Optimizing the depth of the Mottonen state preparation circuit in PennyLane

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[Abstract]

My team worked on an exact method for amplitude embedding. When gate operations can be performed simultaneously on different qubits, the execution speed of a quantum circuit depends on the depth of the circuit. Compared to the `qml.MottonenStatePreparation` circuit implemented in PennyLane, we reduced the depth of the amplitude embedding circuit by $\simeq$25% for >10 qubits by optimizing the placement of the uniformly controlled Pauli-Z rotation (UCRZ) gates based on [1].

[Introduction]

In Möttönen et al.'s paper [2], to convert $|0\rangle$ state into an arbitrary state

$$|x\rangle = |x_0| e^{i\omega_0} |0\rangle + |x_1| e^{i\omega_1} |1\rangle + \cdots + |x_{N-1}| e^{i\omega_{N-1}} |N-1\rangle,$$

first the absolute values $(|x_0|, |x_1|, \ldots |x_{N-1}|)$ are embedded using uniformly controlled Pauli-Y rotation (UCRY) gates, and then the phases $(e^{i\omega_0}, e^{i\omega_1}, \ldots, e^{i\omega_{N-1}})$ are embedded using uniformly controlled Pauli-Z rotation (UCRZ) gates. The set of UCRZ gates can be regarded as a diagonal gate that adds different phases to all computational bases. Zhang et al. proposed an algorithm to reduce the circuit depth by changing the arrangement of UCRZ gates [1], which can also be applied to generating amplitude embedding circuits.

[Methods]

As shown in Fig. 1 from [1], any n-qubit UCRZ gate can commute with the control of CNOT gates.

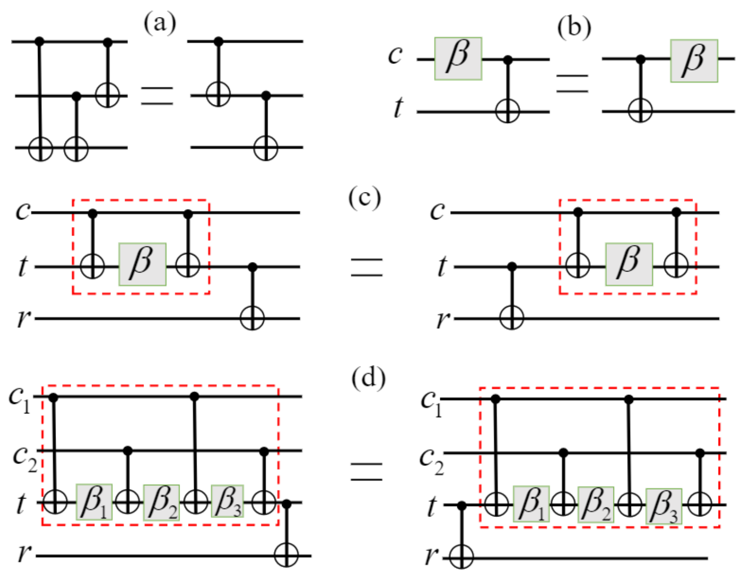

Fig. 1: Commutation and rewriting rules for $\{\mathrm{CNOT}, R_Z\}$ circuits. The box on the circuit with β represents $R_Z(-\beta)$ gates. Image from figure 2 in [1].

By using this property, we can reduce the depth of the circuit by placing all the $(2 \sim N-1)$-qubit UCRZ gates inside the N-qubit UCRZ gate.

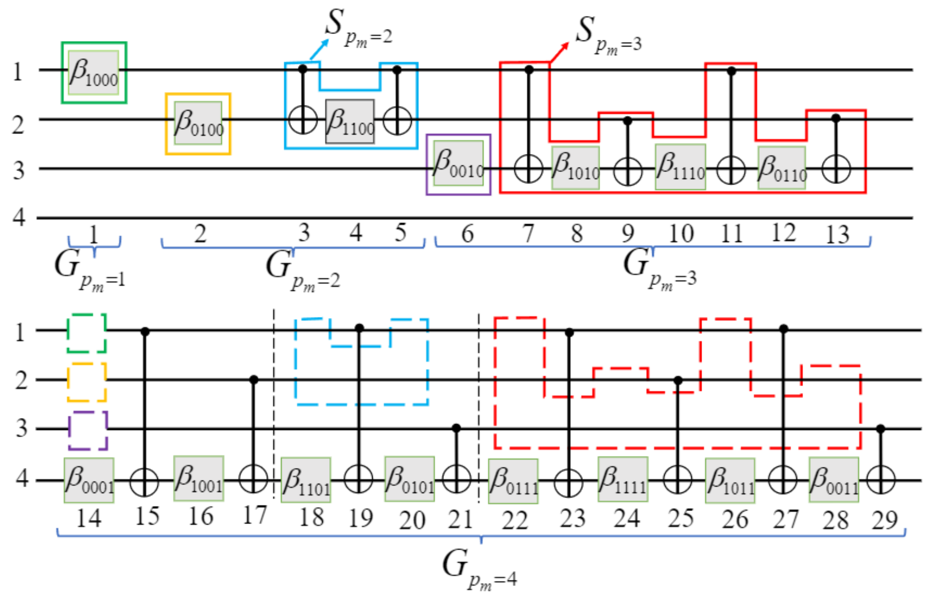

Fig. 2: A 4-qubit circuit representing $2^4 \times 2^4$ diagonal unitary matrix. The box on the circuit with β represents $R_Z(-\beta)$ gates. Image from figure 4 in [1].

To achieve this, we implemented the algorithm proposed in [1] by PennyLane and compared the depth of the circuit with the original `qml.MottonenStatePreparation` circuit. The new implementation is available at `opt.py`.

[Results and Discussion]

We compared the depth of the amplitude embedding circuit for arbitrary $N = 1, 2, \ldots, 11$ qubit state vectors. As shown in Fig. 4, the depth of the circuit is reduced by $\simeq$25% for >10 qubits. Codes for the following figures are available at `make_graph.ipynb`.

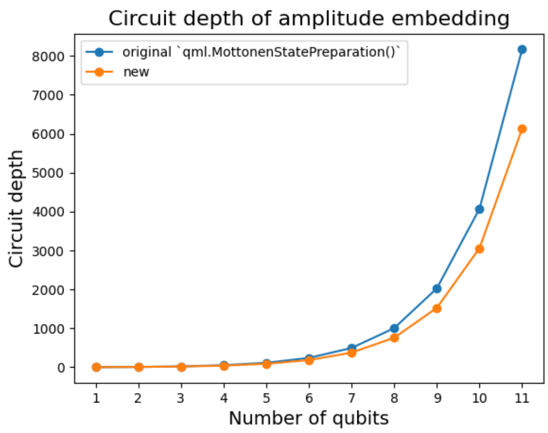

Fig. 3: Comparison of the depth of the circuit.

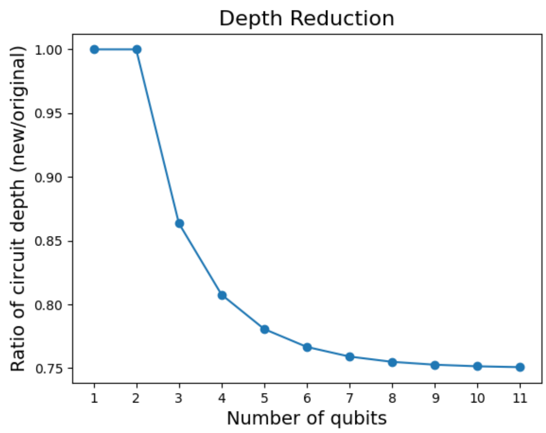

Fig. 4: Ratio of the depth of the circuit.

[References]

[1] S. Zhang, K. Huang, and L. Li, "Automatic Depth-Optimized Quantum Circuit Synthesis for Diagonal Unitary Matrices with Asymptotically Optimal Gate Count," (2022).

[2] M. Mottonen, J. J. Vartiainen, V. Bergholm, and M. M. Salomaa, "Transformation of quantum states using uniformly controlled rotations," (2004).

[Appendix]

To create an adjoint of a UCRZ gate, we can either apply the gates of a UCRZ gate from the last to the first or negate the angles of the UCRZ gates. Thus, reversing the gate sequence and negating the angles of the UCRZ gates simultaneously will give the same unitary matrix up to a global phase. If we place a UCRZ gate next to UCRY using the commutation rule in Fig. 1, we can cancel out the adjacent CNOT gates in between the UCRZ and UCRY gates, but this will not improve the depth of the circuit a lot compared to the previous method. This implementation is available at `cancel_cnots.py`.

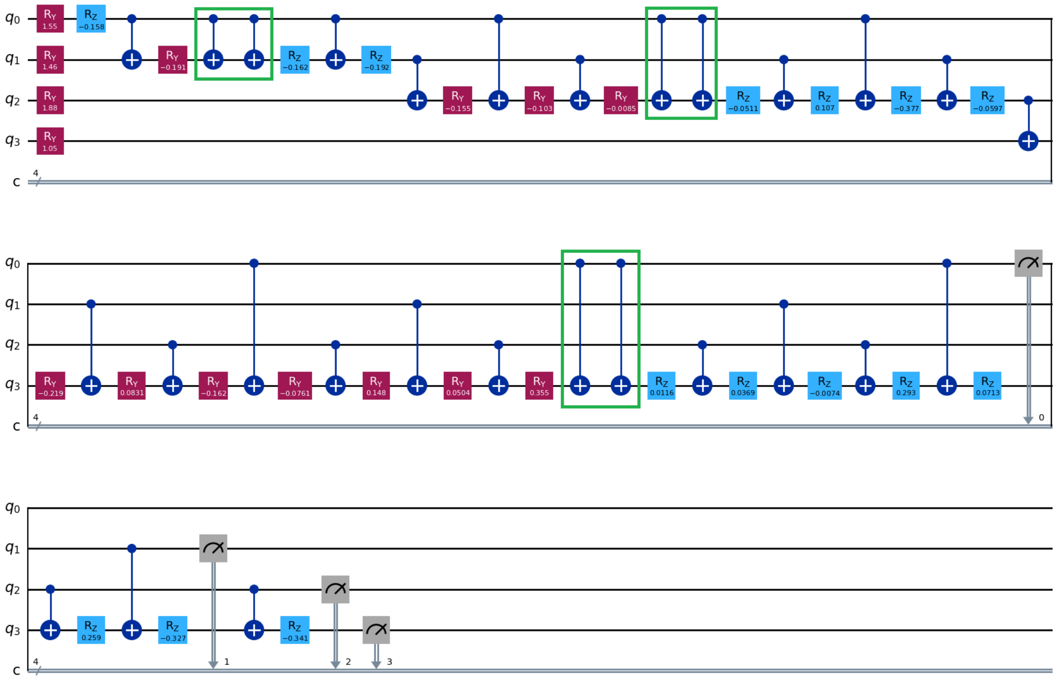

Fig. 5: A quantum circuit to embed a 4-qubit state vector. CNOT gates in the green box can be canceled out.