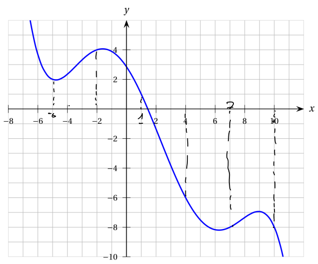
4,1

Intervalle $[-5, 10]$ $\quad G_s \quad D_s$



$$\Delta x = \frac{b-a}{n} = \frac{10-(-5)}{5} = \frac{15}{5} = 3$$

G D

$[-5,-2]$

$[-2,1]$

$[1,4]$

$[4,7]$

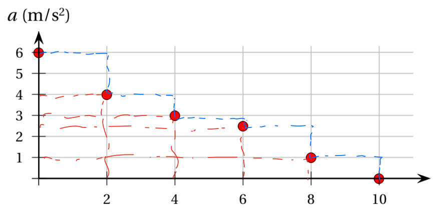
$[7,10]$

$$G_s = 3\cdot[f(-5)+f(-2)+f(1)+f(4)+f(7)]$$
$$= 3(2+4+1+-6-8)$$
$$= 3(7)$$
$$= 21\,u$$

$$D_s = 3\,[f(-2)+f(1)+f(4)+f(7)+f(10)]$$
$$3(4+1-6-8-8)$$
$$= -51\,u$$

n peut être un choix sauf si indiqué
ex: Ds dans l'énoncé

4,2




intervalle $[0, 10]$

$a = 0$

$b = 10$

$$\frac{b-a}{n} = \frac{10-0}{5} = 2$$

dérivé

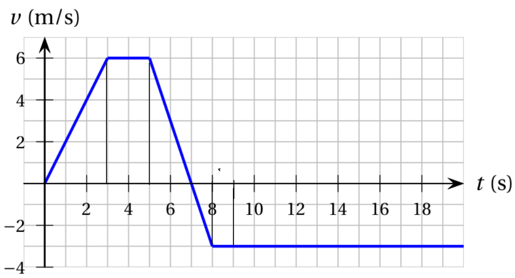
$x(t) \cdot v(t) \quad a(t)$

intégrale

| $n$ | $x_i$ | $f(x_i)$ |
|---|---|---|
| 0 | 0 | 6 |
| 1 | 2 | 4 |
| 3 | 4 | 3 |
| 4 | 6 | 2,5 |
| 5 | 8 | 1 |
| 6 | 10 | 0 |

$$G_s = (6+4+3+2{,}5+1)$$
$$= 33$$

$$D_s = (4+3+2{,}5+1)$$
$$= 21$$




a) après 5

0 à 3s

Δ $\frac{3\cdot 6}{2} = 9m$

3 à 5 se

□ $6\cdot h = 6\cdot 2 = 12$

$9+12 = 21m$

après 9

5 à 7

Δ $\frac{2\cdot 6}{2} = 6$

Δ $\frac{8\cdot 3}{2} = 12$

□ $3\cdot 1 = 3$

21m

b) 7s, car $v_{(7)} = 0$

c) $t = 2s$

entre 0 à 3s

$\frac{6-0}{2} = 3$

$v_{i\,(t=0)} = 0 \quad v_{f\,(t=3)} = 6$

$t = 4$

est cst

$t = 6$

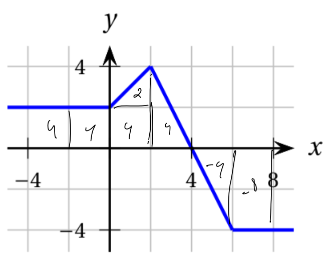
entre 5 à 7

$v_{i\,(t=5)} = 6 \quad v_{f\,(t=7)} = 0$

$\frac{0-6}{2} = -3$

d) Oui, pour que l'objet revient à sa position de départ, l'aire positive (déplacement en avant) doit être annulée par l'air négative

e) non,
Si un objet se déplace vers 0 elle ralentie pente négative
t=7,5s accélère négativement




Δ $\frac{6\cdot h}{2}$

a) $\int_{-4}^{6} f(x)\,dx = 4$

b) $\int_{-4}^{4} f(x)\,dx = 16$

$4+4+2\ 4$

c) $\int_{2}^{6} f(x)\,dx = 0$

$4-4=0$

d) $\int_{-4}^{8} f(x)\,dx = 2$

$4+4+2+5-6-8$

e) $\int_{4}^{6} f(x)\,dx$ donc $-10$

$\frac{6-4}{h} = \frac{6-4}{h}$

f) $\int_{8}^{6} = 8$

4,10

a) $\int_0^{10} f(x)\,dx + \int_{10}^{15} f(x)\,dx = \int_0^{15} f(x)\,dx$

b) $\int_0^{10} f(x)\,dx - \int_0^{5} f(x)\,dx = \int_5^{10} f(x)\,dx$

c) $\int_0^{10} f(x)\,dx + \int_{10}^{5} f(x)\,dx = \int_0^{5} f(x)\,dx$

4,11

a) $\int_1^7 5\,dx = h \cdot l = 5 \cdot 6 = 30$

$[1,7]$ $7-1=6$

b) $\int_2^6 (10-x)\,dx = \frac{(b+B)h}{2} = \frac{(8+4)\cdot 4}{2} = 24$

points $(10,0)$

$(0,10)$

$[2,6]$

b: $10-6=4$

B: $10-2=8$

h: $6-2=4$

c) $\int_0^5 (2x-3)\,dx = -2{,}25 + 12{,}25 = 10$

$0 \leq 1{,}5$ $\quad$ $1{,}5 \leq 5$

$2x-3=0$ $\quad$ $2(1{,}5)-3=0$

$2x=3$ $\quad$ $2(5)-3=7$

$x=1{,}5$

$b=1{,}5,\ h=3$ $\quad$ $\frac{(0+7)\cdot 3{,}5}{2} = 12{,}25$

$\frac{1{,}5 \cdot 3}{2} = -2{,}25$

d) $\int_{-2}^{2} \left(-4-\sqrt{4-x^2}\,dx\right.$ $\quad$ $y=\sqrt{r^2-x^2}$

demi cercle $-\sqrt{4-x^2}$ et $r=2$

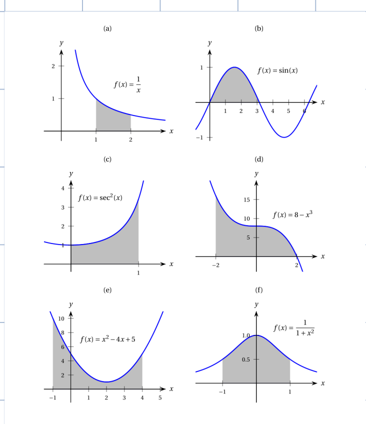
$\frac{r^2\pi}{2} = \frac{\pi 2^2}{2} = -2\pi$ □

f) $\int_{-1}^{3} |x|\,dx = \frac{1}{2} + \frac{9}{2} = 5$

$[-1,0] = -x$ $\quad$ $\frac{b \cdot h}{2} = \frac{1 \cdot 1}{2} = \frac{1}{2}$

$[0,3] = x$ $\quad$ $\frac{b \cdot h}{2} = \frac{3 \cdot 3}{2} = \frac{9}{2}$

4,16




a) $f(x) = \frac{1}{x} \to \mathbb{R}2$

$x=1$ et $x=2$

$\int_1^2 \frac{1}{x}\,dx = [\ln|x|]_1^2$

$\ln(a) = b$ ssi $e^b = a$

$\ln(1) = b$ tq $e^b = 1$

$e^0 = 1$ donc $\ln(1) = 0$

$\ln(2) - \ln(1)$

$= \ln(2) - \ln(0)$

$= \ln(2)$ □

b) $f(x) = \sin(x)$

$x=0$ et $x=\pi$

$\int_0^\pi \sin(x)\,dx = [-\cos(x)]_0^\pi$

$= -\cos(\pi) + \cos(0)$

$= -(-1) + 1$

$= 2$ □

c) $f(x) = \sec^2(x)$

$\int_0^1 \sec^2(x)\,dx = [\tan(x)]_0^1$

$= \tan(1) - \tan(0)$

$= \tan(1)$ □

d) $f(x) = 8 - x^3$

$x=-2$ et $x=2$

$\int_{-2}^{2} (8-x^3)\,dx$

$= 8\,dx - x^3\,dx$

$= 8 \cdot (2-(-2))$

$= 8 \cdot 4$

$= 32$ $\quad$ forme moy

$\frac{1}{b-a} \int_a^b f(x)\,dx$

$= \frac{1}{4}$

$\int_{-2}^{2} x^3\,dx = 0$

$= 2^3 = 8$

$= -2^3 = -8$ $\Big\} = 0$

$\frac{1}{4} \int_{-2}^{2} (8-x^3)\,dx$

$\frac{1}{4} \cdot 32 = 8$

e) $f(x) = x^2 - 4x + 5$

$\int_{-1}^{4} (x^2 - 4x + 5)\,dx$

Primitive

$= \frac{x^3}{3} - 2x^2 + 5x$

$= \frac{-1^3}{3} - 2(-1)^2 + 5(-1)$

$= -\frac{22}{3}$

$= \frac{4^3}{3} - 2(4)^2 + 5(4)$

$= \frac{28}{3}$

$= \frac{28}{3} - \left(-\frac{22}{3}\right)$

$= \frac{50}{3}$ □

f) $f(x) = \frac{1}{1+x^2}$

$x=-1$ et $x=1$

$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

$= \int_{-1}^{1} \frac{1}{x} \arctan\left(\frac{x}{1}\right)$

$= \frac{1}{1} \arctan\left(\frac{x}{1}\right)$

$\arctan(x)$

$= \frac{1}{-1} \arctan\left(\frac{-1}{1}\right)$

$-1 \arctan(-1)$

$\arctan(1) - \arctan(-1)$

$= \frac{\pi}{2}$

4.17 $\quad f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

a)

$f_{avg} = \frac{1}{2-1}\int_1^2 \frac{1}{x} = \ln(2)$

b)

$f_{avg} = \frac{1}{\pi - 0}\int_0^{\pi} \sin(x) = 2$

$\frac{1}{\pi}\cdot 2 = \frac{2}{\pi}$

c) $\tan(1)$

$f_{avg}\ \frac{1}{1-0}$

$\frac{1}{1}\cdot\tan(1)$

$=\tan(1)$

d) $32$

$\frac{1}{2-(-2)}\cdot 32$

$\frac{1}{4}\cdot 32$

$8$

e) $\frac{80}{3}$

$= \frac{1}{4-(-1)} = \frac{1}{5}\cdot\frac{80}{3} = \frac{16}{3}$

f) $\frac{\pi}{2}$

$\frac{1}{1-(-1)} = \frac{1}{2}\cdot\frac{\pi}{2}$

$\frac{\pi}{4}$

4.18

a) R1

$\int 8\,dx = \frac{8x^{0+1}}{0+1} + C$

$= 8x + C$

b) R5

$\int 4\sin(x)\,dx$

$-4\cos(x) + C$

c)

$\int (e^x - \cos(x))\,dx$

$\int e^x = e^x + C$

$\int -\cos(x)$

$-\int \cos(x) = \int \sin(x) + C$

$e^x - \sin(x) + C$

d)

$\int (6x+1)\,dx$

$\int 6x\,dx + \int 1\,dx$

$\int 6x = 6\cdot\frac{x^{1+1}}{1+1} = 6\frac{x^2}{2} = 3x^2$

$\int x\,dx = x$

$= 3x^2 + x + C$

e) $\int (2x^3 - 4x + 9)\,dx$

$\int 2x^3\,dx + \int -4x\,dx + \int 9\,dx$

$\int 2x^3\,dx = 2\frac{x^{3+1}}{3+1} = 2\frac{x^4}{4} = \frac{x^4}{2}$

$\int -4x\,dx = -4\frac{x^{1+1}}{1+1} = -4\frac{x^2}{2} = 2x^2$

$\int 9x\,dx = 9x$

$= \frac{x^4}{2} - 2x^2 + 9x + C$

f) $\int \frac{2}{x}\,dx$

$= 2\int \ln(|x|) + C$

g) $\int \frac{1}{2x^3}\,dx$

$= \frac{1}{2}\int x^{-3}\,dx$

$= \int x^{-3}\,dx = \frac{x^{-3+1}}{-3+1} = \frac{x^{-2}}{-2}$

$= \frac{1}{2}\int x^{3}\,dx = \frac{1}{2}\cdot\frac{x^{-2}}{-2} = -\frac{1}{4}x^{-2}$

$= -\frac{1}{4x^2} + C$

h) $\int \frac{\sqrt{x}}{2}\,dx$

$= \frac{1}{2}\int x^{1/2}\,dx$

$= \frac{1}{2}\int \frac{x^{1/2+1}}{1/2+1} + C$

$= \frac{1}{2}\int \frac{x^{3/2}}{3/2} = \frac{1}{2}\int \frac{1}{3}x^{3/2}$

$= \int \frac{1}{2}\cdot\frac{2}{3}x^{3/2} = \frac{1}{3}x^{3/2} + C$

i) $\int \sqrt[3]{x}\,dx$

$= \int x^{1/3}\,dx$

$= \frac{x^{1/3+1}}{1/3+1}\,dx$

$= \frac{x^{4/3}}{4/3} = \frac{3}{4}x^{4/3} + C$

j) $\int \sec^2(x)\,dx$

$= \int \tan(x) + C$

k) $\int \frac{3}{1+x^2}\,dx \qquad a=1,\ u=x$

$= \int \frac{3}{1+x^2}\,dx = 3\int \frac{1}{1+x^2}\,dx$

$3\int \frac{1}{1}\arctan\left(\frac{x}{1}\right)$

$= 3\arctan(x) + C$

l) $\int \frac{-2}{\sqrt{1-x^2}}\,dx \qquad a=1,\ u=x$

$= -2\int \frac{1}{\sqrt{1-x^2}} = -2\int \arcsin\left(\frac{x}{1}\right) + C$

$= -2\arcsin(x) + C$

m) $\int \frac{x^4 - 3x^2 + 5x + 10}{x^2}\,dx$

$= \int x^2 - 3 - \frac{5}{x} + \frac{10}{x^2}\,dx$

$= \int x^2 = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} + C$

$= \int -3x + C$

$= \int -\frac{5}{x} = -5\int \ln(|u|) + C = -5\ln(|x|) + C$

$= \int \frac{10}{x^2} = \int 10x^{-2}\,dx = 10\int \frac{x^{-2+1}}{-2+1} = 10\int \frac{x^{-1}}{-1} = -\frac{10}{x}$

$= \int \frac{x^3}{3} - 3x + 5\ln|x| - \frac{10}{x} + C$

4.19

a) $\int_2^6 e^x\,dx$

$\int_2^6 e^x\,dx = [e^x]_2^6 = e^6 - e^2$ D

b) $\int_1^2 x^3\,dx = \frac{x^{3+1}}{3+1} + C$

$= \frac{x^4}{4} = \frac{2^4}{4} - \frac{1^4}{4}$

$= 4 - \frac{1}{4} = \frac{15}{4}$ D

c) $\int_0^4 (3x^2 + 2x - 2)\,dx$

$\int_0^4 3x^2\,dx = 3\int \frac{x^{2+1}}{2+1} = 3\frac{x^3}{3} = x^3$

$\int_0^4 2x\,dx = 2\int \frac{x^{1+1}}{1+1} = 2\frac{x^2}{2} = x^2$

$\int_0^4 -2\,dx = -2x$

$= \int x^3 + x^2 - 2x\,dx$

$4^3 + 4^2 - 2(4) = 72$ D

$0^3 + 0^2 + 2(0) = 0$

d) $\int_1^9 \frac{1}{\sqrt{x}}\,dx = x^{-1/2}$

$= \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{1/2}}{1/2} = 2\sqrt{x}$

$\int_1^9 \frac{1}{\sqrt{x}}\,dx = [2\sqrt{x}]_1^9 = \frac{6}{2}$

$6 - 2$

$4$

e) $\int_0^{2\pi} \sin(x)\,dx$

$= [-\cos(x)]_0^{2\pi}\,dx$

$-1 + 1$

$= 0$

f) $\int_4^8 -5\,dx = -5x + C$

$= \int [-5x]_4^8 =$

$-40 + 20$

$-20$ D

4.20

a) $f'(x) = \cos(x)$, $f\left(\frac{\pi}{2}\right) = 4$

$f(x) = \sin(x) + C$

$= \sin\left(\frac{\pi}{2}\right) + C = 4$

b) $f(x)' = 2x + 5$, $f(2) 10$

$2x = 2\int \frac{x^{1+1}}{1+1} = \frac{x^2}{2} = \frac{2x^2}{2} = x^2$

$f(x) = \int x^2 + 5x + C$

$f(2) = 2^2 + 5(2) = 10$

$= 4 + 10 = 10$

$= 10 + 4$

$= -4$ D

c) $f'(x) = \frac{2}{x^5}$, $f(1) = 0$

$= \int 2x^{-5}\,dx = 2\int x^{-5} = \frac{x^{-5+1}}{-5+1} = 2\frac{x^{-4}}{-4}$

$= \int \frac{-1}{2x^4} + C$

$f(1) = \frac{-1}{2(1)^4} = 0$

$= \frac{1}{2} + C$

d) $f''(x) = -\sin(x)$, $f'(0) = 6$, $f(0) = 3$

$= -\int \sin(x) = \int -\cos(x)$

$= \cos(x) + C$

$f(0) = \cos(0) = 6$

$= -5$

done

$f'(x) = \cos(x) + 5$

$f'(x) = \cos + 5$

$\Downarrow$

$f(x) = \sin(x) + 5x + C$

$f(0) = \sin(0) + 5(0) = 3 + C$

$C = -3$

$f(x) = \sin(x) + 5x + 3$ D

e) $f''(x) = 1 - \frac{2}{x^2}$, $f'(1) = 7$, $f(1) = 4$

$= -2\int \frac{1}{x^2} \Rightarrow \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1}$

$= \frac{2}{x}$

$f(x) = x + \frac{2}{x} + C$

$f(1) = 1 + \frac{2}{1} + C = 7$

$= 3 = 7$

$= 4$

$f'(x) = x + \frac{2}{x} + 4 + C$

$\int x\,dx = \frac{x^2}{2}$

$\int 2\frac{1}{x} = 2\ln(x)$

$\int 4\,dx = 4x$

$f(x) = \frac{x^2}{2} + 2\ln(x) + 4x + C$

$f(1) = \frac{1^2}{2} + 2\ln(1) + 4(1) = 4$

$-\frac{1}{2}$

$f(x) = \frac{x^2}{2} + 2\ln(|x|) + 4x - \frac{1}{2}$