# LAB REPORT
for IE1206

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# Contents

# 1 DC Measurements

## 1.1 KVL, KCL and power in resistive net

For the four resistors between 1kOhm and 47kOhm I chose $R_1$=1 kΩ, $R_2$=47 kΩ, $R_3$=2.2 kΩ and $R_4$=22 kΩ. I first drew the circuit in QUCS and simulated the currents and voltages across all resistors and then built it on my breadboard and measured them with a multimeter.

### 1.1.1 Simulation (using QUCS)

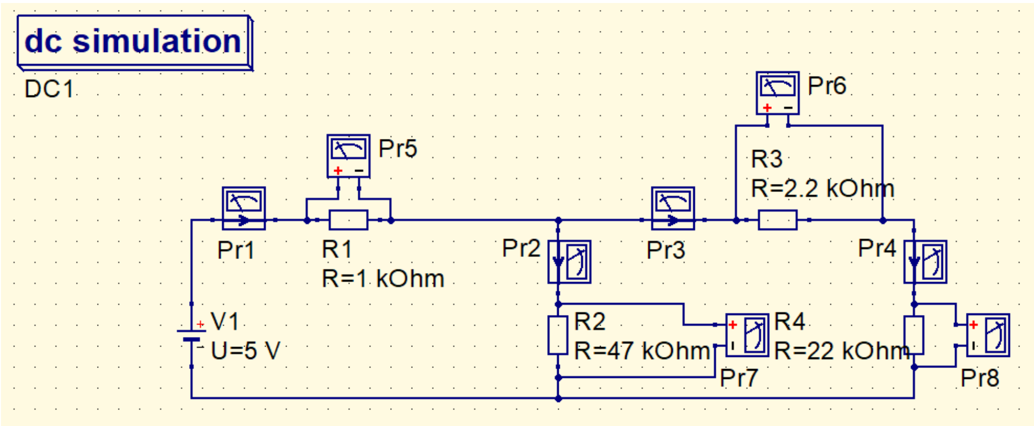



Figure 1: The circuit in QUCS

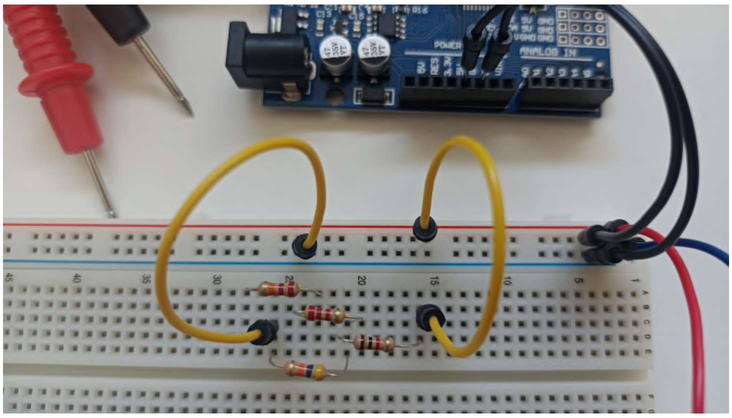
Figure 1 shows the circuit in the simulation and figure 2 shows the simulated measurements.

| number | Pr1.I | Pr2.I | Pr3.I | Pr4.I | Pr5.V | Pr6.V | Pr7.V | Pr8.V | Pr9.V |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.000295 | 0.0001 | 0.000194 | 0.000194 | 0.295 | 0.428 | 4.71 | 4.28 | 5 |

Figure 2: The measurements simulated by QUCS

### 1.1.2 Building the circuit

Based on the drawing I built the circuit on the breadboard and took measurements with the multimeter. For the voltages I used the 20 V setting to get the most precision and for the currents I used 2000 µA.



The circuit on the breadboard

### 1.1.3 Results and comparisons

| **Comparisons** | $R_1$ | $R_2$ | $R_3$ | $R_4$ | Arduino 5V Voltage source |
|---|---|---|---|---|---|
| Meas R Multimeter [kOhm] | 1.010 | 45.700 | 2.160 | 21.600 | not applicable |
| Meas V Multimeter [V] | 0.300 | 4.780 | 0.430 | 4.340 | 5.040 |
| Meas I Multimeter [mA] | 0.302 | 0.105 | 0.202 | 0.200 | -0.296 |
| Calc R=V/I [Ohm] | 0.993 | 45.524 | 2.129 | 21.7 | not applicable |
| Calc P=V*I [mW] | 0.090 | 0.502 | 0.087 | 0.868 | -1.492 |
| Simulated V QUCS [V] | 0.295 | 4.710 | 0.428 | 4.280 | 5.000 |
| Simulated I QUCS [mA] | 0.295 | 0.100 | 0.194 | 0.194 | -0.295 |
| Simulated P QUCS [mW] | 0.087 | 0.471 | 0.083 | 0.830 | -1.475 |

Table 1: Comparing simulated and measured values

Table 1 contains my measurements. All my measurements agree within measurement/simulation accuracy.
Kirchhoff's Voltage Law in the left loop says:

$$V_A - V_1 - V_2 = 0 \Leftrightarrow V_A = V_1 + V_2 \Leftrightarrow 5.040 \approx 0.300 + 4.780 = 5.080$$

which is true within measurements accuracy. Similarly, in the right loop we have:

$$V_2 - V_3 - V_4 = 0 \Leftrightarrow V_2 = V_3 + V_4 \Leftrightarrow 4.780 \approx 0.430 + 4.340 = 4.770$$

Kirchhoff's Current Law says that in a particular node, the sum off all the currents flowing in is equal to the sum of all the currents flowing out. This means for the node $R_1$ - $R_2$ - $R_3$ that:

$$I_1 = I_2 + I_3 \Leftrightarrow 0.302 \approx 0.105 + 0.202 = 0.307$$

which is true as well.
Since we know resistors always consume power, the only component in this circuit that supplies power is the voltage source, which is confirmed by being the only column where power is negative. To see whether power is balanced or not, we need to solve the equation $P_1 + P_2 + P_3 + P_4 = -P_A$:

$$0.090 + 0.502 + 0.087 + 0.868 = 1.547 \quad \approx 1.475$$

The difference (0.072 W) occurs only because of measurement accuracy limitations, the power is balanced in the circuit.

## 1.2 Determine Thevenin equivalent circuits

In this section I will determine the Thevenin equivalent circuit (Figure 3) of two pins of the Arduino.

The rationale behind Thevenin circuits is to hide unnecessary details of a complicated circuit's implementation if we are only interested in the current-voltage relationship of two nodes (often where we want to connect other circuit elements).

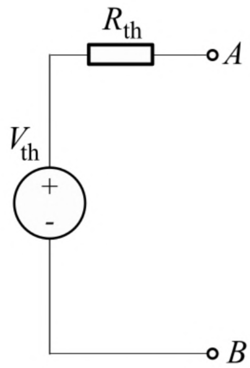



Figure 3: Thevenin equivalent circuit scheme

### 1.2.1 Thevenin equivalent circuit for Arduino 5 V pin

In order to determine the Thevenin equivalent circuit seen from the 5 V pin and GND, I first connected these two to different rows of my breadboard, thus creating an open circuit, so by measuring the voltage I got my $V_{TH}$ (Figure 4). Then I connected these two rows through a resistor to measure the voltage again and calculate $R_{TH}$. First I tried using relatively high resistances (1 kΩ ... 47 kΩ) but the difference from $V_{TH}$ was not measurable. Once I switched to the lowest resistance the Arduino output could still handle (100 Ω), I was able to measure my $V_L$ (Figure 5) and calculate $I_L$ and then $R_{TH}$. I got:

$$V_{TH} = 5.09\,\mathrm{V} \qquad V_L = 5.05\,\mathrm{V} \qquad I_L = \frac{5.05\,\mathrm{V}}{100\,\Omega} = 0.0505\,\mathrm{A}$$
$$\Rightarrow R_{TH} = \frac{V_{TH} - V_L}{I_L} = \frac{0.04}{0.0505} = 0.79\,\Omega$$

This is a very small resistance, which explains why it was so difficult to measure and due to the limitations of the lab equipment, it is not a very precise value.

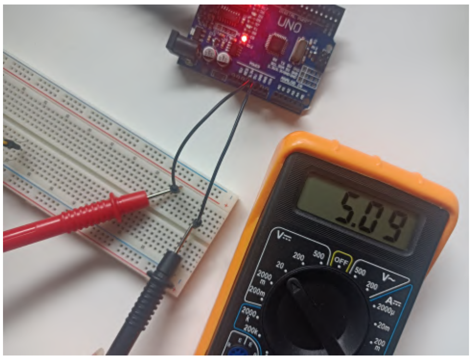



Figure 4: Measuring $V_{TH}$

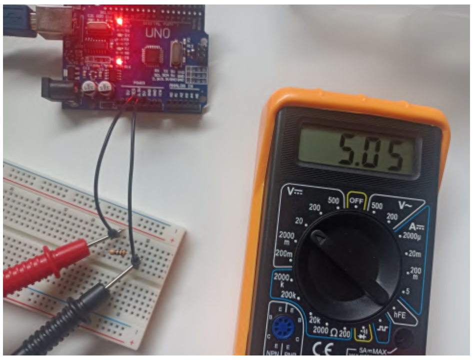



Figure 5: Measuring $V_L$

### 1.2.2 Thevenin equivalent for Arduino digital output pin

After installing the program that sets pin 8 high on the Arduino, I measured the voltage over the pin in an open circuit (as described in the previous section) and got $V_{TH}$=5.05 V and over a 100 Ω resistor I got $V_L$=3.92 V. Through similar calculations as before, I found $R_{TH} = \frac{5.05 - 3.92}{0.0392}$=28.827 Ω.

## 1.3 Light Emitting Diode circuits

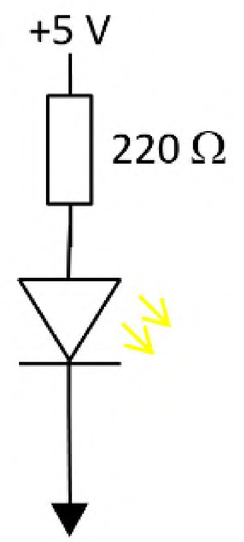
In this task I built and studied 4 circuits containing a 220 Ω resistor and two LEDs, one yellow and one blue, using the Arduino 5 V pin as power supply.

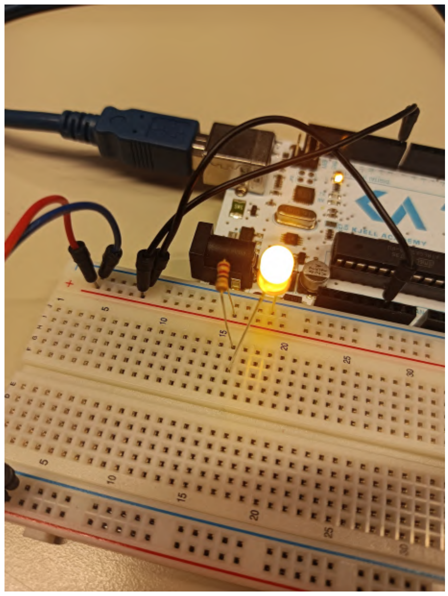
#### 1.3.1 Resistor in series with yellow LED

I connected the yellow LED in series with a resistor and saw that the LED was on. The voltage measured over the resistor was 3.02 V and 2.06 V over the LED. The current flowing in the circuit can be calculated from the resistance:

$$I = \frac{V}{R} = \frac{3.02}{220} = 13.73\,\mathrm{mA}$$

Therefore the power consumed by the LED is $P = V * I \quad = 28.28\,\mathrm{mW}$.

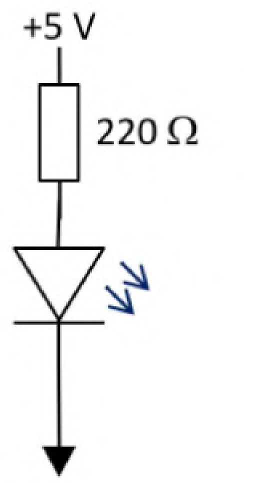


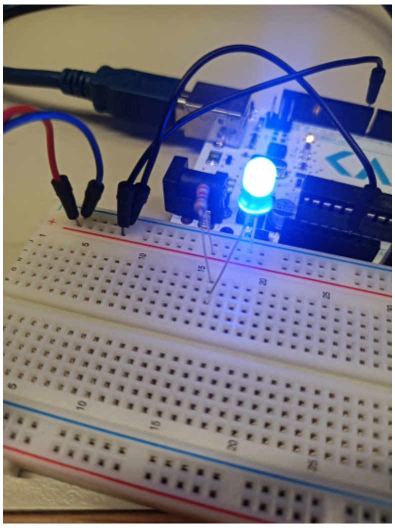



The yellow LED in series with the resistor

#### 1.3.2 Resistor in series with blue LED

Similarly, with the blue LED I measured 2.12 V over the resistor and 2.62 V over the LED, so I=9.64 mA and from there P=25.26 mW.

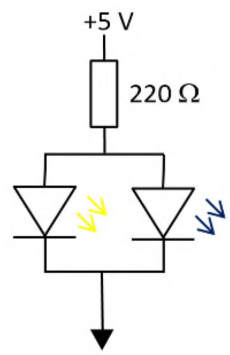


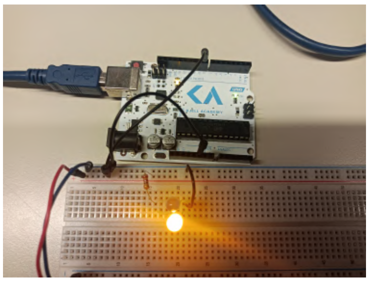



The blue LED in series with the resistor

#### 1.3.3 Resistor in series with a yellow and blue LED connected in parallel

When connecting the two LEDs in parallel, and in series with the resistor I found that only the yellow LED turns on, because they have different forward voltage drops. The yellow LED having a lower $V_F$ forces most of the current to flow in that branch and therefore the other LED with the higher $V_F$ can't turn on.

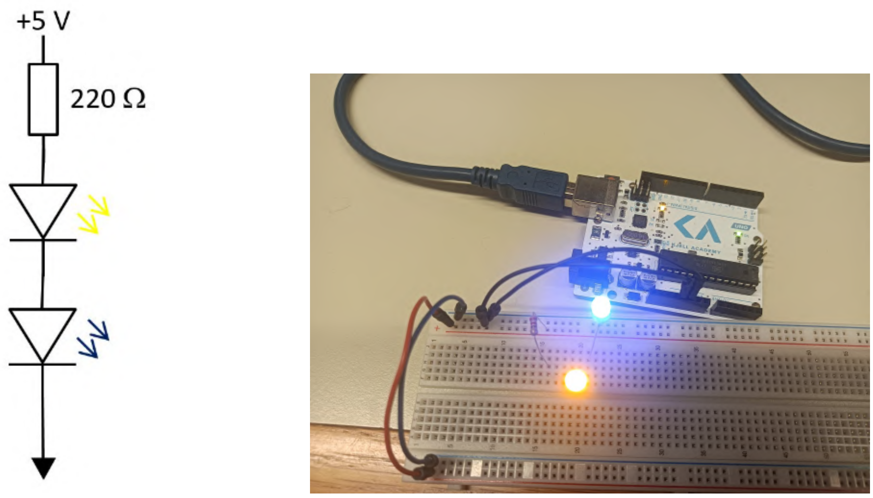


The two LEDs parallel to each other, in series with the resistor

#### 1.3.4 Resistor in series with a yellow and blue LED connected in series

On the other hand, if we connect all of them in series, both LEDs light up. In this case, the only possibilities were both or neither lighting up, but the 5 V from the Arduino provided enough of a voltage drop so that the LEDs conduct current.



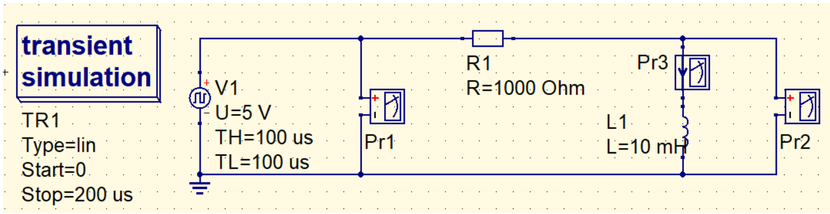

The two LEDs and the resistor in series

# 2 Time dependent circuits

## 2.1 Time dependent behavior of an RL circuit

The time constant of a circuit with an inductance and some resistance is $\tau = \frac{\mathrm{L}}{\mathrm{R_{TH}}}$. Here L is 10 mH and $\mathrm{R_{TH}}$ is just $\mathrm{R_1}$, so we have:

$$\tau = \frac{10 * 10^{-3}}{1 * 10^{3}} = 10^{-5} = 10\,\mu\text{s}$$




At $\mathrm{t} = \tau$ we have $\mathrm{I}(\tau) = \mathrm{I}(0) * (1 - \mathrm{e}^{-1}) = \frac{\mathrm{V_0}}{\mathrm{R_1}} * (1 - \mathrm{e}^{-1}) = \frac{5}{1} * 0.632 = 3.16\,\mathrm{mA}$. By setting a marker to the point on the plot where I=3.16, we can read off $\mathrm{t} = 1 * 10^{-5}$.

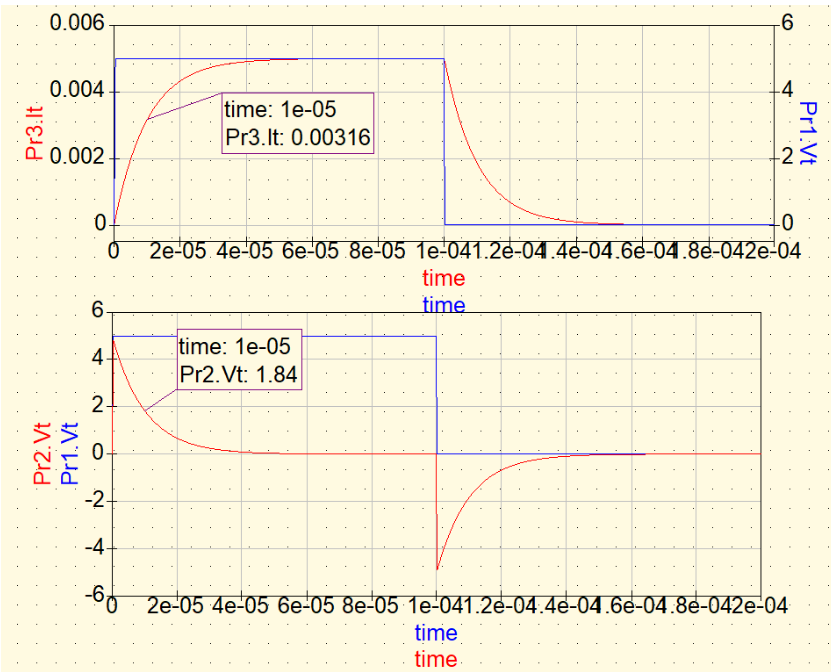
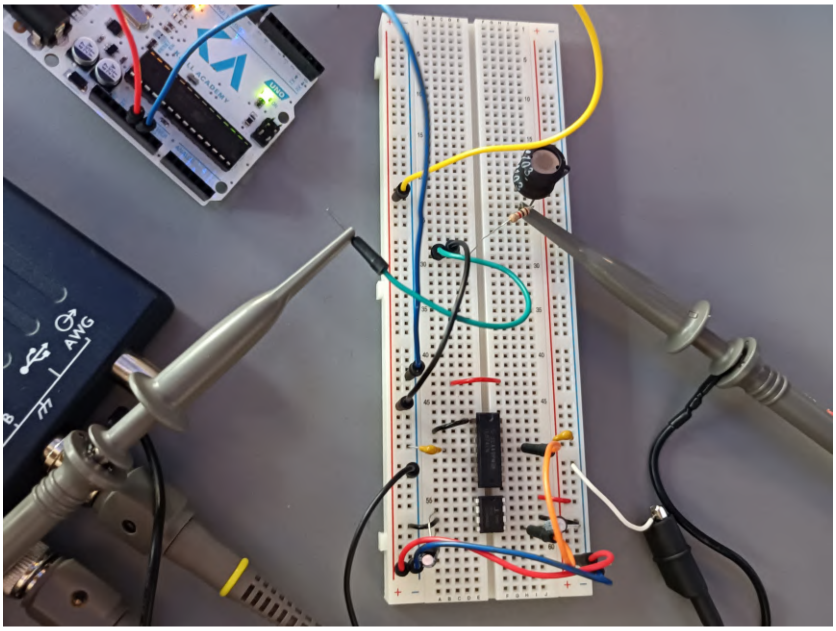



Figure 6: The plots in QUCS

Similarly, for the voltage at $\mathrm{t} = \tau$ we have: $\mathrm{V}(\tau) = \mathrm{V}_0 * \mathrm{e}^{-1} = 5 * 0.368 = 1.84\,\mathrm{V}$. In the simulation we can see that the current $\mathrm{I_L}$ through an inductor is always continuous, since there cannot exist infinite voltage.




The circuit on the breadboard

After building the circuit on my breadboard I measured the voltage from pin 9 (on channel A) and over the inductor (channel B) with the picoscope (shown in Figure 7). The Arduino produces the square wave shown in blue and the voltage across the picoscope varies as shown in red. As we can see

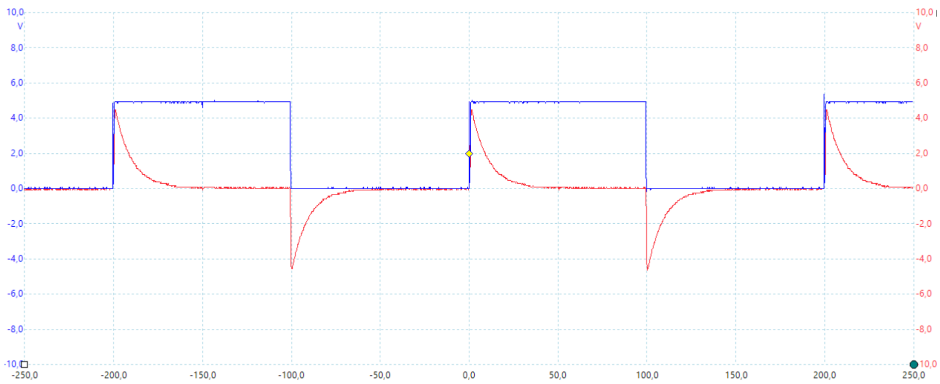



Figure 7: The plots in the picoscope software

on the plot, once the inductor reaches it's steady state, it works as a short circuit i.e. the voltage is zero. When a change occurs in the circuit (an edge in the square wave in this case) during the transient the voltage jumps to almost exactly the source voltage and than slowly settles back to 0 V. The voltage over the inductor is also proportional to the derivative of the current through it, which changes faster in the beginning and slower as it approaches it's steady state value. Same way as before, I can pinpoint the value of $V_L$ at t=$\tau$ and read off the corresponding time which close to 10 µs once again (shown in Figure 8).

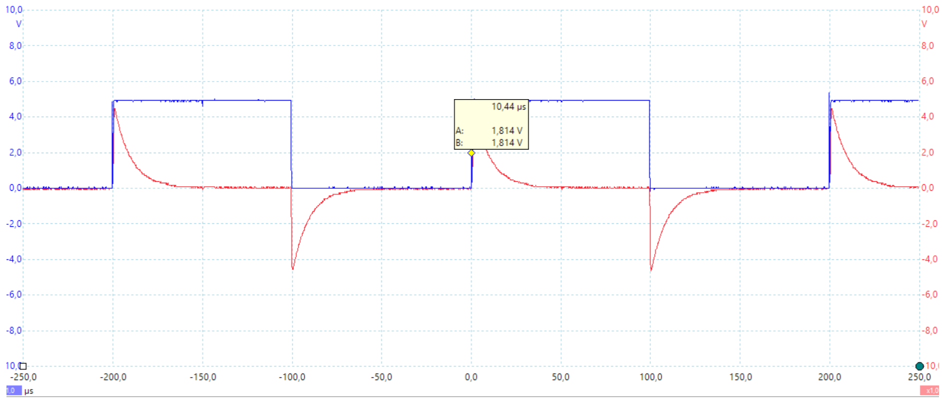



Figure 8: Finding $\tau$

Placing another inductor of the same inductance parallel to the one we already had reduced $\tau$ to half it's previous value. This is because for two parallel inductors we have:

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow \frac{1}{L} = \frac{1}{10} + \frac{1}{10} \Leftrightarrow L = 5$$

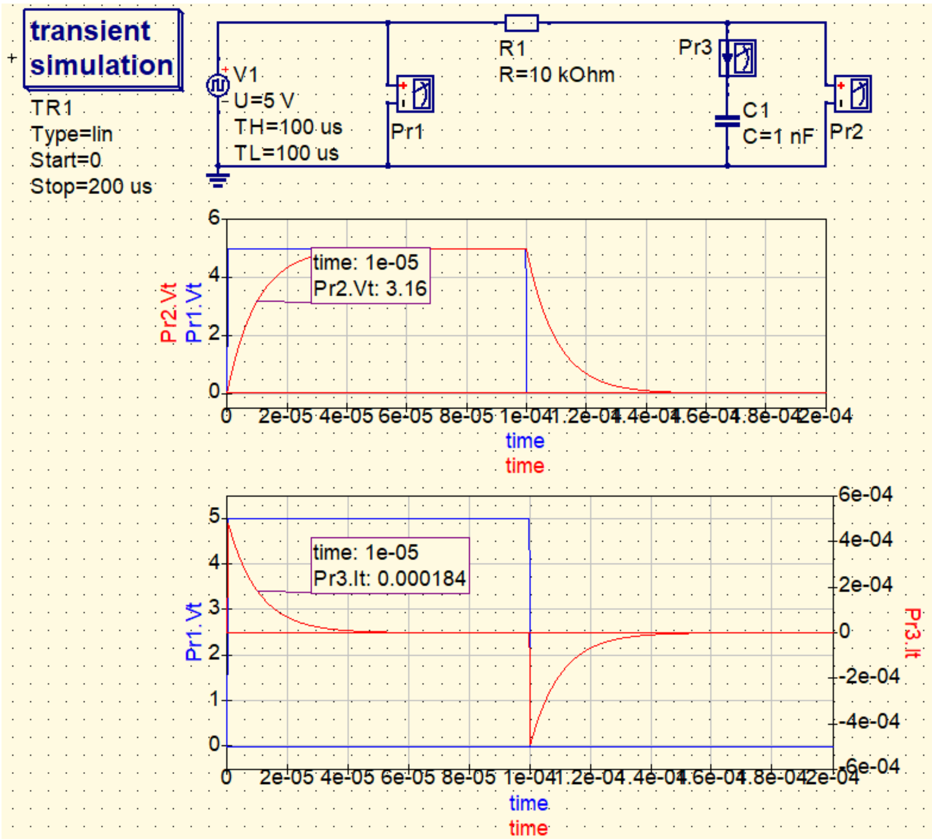
is the new inductance, so $\tau$=5/1=5 µs.
Moreover, for inductors connected in series we have:

$$L_s = L_1 + L_2 \Rightarrow L = 20, \text{ so } \tau\text{=20 µs has doubled.}$$

## 2.2 Time dependent behavior of an RC circuit

For a circuit with a capacitor and some resistance, the time constant is $\tau = C * R_{TH}$. In this case, C=1 nF and $R_{TH} = R_1$=10 kΩ, so

$$\tau = 1 * 10 * -9 * 10 * 10^3 = 10^{-5}\text{=10 µs.}$$




At $t = \tau$ we have $V(\tau) = V(0) * (1 - e^{-1}) = 5 * 0.632 = 3.16\,V$, at which point we can read off that $\tau = 10^{-5}$. Same goes for current with $\frac{V_0}{R_1} * e^{-1} = 1.839\,mA$. We can see that the voltage over the capacitor is a continuous function of time and the current is proportional to it's derivative.

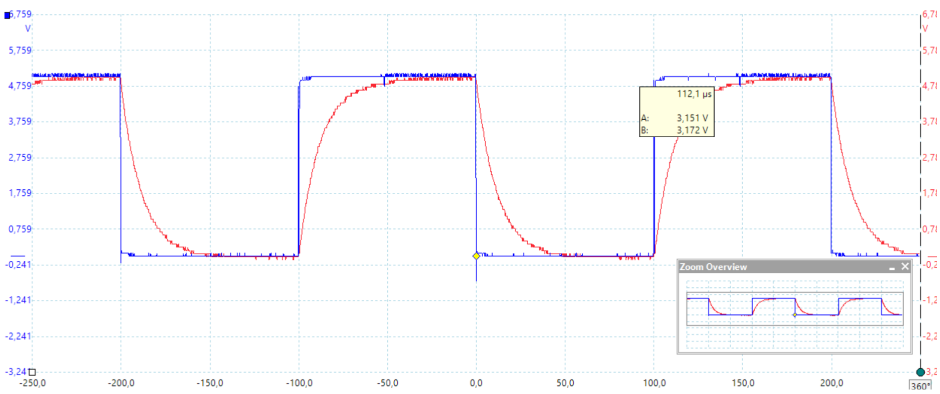
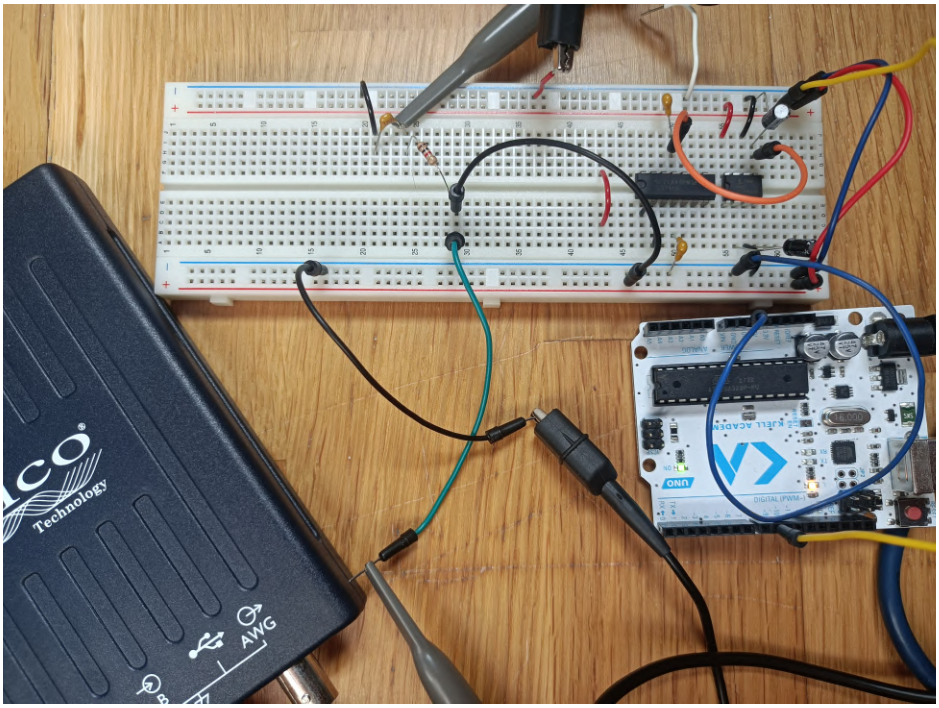



Figure 9: The plot in the picoscope software

Once again, I built the circuit on the breadboard and took measurements with the picoscope (Figure 9). The main difference from the RL circuit's plot is that the capacitor once it reached it's steady state works as an open circuit, i.e. the current is zero and the voltage is whatever value it was (dis)charged to.
By observing the time period it takes for the capacitor to charge up to $(1 - \frac{1}{e}) * 5 = 3.16\,\text{V}$ we can see that $\tau$=112.1-100=12.1 µs $\approx$ 10 µs.




The circuit on the breadboard

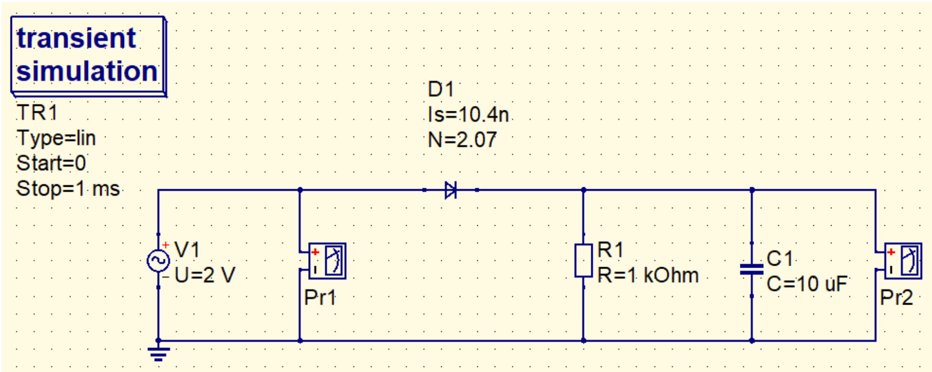
Placing another capacitor of the same size in parallel has doubled $\tau$. This is because for two parallel capacitors we have:

$$C_p = C_1 + C_2 \Rightarrow C = 200\,\text{nF}$$

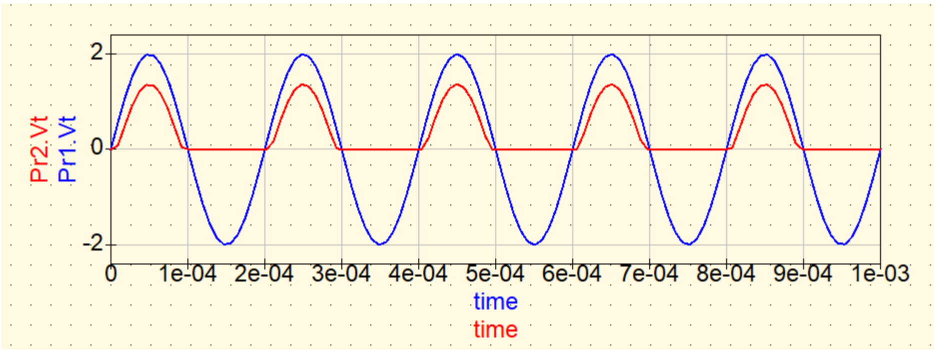
is the new capacitance, so $\tau$=20 µs.
Moreover, for capacitors connected in series we have:

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = 5$, so $\tau$=5 µs has reduced to half it's previous value.

## 2.3 Diode rectifier circuit with resistor and capacitor

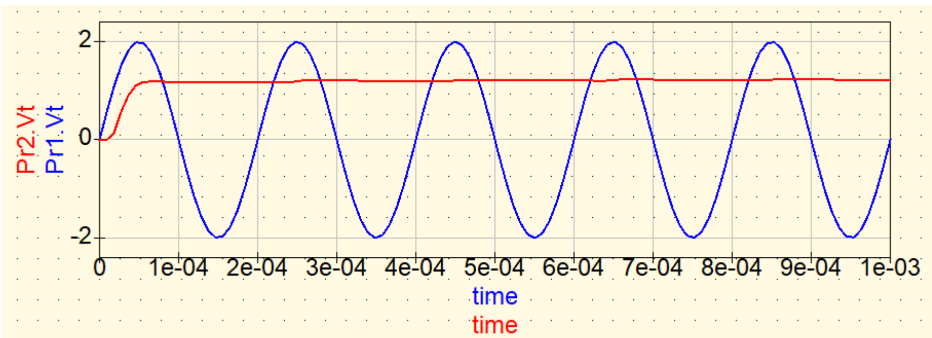



The simulated circuit




The simulated voltage **without** the capacitor

The voltage over the resistor is proportional to the voltage of the sinusoidal voltage source as expected.




The simulated voltage **with** the capacitor

The capacitor charges up with the growing voltage as expected to around 1.2 V but cannot discharge because of the diode blocking the way of current in that direction.

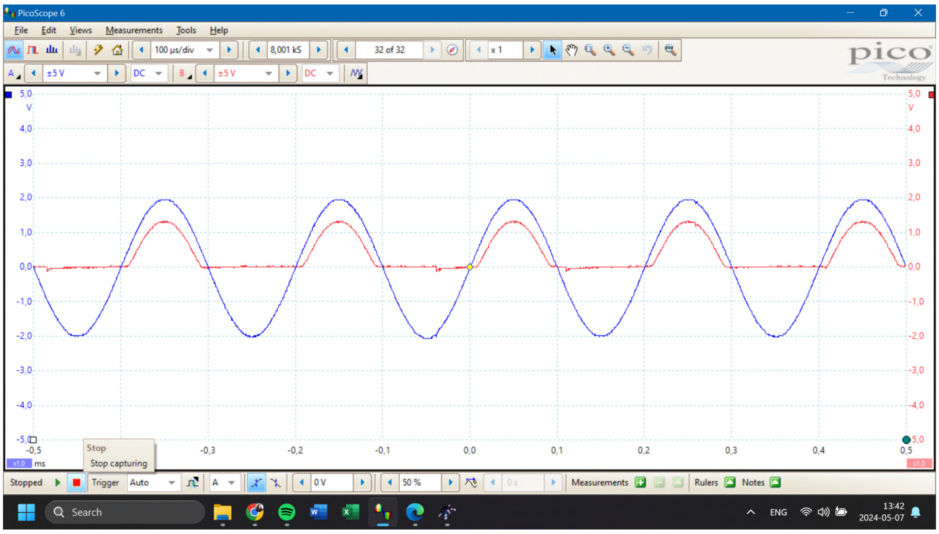



Figure 10: Measuring the voltage **without** the capacitor

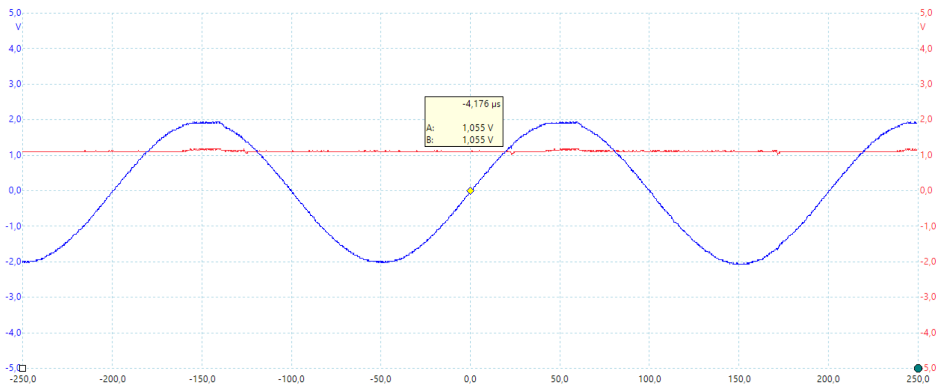
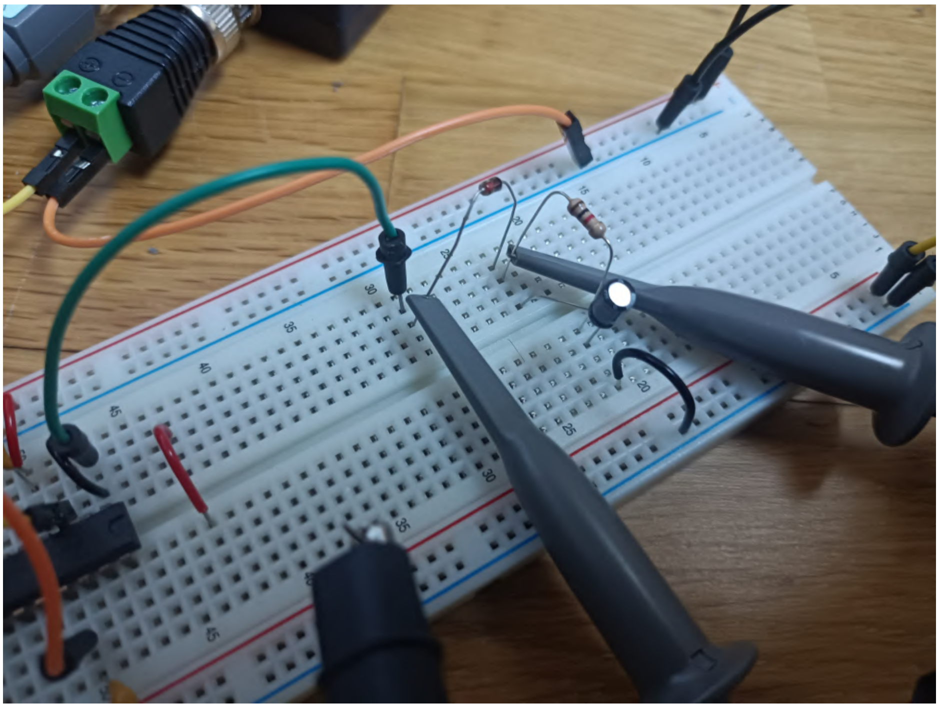


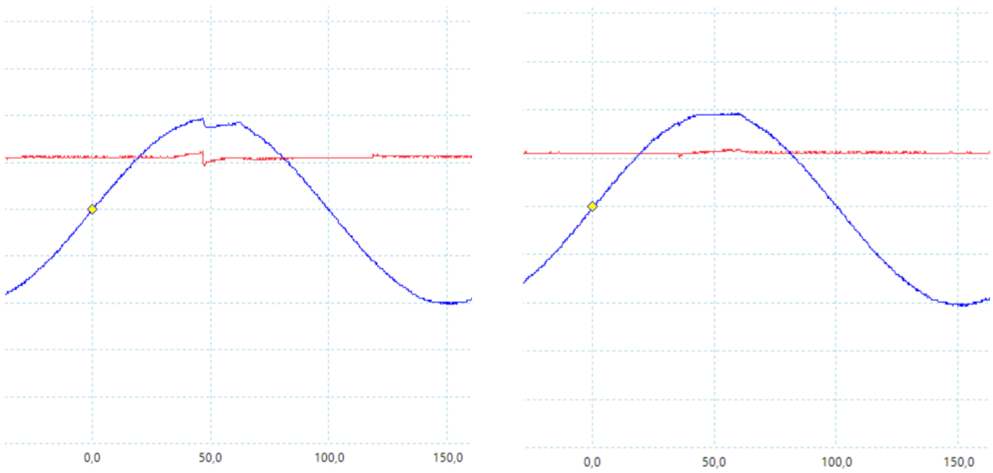

Figure 11: Measuring the voltage **with** the capacitor

Through these measurements I got the same results as in the simulation. Without the capacitor, the voltage grows proportional to the source and with the capacitor, the circuit converts AC voltage to DC voltage. The constant voltage $V_{OUT}$ I got was 1.055 V.
To determine the Thevenin equivalent circuit, I measured the voltage unloaded ($V_{TH} = V_{OUT}$) and the voltage over a resistor of 220 Ω which was 0.549 V, so I calculated $R_{TH}$=202.4 Ω. This is much larger than what I got for the 5 V pin which was less than 1 Ω.



The rectifier circuit on the breadboard

**Remark:** By moving around some of the wires to get a better picture of the circuit, I noticed I got rid of some noise that was happening in my initial picoscope measurements. This seems to have happened due to some additional capacitances between the wires.

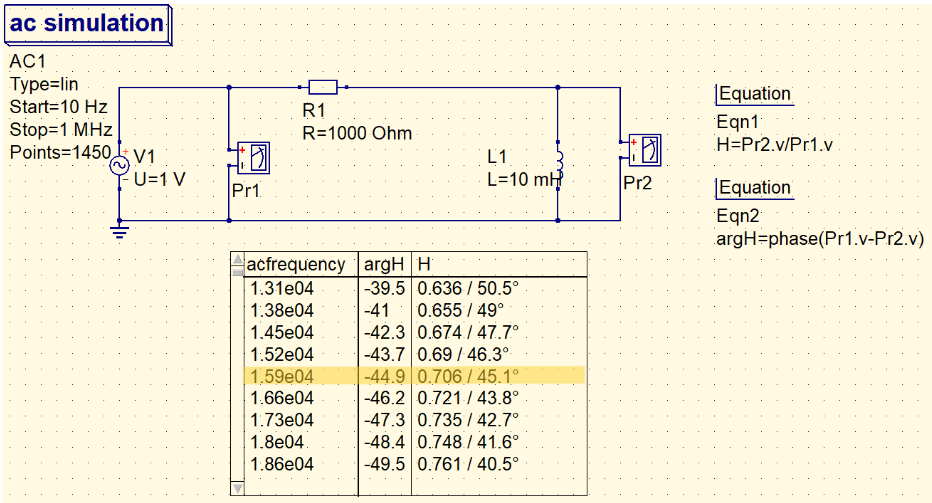



The plot before and after moving the wires

# 3 AC Measurements

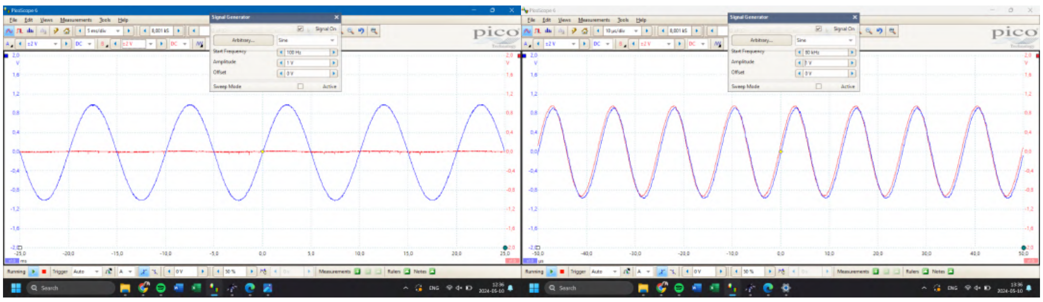
## 3.1 RL filter

We can calculate the cut-off frequency in an RL filter from the component values as $\frac{\mathrm{R}}{2\pi\mathrm{L}} = \frac{1000}{2\pi * 10 * 10^{-3}}$ =15 915 Hz=15.9 kHz.
It is also possible to determine the cut-off frequency from the simulation by finding the frequency related to when $\frac{\mathrm{V_{OUT}}}{\mathrm{V_{IN}}} = \frac{\mathrm{Pr2.v}}{\mathrm{Pr1.v}} = \mathrm{H} = \frac{1}{\sqrt{2}}$=0.707 V.




Determining the cut-off frequency in the simulation

After investigating the response of the filter to sine signals of different frequencies, I found that this circuit is a **high pass filter**, i.e. on higher frequencies (∼ 80 kHz) the output voltage is almost the same as the input voltage whereas on lower frequencies (∼ 100 Hz) it is close to 0 V. Table 2 shows my measurements.

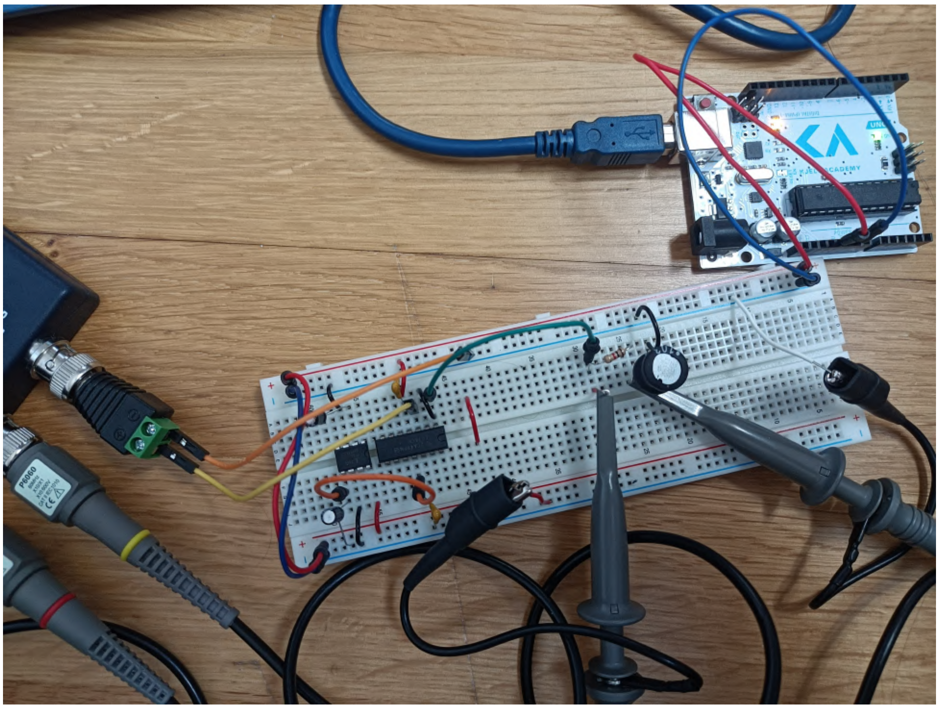


The output voltage (in red) with 100Hz and 80kHz

| Frequency | $V_{OUT}$ |
|---|---|
| 100 Hz | $\sim$ 0 V |
| 1 kHz | $\sim$ 60 mV |
| 5 kHz | $\sim$ 280 mV |
| 10 kHz | $\sim$ 490 mV |
| 16 kHz | $\sim$ 700 mV |
| 30 kHz | $\sim$ 860 mV |
| 50 kHz | $\sim$ 950 mV |
| 80 kHz | $\sim$ 1 V |

Table 2: Frequencies with their corresponding output voltages

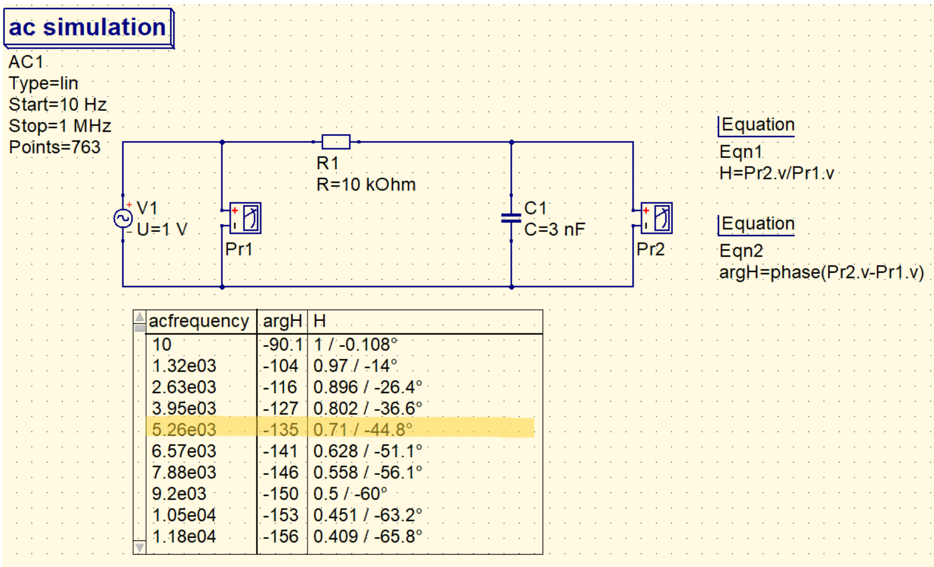
Based on the table we can also confirm that at 16 kHz the output voltage is about 0.707 V, so that is indeed the cut-off frequency.



The RL filter circuit on the breadboard

## 3.2 RC filter

For an RC filter the cut-off frequency can be calculated as $\frac{1}{2\pi \mathrm{RC}} =$
$= \frac{1}{2\pi * 10 * 10^3 * 3 * 10^{-9}} = \frac{10^5}{6\pi} = 5305\,\mathrm{Hz} = 5.3\,\mathrm{kHz}$. From the simulation I determine the cut-off frequency as the frequency when H=0.707 V.




Determining the cut-off frequency in the simulation

In order to get 3 nF I connected three 1 nF capacitors in parallel. Figure 12 shows the circuit on the breadboard and table 3 shows the measured output voltages to different frequencies. Based on the table this is a low pass filter.

| Frequency | $V_{\mathrm{OUT}}$ |
|---|---|
| 100 Hz | ~ 1 V |
| 1 kHz | ~ 950 mV |
| 5 kHz | ~ 710 mV |
| 5.2 kHz | ~ 700.4 mV |
| 10 kHz | ~ 455 mV |
| 50 kHz | ~ 75 mV |
| 80 kHz | ~ 8 mV |

Table 3: Frequencies with their corresponding output voltages

It also confirms that the output voltage is around 0.707 V at 5.2 kHz.

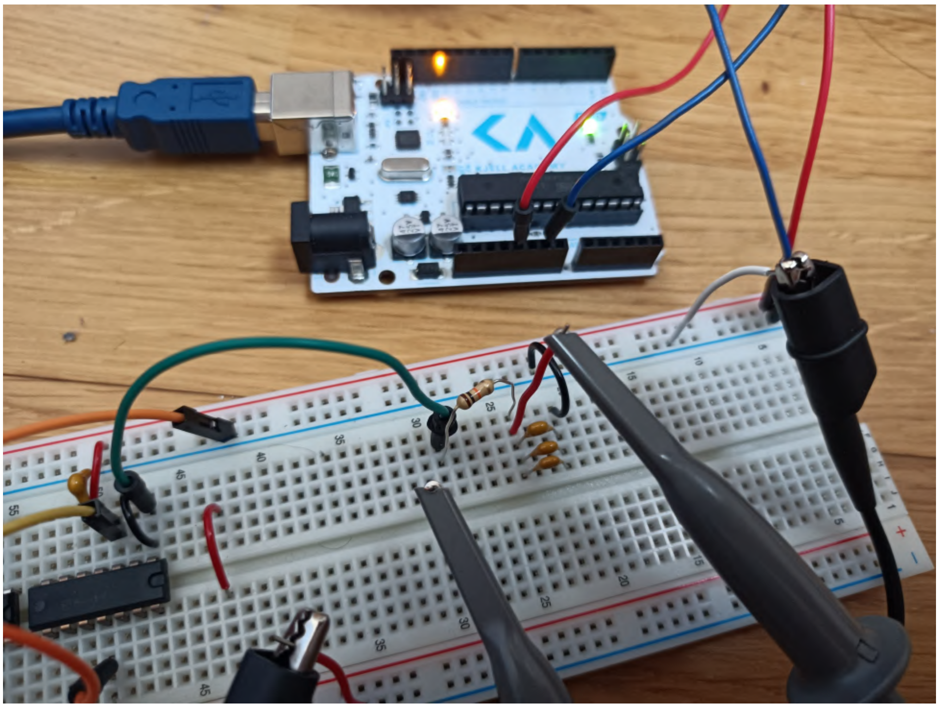
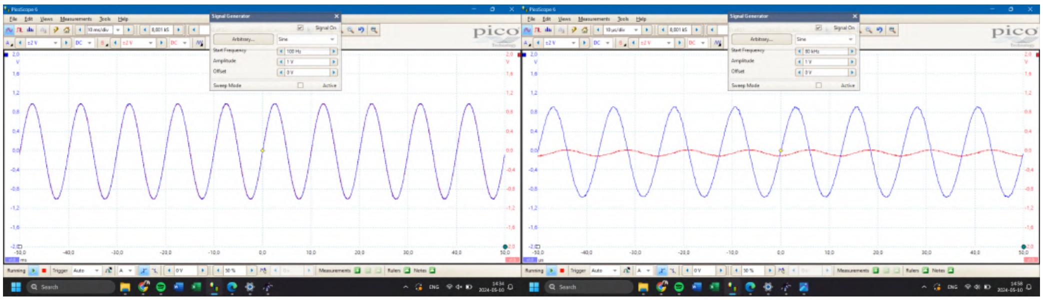


Figure 12: The RC filter circuit on the breadboard



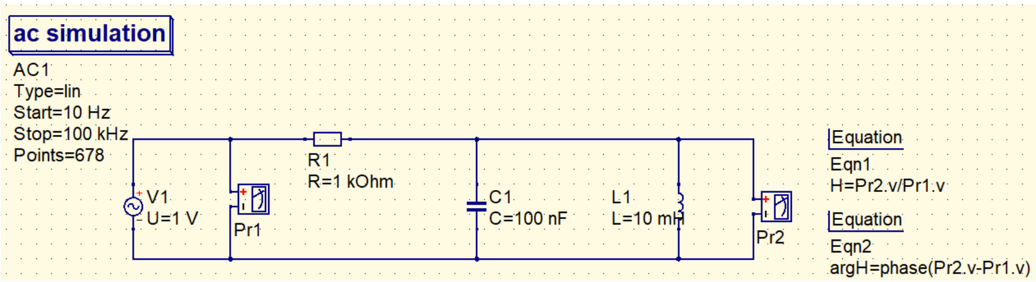
The output voltage (in red) with 100Hz and 80kHz

## 3.3 RLC filter

### 3.3.1 Sine input to the filter and viewing voltage versus time

The center frequency can be calculated as the geometric mean of the two cut-off frequencies: $\frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10*10^{-3}*100*10^{-9}}} = 5033\,\text{Hz}{=}5.03\,\text{kHz}$.

On Figure 13 from the simulation we can see that around 5.03 kHz H=$\frac{\text{Pr2.v}}{\text{Pr1.v}}$ hits it's maximum value 1. The output voltage is also 1 V there.




The simulated circuit

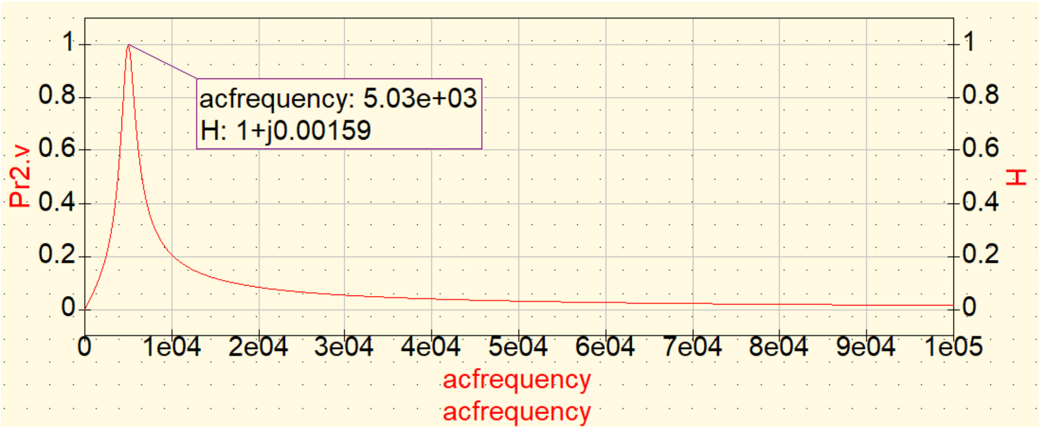
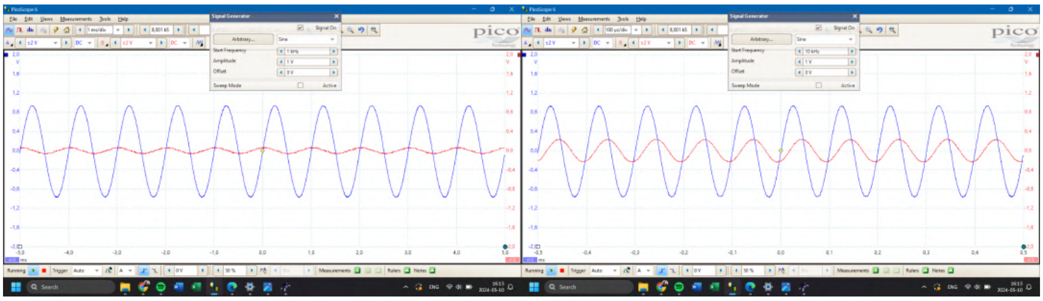



Figure 13: Finding the center frequency



The output voltage (in red) with 1kHz and 10kHz

Both at 1 kHz and 10 kHz the output voltage is close to zero. This (and the fact that in between it increases) means that this circuit is a **band pass filter**. To determine the center frequency I found the frequency at which the output voltage is closest to 1 V at 5.4 kHz.

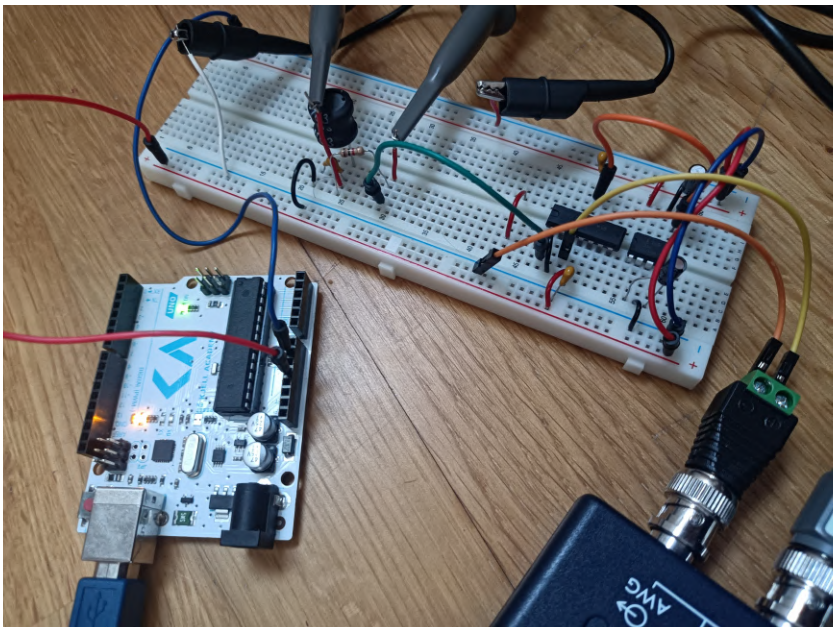


Figure 14: The circuit on the breadboard

### 3.3.2 Sinusoidal and square wave (5 kHz) input to the filter

**Sinusoidal input**

On Figure 15 we can see that the output voltage is almost the same as the input voltage. That is, because the filter circuit let's waves near the center frequency (5.03 kHz) pass through almost entirely.

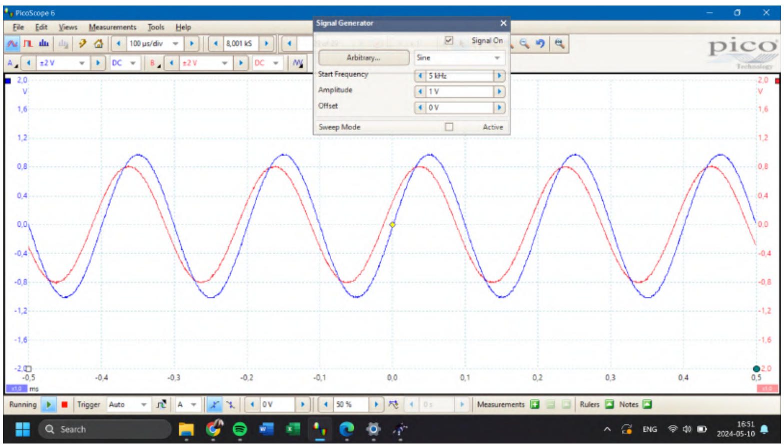
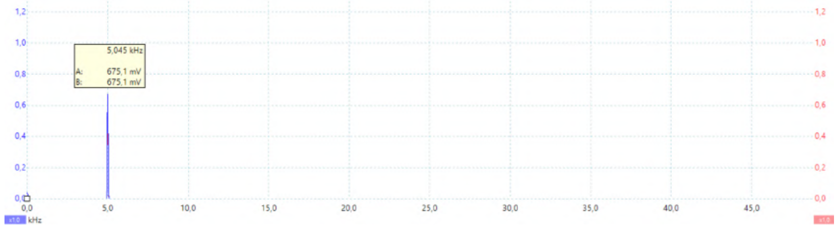



Figure 15: The plots in the picoscope software

By entering spectrum mode in the picoscope software, we can see a peak at 5 kHz of about 675 mV.




The peaks in spectrum mode

**Square wave input**

A square wave is a sum of sinusoidal waves of different phase and amplitude.

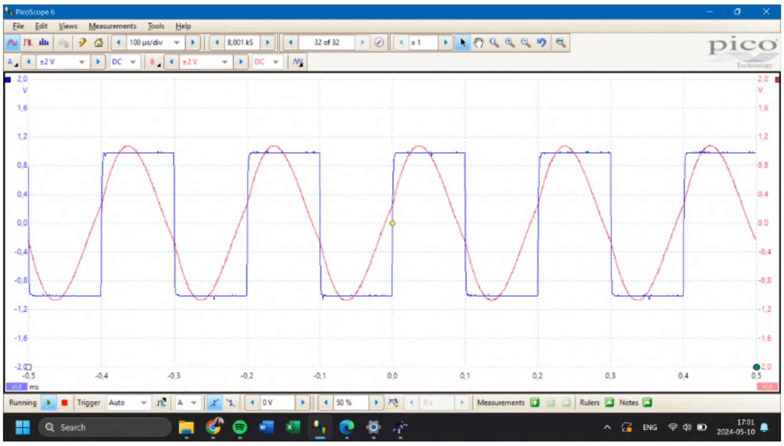


Figure 16: The plots in the picoscope software

This filter circuit lets through the 5 kHz sine wave component the most, so on Figure 16 the red line looks rather similar to what we had before. That is also the reason for the most noticeable peak being around 5 kHz in spectrum mode (see Figure 17) with an output voltage of around 780 mV. However, this time other peaks occur as well around multiples of 5 kHz.

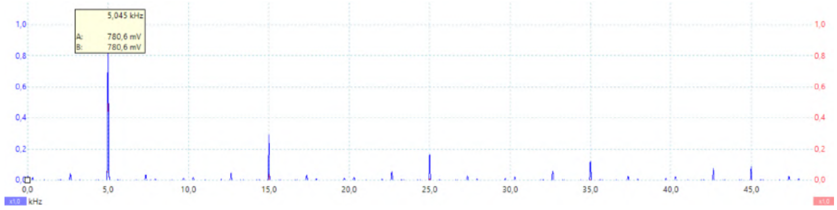



Figure 17: The peaks in spectrum mode