Rappel :

Si f est continue sur $]a,b[$, alors $\int_a^b f(x)\,dx = F(b) - F(a)$ où F : primitive de f
c.à.d $F' = f$

Si $a, b \in \{-\infty, +\infty[$

ou bien f est discontinue sur les points isolés de $]a,b[$ on parle d'intégration impropre

type 1 type 2

ex : ① $\int_1^\infty \frac{dx}{dx}$: intégrale impropre ② $\int_0^3 \frac{2x^2}{x^2-1}\,dx$ est impropre

où f n'est pas continue en $x = 1$

Solution : Se base sur la propriété de chardes pour le type 2

$$\int_0^3 \frac{x^2}{x^2-1}\,dx = \int_0^1 \frac{2x^2}{x^2-1} + \int_1^3 \frac{x^2}{x^2}\,dx = \lim_{s \to 1^-} \int_0^s \frac{x^2}{x^2-1} + \lim_{t \to 1^+} \int_t^3 \frac{x^2}{x^2-1}\,dx$$

Pour le types deux :

$$\int_1^\infty \frac{dx}{x} = \lim_{s \to \infty} \int_1^s \frac{dx}{x} = \lim_{s \to \infty} [\ln x]_1^s = \lim_{s \to \infty} (\ln(s) - \ln 1) = \lim_{s \to \infty} \ln s = \infty$$ Donc la fonction diverge

4.54

a)

$$\int_2^6 \frac{3}{\sqrt{t-2}} \qquad I = \lim_{s \to 2^+} \int_s^6 \frac{3}{\sqrt{t-2}}\,dt = \lim_{s \to 2^+} I_s$$

$T = 2$ impropre

$$I_s = \int_s^6 3(t-2)^{-1/2}\,dt = \left[3 \frac{1}{-1/2+1} (t-2)^{-1/2+1}\right]_s^6 = \left[6(t-2)^{1/2}\right]_s^6 = 6\left(2 - (s-2)^{1/2}\right)$$

$$I_s = \lim_{s \to 2^+} 6\left(2 - (s-2)^{1/2}\right) = 12$$ □

e) $\int_4^\infty \frac{3}{t^2}\,dt = \lim_{s \to \infty} \int_4^s \frac{3}{t^2}\,dt$ or $\int_4^s 3t^{-2}\,dt = 3\left[\frac{t^{-1}}{-1}\right]_4^s = 3\left(\frac{1}{4} - \frac{1}{s}\right)$ $\qquad I = \lim_{s \to \infty} 3\left(\frac{1}{4} - \frac{1}{s}\right) = \frac{3}{4}$ □

d) propre

f) $\int_{-1}^\infty \frac{3}{t^2}\,dt$

impropre $t = 0$

$$I = \int_{-1}^0 \frac{1}{t^2}\,dt + \int_0^2 \frac{dx}{t^2} \qquad \int \frac{dt}{t^2} = \int t^{-2}\,dt = \frac{t^{-2+1}}{-2+1} = -t^{-1} + C$$

$$= \lim_{s \to 0^-} \int_{-1}^s \frac{dt}{t^2} + \lim_{t \to 0^+} = \lim_{s \to 0^-} [-t^{-1}]_{-1}^s + \lim_{t \to 0^+} [-t^{-1}]_t^2 = \lim_{s \to 0^-} \left(-1 - \frac{1}{s}\right) + \lim_{t \to 0^+} \left(\frac{1}{t} - \frac{1}{2}\right) = +\infty$$ I diverge

$+\infty + +\infty$

4.55

$$\int_0^\infty \frac{2x}{x^2-1}\,dx = \int_0^1 \frac{2x}{x^2-1}\,dx + \int_1^\infty \frac{2x}{x^2-1}\,dx = \int_0^1 \frac{2x}{x^2-1}\,dx + \int_1^3 \frac{2x}{x^2-1} + \int_3^\infty \frac{2x}{x^2-1}\,dx$$

$$\int \frac{2x}{x^2-1}\,dx = \int \frac{du}{u} \text{ ou } u = x^2-1$$
$$= \ln u + C$$
$$= \ln(|x^2-1|) + C$$

$$= \lim_{s \to 1^-} \int_0^s \frac{2x}{x^2-1}\,dx + \lim_{t \to 1^+} \int_t^3 \frac{2x}{x^2-1} + \lim_{u \to \infty} \int_3^u \frac{2x}{x^2-1}\,dx = \lim_{s \to 1^-} \left[\ln(|x^2-1|)\right]_0^s + \lim_{s \to 1^+} \left[\ln |x^2-1|\right]_t^3 + \lim_{u \to \infty} \left[\ln |x^2-1|\right]_3^u$$

$$= \lim_{s \to 1^-} \ln|s^2-1| + \cancel{\ln(8)} - \lim_{t \to 1^+} (\ln|t^2-1|) + \lim_{u \to \infty} \ln|u^2-1| - \cancel{\ln(8)} = \lim_{s \to 1} \ln\left|\frac{s^2-1}{s^2-1}\right| + \infty = 0$$ diverge

4,56

$v(x)^2 - v_0^2 = \frac{2}{m}\int_{x_0}^{x} F(u)\,du$

$x = R_t + h$

$h = 2c$ [illegible]

v: vitesse à laquelle, il touche la terre

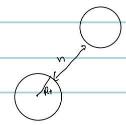
$v^2 - 0^0 = \frac{2}{m}\int_{R_t+h}^{R_t} -\frac{GM_m}{x^2}\,dx = 2GM\int_{R_t+h}^{R_t} -\frac{1}{x^2}\,dx$

$= 2GM\left[x^{-1}\right]_{R_t+h}^{R_t}$

$v^2 = 2GM\left(\frac{1}{R_t} - \frac{1}{R_t+h}\right)$

Lorsqu'il touche la terre

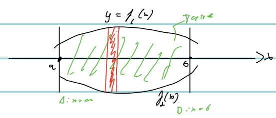
$x = R_t$



4.98

$W = \int_0^3 w(x)\,dx = \int_0^3 c\,y(x)\,dx = \int_0^3 c\left(\cosh\left(\frac{x}{3}\right) - 1\right)dx$

5.1 Aires



Surface située entre les courbes $f_2(x)$, $f_1(x)$ et la droite $\Delta: x=a$ et $D: x=b$

$\text{Aire} = \int_a^b \left(f_1(x) - f_2(x)\right)dx$

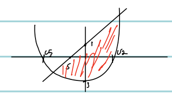
L'unité de surface $ds = \underbrace{(f_1(x) - f_2(x))}_{\text{longueur}}\underbrace{dx}_{\text{largeur}}$

d'où $S = \sum ds$

$= \int_a^b \left(f_1(x) - f_2(x)\right)dx$

Calculer l'aire de la surface contenue entre les courbes

$y = x^2 - 3$ et $d: y = x+1$



$\text{Aire} = \int_a^b \left((x+1) - (x^2-3)\right)dx$, a et b s'obtiennent par la résolution de

$x + 1 = x^2 - 3$

$x^2 - x - 4 = 0$

$x_1 = \frac{1-\sqrt{17}}{2} = a$ (propriété)

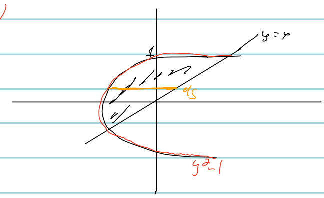
$x_2 = \frac{1+\sqrt{17}}{2} = b$

$\Delta = 1 - 4(1)(-4)$

$= 17 > 0$

$x = y^2 - 1$

$y = x$



$ds = \quad dy$

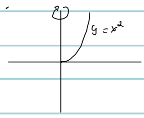
$= (y - y^2 + 1)\,dy$

$S = \int_{y_{m_1}}^{y_{m_2}} (y - y^2 + 1)\,dy$

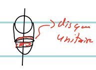
5.2 Solide de revolution

def: Un solide de révolution est obtenu par un tour complet d'une région autour d'un axe

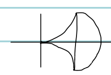
ex:



tour complet autour de l'axe $\vec{Oy}$ donne



Tour complet de $y = x^2$ autour de l'axe $\vec{Ox}$

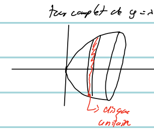


Q: Cmt calculer le volume du solide de revolution ?

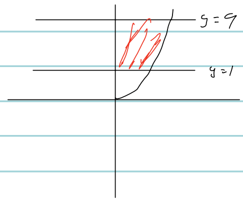
R: On a deux méthodes

* Méthode des disques
* Méthode des tubes

tour complet de $y = x^2$

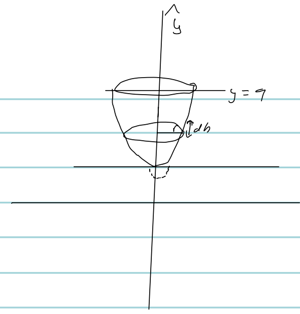


Volume du disque $= dv = \pi r^2 dh$



R : définie par

$y=x^2$, $x=0$, $y=1$, $y=9$



disque horizontale

$dv = \pi r^2\, dh$

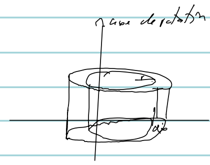
$= \pi\sqrt{y}\; dy$

$r = x$

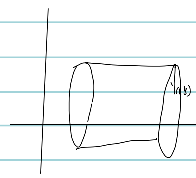
Or $y = x^2 \Rightarrow x = \sqrt{y}$

$v_{dm} = \int dv = \int_1^9 \pi y\, dy = \pi\left[\frac{y^2}{2}\right]_1^9 = \pi 40$

Méthode des tubes



$dv = 2\pi\, r(x)\; h(x)\, dx$



$dv = 2\pi\, r(y)\, h(y)\, dy$

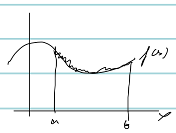
5.2

$r(x) = 2-x$

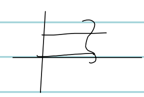
$h(x) = y-1 = x^3+2x+1)-1$

$v = \int_0^1 2\pi(2-x)(x^3+2x)\,dx$

5.3 Longueurs d'arc



$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$



$L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$y = x^2 + 4x - 1$

entre les points $P(1,4)$ et $Q(3,20)$

$L = \int_1^3 \sqrt{1+y'^2}\, dx$ Or $y' = 2x+4$

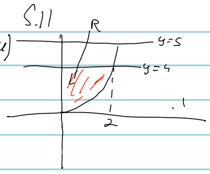
D'où $L = \int_1^3 \sqrt{1+(2x+4)^2}\, dx$

$x = \frac{y^6}{3} - y - 2$ On résoud $x=0 \Rightarrow$ solve$(\frac{y^6}{3} - y - 2 = 0, y)$

$L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

5.11

a)

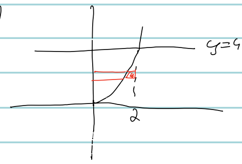


$dv = \pi\,(r_2^2 - r_1^2)\,dx$

$r_2 = 5 - x^2$

$r_1 = 1$

b)



$dv = \pi(r_2^2 - r_1^2\,dy$

$r_2 = 2$

$r_1 = 2 - \sqrt{y}$