Airplane Wing Analysis

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1 Introduction

Vibrations caused on airplane is not an unusual phenomenon. There are many different causes of vibration, which include landing gears extensions, speed breaks and movement on a free surface. There are also different types of vibrations, including buffet, flutter, and noise. Vibration itself is an oscillating motion, when the frequency and magnitude are constant, the vibration is constant. If frequency and magnitude vary with respect to time, it is random.Modal analysis is a common method to understand certain dynamic behaviors on mechanical structures. They help us find the reasons for vibration and initiate ways to reduce it. This type of analysis helps us improve performance. This is what we want to learn from this experiment.

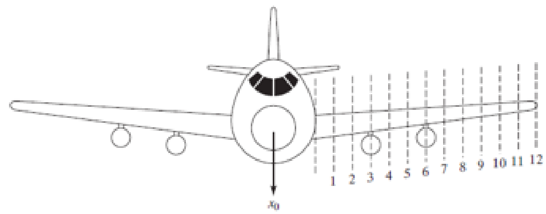
The vibration analysis of an airplane wing is one of the toughest problems in designing an aircraft. This paper focuses strictly on vibrations on the wing of an airplane, and how movements caused on the fuse luge affect it. For the sake of simplicity we assume the movement on the fuse luge as a harmonic force.

The question at hand is given to us in the form of 12 degrees of freedom; where the wing is split up of. We are also given the first 3 vibration modes of the system and we must find the dynamic response of the wing as a linear combination of the first 3 modes. By keeping certain variables constant for each mode, we can derive the 12 responses $x(t)$ with our given information. Figure 1 includes the airplane wing in question and the given values of the modes.

We approach this question using a form of modal analysis, where we are only given partial information by the modal matrix. The wing of the aircraft is analogously considered to be a cantilever beam where one end is fixed. This helps us to visually understand how the wing behaves in different vibration modes.

2 Literature

A handful of literature is seen in order to conduct vibrations on a plane. This is seen to increase performance and safety to an otherwise critical component on an airplane. One study related a NACA 4415 airplane wing exactly like a cantilever beam. The study split a beam and a wing into 6 degrees of freedom



Degrees of Freedom													
Mode Shape	0	1	2	3	4	5	6	7	8	9	10	11	12
$\overrightarrow{X}^{(1)}$	0.0	0.126	0.249	0.369	0.483	0.589	0.686	0.772	0.846	0.907	0.953	0.984	1.000
$\overrightarrow{X}^{(2)}$	0.0	−0.375	−0.697	−0.922	−1.017	−0.969	−0.785	−0.491	−0.127	0.254	0.599	0.860	1.000
$\overrightarrow{X}^{(3)}$	0.0	0.618	1.000	1.000	0.618	0.000	−0.618	−1.000	−1.000	−0.618	0.000	0.618	1.000

Figure 1: Airplane wing and modal matrix

with similar properties, such as stiffness and mass, and provided numerical and experimental results of low percent error between the two (Demetras, Bayraktar, 2019). Another paper studies the static and dynamic characteristics of an aircraft wing in order to see the effects of fuel stores(Rajapan and Pugazhenthi,2013). It was concluded that natural frequency actually decreased about 66 percent given from mode 1 to mode 4 of their analysis. Vibration analysis of a beam has also been heavily conducted, in its geometry is very similar to many structures and applications in engineering. Papers have reported different natural frequencies with different materials such as aluminum, brass, or steel and different parameters(Nirmall and Vimala,2016). Unfortunatly for an airplane wing, different materials and different shapes are used for its construction, so for the sake of simplicity it is critical to treat it as a beam to understand its vibration modes.

3 Cantilever Beam Comparison

We will take a hypothetical example, taken from previous literature, to mask out our problem. Our question gives no information on critical components, such as mass,stiffness, or damping, even so we do not have information on the wings elastic modulus, length or mass density. Fortunately, we do have the full modes, and we want to see how the wing reacts in these modes. For this reason, we take a cantilever beam example.

To understand our question, we set up a cantilever beam comparison for 3 modes of vibration in parallel for our question, were we are given 3 modes. Its imperative to understand physically what is going on with the wing. Since we only care about vibration mode, our parameters for a hypothetical beam do not have to be the same as our airplane wing.

Lets take for example a beam such as Figure 2.

We apply boundary conditions by stating that the left of the beam is fixed on the wall and the right is free. These boundary conditions go hand to hand

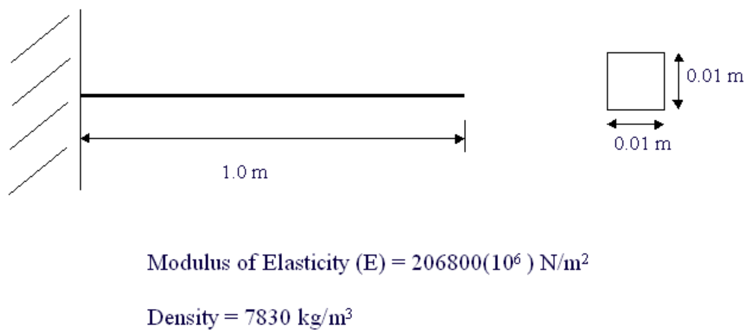


Figure 2: Note that this is a previously derived example

with our airplane wing.

The mass can be found by multiplying density area and length $m = \rho AL$ and the stiffness is $k = \frac{3EI}{l^3}$. The length L can be divided respectively to the degrees of freedom.

Now by using simple ANSYS features, we can conduct a modal analysis by inserting our number of modes, and in this specific example the modes chosen is 5, and using subspace modal analysis to conduct on all degrees of freedom. We also want to make sure its normalized against the mass, which is taken as a lump sum. By analyzing the results, we receive the modes and there corresponding frequency

Mode	Frequency
1	8.3
2	52.01
3	145.64
4	285.51
5	472.54

Note that the frequency is increasing in respect to mode.

With this information we arrive at the vibration modes that we are interested in. The first 3 modes that we want to observe are given in Figure 3.

With the images given in Figure 3, we can comparatively see how the airplane wing will act in the respective modes 1,2,and 3 given in the experimental modal matrix. We can also assume that natural frequencies will increase with increasing modes, as seen in our cantilever example.

4 Theoretical Analysis

We start off by determining the dynamic response of the wing. By using modal analysis, this can be written as

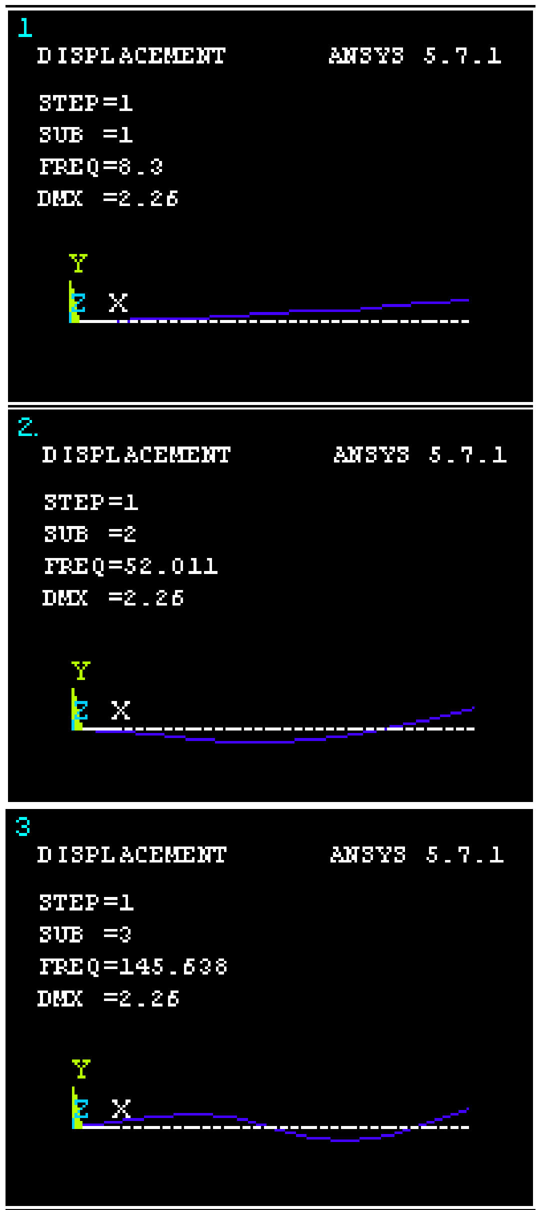


Figure 3: The 3 vibration modes of our cantilever example

$$x(t)_{12x1} = [X]_{12x3} q(t)_{3x1}$$

Where *x(t)* is the response, $[X]$ is our experimentally derived modal matrix and *q(t)* is our response in principle coordinates. We also wrote the dimensions of the variables in the equation to determine how many responses we would get. Fortunately, we get the same number of responses as the degrees of freedom split on the wing, even though the question only gives the first 3 modes in the modal matrix.

To derive our principle coordinate system, certain factors must be taken into consideration. The first thing to consider is that the equation of motion is considered force excited from the base. This makes sense, since an airplane is subjected to some force given on an uneven runway or downward drag. For the sake of simplicity, we will assume that this base force is harmonic, which will be seen later on. If this is the case our equation of motion is written as:

$$[m]\ddot{\vec{x}} + [c](\dot{\vec{x}} - \dot{x}_0 \vec{u_1}) + [k](\vec{x} - x_0 \vec{u_1}) = 0$$

where u is a unit vector $\vec{u} = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

The next thing to consider is that there is an initial displacement $x_0(t)$ but an initial velocity is not given so it is assumed zero. With a couple linear algebra manipulations, our equation becomes:

$$[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = -x_0[m]\vec{u}$$

where we can consider $-x_0[m]\vec{u}$ our vertical force *F(t)* to the fuselage.

To find the dynamic response we turn to modal analysis, which allows us to find n uncoupled equations in principle space *q(t)* where n is the number of degrees of freedom. In this case we are only given 3 modes so only 3 uncoupled equations can be derived. This is still okay, since we can still derive 12 responses as a linear combination of the first 3 modes and their principle space. To derive these uncoupled equations, we multiply the transpose of our modal matrix to our equation of motion, and we also assume orthogonality against the mass matrix:

$$[X]^T[m]\ddot{\vec{x}} + [X]^T[c]\dot{\vec{x}} + [X]^T[k]\vec{x} = [X]^T F\vec{(t)}$$

which leads to:

$$\ddot{\vec{q}}_i + 2\zeta_i \omega_i^2 \dot{\vec{q}}_i + \omega_i^2 \vec{q}_i = Q\vec{(t)}_i$$

i=1,2,3 for each mode and we can derive $\vec{q}$.

To get started we first find the force $-x_0[m]\vec{u}$,
we assume that the mass matrix is dynamically uncoupled and diagonal:

$$-x_0(t)\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0... \\ 0 & m_2 & 0 & 0 & 0 & 0... \\ 0 & 0 & m_3 & 0 & 0 & 0... \\ 0 & 0 & 0 & m_4 & 0 & 0... \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ ...0 & 0 & 0 & 0 & 0 & m_{12} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} = -x_0(t)\begin{bmatrix} m_1 \\ 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} = -x_0(t)m_1\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}$$

Now to find $\vec{Q(t)}_i$ we multiply this by the transpose of our given modal matrix, disregarding mode 0, given by:

$$[X] = \begin{bmatrix} 0.126 & -0.375 & 0.618 \\ 0.249 & -0.697 & 1 \\ 0.369 & -0.922 & 1 \\ 0.483 & -1.017 & 0.618 \\ 0.589 & -0.969 & 0 \\ 0.686 & -0.785 & -0.618 \\ 0.772 & -0.491 & -1 \\ 0.846 & -0.127 & -1 \\ 0.907 & 0.254 & -0.618 \\ 0.953 & 0.599 & 0 \\ 0.984 & 0.86 & 0.618 \\ 1 & 1 & 1 \end{bmatrix}$$

to

$$\vec{Q(t)} = \begin{bmatrix} 0.126 & -0.375 & 0.618 \\ 0.249 & -0.697 & 1 \\ 0.369 & -0.922 & 1 \\ 0.483 & -1.017 & 0.618 \\ 0.589 & -0.969 & 0 \\ 0.686 & -0.785 & -0.618 \\ 0.772 & -0.491 & -1 \\ 0.846 & -0.127 & -1 \\ 0.907 & 0.254 & -0.618 \\ 0.953 & 0.599 & 0 \\ 0.984 & 0.86 & 0.618 \\ 1 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} (-x_0(t)m_1) = \begin{bmatrix} 0.126 \\ -0.375 \\ 0.618 \end{bmatrix} (-x_0(t)m_1)$$

Now we must compute $q(t)$ but this becomes difficult since we are not given any information about damping, stiffness or mass matrix. Also to initiate initial condition in principle space, modal analysis insist we use $q_{x_0} = [X]^T * [m]\vec{u} * x_0(t)$ but this derivation is difficult since $[m]$ is not diagonal nor symmetric, and matrix multiplication with initial displacement becomes impossible.

A unique approach to the solution is to assume harmonic force from the base, i.e. $x_0(t) = cos(\omega t)$, and refer to the steady state solution for the response of a damped system under harmonic excitation. Since $q(t)$ is in principle space resulting in 3 individual uncoupled vibration system, we can analogously use 3 individual systems that are harmonically excited. This solution is written as:

$$m\ddot{\vec{x}} + c\dot{\vec{x}} + k\vec{x} = F_0 cos(\omega t)$$

and

$$x_p(t) = X_p cos(\omega + \phi)$$

where $X_p = \frac{F_0/k}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ and $\phi = tan^{-1}[\frac{2\zeta r}{1-r^2}]$; r= $\frac{\omega}{\omega_n}$; $\zeta = \frac{c}{c_c}$

In parallel we can write this in principle space:

$$q_i(t) = q_{i0} cos(\omega + \phi_i)$$

where $q_{i0}(t) = \frac{Q_{i0}/\omega_{ni}^2}{\sqrt{(1-r^2)^2+(2\zeta_i r)^2}}$ and $\phi_i = tan^{-1}[\frac{2\zeta_i r}{1-r^2}]$; r= $\frac{\omega}{\omega_{ni}}$

An important thing to notice is that $F_0 = Q_0$ and $k = \omega_{ni}^2$ due to orthogonality. Since we do not know the damping frequency, forcing frequency nor the damping ratio for any of the modes its, best to keep these variables a constant for each principle mode. Hence:

$$Y_i = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta_i r)^2}}; q_i(t) = \frac{Q_{i0}}{\omega_{ni}} Y_i cos(\omega + \phi_i)$$

The 3 principle coordinates can now be determined, with natural frequency $\omega_1 = 225$, $\omega_2 = 660$, and $\omega_3 = 1100$ as:

$$q(t) = \begin{bmatrix} \frac{0.126}{\omega_1^2} \\ \frac{-0.375}{\omega_2^2} \\ \frac{0.618}{\omega_3^2} \end{bmatrix} (-m_1 Y_i cos(\omega+\phi_i)); \begin{bmatrix} \frac{0.126}{(225)^2} \\ \frac{-0.375}{(660)^2} \\ \frac{0.618}{(1100)^2} \end{bmatrix} (-m_1 Y_i cos(\omega+\phi_i)); \begin{bmatrix} 0.2489 \\ -0.0861 \\ 0.0511 \end{bmatrix} x10^{-5}(-m_1 Y_i cos(\omega+\phi_i))$$

We return to our original equation to find the dynamic response, now that we have our 3 uncoupled equations and 3 modes in our modal matrix:

$$x(t)_{12x1} = [X]_{12x3} q(t)_{3x1}$$

to:

$$x(t) = \begin{bmatrix} 0.126 & -0.375 & 0.618 \\ 0.249 & -0.697 & 1 \\ 0.369 & -0.922 & 1 \\ 0.483 & -1.017 & 0.618 \\ 0.589 & -0.969 & 0 \\ 0.686 & -0.785 & -0.618 \\ 0.772 & -0.491 & -1 \\ 0.846 & -0.127 & -1 \\ 0.907 & 0.254 & -0.618 \\ 0.953 & 0.599 & 0 \\ 0.984 & 0.86 & 0.618 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.2489 \\ -0.0861 \\ 0.0511 \end{bmatrix} x10^{-5}(-m_1 Y_i cos(\omega + \phi_i))$$

Its important to notice that the i values correspond to the first 3 modes, since the answer should be a linear combination of the first 3 modes, but if we want to further simplify our answer, with simple matrix multiplication done with MAT LAB software, we come to the answer:

$$x(t) = \begin{bmatrix} 0.0952 \\ 0.1731 \\ 0.2223 \\ 0.2393 \\ 0.23 \\ 0.2068 \\ 0.1833 \\ 0.1704 \\ 0.1723 \\ 0.1856 \\ 0.2024 \\ 0.2139 \end{bmatrix} x10^{-5}(-m_1 Y_i cos(\omega + \phi_i))$$

5 Numerical Analysis

In addition to what we have derived in our theoretical analysis, we must also must see how each response may look like with respect to time. We can do this by using software such as mathamatica and imputing values for our mass matrix. Since we do not have any information about damping, we may not derive a damping ratio and set it to zero, so we analyze a free response with no damping.

As the question states, we have a diagonal mass matrix, and to keep our process simple, we leave $m = 1$ and we also assign $m_3, m_6 = 2$, since at degrees 3 and 6, the airplane wing has extra weight due to the turbines. Our mass matrix is a diagonal 12x12 matrix. Next we input our experimental modal matrix and 3 corresponding natural frequency.

Now we input our force, which we derived from the theoretical section, were $m_1 = 1$ and $x_0 = cos(\omega t)$ (harmonic excitation) and we will also set driving force $\omega = 1$.

$$-x_0(t)m_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = -1cos(t) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -cos(t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and when we find principle force we arrive at:

$$\vec{Q(t)} = \begin{bmatrix} -0.126cos(t) \\ 0.375cos(t) \\ -0.618cos(t) \end{bmatrix}$$

which gives us a nice 3x1 matrix to work with. We also allow no initial displacement or velocity in principle space, so:

$$q_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \dot{q_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We now can find $q(t)$ given by the equation:

$$q_i(t) = e^{-\zeta_i \omega_i t}\left[q_i(0)\cos(\omega_{di}t) + \frac{\zeta_i q_i(0)}{\sqrt{1-\zeta_i^2}}\sin(\omega_{di}t) + \frac{\dot{q}_i(0)}{\omega_{di}}\sin(\omega_{di}t) \right]$$

$$+\frac{1}{\omega_{di}}\int_0^t Q_i(\tau)e^{-\zeta_i\omega_i(t-\tau)}\sin[\omega_{di}(t-\tau)]d\tau \qquad (i=1,2,\cdots,n)$$

$$(i=1,2,\cdots,n)$$

$$\omega_{di} = \omega_i\sqrt{1-\zeta_i^2}$$

Figure 4: General solution

and we end with:

$$q_1(t) = -2.48894x10^{-6}cos(t) + 2.48894x10^{-6}cos(225t)$$

$$q_2(t) = 1.04167x10^{-6}cos(t) - 1.04167x10^{-6}cos(600t)$$

$$q_3(t) = -5.10744x10^{-7}cos(t) + 5.10744x10^{-7}cos(1100t)$$

We arrive at our dynamic responses, which we get from $x(t) = [X]q(t)$, and our 12 responses for 50 seconds are seen in figure 5:

One thing to notice is that as we increase in the mode, there is much more noise present in our response. This could be since they are linearly combined from the first 3 modes and are undamped. A link to the code is also available to test the results and run with new parameters.

6 Discussion and Conclusion

In this analysis, we compared the vibration of an airplane wing to a cantilever beam and try to derive a dynamic response. What we found was a similar vibration mode of the beam that corresponded to the mode of the wing. In our theoretical analysis, we set up a dynamic response by assuming harmonic motion vertically on the fuse luge and, by keeping damping ratio and damping

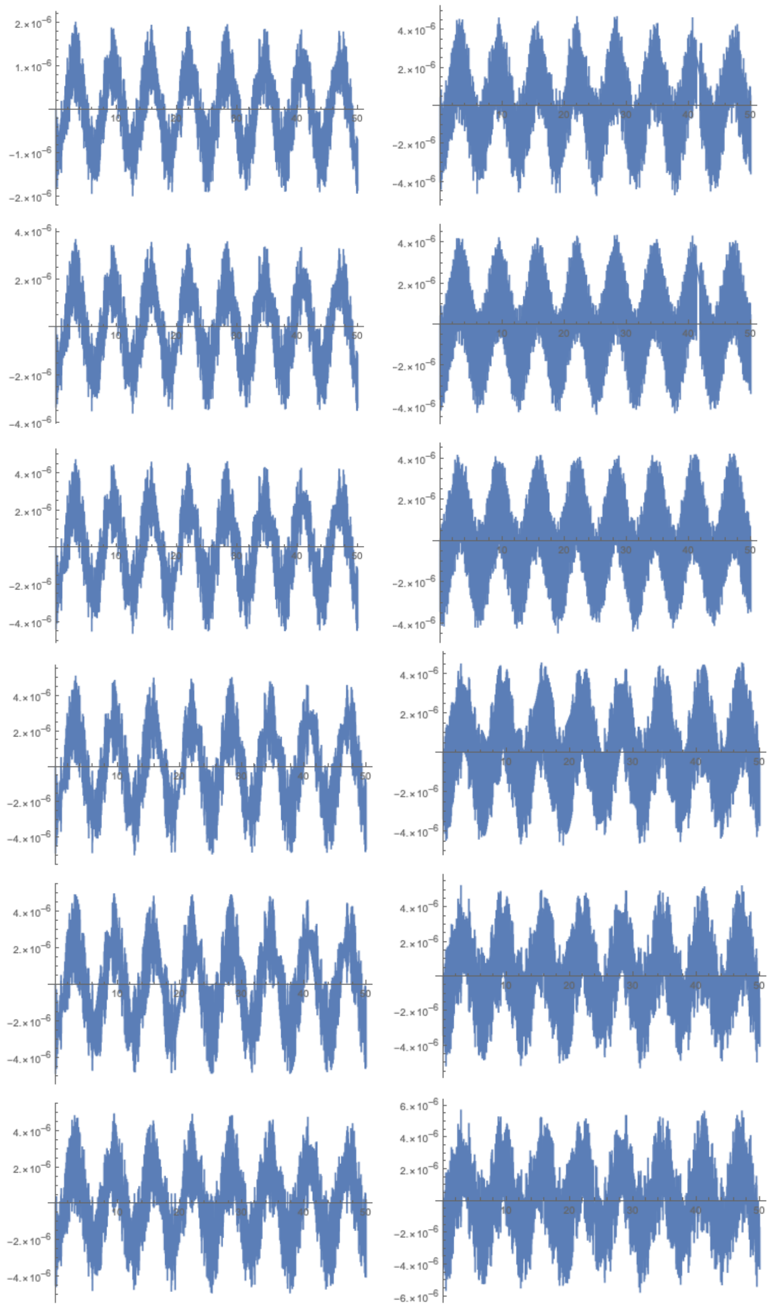


Figure 5: Dynamic Responses

frequency constant with respect to its principle mode, we were able to derive a dynamic response for the wing. We also try to model a free response analysis using a mathamatica that gave a more or less accurate measure of how the dynamic responses would look like. In comparing the theoretical and numerical results, we see that all the coefficient values are negative, which is a good sign since our vertical force is negative. We can also notice that the coefficients are in the range of the same magnitude. Its also important note that these dynamic responses are all a linear combination of the first 3 modes.

We notice quite bit of noise given in the dynamic free response which makes sense since we did not include any form of damping. As the responses increase with the degree of freedom, it becomes much noiser, which makes sense since it is further away from the initial force excitation. A plot of the first response from time 0 to 0.1, (Figure 6), shows us just how much noise our responses have but overall seem to oscillate.

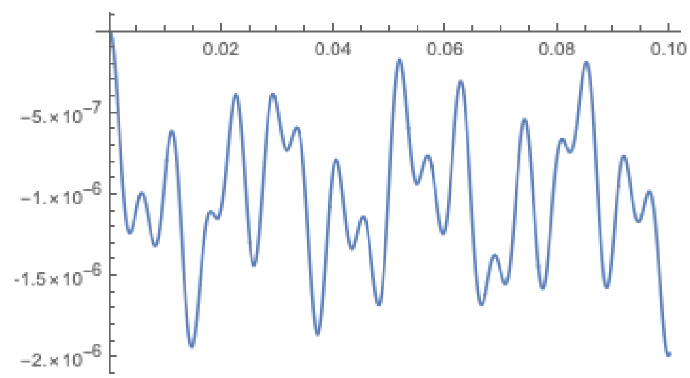


Figure 6: first dynamic respond time 0-0.1 sec

This analysis is very important to understand and initiate, since by observing the vibrations of an airplane wing can improve future prototypes and increase performances. It is also critical for implementing safety measurements, to prevent accidents and failures.

7 References

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