Practice 1

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In [23]:
```python
import numpy as np
import pandas as pd

from scipy.optimize import linprog

import matplotlib.pyplot as plt

from functools import reduce
```

In [24]:
```python
# 1.4.3
C1 = np.array([
    [2, 2, 6, 5],
    [3, 3, 7, 7],
    [4, 3, 4, 2],
    [5, 6, 2, 4],
])

# 1.5.3
C2 = np.array([
    [2, 7],
    [4, 3],
])

# 1.6.3
C3 = np.array([
    [3, 6, 1, 4, 2],
    [5, 2, 4, 2, 7],
])

# 1.7.3
C4 = np.array([
    [4, 8],
    [3, 4],
    [6, 5],
    [7, 2],
    [6, 3],
])

# 1.8.3
C5 = np.array([
    [2, 1, 3],
    [5, 2, 4],
    [3, 7, 5],
])


def intersection(a1, a2, b1, b2):
    x1, x2, x3, x4 = a1[0], a2[0], b1[0], b2[0]
    y1, y2, y3, y4 = a1[1], a2[1], b1[1], b2[1]

    d = (x1-x2)*(y3-y4)-(y1-y2)*(x3-x4)
    t = ((x1-x3)*(y3-y4)-(y1-y3)*(x3-x4))/d
    u = ((x1-x3)*(y1-y2)-(y1-y3)*(x1-x2))/d

    if 0.0<=t<=1.0 and 0.0<=u<=1.0:
        return (x1+t*(x2-x1), y1+t*(y2-y1))
    return None



def draw_to_x_axes(point, fmt="-"):
    plt.plot((point[0], point[0]), (point[1], 0), fmt)

def draw_to_y_axes(point, fmt="-"):
    plt.plot((point[0], 0), (point[1], point[1]), fmt)

def point_side(a1, a2, point):
    v1 = (a2[0] - a1[0], a2[1] - a1[1])
    v2 = (point[0] - a1[0], point[1] - a1[1])
    xp = v1[0]*v2[1]-v1[1]*v2[0]
    if abs(xp) < 0.00001: return 0
    elif xp > 0: return 1
    elif xp < 0: return -1
    return 0

def calculate_error(approx, real):
    return np.abs(approx - real) / real
```

Определеить границы выигрыша и наличие седловой точки C1

In [25]:
```python
C1
```

Out[25]:
```
array([[2, 2, 6, 5],
       [3, 3, 7, 7],
       [4, 3, 4, 2],
       [5, 6, 2, 4]])
```

In [26]:
```python
maxmin = max(np.apply_along_axis(lambda x: min(x), axis=1, arr=C1))
print(f"{maxmin=}")
```

```
maxmin=3
```

In [27]:
```python
minmax = min(np.apply_along_axis(lambda x: max(x), axis=0, arr=C1))
print(f"{minmax=}")
```

```
minmax=5
```

Седловой точки нет. Границы выигрыша (3, 5)

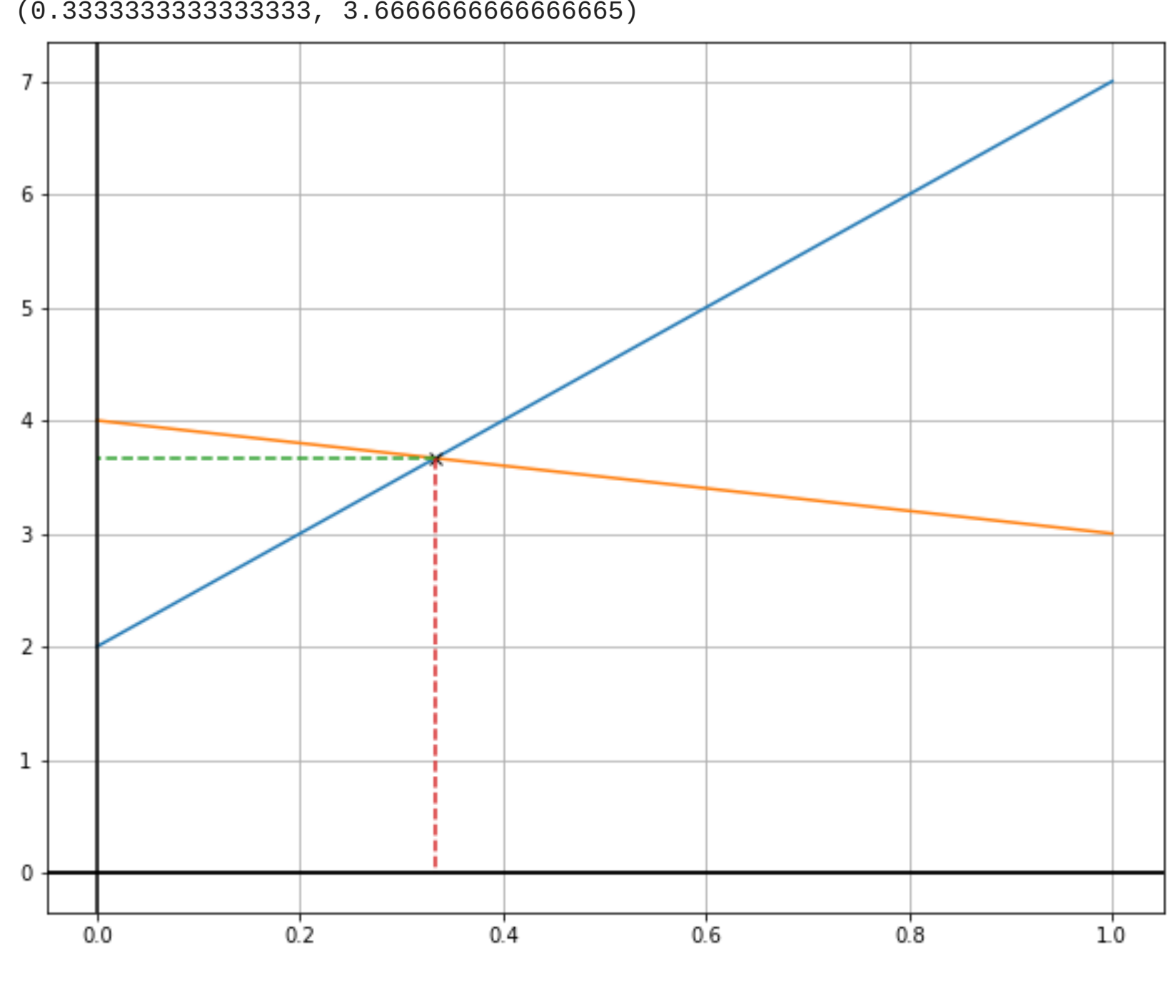
Графически и аналитически решить матричную игру 2x2 для матрицы C2

In [28]:
```python
C2
```

Out[28]:
```
array([[2, 7],
       [4, 3]])
```

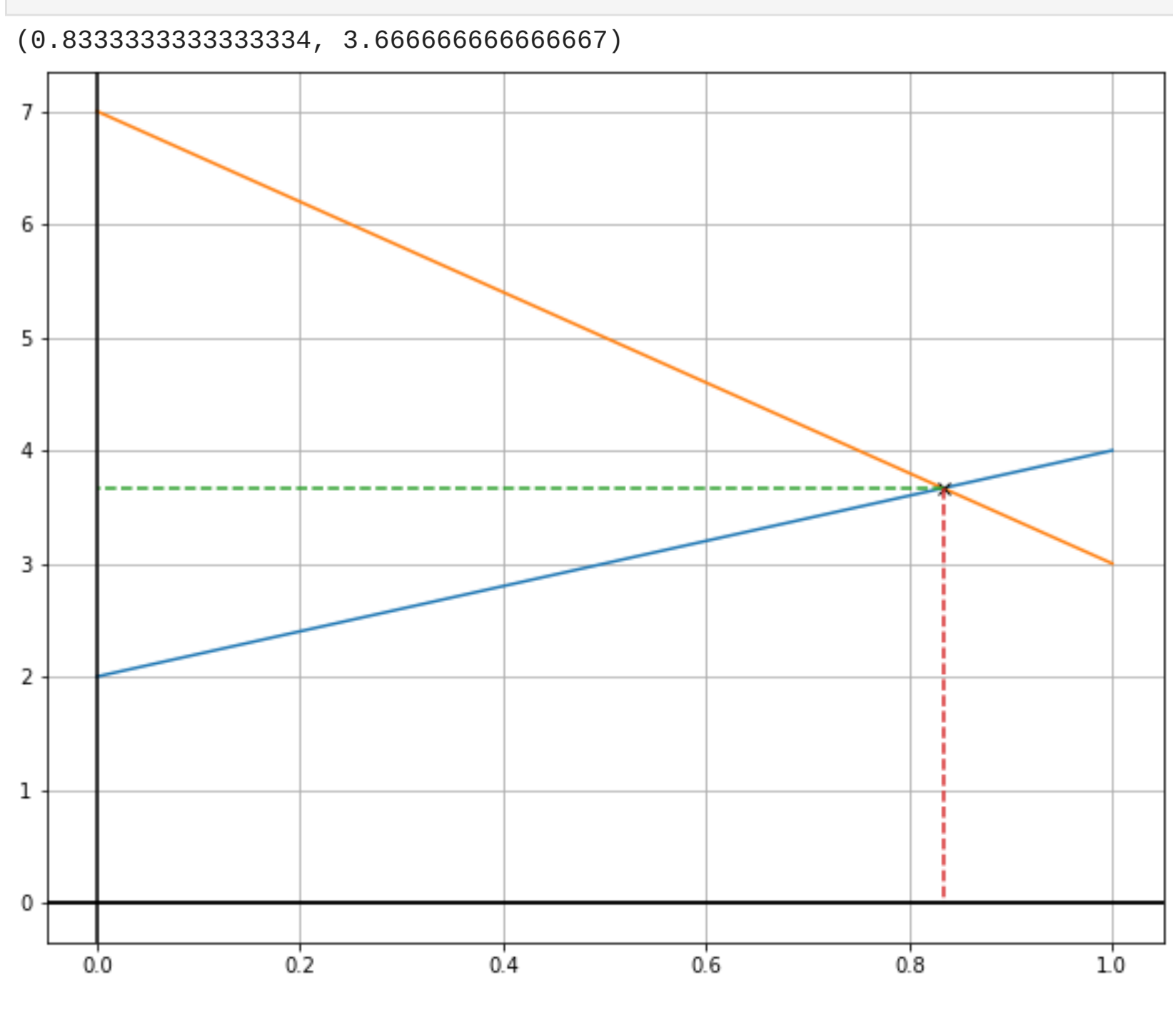
In [29]:
```python
plt.figure(figsize=(10, 8))

# draw lines
plt.plot([0, 1], C2[0])
plt.plot([0, 1], C2[1])

# find intersection
cross = intersection(
    (0, C2[0][0]),
    (1, C2[0][1]),
    (0, C2[1][0]),
    (1, C2[1][1]),
)
print(cross)

# draw intersection
plt.plot(cross[0], cross[1], "kx")
draw_to_y_axes(cross, '--')
draw_to_x_axes(cross, '--')

# beutify plot
plt.axhline(0, color='black')
plt.axhline(0, color='black')
plt.axvline(0, color='black')
plt.grid(True, "both")

# shot plot
plt.show()
```

```
(0.3333333333333333, 3.6666666666666665)
```



In [30]:
```python
plt.figure(figsize=(10, 8))

# draw lines
plt.plot([0, 1], C2.T[0])
plt.plot([0, 1], C2.T[1])

# find intersection
cross = intersection(
    (0, C2.T[0][0]),
    (1, C2.T[0][1]),
    (0, C2.T[1][0]),
    (1, C2.T[1][1]),
)
print(cross)

# draw intersection
plt.plot(cross[0], cross[1], "kx")
draw_to_y_axes(cross, '--')
draw_to_x_axes(cross, '--')

# beutify plot
plt.axhline(0, color='black')
plt.axhline(0, color='black')
plt.axvline(0, color='black')
plt.grid(True, "both")

# shot plot
plt.show()
```

```
(0.8333333333333334, 3.666666666666667)
```



In [31]:
```python
c2_p1 = (C2[1][1]-C2[1][0])/(C2[0][0]+C2[1][1]-(C2[0][1]+C2[1][0]))
c2_p2 = (C2[0][0]-C2[0][1])/(C2[0][0]+C2[1][1]-(C2[0][1]+C2[1][0]))
c2_v  = (C2[0][0]*C2[1][1]-C2[0][1]*C2[1][0])/(C2[0][0]+C2[1][1]-(C2[0][1]+C2[1][0]))
c2_q1 = (C2[1][1]-C2[0][1])/(C2[0][0]+C2[1][1]-(C2[0][1]+C2[1][0]))
c2_q2 = (C2[0][0]-C2[1][0])/(C2[0][0]+C2[1][1]-(C2[0][1]+C2[1][0]))

print("v ~", 3.7)
print("p1 ~", 1 - 0.82)
print("p2 ~", 0.82)
print("delta(v) =", calculate_error(3.7, c2_v))
print("delta(p1) =", calculate_error(1 - 0.82, c2_p1))
print("delta(p2) =", calculate_error(0.82, c2_p2))

pd.DataFrame(data={
    "p1": [c2_p1],
    "p2": [c2_p2],
    "q1": [c2_q1],
    "q2": [c2_q2],
    "v":  [c2_v],
}).round(3)
```

```
v ~ 3.7
p1 ~ 0.18000000000000005
p2 ~ 0.82
delta(v) = 0.00909090909090918
delta(p1) = 0.08000000000000035
delta(p2) = 0.0160000000000001
```

Out[31]:

	p1	p2	q1	q2	v
0	0.167	0.833	0.667	0.333	3.667

Графически и аналитически решить матричную игру 2xN для матрицы C3

In [32]:
```python
C3
```

Out[32]:
```
array([[3, 6, 1, 4, 2],
       [5, 2, 4, 2, 7]])
```

In [33]:
```python
pd.DataFrame(data={
    "maxmin": [max(np.apply_along_axis(lambda x: min(x), axis=1, arr=C3))],
    "minmax": [min(np.apply_along_axis(lambda x: max(x), axis=0, arr=C3))],
})
```

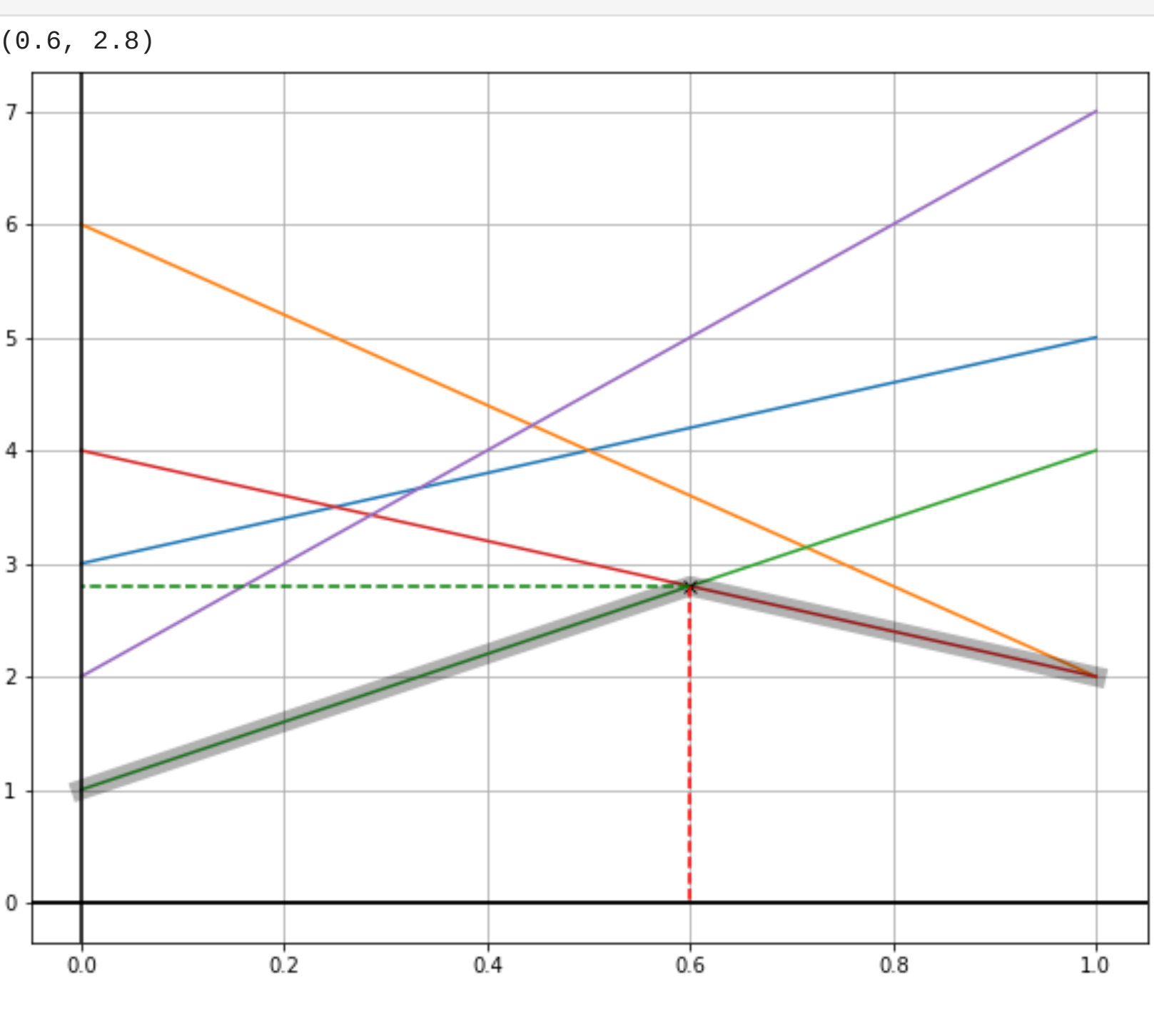
Out[33]:

	maxmin	minmax
0	2	4

In [34]:
```python
def minimal_pairs(pairs):
    minimal = []
    for vec in pairs:
        ignore = False
        remove_indices = []
        for index, min_vec in enumerate(minimal):
            if vec[0] >= min_vec[0] and vec[1] >= min_vec[1]:
                ignore = True
                break
            elif vec[0] <= min_vec[0] and vec[1] <= min_vec[1]:
                remove_indices.append(index)
        for index in sorted(remove_indices, reverse=True):
            del minimal[index]
        if not ignore:
            minimal.append(vec)
    return minimal
```

In [35]:
```python
plt.figure(figsize=(10, 8))

minimal = minimal_pairs(C3.T)
first = (0, min(vec[0] for vec in minimal))
last = (1, min(vec[1] for vec in minimal))

for line in C3.T:
    plt.plot([0, 1], line)

added = []
crosses = []
for line in minimal:
    for line__ in added:
        cross = intersection((0, line[0]),(1, line[1]),(0, line__[0]),(1, line__[1]))
        if cross:
            crosses.append(cross)
            plt.plot(cross[0], cross[1], "kx")
    added.append(line)

def check_point(point):
    for line in minimal:
        if point_side((0, line[0]), (1, line[1]), point) > 0:
            return False
    return True

crosses = list(filter(check_point, crosses))
crosses.sort(key=lambda v: v[0])

def reducer(a, b):
    if a[1] > b[1]: return a
    else: return b

v_cross = reduce(reducer, crosses)

crosses.insert(0, first)
crosses.append(last)

print(cross)

plt.plot(
    [vec[0] for vec in crosses],
    [vec[1] for vec in crosses],
    "black", linewidth=10, alpha=0.3
)

draw_to_x_axes(v_cross, "r--")
draw_to_y_axes(v_cross, "g--")

plt.axhline(0, color='black')
plt.axhline(0, color='black')
plt.axvline(0, color='black')
plt.grid(True, "both")
plt.show()
```

```
(0.6, 2.8)
```



In [36]:
```python
# 2 and 3 form a mixed strategy
c3_p1 = (C3[1][2]-C3[1][3])/(C3[0][3]+C3[1][2]-(C3[0][2]+C3[1][3]))
c3_p2 = (C3[0][3]-C3[0][2])/(C3[0][3]+C3[1][2]-(C3[0][2]+C3[1][3]))
c3_v  = (C3[0][3]*C3[1][2]-C3[0][2]*C3[1][3])/(C3[0][3]+C3[1][2]-(C3[0][2]+C3[1][3]))

print("v ~", 2.8)
print("p1 ~", 1 - 0.6)
print("p2 ~", 0.6)
print("delta(v) =", calculate_error(2.8, c3_v))
print("delta(p1) =", calculate_error(1 - 0.6, c3_p1))
print("delta(p2) =", calculate_error(0.6, c3_p2))

pd.DataFrame(data={
    "p1": [c3_p1],
    "p2": [c3_p2],
    "v":  [c3_v],
}).round(3)
```

```
v ~ 2.8
p1 ~ 0.4
p2 ~ 0.6
delta(v) = 0.0
delta(p1) = 0.0
delta(p2) = 0.0
```

Out[36]:

	p1	p2	v
0	0.4	0.6	2.8

Графически и аналитически решить матричную игру Nx2 для матрицы C4

In [37]:
```python
C4
```

Out[37]:
```
array([[4, 8],
       [3, 4],
       [6, 5],
       [7, 2],
       [6, 3]])
```

In [38]:
```python
pd.DataFrame(data={
    "maxmin": [max(np.apply_along_axis(lambda x: min(x), axis=1, arr=C4))],
    "minmax": [min(np.apply_along_axis(lambda x: max(x), axis=0, arr=C4))],
})
```

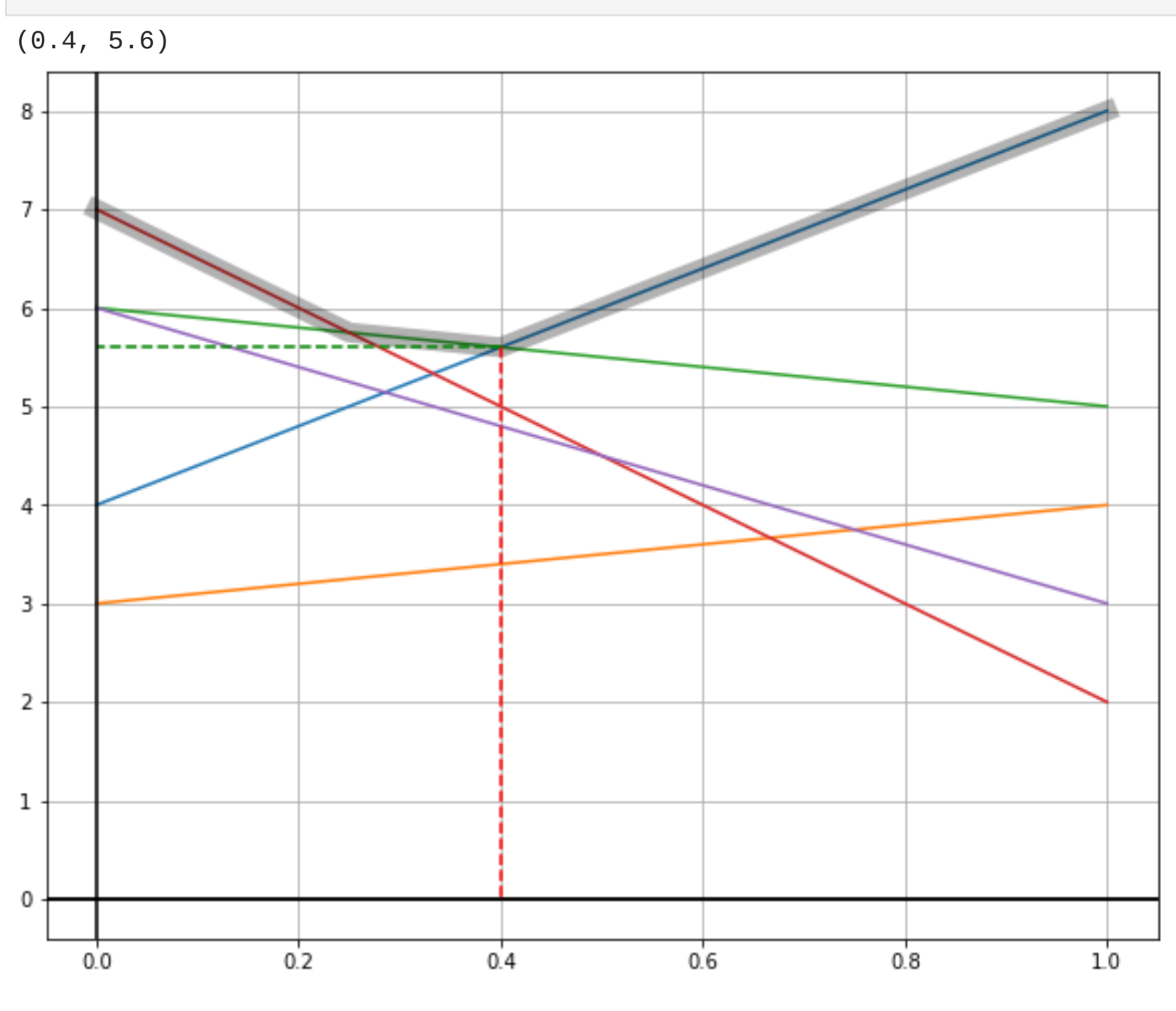
Out[38]:

	maxmin	minmax
0	5	7

In [39]:
```python
def maximal_pairs(pairs):
    maximal = []
    for vec in pairs:
        ignore = False
        remove_indices = []
        for index, min_vec in enumerate(maximal):
            if vec[0] <= min_vec[0] and vec[1] <= min_vec[1]:
                ignore = True
                break
            elif vec[0] >= min_vec[0] and vec[1] >= min_vec[1]:
                remove_indices.append(index)
        for index in sorted(remove_indices, reverse=True):
            del maximal[index]
        if not ignore:
            maximal.append(vec)
    return maximal
```

In [40]:
```python
plt.figure(figsize=(10, 8))

maximal = maximal_pairs(C4)
first = (0, max(vec[0] for vec in maximal))
last = (1, max(vec[1] for vec in maximal))

for line in C4:
    plt.plot([0, 1], line)

added = []
crosses = []
for line in maximal:
    for line__ in added:
        cross = intersection((0, line[0]),(1, line[1]),(0, line__[0]),(1, line__[1]))
        if cross: crosses.append(cross)
    added.append(line)

def check_point(point):
    for line in maximal:
        if point_side((0, line[0]), (1, line[1]), point) < 0:
            return False
    return True

crosses = list(filter(check_point, crosses))
crosses.sort(key=lambda v: v[0])

def reducer(a, b):
    if a[1] < b[1]: return a
    else: return b

v_cross = reduce(reducer, crosses)

crosses.insert(0, first)
crosses.append(last)

print(v_cross)

plt.plot(
    [vec[0] for vec in crosses],
    [vec[1] for vec in crosses],
    "black", linewidth=10, alpha=0.3
)

draw_to_x_axes(v_cross, "r--")
draw_to_y_axes(v_cross, "g--")

plt.axhline(0, color='black')
plt.axhline(0, color='black')
plt.axvline(0, color='black')
plt.grid(True, "both")
plt.show()
```

```
(0.4, 5.6)
```



In [41]:
```python
# 0 and 2 form a mixed strategy
c4_q1 = (C4[0][1]-C4[2][1])/(C4[2][0]+C4[0][1]-(C4[2][1]+C4[0][0]))
c4_q2 = (C4[2][0]-C4[0][0])/(C4[2][0]+C4[0][1]-(C4[2][1]+C4[0][0]))
c4_v  = (C4[0][1]*C4[2][0]-C4[2][1]*C4[0][0])/(C4[2][0]+C4[0][1]-(C4[2][1]+C4[0][0]))

print("v ~", 5.6)
print("q1 ~", 1 - 0.4)
print("q2 ~", 0.4)
print("delta(v) =", calculate_error(5.6, c4_v))
print("delta(q1) =", calculate_error(1 - 0.4, c4_q1))
print("delta(q2) =", calculate_error(0.4, c4_q2))

pd.DataFrame(data={
    "q1": [c4_q1],
    "q2": [c4_q2],
    "v":  [c4_v],
}).round(3)
```

```
v ~ 5.6
q1 ~ 0.6
q2 ~ 0.4
delta(v) = 0.0
delta(q1) = 0.0
delta(q2) = 0.0
```

Out[41]:

	q1	q2	v
0	0.6	0.4	5.6

С помощью симплекс-метода решить матричную игру MxN для матрицы C5

In [42]:
```python
C5
```

Out[42]:
```
array([[2, 1, 3],
       [5, 2, 4],
       [3, 7, 5]])
```

In [43]:
```python
pd.DataFrame(data={
    "maxmin": [max(np.apply_along_axis(lambda x: min(x), axis=1, arr=C5))],
    "minmax": [min(np.apply_along_axis(lambda x: max(x), axis=0, arr=C5))],
})
```

Out[43]:

	maxmin	minmax
0	3	5

$2p_1 + 5p_2 + 3p_3 \geq V$

$1p_1 + 2p_2 + 7p_3 \geq V$

$3p_1 + 4p_2 + 5p_3 \geq V$

$1p_1 + 1p_2 + 1p_3 = 1$

---

divide both sides by V, replace: $x_i = p_i / V$

$2x_1 + 5x_2 + 3x_3 \geq 1$

$1x_1 + 2x_2 + 7x_3 \geq 1$

$3x_1 + 4x_2 + 5x_3 \geq 1$

$1x_1 + 1x_2 + 1x_3 = 1 / V$

---

maximize V i.e. minimize $Z = 1 / V$

$2x_1 + 5x_2 + 3x_3 \geq 1$

$1x_1 + 2x_2 + 7x_3 \geq 1$

$3x_1 + 4x_2 + 5x_3 \geq 1$

$1x_1 + 1x_2 + 1x_3 = Z$

$Z \rightarrow min$

Linear programming problem

In [44]:
```python
objective_funciton_coefs = np.array([1, 1, 1])
constraint_matrix = C5.T * -1
constraint_vector = np.array([-1, -1, -1])
np.warnings.filterwarnings('ignore', category=np.VisibleDeprecationWarning)
result = linprog(
    objective_funciton_coefs,
    A_ub=constraint_matrix,
    b_ub=constraint_vector
)

print("V =", 1/result.fun)
print("p =", result.x/result.fun)
```

```
V = 4.142857140161906
p = [2.96616295e-10 5.71428571e-01 4.28571429e-01]
```