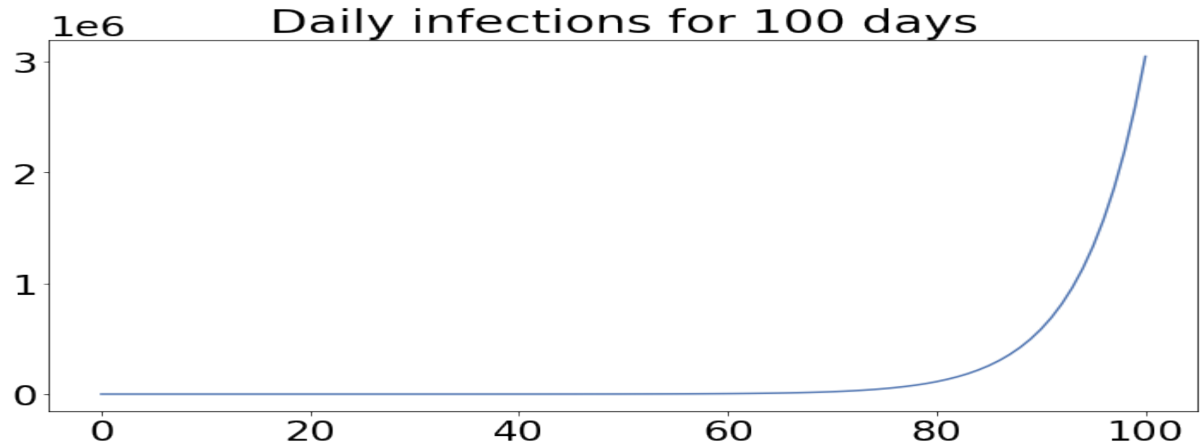

Figure 4: Daily infections plot for first 100 days using the new output function

2.2

Number of days to reach 1 million daily cases= 95 days when daily cases shall be 1.13 million.

As it can clearly be observed that number of days taken for daily infections to cross the 1 million mark has increased rapidly. While it was just 16 days for the 1st case assumption of function it stood at 96 days for the second case function. It happens so because the filter used now also considers the fact of decreasing probability of infecting some other person. While the function in problem 1 considers every person spreading the virus the very same day. So an improved assumption in function helps getting a more accurate result.

3 Problem-3

3.1

ρ refers to the fraction of people which follow social distancing.When applied to the coefficients of the above filter as $1-\rho$ it improves the considerations and decreases the assumptions.

$\rho = 1$ would be an ideal case when a person shall affect no one because everyone follows social distancing. In such a case everyone shall remain unaffected and the virus shall not spread at all.

3.2

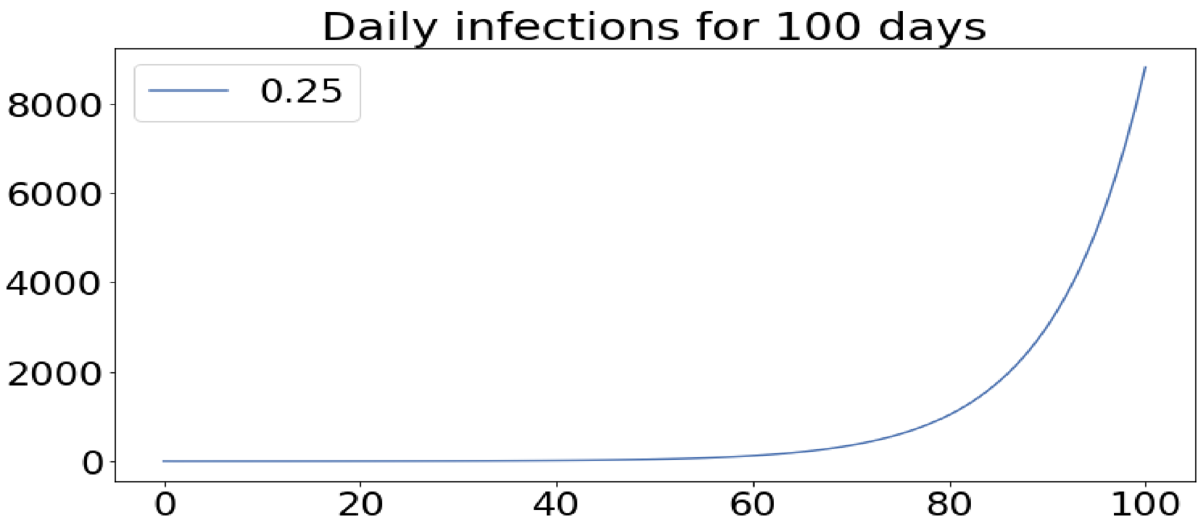

Figure 5: Daily infections plot for first 100 days for $\rho = 0.25$

Using 1.7 and applying it to the given function we get the integration upto each day. So integration for 100 days is as follows :

Total infections upto 100 days = 77,745

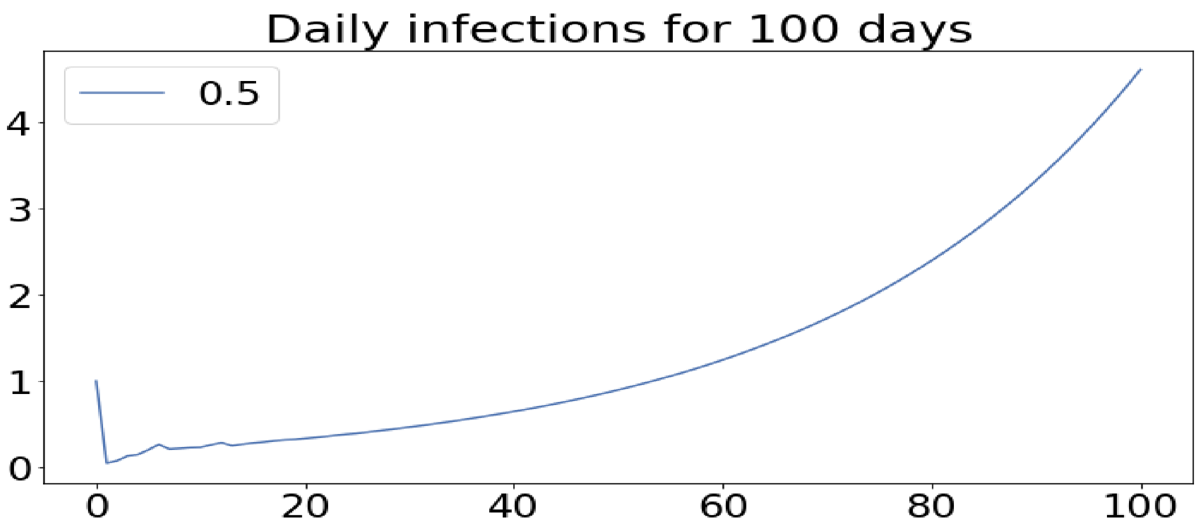

Figure 6: Daily infections plot for first 100 days for $\rho = 0.5$

Using 1.7 and applying it to the given function we get the integration upto each day. So integration for 100 days is as follows :

Total infections upto 100 days = 134

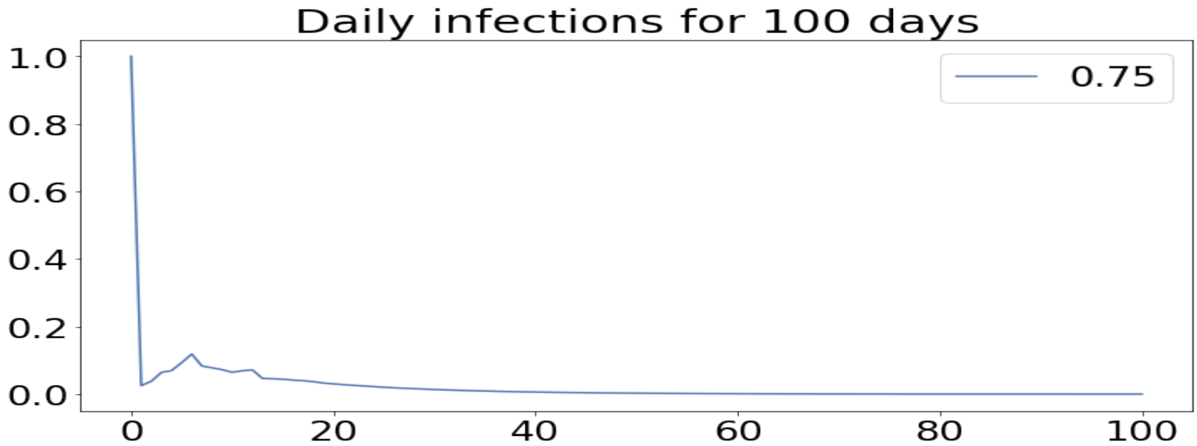

Figure 7: Daily infections plot for first 100 days for $\rho = 0.75$

Using 1.7 and applying it to the given function we get the integration upto each day. So integration for 100 days is as follows :

Total infections upto 100 days = 3

3.3

As observed in the above plots social distancing helps a great deal in reducing daily infections as well as total infections by a huge number. Social distancing means maintaining distance from an infected person and reducing the risk of getting covid to zero. As more and more number of people maintain social distancing the spread of the virus reduces. Each day the new number of infected person increases by a lower number and it brings a compound effect over a long period of time.

4 Problem-4

Up until now we have considered a non-ideal scenario of infections following an unbounded plot. In reality after some period of time the daily new infections shall reduce to zero and the cumulative number of cases shall become constant. In this problem we consider this scenario of saturation.

$$H(z) = \frac{z^100}{1 + z + z^2 + z^3 + - - - - - - - - - - - + z^{100}} \tag{4.1}$$

The input function for cumulative cases over a period of time is given by the function :

$$X[n] = \frac{R_o^{n+1}}{R_o^{n+1} + K(R_o - 1) - R_o} - \frac{1}{R_o - 1} \tag{4.2}$$

4.1

$R_o = 1.15$

$K = 1000000$

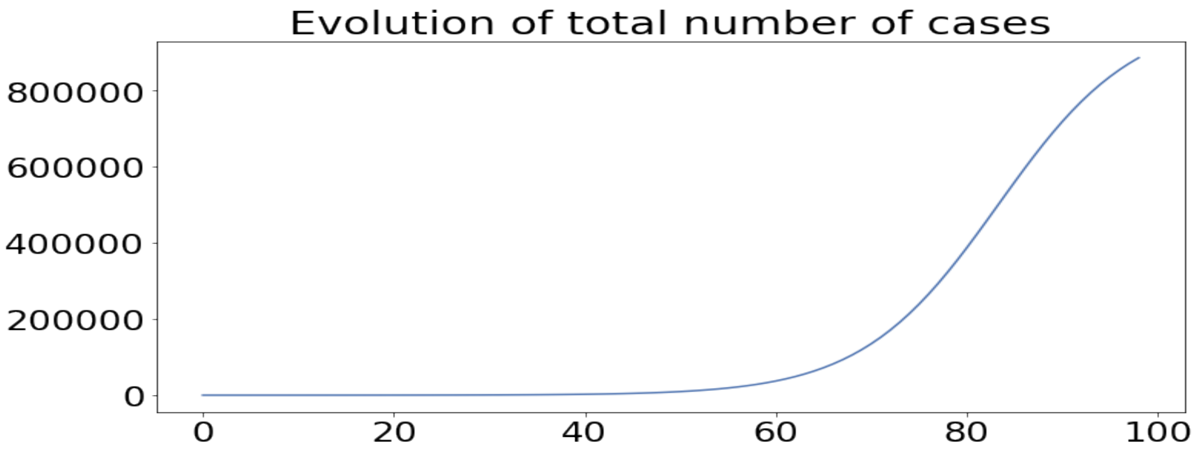

Figure 8: Total infections plot for first 100 days

Total infections upto 100 days = 9,00,025

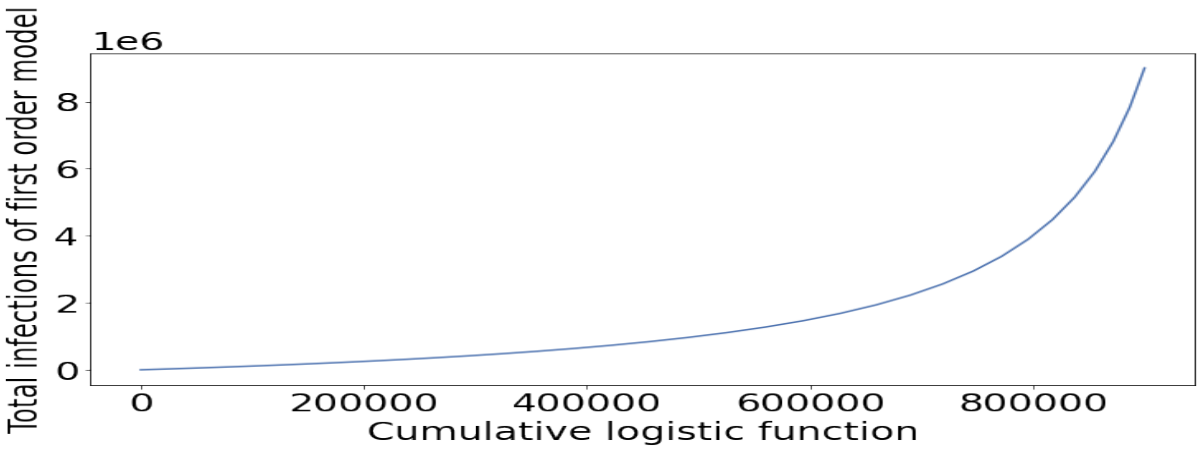

Figure 9: Total infections plot for first 100 days

The above figure shows that the two functions vary and do not bring a common result.

4.2

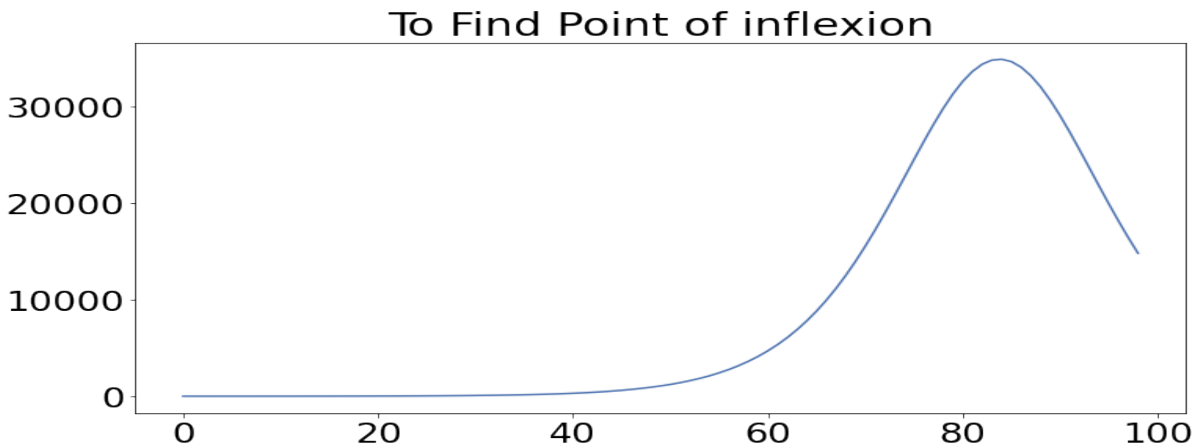

Figure 10: First derivative of the function output

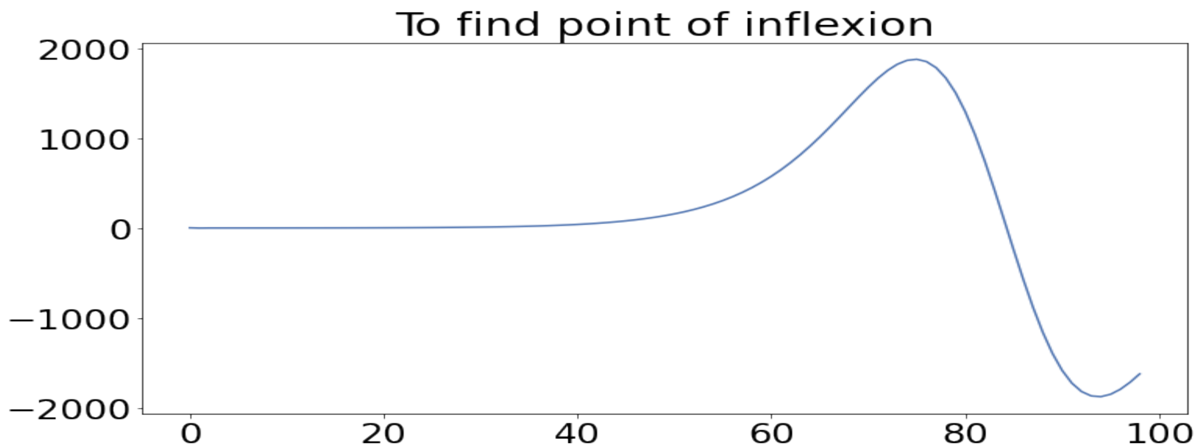

Figure 11: Second derivative of the function output

It can be observed from the above two figures that point of inflexion is 84^{th} **day.** It is calculated using the find-peaks function in python.

Note: Saturation of the total number of cases can clearly be seen for a period of 200 days, i.e. after this no more new cases are added.