4,94

a) $I_a = \int_2^6 \frac{3}{\sqrt{t-2}}\,dt$

$= 3 \cdot 2\sqrt{t-2}\,\Big]_2^6$

$= \lim_{x \to 2^+} 6\sqrt{t-2}\,\Big|_x^6$

$= 6\sqrt{4} - \lim_{x \to 2^+} 6\sqrt{x-2}$

$= 12$

b) $\int_0^\infty e^{-x}\,dx$

$= \int e^{-x}\,dx = -e^{-x} + C$

$I_b = (\lim -e^{-x}) - (-e^{-0})$ $\;\to 0$

$= 1$

g) $I_g = \int_{-1}^{2} \frac{1}{x^2}\,dx$

$= \int_{-1}^{0^-} \frac{1}{x^2} + \int_{0^+}^{2} \frac{1}{x^2}\,dx$

$= -\frac{1}{x}\Big]_{-1}^{0^-} + -\frac{1}{x}\Big]_{0^+}^{2}$

On a $-\frac{1}{x}\Big]_{-1}^{0^-}$ $= -\frac{1}{1} - \lim_{x \to 0} -\frac{1}{x}$

$= -\infty$

donc I_g diverge

f) $\int_2^\infty \frac{1}{x\ln(x)}\,dx$

$= \lim_{t \to \infty} \int_2^t \frac{1}{x\ln(x)}\,dx$

$u = \ln(x)$, $du = \frac{1}{x}dx$

$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du = \ln(u) + C$

$= \ln(\ln(x)) + C$

On a $I_f = \lim_{t \to \infty} \ln(\ln(x))\Big|_2^t$

$= \lim_{t \to \infty} \ln(\ln(t)) - \ln(\ln(2))$

$= \infty$

donc I_f diverge

r) $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$

$= \int_{-\infty}^{0} \frac{e^x}{1+e^{2x}}\,dx + \int_0^\infty \frac{e^{2x}}{1+e^{2x}}\,dx$

$= \lim_{b \to -\infty} \int_t^0 \frac{e^x}{1+e^{2x}}\,dx + \lim_{t \to \infty} \int_0^t \frac{e^x}{1+e^{2x}}\,dx$

$1+e^{2x} = 1+(e^x)^2 = 1+u^2 \Rightarrow a=1 \quad u=e^x$

on a $du = e^x dx$

$= \int \frac{1}{1+u^2}\,du = \arctan(u) + C$

$= \arctan(e^x) + C$

$= \lim_{t \to -\infty} \int_t^0 \frac{e^x}{1+e^{2x}}\,dx = \lim_{t \to -\infty} \arctan(e^x)\Big|_t^0$

$= \arctan(e^0) - \lim_{t \to -\infty} \arctan(e^t)$

$= \arctan(1) - \arctan(0)$

$= \frac{\pi}{4} - 0$

$= \lim_{t \to \infty} \arctan(e^t) - \arctan(e^0)$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

d'où $I_r = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

5,2

$f(x) = -2x^2 + 1$

$g(x) = \sin(2x - 0{,}5)$

solve $(f(x) = g(x), x)$

$x_1 = -0{,}925 \quad x_2 = 0{,}11$

$A = \int_a^b (x_{1t} - x_0)\,dx$

$\int_{x_1}^{x_2} (f(x) - g(x))\,dx$

$= 1{,}6042$

5,3

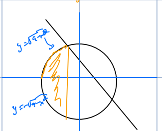
j) $x^2 + y^2 = 9$ et $y = -2x + 1$

mettre dans la fx calc graph

ou

solve$\left(\begin{cases} x^2+y^2=9 \\ y=-2x+1 \end{cases} \{x, y\}\right)$

$I_2 = \int_{-0,9}^{1,7} (-2x+1 - (-\sqrt{9-x^2}))\,dx$



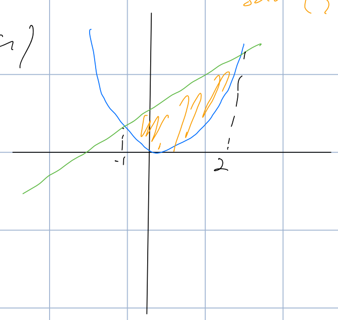
$x^2 + y^2 = 9$

$y = \pm\sqrt{9-x^2}$

$I_1 = \int_{-3}^{-0,9} (\sqrt{9-x^2} - (-\sqrt{9-x^2}))\,dx$

S.4

a)



Solve $\left(\left\{ \begin{matrix} y = x^2 \\ y = x+2 \end{matrix}, (x,y) \right\}\right)$

$x = -1$
$y = 1$
$x = 2$
$y = 4$

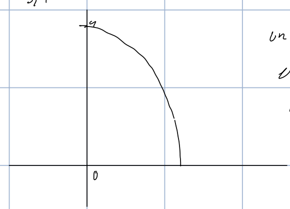
CAS $\int_{-1}^{2} (x+2-x^2)\,dx$ $\frac{9}{2}$

On cherche a tel que

$\int_{-1}^{a} (x+2-x^2)\,dx = \int_{a}^{2} (x+2-x^2)\,dx, a) \mid -1 < a < 2$

$a = 1/2$

S.9



$V = \int_{c_1}^{c_2} \pi r^2 \, dc$

Ona $c_1 = 0$ $c_2 = 9$ $dc = dy$

et $r = x_a - x_D =$

$y = 9 - \frac{4x^2}{9} \rightarrow \frac{4x^2}{9} = 9 - y \Leftrightarrow x = \sqrt{\frac{36-9y}{4}} = r$

et $x_D = 0$

Solve $\left(y = \frac{9-4b^2}{9}, x\right)$ = r

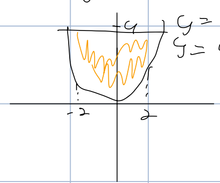
on choisit

r $\int_0^{4} \left(\pi\left(\frac{3-\sqrt{4y}}{2}\right)\right) dy$

$= 18\pi$

S.11

a) $y = 4$



Le disque sont verticaux

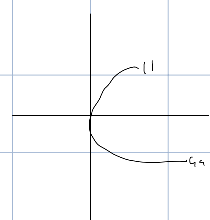
$V = \int_{c_1}^{c_2} \pi r^2 \, dc$ $r = 4 - x^2$

$c_1 = -2$
$c_2 = 2$ $dc = dx$

$\int_{-2}^{2} (\pi \cdot 4 \cdot \pi)^2 \, dx$ $\frac{512\pi}{15}$

S.21

$x = y^2$ $(1;1)$ $(4;-2)$



$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$

$\int_{-2}^{1} \sqrt{1 + \left(\frac{d}{dy}(y^2)\right)^2} \, dy$

9,263

S.20

$\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)} \, dx$

$\int_0^{3} \sqrt{1 + \left(\frac{d}{dx}(b^x)\right)^2} \, dx$

S.23

$f(x) := 2\cos(3x-4)$

$g(x) := 1,8 - e^{(x-3,8)^2}$

Solve $(f(x) = g(x), x)$
x_1 x_2

$gh(x) := g(x)$

$gb(x) := f(x)$ $\Rightarrow \int_a^b (gh(x) - gb(x))\,dx$

$a := x_1$

$b := x_2$

① longueur 1 + longueur 2

$$f[\,]:=\int_a^b \sqrt{\left(1+\frac{d}{dx}(f(x))\right)^2}$$