MAT145

DEVOIR 4

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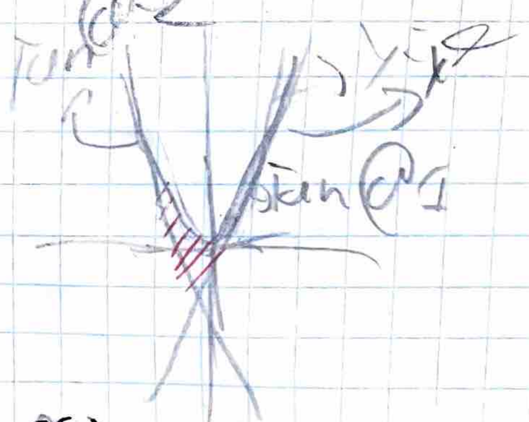
Question 1

$y = x^4$




Trouver les tangentes

$f'(x) = (x^4)'$

$y = ax + b$

$y = f'(a)(x-a) \pm f(a)$

$y_1 = (f'(-2) \cdot (x - (-2)) + f(2))$

$f'(-2) = -32$

$y_1 = -32(x+2) + 16$

$x - -2 = x + 2$

$f(-2) = 16$

$y_1 = -32x - 64 + 16$

$y_1 = -32x - 48$

$f'(a) = 4(1)^3 = 4$

$y_2 = f'(a) \cdot (x-a) + f(a)$

$x - a = x - 1$

$f(a) = 1^4 = 1$

$= 4(x-1) + 4$

$y_2 = 4x - 4 + 1$

$y_2 = 4x - 3$

intersection des Tangentes

$y_1 = y_2$

$-32x - 48 = 4x - 3$

$-48 + 3 = 4x + 32x$

$-45 = 36x$

$\frac{-45}{36} = x$

$\frac{-5}{4} = x$

Question1 suite

Air dela région

$$A = \int_{-2}^{-\frac{5}{4}} x^4 - y_1 + \int_{-\frac{5}{4}}^{1} x^4 - y_2$$

$$\int_{-2}^{-\frac{5}{4}} x^4 - (-32x - 48) + \int_{-\frac{5}{4}}^{1} x^4 - (4x - 3)$$

2,7897 + 8,6854

Air de la région = 11,475

Question 2

a) $\int_0^{\infty} \frac{x}{4-x^2} \Rightarrow \int \frac{x}{4-x^2} = -\frac{\ln(|x^2-4|)}{2}$

quand $x=2$ la Fonction n'existe pas

$$\int_0^{2^-} \frac{x}{4-x^2} + \int_{2^+}^{\infty} \frac{x}{4-x^2}$$

$$\lim_{x \to 2^-} -\frac{\ln(|x^2-4|)}{2} \notin \mathbb{R} \Rightarrow \text{diverge}$$

$$\lim_{x \to 2^+} -\frac{\ln(|x^2-4|)}{2} \notin \mathbb{R} \Rightarrow \text{diverge}$$

b)

$$\int_2^5 \frac{1}{(4-x)^{2/5}} \Rightarrow \int \frac{1}{(4-x)^{2/5}} = \frac{5\cdot(x-4)^{3/5}}{5}$$

lafonction n'existepas en $x = 4$.

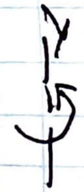
$$\int_2^4 \frac{1}{(4-x)^{2/5}} + \int_{4^+}^5 \frac{1}{(4-x)^{2/5}}$$

$$\lim_{x\to 4^-} \frac{5\cdot(x-4)^{3/5}}{5} = 0^- \Rightarrow \text{Converge}$$

$$\lim_{x\to 4^+} \frac{5\cdot(x-4)^{3/5}}{5} = 0^+ \Rightarrow \text{converge}$$

Question3

$y = 36 - x^2$



a)

**périmètre**

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dy}\right)^2}\, dx$$

bornes a-b

$0 = 36 - x^2$

$x = \pm 6$

$$\frac{dy}{dx} = (36 - x^2)' = -2x$$

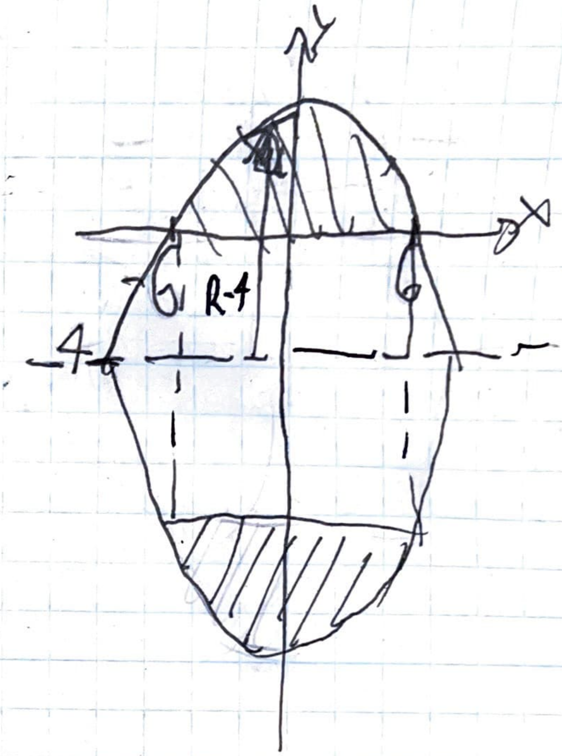
$$L = \int_{-6}^{6} \sqrt{1 + (-2x)^2}\, dx$$

$L = 73{,}8395 + (b-a)$

$L = 85{,}839$

b) $\text{volume} = \int_{e^1}^{e^2} \pi \cdot r^2 \, dx$

$= \int_{-6}^{6} \pi \cdot (36 \cdot x^2 - 4)^2$




$\text{Volume} = 19\ 422{,}6$

Question 4

a) $\sum_{n=0}^{\infty} \frac{(n+1)^2 (x+2)^n}{3^n}$

$a_n = \frac{(n+1)^2 \cdot (x+2)^n}{3^n}$

$R = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$R = \lim_{n\to\infty} \left|\frac{(n+2)^2 \cdot (x+2)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{(n+1)^2 \cdot (x+2)^n}\right|$

$R = \lim_{n\to\infty} \left|\frac{(n+2)^2}{(n+1)^2} \cdot \frac{x+2}{3}\right| = \frac{|x+2|}{3} \Rightarrow -1 < \frac{x+2}{3} < 1$

$-3 < x+2 < 3$

$-5 < x < 1$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{2n+1} (x-5)^n}{n}$

Simplife

$\sum_{n=1}^{\infty} \frac{-1 \cdot (x-5)^n}{n}$

$a_n = \frac{(x-5)^n}{n}$ $\lim_{n\to\infty} \frac{\infty}{\infty} = 1$

$a_n \geq a_{n+1}$

$a_{n+1} = \frac{(x-5)^{n+1}}{n+1}$

$\sum_{n=1} (-1)^n \cdot \frac{(x-5)^n}{n}$

$R = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \Rightarrow \left|\frac{(x-5)^{n+1}}{n+1} \cdot \frac{n}{(x-5)^n}\right| = |x-5| \cdot \frac{n}{n+1}$

$\lim_{n\to\infty} = |x-5|$ $-1 < x-5 < 1 \Rightarrow 4 < x < 6$

Question 5

$a=0$

$F(x) = x^3 \cdot \ln(1+2 \cdot x^4)$

$\sum_{n=0}^{\infty} C_n (x-a)^n \Rightarrow C_n = \frac{f^{(n)}(a)}{n!}$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \cdot x^n$

Série de base #6

$\ln(1+u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \dots$

$\ln(1+2x^4) = 2x^4 - \frac{(2x^4)^2}{2} + \frac{(2x^4)^3}{3} - \frac{(2x^4)^4}{4} + \dots$

$= 2x^4 - 2x^8 + \frac{8x^{12}}{3} - \frac{16x^{16}}{4} \dots$

$F(x) = x^3 \left(2x^4 - 2x^8 + \frac{8x^{12}}{3} - \frac{16x^{16}}{4} + \dots\right)$

$= 2x^7 - 2x^{11} + \frac{8x^{15}}{3} - \frac{16x^{19}}{4} + \dots$ > que 16

$f(x) = 2x^7 - 2x^{11} + \frac{8x^{15}}{3}$

b)

$\ln(1+u) = x^3 \sum_{n=0}^{\infty} (-1)^n \frac{u^{n+1}}{n+1} \Rightarrow x^3 \ln(1+2x^4) = x^3 \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^n x^{4n}}{n}$

$\sum_{n=0}^{\infty} - \frac{(-1)^{n+1} 2^n x^{4n+3}}{n}$

$|u| \in [-1; 1] \Rightarrow -1 < 2x^4 < 1$

$2x^4 > 1$

$x > \sqrt[4]{\frac{1}{2}}$

Question 6₂

$g(x) = e^{-x^2}$

a) $e^{u} = \sum_{n=0}^{\infty} \frac{u^n}{n!} \Rightarrow \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = R$

$a_n = \frac{(-1^{n})\, x^{2n}}{n!}$

$a_{n+1} = \frac{(-1)^{n+1}\, x^{2(n+1)}}{(n+1)!}$

$\lim_{n\to\infty} \left|\frac{(-1)^{n+1} \cdot x^{2(n+1)}}{n-1} \cdot \frac{n!}{(-1)^n \cdot x^{2n}}\right| = R$

$\lim_{n\to\infty} \left|x^2 \cdot \frac{1}{n+1}\right| = \frac{x^2}{\infty} = 0 \quad R = \infty$

$g(x) = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$

$g(1) = 1 - 1 + \frac{1^4}{2} - \frac{1^6}{6} = 0 + \frac{1}{2} - \frac{1}{6}$

$g(1) \simeq \frac{3}{6} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

b) $|\text{Erreur}| = |S - S_k| < a_{k+1}$

$S_k = g(1) \simeq 0{,}\overline{333}$

Erreur $= 0{,}03455$

$S = e^{-1} = 0{,}3674$