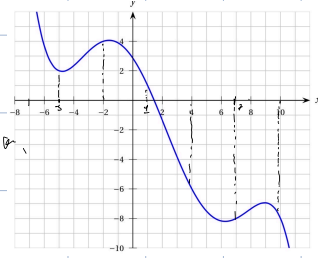


4.1



Intervalle $[-5, 10]$ G_S D_S

$$\Delta x = \frac{b-a}{n} = \frac{10-(-5)}{8} = \frac{15}{8} \approx 1.875$$

1: $[-5, -2]$

2: $[-2, 0]$

3: $[0, 2]$

4: $[2, 4]$

5: $[4, 6]$

6: $[6, 8]$

7: $[8, 10]$

G : les valeurs gauche
 D : les valeurs droite

$$G_S = 3 \cdot [f(-5) + f(-2) + f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$$

$$= 3 \cdot (2 + 4 + 1 + -6 + 8)$$

$$= 3 \cdot (-2)$$

$$= -6$$

$$D_S = 3 \cdot [f(-2) + f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$$

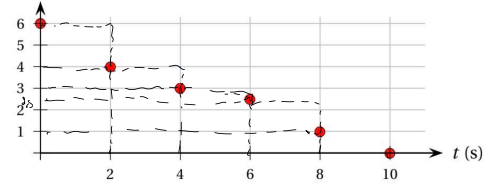
$$= 3 \cdot (4 + 1 + -6 + 8 + -8)$$

$$= -9$$

4.2

n est un choix, souvent indicé par $n = 5$ ou $n = 10$

a (m/s²)



$$a = 6$$

$$b = 0$$

$$\frac{b-a}{n} = \frac{0-6}{3} = -2$$

n	Δt	$f(t_i)$
0	6	6
1	2	4
2	4	3
3	6	2.5
4	8	1
5	10	0

$$G_S = 2 \cdot (6 + 4 + 3 + 2.5 + 1)$$

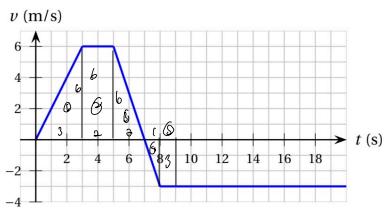
$$= 33$$

$$D_S = 2 \cdot (4 + 3 + 2.5 + 1 + 0)$$

$$= 21$$

$$x(t) \xrightarrow{\text{valeur}} v(t) \xrightarrow{\text{intégrale}} a(t)$$

4.3



$$① \quad \frac{6 \cdot 6}{2} = \frac{36}{2} = 18$$

$$② \quad \frac{6 \cdot 6}{2} = 18$$

$$③ \quad \frac{6 \cdot 6}{2} = 18$$

$$④ \quad 3 \cdot 6 = 18$$

$$9 \cdot 2 = 18$$

$$⑤ \quad 3 \cdot 6 = 18$$

4.9

a) $\int_{-2}^0 f(x) dx \quad a = \frac{1}{2} = 4$

b) $\int_{-2}^4 f(x) dx = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$
 $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$
 $4 + 4 + 4$

4.18
 k) $\int \frac{3}{1+x^2} dx = 3 \arctan(x) + C$
 $= 3 \arctan(x) + C$

e) $\int \frac{-2}{1+x^2} dx = -2 \arctan(x) + C$
 $F(x) = \arctan(x)$

z) $\int \frac{1}{\sqrt{4x^2+1}} dx = \ln(\sqrt{4x^2+1} + 2x) + C$
 $\left(\begin{array}{l} u = 2x \Rightarrow u = 2x \\ u^2 = 4x^2 \Rightarrow u = 2 \\ du = 2 dx \Rightarrow dx = \frac{1}{2} du \end{array} \right)$
 $\int \frac{1}{2\sqrt{u^2+1}} du = \frac{1}{2} \ln(\sqrt{u^2+1} + u) + C$

z1) $\int \sin(4x) dx = -\frac{1}{4} \cos(4x) + C$

$\frac{1}{4} \int \sin(4x) dx$
 $u = 4x \Rightarrow du = 4 dx \Rightarrow dx = \frac{1}{4} du$

z2) $\int x e^{x^2} dx$

$\int e^u du = e^u + C \quad \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$
 $u = x^2 \Rightarrow du = 2x dx \Rightarrow x dx = \frac{1}{2} du$

$\frac{du}{dx} = 2x \Rightarrow du = 2x dx$
 $x dx = \frac{1}{2} du$
 $\int x e^{x^2} dx = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$

z3) $\int \frac{1}{x^2+2x+5} dx$
 $= \int \frac{1}{(x+1)^2+4} dx = \frac{1}{4} \arctan\left(\frac{x+1}{2}\right) + C$
 $= \frac{1}{4} \arctan\left(\frac{x+1}{2}\right) + C$
 $u = x+1 \Rightarrow du = dx$
 $u^2+4 \Rightarrow u = 2$

4.24

$U(t) = \frac{t^2}{2} - 2, \quad h(t) = 5$

a) $h(t) - h(t) = \int_0^t \left(\frac{t^2}{2} - 2 \right) dt$
 $h(t) = 5 + \int_0^t \left(\frac{t^2}{2} - 2 \right) dt = \frac{t^3}{6} - 2t + 5$

$h(0) = 5$
 $h(5) = \frac{125}{6} - 10 + 5 = \frac{125}{6} - 5$

$a(t) = \frac{d^2}{dt^2} h(t)$

$a(0) = 0$

$a(5) = 0$

$v(0) = 0$

$v(5) = \frac{d}{dt} h(t) = \frac{t^2}{2} - 2$

$\left. \begin{array}{l} h(0) \\ v(0) \\ a(0) \end{array} \right\} \text{ Anfangswerte}$