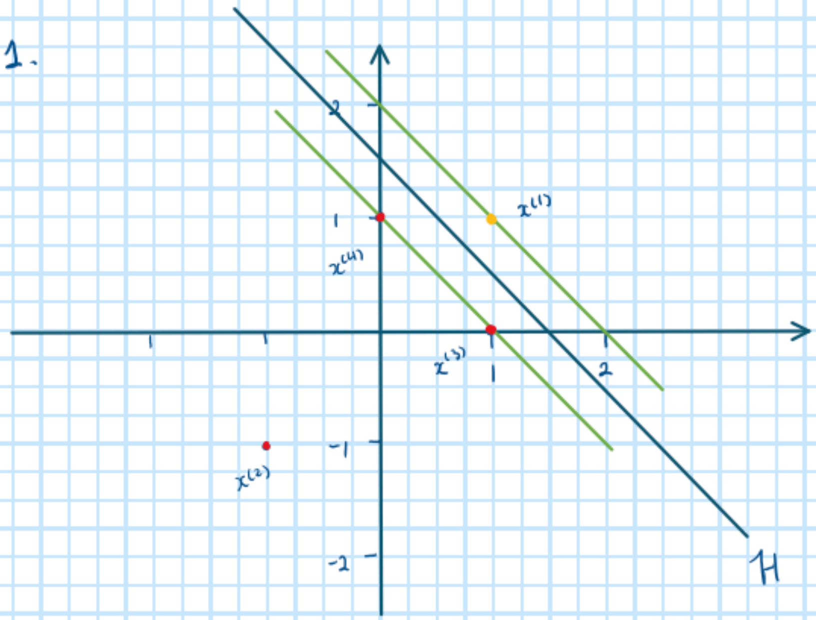
1.



• Positive: $x^{(1)}$

• Negative: $x^{(2)}, x^{(3)}, x^{(4)}$

H passes through:

y-intersection: 1.5

points: $(1.5, 0)$, $(0, 1.5)$ $\Rightarrow$ gradient $= 1$

$\therefore \boxed{y = -x + 1.5}$

2. support vectors are

$$x^{(1)}, x^{(3)}, x^{(4)}$$

3. for $\min f_0(x)$

subject to $f_i(x) \le 0 \quad i \in I$

$$\Rightarrow \mathcal{L}(x,\lambda) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) \quad , \quad p^* = \min_x \max_\lambda \mathcal{L}(x,\lambda)$$

For $\min_{\bar{w}, b} \frac{1}{2}\|\bar{w}\|^2$ s.t $\forall (\bar{w} \cdot \bar{x^{(j)}} + b) y^{(j)} \ge 1$: SVM hard margin

$$\Rightarrow \mathcal{L}(\bar{W}, b, \bar{\alpha}) = \frac{1}{2}\|\bar{w}\|^2 + \underbrace{\sum_{j}^{m} \alpha \left(1 - \bar{w} \cdot \bar{x^{(j)}} y^{(j)} - b \cdot y^{(j)}\right)}_{\circledast}$$

4. We know that maximizing $\circledast$ which is negative with parameters that minimize the function will give the p^*.

$$\therefore p^* = \min_{\bar{w}, b} \max_{\bar{\alpha}} \mathcal{L}(\bar{w}, b, \bar{\alpha})$$

$\Rightarrow$ from slater's condition

$$p^* = d^* = \max_{\bar{\alpha}} \min_{\bar{w}, b} \mathcal{L}(\bar{w}, b, \bar{\alpha})$$

minimize $\mathcal{L}(\bar{w}, b, \bar{\alpha})$ over $(\bar{w}, b)$

$$\mathcal{L}(\bar{W}, b, \bar{\alpha}) = \frac{1}{2}\|\bar{w}\|^2 + \sum \alpha^{(j)}\left(1 - \bar{w} \cdot \bar{x^{(j)}} y^{(j)} - b \cdot y^{(j)}\right) = \frac{1}{2}\|\bar{w}\|^2 + \sum \alpha^{(j)} \left(1 - y^{(j)} (\bar{w} \cdot \bar{x^{(j)}} + b)\right)$$

$$\frac{\partial \mathcal{L}}{\partial \bar{w}} = \bar{w} - \sum \alpha_j \bar{x^{(j)}} y^{(j)} = 0 \qquad \frac{\partial \mathcal{L}}{\partial b} = -\sum \alpha_j y^{(j)} = 0$$

$$\therefore \bar{w} = \sum_j a_j \bar{x^{(j)}} y^{(j)} \qquad \therefore \sum_j \alpha_j \cdot y^{(j)} = 0$$

$$\Rightarrow A = \bar{w} \quad B = a_j \quad C = y^{(j)} \quad D = \bar{x^{(j)}} \quad E = \alpha_j \quad F = y^{(j)} \quad G = 0$$

5. Substituting $\overline{w} = \sum_j \alpha_j x^{(j)} y^{(j)}$ into $\mathcal{L}(\overline{w}, b, \overline{\alpha})$

$$\mathcal{L}(\overline{w}, b, \overline{\alpha}) = \underbrace{\frac{1}{2}\|\overline{w}\|^2}_{⊛} + \underbrace{\sum \alpha_j \left(1 - \overline{w} \cdot \overline{x^{(j)}} y^{(j)} - b \cdot y^{(j)}\right)}_{⊛}$$

⊛ $\frac{1}{2}\|\overline{w}\|^2$; $\overline{w} = \sum_j \alpha_j \overline{x^{(j)}} y^{(j)}$

$$\frac{1}{2}\left\| \sum_j a^{(j)} \overline{x^{(j)}} y^{(j)} \right\|^2$$

$$= \frac{1}{2}\left(\sum_j \alpha_j \overline{x^{(j)}} y^{(j)}\right)^T \left(\sum_k a_k \overline{x^{(k)}} y^{(k)}\right)$$

$$= \frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} (\overline{x^{(j)}})^T \overline{x^{(k)}}$$; transpose only influences $\overline{x}$ term which is vector

$$= \frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} (\overline{x^{(j)}}) \cdot (\overline{x^{(k)}})$$

⊛ $\sum_j \left\{\alpha_j \left[1 - \left(\sum_i \alpha_i \overline{x^{(i)}} y^{(i)}\right) x^{(j)} y^{(j)} - b \cdot y^{(j)}\right]\right\}$

$$= \sum_j \left\{\alpha_j - \alpha_j \left(\sum_i \alpha_i \overline{x^{(i)}} y^{(i)}\right) \overline{x^{(j)}} y^{(j)} - \alpha_j y^{(j)} \cdot b\right\}$$

$$= \sum_j \alpha_j - \sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \overline{x^{(j)}}^T \overline{x^{(k)}} - \sum_j \alpha_j y^{(j)} \cdot b$$

$\because$ from 4, we know $\sum_j a_j y^{(j)} = 0$

$$\Rightarrow \sum_j \alpha_j - \sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \overline{x^{(j)}}^T \overline{x^{(k)}}$$

$\therefore$ putting back together,

$$\Rightarrow \sum_j \alpha_j + \frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \overline{x^{(j)}} \cdot \overline{x^{(k)}} - \sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \overline{x^{(j)}}\ \overline{x^{(k)}}$$

$$= \sum_j \alpha_j - \frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \overline{x^{(j)}}\ \overline{x^{(k)}}$$

$A_j = \alpha_j$ $\quad B_j = \alpha_j$ $\quad C_k = \alpha_k$ $\quad \vec{F} = \overline{x^{(j)}}$

$D_j = y^{(j)}$ $\quad E_k = y^{(k)}$ $\quad G = \overline{x^{(k)}}$

6. $j = 2$ is not a support vector.
This can be seen in plot in ①.
Therefore, a_2 has no effect on the margin.

In otherwords, as long as the point is inside the margin or on the margin.
Therefore, the x_2 which is not a support vecter has $\underline{\underline{a_2 = 0}}$.

7. $\sum_j E F_j = G \quad \Rightarrow \quad \sum_j \alpha_j y^{(j)} = 0$

for $j \neq j^* = 2$, $\sum_j \alpha_j y^{(j)} = \alpha_1(1) + \alpha_3(-1) + \alpha_4(-1) = 0$

$\therefore \alpha_1 - \alpha_3 - \alpha_4 = 0$

8.

$$\mathcal{L}(\bar{w}, b, \bar{\alpha}) = \max_{\alpha} \sum_j \alpha_j - \frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \bar{x}^{(j)T} \bar{x}^{(k)}$$

$$\underbrace{\sum_j \alpha_j \cdot y^{(j)} = 0}$$

$$\Downarrow$$

$\alpha_1 - \alpha_3 - \alpha_4 = 0$; from (17)

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial \bar{\alpha}} = 0$$

$$\frac{\partial}{\partial \bar{\alpha}} \left\{ \sum_j \alpha_j - \underbrace{\frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y^{(j)} y^{(k)} \bar{x}^{(j)T} \bar{x}^{(k)}}_{*} \right\}$$

$$* = \frac{1}{2} \begin{bmatrix} 2\alpha_1^2 & 0 & -\alpha_1\alpha_3 & -\alpha_1\alpha_4 \\ 0 & 0 & 0 & 0 \\ -\alpha_1\alpha_3 & 0 & \alpha_3^2 & 0 \\ -\alpha_1\alpha_4 & 0 & 0 & \alpha_4^2 \end{bmatrix}$$

$j=1$:	$(1,1)$	$y=1$
$j=2$:	$(-1,-1)$	$y=-1$; $\alpha_2=0$
$j=3$:	$(1,0)$	$y=-1$
$j=4$:	$(0,1)$	$y=-1$

SAMPLE CALCULATION:

$$\alpha_4\alpha_1 y^{(4)} y^{(1)} \bar{x}^{(4)T} \bar{x}^{(1)}$$

$$= \alpha_4\alpha_1 (-1)(1) [0, 1]\begin{bmatrix}1\\1\end{bmatrix}$$

$$= -\alpha_1\alpha_4$$

$$= \frac{\partial}{\partial \bar{\alpha}} \left\{ (\alpha_1 + \cancelto{0}{\alpha_2} + \alpha_3 + \alpha_4) - \left(\alpha_1^2 - \alpha_1\alpha_3 - \alpha_1\alpha_4 + \frac{\alpha_3^2}{2} + \frac{\alpha_4^2}{2}\right) \right\}$$

$$= \frac{\partial}{\partial \bar{\alpha}} \left\{ \alpha_1 + \alpha_3 + \alpha_4 - \alpha_1^2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 - \frac{\alpha_3^2}{2} - \frac{\alpha_4^2}{2} \right\}$$; $\alpha_1 - \alpha_3 - \alpha_4 = 0$, $\alpha_1 = \alpha_3 + \alpha_4$.

$$= \frac{\partial}{\partial \bar{\alpha}} \left\{ 2(\alpha_3 + \alpha_4) - \cancel{\alpha_1^2 + \alpha_1(\alpha_3 + \alpha_4)} - \frac{\alpha_3^2}{2} - \frac{\alpha_4^2}{2} \right\}$$

$$= \frac{\partial}{\partial \bar{\alpha}} \left\{ 2\alpha_3 + 2\alpha_4 - \frac{\alpha_3^2}{2} - \frac{\alpha_4^2}{2} \right\}$$

$$\frac{\partial}{\partial \alpha_3} = 2 - \alpha_3 = 0 \quad \therefore \alpha_3 = 2$$

$$\alpha_1 - \alpha_3 - \alpha_4 = 0 \Rightarrow \therefore \alpha_1 = 4$$

$$\frac{\partial}{\partial \alpha_4} = 2 - \alpha_4 = 0 \quad \therefore \alpha_4 = 2$$

$$\therefore \bar{\alpha} = \begin{bmatrix} 4 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

9. $\overline{w} = \sum_j \alpha_j \overline{x^{(j)}} y^{(j)}$ from (4)

$$= (4)\begin{bmatrix}1\\1\end{bmatrix}(1) + (2)\begin{bmatrix}1\\0\end{bmatrix}(-1) + (2)\begin{bmatrix}0\\1\end{bmatrix}(-1)$$

$$= \begin{bmatrix}4\\4\end{bmatrix} + \begin{bmatrix}-2\\0\end{bmatrix} + \begin{bmatrix}0\\-2\end{bmatrix}$$

$$= \begin{bmatrix}2\\2\end{bmatrix}$$

10.

$(\overline{w}^T x^{(j)} + b)\, y^{(j)} \geq 1$; choosing a support vector, I can set the inequality to equality

$\Rightarrow$ let $j=1$.

$$\left([2\ 2]\begin{bmatrix}1\\1\end{bmatrix} + b\right)(1) = 1$$

$\because$ support vector $\therefore$ equals 1.

$$4 + b = 1$$

$$b = -3$$

verify using different j

$$\left([2\ 2]\begin{bmatrix}1\\0\end{bmatrix} + b\right)(-1) = 1$$

$$(2+b)(-1) = 1$$

$$b = -3.$$

$$\therefore \underbrace{W = \begin{bmatrix}2\\2\end{bmatrix}}_{\text{orthogonal to hyperplane}} \qquad b = -3$$

11. From (10), I know $\overline{w} = \begin{bmatrix}2\\2\end{bmatrix}$ and $b = 3$.

$$w_1 x + w_2 y + b = 0$$

$$2x + 2y + 3 = 0$$

$$2y = -2x - 3$$

$$\boxed{y = -x - 1.5}$$ ✓ confirmed.