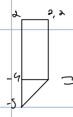
4,26

a) $\frac{dv}{dt}$ en cm^3/s

b) $\int_2^{2.2} \frac{dv}{dt} \cdot dt = V(2.2) - V(2)$ en cm^3

c)



$= 0{,}2 \cdot 4 + \frac{0{,}2 \cdot 1}{4}$

$= 0{,}81$

donc

$\int_2^{2.2} \frac{dv}{dt}\, dt = -0{,}81$

d'où $V(2.2) - \overset{20}{V(2)} = -0{,}81$

on obtient $V(2{,}2) = 19{,}19$

g) $V(b) - V(0) = \int_0^b \frac{dv}{dt}\, dt$

$V(b) = V(0) + \int_0^b \frac{dv}{dt}\, dt$

$V(b) = 20 + \int_0^b \frac{dv}{dt}\, dt$

d) on a $f'(3) = 0$ ← extremum

on doit étudier $x=3$ et ($x=0$ et $x=4$)
borne

$V(3) - V(0) = \int_0^3 \frac{dv}{dt}\, dt < 0$

$V(3) < V(0)$

on a aussi $V(4) - V(0) < V(0)$

donc max en $V(0)$

e) on doit étudier $t=0$ et $t=4$
(on a déjà traité $t=0$)

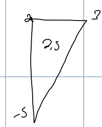
$\int_2^3 \frac{dv}{dt}\, dt$ vs $\int_2^4 \frac{dv}{dt}\, dt$

$-S_1$ $\quad < \quad S_1 + S_2$

donc min en $t=0$

d) et e) avec tableau variation

f) $\int_2^3 \frac{dv}{dt}\, dt = -2{,}5$



donc $V(3) - V(0) = -2{,}5$

d'où $V(3) = 17{,}5\, cm^3$

4,28

a) $\int_0^6 \frac{t^2}{5} - 2$

b) $h(t) - h(0) = \int_0^t v(s)\, ds$

$h(t) = h(0) + \int_0^t v(s)\, ds$

b) $h(t)$ $\quad h(5)$

c) résoudre $v(t) = 0$

d) $a(t) = v'(2)$

e) résoudre $h(t) = h(0) \iff h(t) = 5$?

f) $v(t)$ vs $a(t)$

g) signe de $v(2)$

ti

$v(t) := \frac{t^2}{5} - 2$

$h(t) := 5 + \int_0^t v(s)\, ds$

$h(t)$ $\quad$ rep $\frac{t^3}{15} - 2t + 5$

$h(2)$

$h(5)$

solve$(v(t) = 0, t)$

$a(t) := \frac{d}{dt}(v(t))$

$a(2)$

solve$(h(t) = 5, t)$

4,36

$a(t) = 6t - t^2$ et $v(0) = 5$ cm/s $h(0) = 10$ cm

op1 : $v(t) = \int a(t) = 3t^2 - \frac{t^3}{3} + c_1$

on a $v(0) = 5 \Rightarrow c_1 = 5$

op2 $v(t) - v(0) = \int_0^t a(x)\,dx$

donc $v(t) = v(0) + \int_0^t a(x)\,dx$

$= 5 + 3t^2 - \frac{t^3}{3}$

ti $a(t) = 6 \cdot t - t^2$

$v0 := 5$

$h0 := 10$

$v(t) := v0 + \int_0^t a(x)\,dx$

$v(t$

$h(t) := h0 + \int_0^t v(x)\,dx$

$h(t)$

solve$(v(t) = 0, t)$

4,43

h) $e^x + c$

b) $\int \sin(x^4)\, x^3\,dx$ ou $\frac{1}{4}\int \sin(u)\,du$

$u = x^4$

$\frac{du}{dx} = 4x^3\,dx$

on a $x^3\,dx$

$= \frac{1}{4}du$

$= -\frac{1}{4}\cos(u) + c$

$= -\frac{1}{4}\cos(x^4) + c$

j) $\int e^{\sin(x)} \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int e^u\,du = e^u + c$

$= e^{\sin x} + c$

c) $\int \frac{2x}{\sqrt{3x^2+1}}\,dx$

$= \int (3x^2+1)^{-1/2} \cdot 2x\,dx$

$u = 3x^2 + 1$

$du = 6x\,dx$

$2x\,dx = \frac{1}{3}du$

$= \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3} \cdot \frac{1}{2} u^{1/2} + c$

$= \frac{1}{6}\sqrt{u} + c$

$= \frac{1}{6}\sqrt{3x^2+1} + c$

4.44

b) R.14

$$\frac{\cos x}{9-\sin x}\,dx$$

$u^2=\sin^2 x \rightarrow u=\sin x$

ave $u=\sin x$ on a $du=\cos x\,dx$

$$= \int \frac{1}{a^2-u^2}\,du \quad \text{ave } u=\sin x \text{ og } a=3$$

$$= \frac{1}{2a}\ln\left(\left|\frac{u+a}{u-a}\right|\right)+C$$

$$= \frac{1}{6}\ln\left(\left|\frac{\sin x+3}{\sin x-3}\right|\right)+C$$

$$= \frac{1}{6}\ln\left(\frac{3+\sin x}{3-\sin x}\right)+C$$

c) R15 $a=5$ $u=\ln(x)$

d)

$$\int \frac{x^2}{1+x^6}\,dx$$

$a=1,\ u^2=x^6 \rightarrow a=1,\ u=x^3$

$du = 3x^2\,dx$

$x^2\,dx = \frac{1}{3}\,du$

$$\frac{1}{3}\int \frac{1}{1+u^2}\,du = \frac{1}{3}\arctan(u)\,du$$

$$= \frac{1}{3}\arctan(u^3)+C$$

l) $\int 3(y-2)^5\,dy$

ave $u=y-2$

$\frac{du}{dy}=1 \rightarrow du=dy$

$\int 3u^5\,du$

$= 3\cdot\frac{1}{6}u^6+C = \frac{1}{2}u^6+C$

$= \frac{1}{2}(y-2)^6+C$

k) $u=4x^2+4$

$du = 8x\,dx$

$3x\,dx = \frac{1}{6}\,du$

$= \frac{1}{6}\int u^{1/2}\,du$

$= \frac{1}{6}\cdot\frac{1}{3/2}u^{3/2}+C$

$= \frac{1}{4}\sqrt{(4x^2+4)^3}+C$

4.45

f) $\int (3t+17)^{10}\,dt$

$u=3t+17$

$du = 3\,dt$

$dt = \frac{1}{3}\,du$

$\int (3t+17)^{10}\,dt = \frac{1}{3}\int u^{10}\,du$

$= \frac{1}{3}\cdot\frac{1}{11}u^{11}+C$

$= \frac{1}{33}(3t+17)^{11}+C$

g) $u=x^3+4$

$du = 3x^2\,dx$

on a $4x^2\,dx = \frac{4}{3}\,du$

$= \frac{4}{3}\int u^3\,du$

$= \frac{4}{3}\cdot\frac{1}{4}u^4+C$

$= \frac{1}{3}(x^3+4)^4+C$

p) $\int 3e^{-4y}\,dy$

$= \frac{3}{-4}e^{-4y}+C$

$= -\frac{3}{4}e^{-4y}+C$

s) $\frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{2}\cdot\sqrt{x}} = \frac{1}{\sqrt{x}}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$

$= \left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)\int \frac{1}{\sqrt{x}}\,dx$

$= \left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)\cdot 2\sqrt{x}+C$

$= (1+\sqrt{2})\sqrt{x}+C$

2) $\int \frac{5}{\sqrt{4-x^2}}\, dx$

$a = 2$
$u = x$

$= 5 \arcsin\left(\frac{x}{2}\right) + C$

4) $\int \frac{3x}{\sqrt{4-x^2}}$

$u = 4 - x^2$

$du = -2x\, dx$

$x\, dx = -\frac{1}{2}\, du$

$3\int \frac{x}{\sqrt{4-x}}\, dx = -\frac{3}{2}\int u^{-1/2}\, du$

$= -\frac{3}{2} \cdot \frac{1}{\frac{1}{2}} \cdot u^{1/2} + C$

$= 3\sqrt{4-x^2}\; C$

6) $\int \tan^4(3x) \sec^2(3x)\, dx$

$(\tan(x))' = \sec^2(x)$

$u = \tan(3x)$

$du = 3\sec^2(3x)\, dx \longrightarrow \sec^2(3x)\, dx = \frac{1}{3} du$

$= \frac{1}{3}\int u^4\, du$

$= \frac{1}{3} \cdot \frac{1}{5} u^5 + C$

$= \frac{1}{15}\left(\tan(3x)^5\right) + C$

4,38

$g(x) = f(t)$

a) $=$
b) $>$
c) $>$
d) $<$
e) $>$
f) $<$
g) $>$
h) $=$

4,39 $\quad Li(x) = \int_2^x \frac{1}{\ln(t)}\, dt$

$\int_2^x \frac{1}{\ln(t)}\, dt$

a) $Li(2) = \int_2^2 \frac{1}{\ln(t)} = 0$

b) $Li(x)' = F(x) - F(2)$ si $F(x) = \int \frac{1}{\ln(t)}\, dt$

$= \frac{1}{\ln(x)}$

$Li''(x) = \left(\frac{1}{\ln(x)}\right)' = \frac{1' \ln x - 1 \cdot \ln(x)'}{\ln^2(x)} = \frac{-1}{x \ln^2(x)}$

e) $f'' < 0$ concave
cubs

c) $x \geq 2 \quad \ln(x) \geq 1$

On a $\frac{1}{\ln(x)} > 0$

$Li'(x) > 0 \Rightarrow li(x) \nearrow$
$Li''(x) < 0$

d)

4,52

a) $\int \frac{1}{x^2-6x+13}\,dx$

$x^2-6x+13 = x^2-6x+9-9+13$

$=(x-3)^2+4$

f) $=\int \frac{4x-2}{\sqrt{x^2+10x+16}}\,dx$

$(x^2+10x+16)' = 2x+10$

on a $4x-2 = 2(2x+10)-22$

$x^2+10x+16 = x^2+10x+25-25+16$

$=(x+5)^2-9$

$$\int \frac{2(2x+10)-22}{\sqrt{x^2+10x+16}}\,dx$$

$$=2\int \frac{2x+10}{\sqrt{x^2+10x+16}}\,dx - 22$$

$=\int \frac{1}{\sqrt{u}}\,du$ avec $u = x^2+10x+16$

$=2\sqrt{u}+c_1 = 2\sqrt{x^2+10x+16}+c$

$=\int \frac{1}{\sqrt{x^2+10x+16}}\,dx$

$=\int \frac{1}{\sqrt{(x+5)^2-3^2}}\,dx$ avec $u = x+5$, $a=3$

$=\ln(|(x+5)+\sqrt{(x+5)^2-3^2}|)+c$

$=4\sqrt{x^2+10x+16}-22\ln(|\sqrt{x^2+10x+16}+x+5|)+c$

$$\int = \frac{3x-1}{4x^2-4x+5}$$

$\to (4x^2-4x+5)' = 8x-4$

$=3x-1 = \frac{3}{8}(8x-4)$

$\to 4x^2-4x+5 = 4(x^2-x+5/4)$

$=4(x^2-x+1/4-1/4+5/4)$

$=4((x-1/2)^2+1)$

avec $u = 4x^2-4x+5$

on a $du = (8x-4)dx$

$\int \frac{1}{u}du = \ln(|u|)+c$

$=\ln(|4x^2-4x+5|)+c$

$u = x-1/2$, $a=1$, $du=dx$

$=\int \frac{1}{4(u^2+a^2)}\,du$

$=1/4 \arctan(u)+c_2$

$=1/4 \arctan(x-1/2)+c_2$

$=\frac{3}{8}+1/2$

$=3/8\ln(|4x^2-4x+5|)+1/8\arctan(x-1/2)+c$

4,5

a) $\int x e^x\,dx$