Artificial Intelligence Foundation – JC3001

Lecture 44: Reinforcement Learning - I

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Material adapted from:

Russell and Norvig (AIMA Book): Chapter 22

Sutton and Barto (Reinforcement Learning: An Introduction 2nd ed.)

David Silver (UCL)

Michael Littman (Brown University) and Charles Isbell (GA Tech)

Course Progression

- Part 1: Introduction
  1. Introduction to AI ✓
  2. Agents ✓
- Part 2: Problem-solving
  1. Search 1: Uninformed Search ✓
  2. Search 2: Heuristic Search ✓
  3. Search 3: Local Search ✓
  4. Search 4: Adversarial Search ✓
- Part 3: Reasoning and Uncertainty
  1. Reasoning 1: Constraint Satisfaction ✓
  2. Reasoning 2: Logic and Inference ✓
  3. Probabilistic Reasoning 1: BNs ✓
  4. Probabilistic Reasoning 2: HMMs ✓
- Part 4: Planning
  1. Planning 1: Intro and Formalism ✓
  2. Planning 2: Algorithms & Heuristics ✓
  3. Planning 3: Hierarchical Planning ✓
  4. Planning 4: Stochastic Planning ✓
- Part 5: Learning
  1. Learning 1: Intro to ML ✓
  2. Learning 2: Regression ✓
  3. Learning 3: Neural Networks ✓
  4. **Learning 4: Reinforcement Learning**
- Part 6: Conclusion
  1. Ethical Issues in AI
  2. Conclusions and Discussion

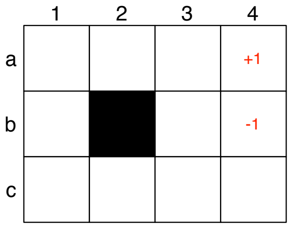
Objectives

- Bandit Problems
- Reinforcement Learning based Agents
- Tabular Reinforcement Learning
- Function Generalization

Outline

- Recap and Motivation
- Multi-armed Bandits
- Reinforcement Learning based Agents

Motivation



- Recall the grid world example we saw for MDPs

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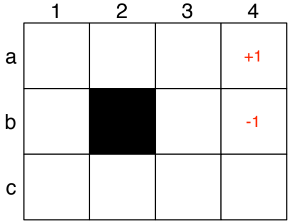

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- We learned how to calculate the optimal policy using transition **probabilities** and **rewards**

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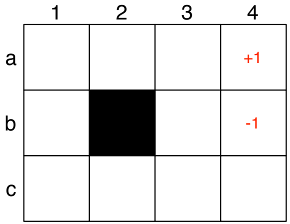

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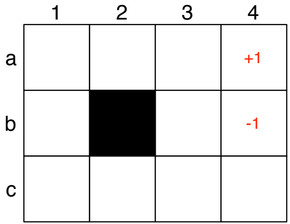

- Recall the grid world example we saw for MDPs
- We learned how to calculate the optimal policy using transition **probabilities** and **rewards**
- But what if we do not know the rewards, probabilities or both?
- We should be able to explore the environment and discover the best policy

Main methods of learning:

- Supervised learning:
  From $(\vec{x}^{(1)}, y^{(1)}), (\vec{x}^{(2)}, y^{(2)}), \ldots (\vec{x}^{(m)}, y^{(m)})$, derive a function $f(X^{(m)}) = Y^{(m)}$

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- Reinforcement learning:
  From a sequence of state transitions (i.e. states and actions)
  and occasional rewards $[s^{\rightarrow r}, a, s^{\rightarrow r}, a, \ldots]$,
  discover the optimal policy $\pi^*(s)$ for this environment.

Machine Learning Problems

Which of the previous methods of learning would be more suitable to solve the following problems:

Sup.	Unsup.	Reinf.	
			Spam Filtering
			Speech Recognition
			Starlight classification
			Playing SFΞ against an unknown player
			Navigating an unknown environment
			Elevator controller

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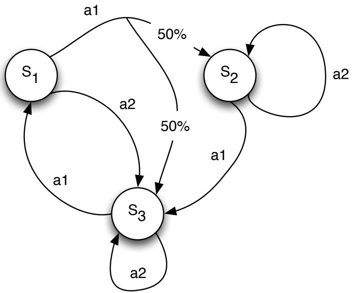
Markov Decision Process (MDP)



- An MDP is defined in terms of
  1. An initial state S_0
  2. A transition model $T(s, a, s') - P(s'|a, s)$ (Markovian)
  3. A reward function $R(s, a, s')$
- A solution to a MDP must specify what the agent should do for any state. Such a solution is called a policy

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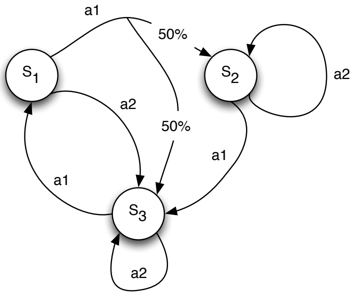

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  1. An initial state S_0
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  3. A reward function $R(s, a, s')$ — sometimes expressed as $R(s')$
- A solution to a MDP must specify what the agent should do for any state. Such a solution is called a policy

Finding the Optimal Policy

$$V(s) \leftarrow \left[\max_a \gamma \sum_{s'} P(s'|s,a) * V(s')\right] + R(s)$$

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$$\pi^*(s) = \arg\max_a \sum_{s'} P(s'|s,a) * V(s')$$

- Recap and Motivation
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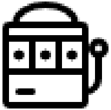
k-armed Bandit Problem

- Repeated choices among k different options or actions
- Each choice has a reward drawn from a stationary probability distribution
- Goal is to maximize rewards over a period of time



Thus, the value is the expected reward:

$$\begin{aligned} q_*(a) &\doteq \mathbb{E}\left[R_t \mid A_t = a\right] \forall a \in \{1, \ldots, k\} \\ &= \sum_r p(r \mid a) r \end{aligned}$$

And to maximize it over time

$$\arg\max_a q_*(a)$$

Computing the expected reward

Consider a doctor with 3 possible treatments to offer a patient.

- Each treatment can either cure the patient ($r = 1$), do nothing ($r = 0$), or make the patient sicker ($r = -1$), with probabilities ($p(r \mid drug)$) as follows:

  d_1 $p(r = 1 \mid d = d_1) = 0.1$, $p(r = 0 \mid d = d_1) = 0.8$, and $p(r = -1 \mid d = d_1) = 0.1$
  d_2 $p(r = 1 \mid d = d_2) = 0.3$, $p(r = 0 \mid d = d_2) = 0.1$, and $p(r = -1 \mid d = d_2) = 0.6$
  d_3 $p(r = 1 \mid d = d_3) = 0.3$, $p(r = 0 \mid d = d_3) = 0.5$, and $p(r = -1 \mid d = d_3) = 0.2$

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Compute q_* for all drug options and help the doctor choose:

d_1 $q(d = d_1) = (0.1 * 1) + (0.8 * 0) + (0.1 * -1) = 0$

d_2 $q(d = d_2) = (0.3 * 1) + (0.1 * 0) + (0.6 * -1) = -0.3$

d_3 $q(d = d_4) = (0.3 * 1) + (0.5 * 0) + (0.2 * -1) = 0.1$

Sample-Average Method

$$Q_t(a) \doteq \frac{\text{sum of rewards for } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t} = \frac{\sum_{i=1}^{t-1} R_i}{t-1}$$

$$\frac{\sum_{i=1}^{t-1} R_i}{t-1}$$

$Q_0(\;) = 0$

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$$\frac{\sum_{i=1}^{t-1} R_i}{t-1}$$

1

$Q_1($$) = 1$

0

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0

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$$\frac{\sum_{i=1}^{t-1} R_i}{t-1}$$

1

0

$Q_2($$) = 0.5$

0

0

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0

0

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$$\frac{\sum_{i=1}^{t-1} R_i}{t-1}$$

		
1	0	0
0	0	0
0	-1	0
$Q_3($$) = 0.5$	$Q_3($$) = -1$	$Q_3($$) = 0$

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$$\frac{\sum_{i=1}^{t-1} R_i}{t-1}$$

			
1	0		0
0	0		0
0	-1		0
0	0		1
0	0		0
$Q_5($$) = 0.5$	$Q_5($$)$ -0.5	$=$	$Q_5($$) = 1$

Incremental Update Rule

$$Q_{n+1} = \frac{1}{n} \sum_{i=1}^{n} R_i$$

Incremental Update Rule

$$
\begin{aligned}
Q_{n+1} &= \frac{1}{n}\sum_{i=1}^{n} R_i \\
&= \frac{1}{n}(R_n + \sum_{i=1}^{n-1} R_i)
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Recall: $Q_n = \frac{1}{n-1}\sum_{i=1}^{n-1} R_i$

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\end{aligned}
$$

Recall: $Q_n = \frac{1}{n-1}\sum_{i=1}^{n-1} R_i$

Incremental Update Rule

We will see this form of update throughout this lecture:

$$\text{NewEstimate} \leftarrow \text{OldEstimate} + \text{StepSize}\,[\text{Target} - \text{OldEstimate}]$$

$$Q_{n+1} = Q_n + \frac{1}{n}(R_n - Q_n)$$

$$Q_{n+1} = Q_n + \alpha_n(R_n - Q_n)$$

Outline

Agents execute **trials** in the environment, using some policy π.
Typical trials look like:

$$(1,1)_{-.04} \rightsquigarrow (1,2)_{-.04} \rightsquigarrow (1,3)_{-.04} \rightsquigarrow (1,2)_{-.04} \rightsquigarrow (1,3)_{-.04} \rightsquigarrow (2,3)_{-.04} \rightsquigarrow (3,3)_{-.04} \rightsquigarrow (4,3)_{+1}$$

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$$(1,1)_{-.04} \rightsquigarrow (2,1)_{-.04} \rightsquigarrow (3,1)_{-.04} \rightsquigarrow (3,2)_{-.04} \rightsquigarrow (4,2)_{-1}$$

What to do if we do not know R

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Agent Types for RL

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- Besides these designs, we decide how “adventurous” our agent will be

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Passive RL The agent simply follows a fixed policy, and learns the components above as he goes
The goal is to learn how good that policy is

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Active RL After learning the components for a while,
the agent changes policy to "cash in" on the learned components

To continue in the next session.