Q1

$u = e^x \quad du = e^x = (e^x)$

a) $$\int \frac{5e^x}{\sqrt{1-e^{2x}}}\,dx = 5\int \frac{e^x}{\sqrt{1-e^{2x}}} = 5\int \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{u}$$

$$= 5\int \frac{u}{\sqrt{1-u^2}} \cdot \frac{1}{u}\,du = 5\int \frac{1}{\sqrt{1-u^2}} = 5\int \arcsin(u)$$

$$= 5\arcsin(u) + C \quad \text{Return to } x \Rightarrow \int 5\arcsin(e^x) + C \;\square$$

b) $$\int \frac{2x-1}{x^2-6x+18}\,dx = \int \frac{2x-1}{x^2-6+18}\,dx = \int \frac{2x-6}{x^2-6+18}\,dx + \int \frac{5}{x^2-6+18}\,dx$$

$$\int \frac{2x-6}{x^2-6+18} = \int \ln(x^2-6+18)$$

$$\int \frac{5}{x^2-6+18} = \int \frac{5}{(x-3)^2+3^2}\,dx = \int \frac{5}{3}\arctan\left(\frac{x-3}{3}\right)$$

$$\int \ln|x^2-6x+18| + \frac{5}{3}\arctan\left(\frac{x-3}{3}\right) + C \;\square$$

c) $$\int_1^4 \frac{1}{\sqrt{t}\,(\sqrt{t}+1)^3}\,dt \qquad \begin{aligned}\sqrt{t} &= u-1\\ (\sqrt{t}+1)^3 &= u^3\\ dt &= 2(u-1)\,du\end{aligned} \Rightarrow \int_2^3 \frac{1}{(u-1)u^3} \cdot 2(u-1)\,du$$

$$= \int_2^3 \frac{2}{u^3}\,du = 2\int u^{-3}\,du = \frac{u^{-3+1}}{-3+1} = -\frac{1}{2u^2} \Big| \cdot 2$$

$$= \left[-\frac{1}{2u^2}\right]_2^3 = -\frac{1}{3^2} + \frac{1}{2^2} = -1/9 + 1/4 = \frac{5}{36} \;\square$$

d) $\int \frac{e^{\sqrt{x}}}{5\sqrt{x}}dx = \frac{1}{5}\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ $\quad u=\sqrt{x}$, $x=u^2$, $dx = 2u\,du$ $\Rightarrow \frac{1}{5}\int \frac{e^u}{u}\cdot 2u\,du$

$= \frac{1}{5}\int 2e^u du = \frac{2}{5}\int e^u du = \int \frac{2}{5} e^u du \overset{\text{rapport}}{\Rightarrow} \int \frac{2}{5} e^{\sqrt{x}} + C$ $\square$

e) $\int (3x+1)e^{2x+1}dx$ $\quad u = 2x+1$, $dx = \frac{du}{2}$ $\quad 3x+1 = \frac{3}{2}(u-1)+1 = \frac{3u}{2} - \frac{1}{2}$

$\int \left(\frac{3u}{2} - \frac{1}{2}\right) e^u \cdot \frac{du}{2} \Rightarrow \frac{1}{2}\int \left(\frac{3u}{2} - \frac{1}{2}\right) e^u du$

$= \frac{1}{2}\left[\frac{3}{2}\int u e^u du - \frac{1}{2}\int e^u du\right] \Rightarrow \frac{1}{2}\left[\frac{3}{2}\int u e^u du - \frac{1}{2}\int e^u du\right]$

$\int \frac{3}{4}(2x+1)e^{2x+1} - e^{2x+1} + C = \int\left(\frac{3}{4}(2x+1)-1\right)e^{2x+1} + C$ $\square$

Q2

$f(x) = e^{-x^2}\int_0^x e^{t^2}dt$

a) $\frac{d}{dx}\left(e^{-x^2}\right) = -2xe^{-x^2}$ $\quad \frac{d}{dt}\left(\int_0^x e^{t^2}dt\right) = e^{x^2}$

donc,

$$f'(x) = \left(-2xe^{-x^2}\right)\int_0^x e^{t^2}\,dt + e^{-x^2}e^{x^2}$$

$$\Rightarrow f'(x) = -2xe^{-x^2}\int_0^x e^{t^2}\,dt + 1$$

$$f'(x) + 2xf(x) = \left[-2xe^{-x^2}\int_0^x e^{t^2}\,dt + 1\right] + 2xe^{-x^2}\int_0^x e^{t^2}\,dt$$

$$f'(x) + 2xf(x) = 1 \quad \square$$

$$f(0) = e^{-0^2}\int_0^0 e^{t^2}\,dt$$

$$= 1\cdot 0$$

$$= 0$$

Q3

a) $$\int_2^5 G(t)\,dt = \int_2^5 \frac{2500}{(1+2t)^2}\,dt$$

$u = 1+2t$

$du = 2dt$ $\qquad t=2,\ u = 1+2(2) = 5$

$dt = \frac{du}{2}$ $\qquad t=5,\ u = 1+2(5) = 11$

$$\int_5^{11} \frac{2500}{u^2}\cdot\frac{du}{2} = 1250\int_5^{11} u^{-2}\,du = 1250\left[-\frac{1}{u}\right]_5^{11} = 1250\left[-\frac{1}{11}+\frac{1}{5}\right]$$

$$1250\left(\frac{6}{55}\right) = \frac{7500}{55} \simeq 136{,}36 \text{ Litres} \quad \square$$

b) $\int_0^4 Q(t)\,dt$ $\quad u = 1+2t$, $du = 2dt \Rightarrow$ $t=0 \Rightarrow u=1$, $t=4 \Rightarrow u=9$

$$\int_0^4 \frac{2500}{(1+2t)^2}\,dt = 1250\int_1^9 u^{-2}\,du \Rightarrow 1250\left[-\frac{1}{u}\right]_1^9 = 1250\left[-\frac{1}{9}+\frac{1}{1}\right] = 1250\left(\frac{8}{9}\right)$$

$\simeq 1111,11$ litres $\square$

c)

$$\text{Débit moy} = \frac{V_{totale}}{t}$$

$t_i = 0 \quad t_f = 4 \Big\} \; 1111,11 \quad$ donc, $\dfrac{1111,11}{4} = 277,78 \; L/h \; \square$

d)

$V_{init} = 400, \; + 1111,11 = 1511,11\,L > 1500\,L$

$$400 + \int_0^t \frac{2500}{(1+2t)^2}\,dt = 1500$$

avec t_i

$$t = \frac{11}{3} \rightarrow 3h \text{ et } 40min$$