Predicting Turning Points of Bond Price

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1 Introduction

This project is about predicting the future turning points, both peaks and troughs, in order to help make decisions. The project is motivated by the fact that many of the successful trading decisions are made during the period near a turning point of a financial time series. The ability to identify current and future turning points of financial data allows traders to sell at high price and buy at low price, therefore obtaining profit or avoiding potential loss.

Generally speaking, a turning point here is defined as a local maximum or a local minimum. We will try to generate the posterior distributions of future turning points based on different models. In this way we are able to not only predict the positions of the future turning points, but also evaluate the volatility of them, which in other words, the risk of the model.

We will follow the general procedure first proposed by Wecker, improved by Kling and explored further by Podding and Huber. We will fit models for proper series and forecast the future maximum and minimum accordingly. The models we will analyze are AR, TVAR, ARMA, and trend and seasonal DLM. We will compare between different models based on their profitability and risk.

2 Data

The data we will evaluate are monthly data of US10Y, which is the 10-year government bonds of United States. The data set is available on `https://www.investing.com`. It contains the date, prices, return rates on specific months. The data set covers the period of time from Jan 2009 to Dec 2019. It is separated into train and test set, which respectively are from Jan 2009 to Dec 2018 and Jan 2019 to Dec 2019. Below is the descriptive plots of the prices and return rates:

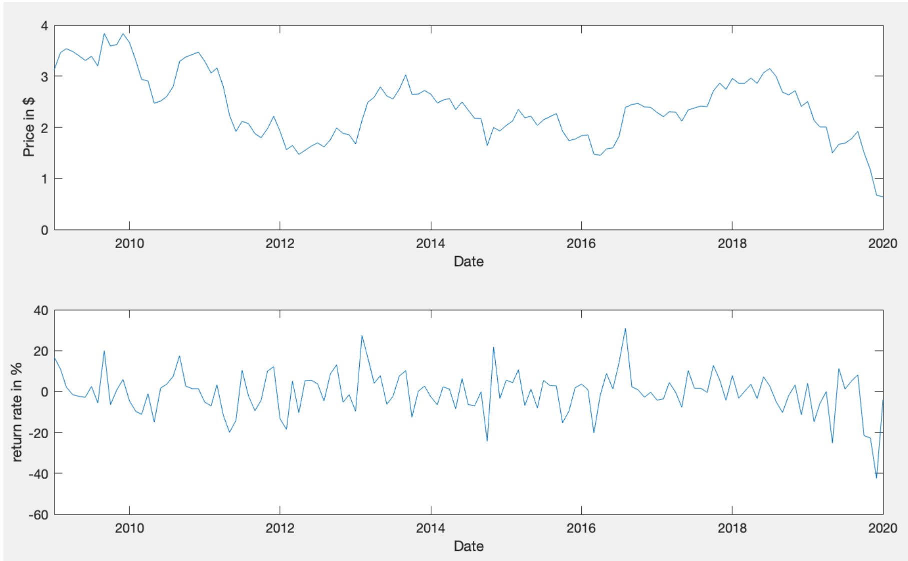

Figure 1: Data Summary

Exploratory data analysis has been done but not been presented due to page limit.

3 Algorithm

The ultimate goal of this project is to find the models that will engender high profit with low risk via trading US10Y bonds. We may achieve this by forecasting the distribution of the turning points, both peaks and troughs, of US10Y bonds. For this purpose we need to predict the future bond prices. However, consider the non-stationarity of price series, we use return rate series for AR model and recover the future prices accordingly. The general procedure is as in **Algorithm 1**.

4 Evaluation

We evaluate the models based on its profitability and risk. The profitability is calculated based on the trading strategy depending on the locations of turning points, as well as the models that determines the future turning points. The risk represents the variance of the model. We will create a profit-risk ratio index (see Appendix) to mimic Sharpe ratio, which represents the profitability by taking an extra unit of risk.

Here I propose a simple trading strategy that we sell bonds at the peaks of price and buy bonds at the troughs. If the model believes that a particular time is neither peak nor trough, or both peak and trough, we then hold our assets until next transaction time. Since the data are monthly, the trading decisions will be made once per month.

Algorithm 1

1: Denote the proper data series to be Y. Normalize Y if needed. Divide the data into train set, from Jan 2009 to Dec 2018, and test set, from Jan 2019 to Dec 2019.
2: Fit the training set with a time series model. Use Bayesian analysis to find out the posterior distributions of the parameters for the particular model. Sample n groups of parameters from the posterior distributions.
3: Each sampled group of parameters defines a specific model. We then decide the width for local extrema, r(see Appendix). Fit the training set with the particular model. Predict $(12 + r)$-step ahead and use the predictions to compute the monthly prices in 2019. We obtain n sample of predicted prices.
4: For each of the n sample of predicted prices, find peaks and troughs with chosen r. The resulting samples represent the empirical distributions of the turning points' locations.
5: Repeat the above procedure for models we want to evaluate. Compare between models based on their profitability and risk.

For simplicity, assume that a trader holds 1000 US10Y bonds on Jan 2019. We need to help him make trading decisions that maximizing his profit over 2019. Note that if the trader holds his bonds for the whole year, he will lose $1836 at the end of 2019. Set this to be the baseline, we will apply four models and decide which one is most profitable or avoids the most loss.

4.1 AR Model

The first model will be the simplest AR model. We first normalize the return rates data in train set, and then fit AR model. After adjusting the predicted return rates, we recover the predicted prices by iteratively counting the return rates. We then decide the locations of turning points based on the predicted prices.

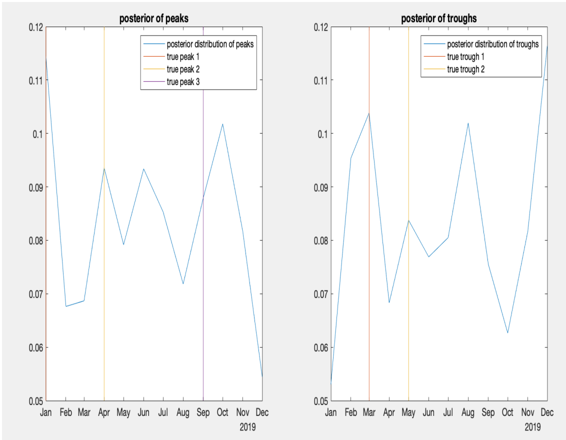

Figure 2: Posterior Distribution of Turning Points with AR

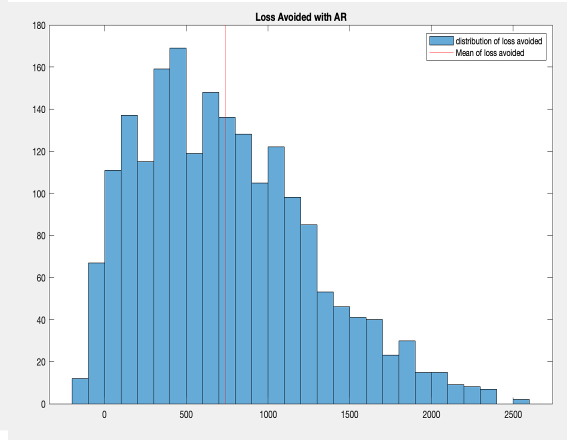

Figure 3: Loss Avoided with AR

Figure 2 displays the posterior distribution of turning points according to an AR(18) model. The AR parameter p is 18 based on likelihood and predictive accuracy on test set. The vertical lines in the plots are the locations of true turning points. It is evident that AR(18) model is doing a reasonable job. It successfully recognizes the peaks on Jan, Apr and troughs on Mar. Nevertheless, the distribution has large mass on the non-turning point month.

Figure 3 displays the avoided loss (comparing to loss of \$1836) based on the simulated turning points' locations. Most of the trading decisions avoids loss over 0, which means that the they are better than simply holding the asset over the year. The mean of the avoided loss is about \$740. The profit-risk ratio is 1.4841.

4.2 TVAR Model

For the second model, we fit a TVAR for the mean-subtracted prices of US10Y bonds. Next, we simulate 12+r-step ahead predicted bond prices. Again, we decide the locations of turning points based on the predicted prices.

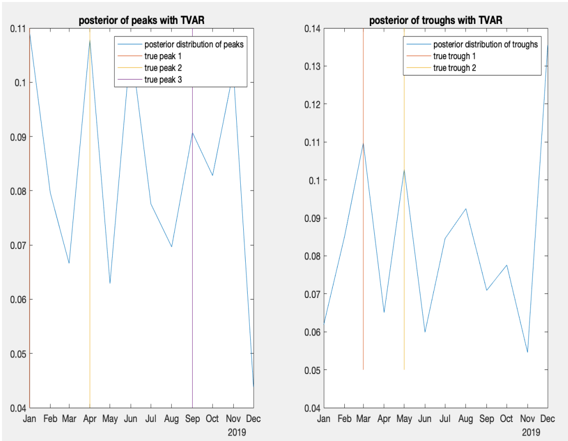

Figure 4: Posterior Distribution of Turning Points with TVAR

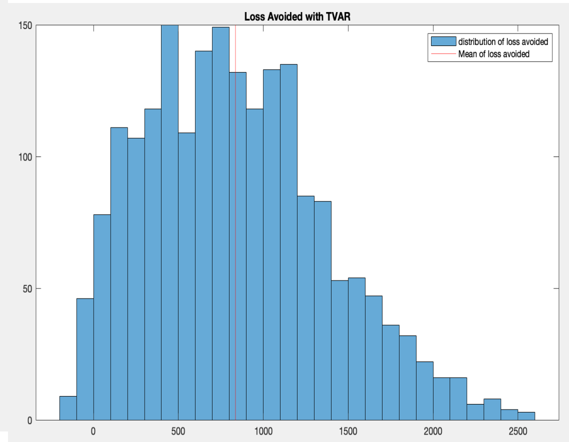

Figure 5: Loss Avoided with TVAR

Figure 4 illustrates the resulting distribution of turning points based on TVAR(18) model with discount factor (0.995,0.965) determined by maximizing likelihood. We observe that TVAR model successfully predicts 4 out of 5 turning points on Jan, April, Mar and May. It also gives substantial probability mass on Sep, which is also a true peak. However, it misclassifies Jun and Nov as a peak (Not a misclassification on Dec since it has the minimum price over the whole 2019 and is on the boundary of test set). This exhibits the inaccuracy of long-term predictions.

Figure 5 shows the avoided loss by the TVAR model. The distribution has a similar shape to that of the AR model though, it has a higher mean. It on average avoids $830 loss. The profit-risk ratio is 1.5608.

4.3 ARMA Model with MCMC

This is the ARMA model that is estimated with a Gibbs Sampler. The procedure of the Gibbs sampler is inspired by the 2-step regression estimation of ARMA parameters (see Appendix).

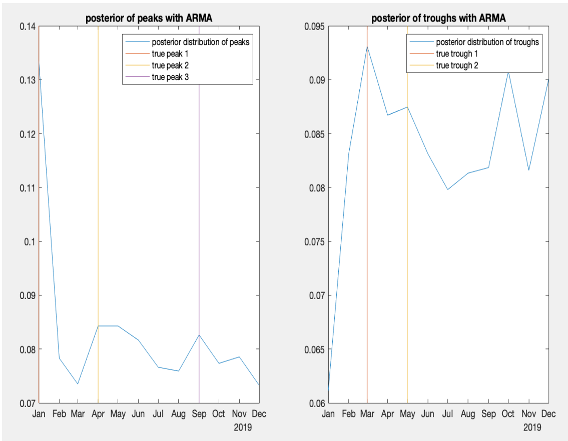

Figure 6: Posterior Distribution of Turning Points with ARMA

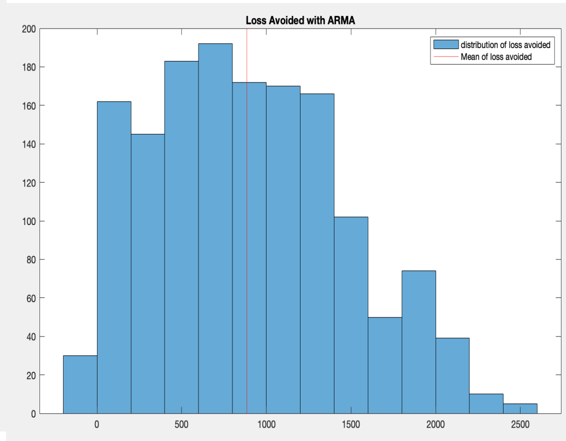

Figure 7: Loss Avoided with ARMA

Figure 6 displays the resulting distribution of turning points based on an ARMA(2,2) model. Compare to previous models, it is more certain about the peak locations at Jan since the distribution concentrates most of the mass on Jan in the peak plot. In general, the model provides similar results to those of AR and TVAR model. It does a reasonable job on turning points recognition and put salient probability mass on each of the turning points. Nevertheless, it incorrectly labels Oct as a trough.

Figure 7 indicates that the profitability of ARMA model is higher than the previous two models. It has an avoided loss of around \$875 on average. The profit-risk ratio is 1.5619. This agrees with the results in Poddig & Huber's paper, which proves the strength of ARMA model in predicting turning points.

4.4 Local Trend and Seasonal DLM

A synthetic DLM with local trend and seasonal components are constructed since we observe a decreasing trend currently and a potential seasonal pattern in return rates. The model is described in Appendix. The predictions are as below:

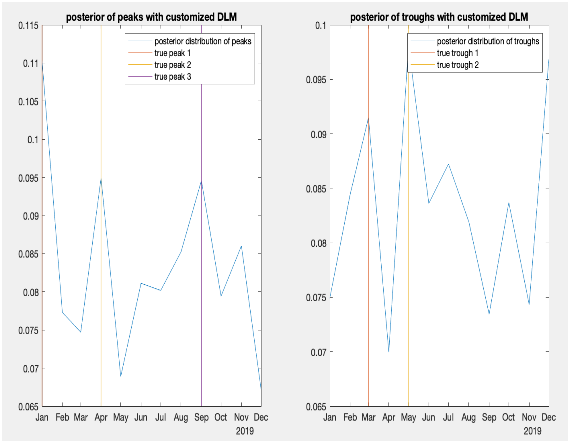

Figure 8: Posterior Distribution of Turning Points with DLM

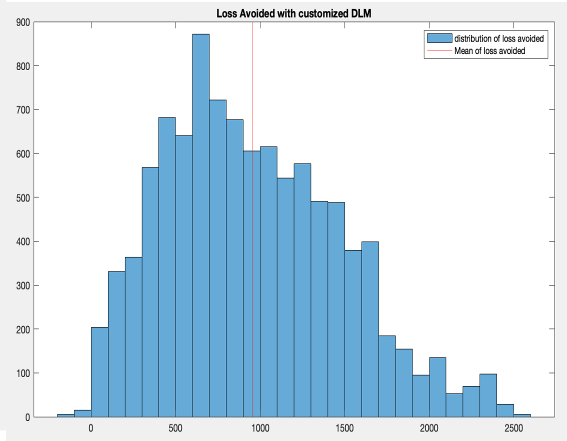

Figure 9: Loss Avoided with DLM

Based on **Figure 8**, all the turning points have been recognized by this DLM model. The generated posterior distributions of turning points put considerable probability mass on each real turning points, while all the other months possess low probability mass.

Figure 9 confirms the high avoided loss of the DLM. Trading decisions based on this model avoid loss of \$970 on average. Which is significantly higher than the other three. The profit-risk ratio is 1.8201.

4.5 Summary

There are several observations in analysis that are worth to be mentioned. First of all, in deciding the month with maximum price during 2019, all the models agree with the answer that bond price is maximized in Jan. This is due to the fact that price in Jan is a 1-step ahead prediction, and all the models are relatively robust in predicting for a short period of time; On the other hand, models disagree with the location of the minimum, and all of them except the trend and seasonal DLM fail to claim that Dec is the minimum. This means that these models are not suitable for long-term predictions. For the same reason, since the next peak and trough are both in the first few months of 2019, all the models do agree with the amount of time needed for the next turning point.

Another observation is that all the models manage to recognize Mar as a trough, despite the fact that price in Mar is only slightly lower than its neighbors. This is another indicator of these models' strength in short-term prediction. However, if the trough moves to Oct or Nov, most of the above models may not able to discover it. This also rises question to the definition of turning points. If the price change at a local extrema is negligible, does it worth to take the risk and perform the transaction? If not, in addition to local extrema, we may need to make sure the change is significant by setting a threshold.

Based on the above evaluation of each models, we conclude the results in **Table 1**.

Table 1: Model Comparison

Model	AR	TVAR	ARMA	DLM
PR Ratio	1.4841	1.5608	1.5619	1.8201

In this case, it is clear that trend and seasonal DLM is the best among 4 models. The TVAR and ARMA models are little bit worse, and the AR model is the least profitable. This matches the intuition that more complex model tends to posses higher predictive accuracy.

5 Further Investigation

There are a lot more to explore about this topic. For example, originally I was planning to fit a TV-VAR model, but I have not managed to create a proper MCMC smapler to estimate the unknown parameters in the model. If I have 3 more months, the first thing I would do is to construct multivariate version of the models. In this way, we are able to not only observe the inter-correlation between different series (different bonds), but also evaluate trading strategies on portfolio allocation, which is more realistic.

Besides, extended definition of turning point (mentioned above) and Sharpe Ratio (instead of the mimic one) could be applied for the portfolio allocation problem. Adding a benchmark, transaction cost could make the scene even more realistic.

6 Appendix

6.1 Width of local extrema, r

Suppose we have a time series $\{x_t\}, t = 1, ..., M$. A turning point is a local maxima or minima over periods $\{x_{t-r}, ..., x_{t+r}\}$ with some positive integer r. The following is the definition of the turning point indicator at time t, where the notation of P and T denotes for peak and trough:

$$z_t^P = \begin{cases} 1, & \text{if } x_t > x_{t+i}, i \in \mathbb{Z} \cap [-r, ..., r] \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Similar definition holds for z_t^T.
In this project, $r = 1$ as recommended in Poddig & Huber's paper.

6.2 Profit-Risk Ratio

$PR\ Ratio = \frac{E[avoided\ loss]}{Var(avoided\ loss)}$

This index should be consistent since we apply the same trading strategy and start with the same assets for each model.

6.3 MCMC of ARMA

The ARMA model is fitted on the normalized return rate series. We may estimate the parameters of the ARMA(p,q) model based on 2-step regression estimation. In order to explore the Bayesian aspects of the series, I develop a Gibbs sampler, as shown in **Algorithm 2**. The first 300 samples are burn-in. 2 steps are used for thinning the MCMC.

Algorithm 2

1: Suppose we have a time series $\{x_1, ..., x_T\}$. Take proper m greater than p and q. We want to regress x_t over $x_{t-1}, ..., x_{t-m}$, for $t = m+1, ..., T$.

2: Denote $Y = \begin{bmatrix} x_T & x_{t-1} & \cdots & x_{m+1} \end{bmatrix}^T$,X= $\begin{bmatrix} x_{T-1} & x_{T-2} & \cdots & x_{T-m} \\ x_{T-2} & x_{T-3} & \cdots & x_{T-m-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_m & x_{m-1} & \cdots & x_1 \end{bmatrix}$ .Y |X,$\beta, \sigma^2 \sim MVN(X\beta, \sigma^2 I)$.

3: Prior: $\beta \sim MVN(\beta_0, \Sigma_0), \sigma^2 \sim invGa(v_0/2, v_0\sigma_0^2/2)$.

4: Updating β: $V = (\Sigma_0^{-1} + X^T X/\sigma^{2(s)})^{-1}, m = V(\Sigma_0^{-1}\beta_0 + X^T Y/\sigma^{2(s)})$; Sample $\beta^(s+1) \sim MVN(m, V)$.

5: Updating σ^2: $SSR(\beta^{(s+1)}) = Y^T Y - 2\beta^{(s+1)T} X^T Y + \beta^{(s+1)T} X^T X \beta^{(s+1)}$; Sample $\sigma^{2(s+1)} \sim invGa((v_0 + n)/2, (v_0\sigma_0^2 + SSR(\beta^{(s+1)}))/2)$.

6: Compute $U = \begin{bmatrix} u_T & u_{t-1} & \cdots & u_{m+1} \end{bmatrix} = Y - X\beta$.

7: Repeat the above sampler with $Y_2 = \begin{bmatrix} x_T & x_{t-1} & \cdots & x_{m+q+1} \end{bmatrix}$, X_2= $\begin{bmatrix} x_{T-1} & \cdots & x_{T-p} & u_{T-1} & \cdots & u_{T-q} \\ x_{T-2} & \cdots & x_{T-p-1} & u_{T-2} & \cdots & u_{T-q-1} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ x_{m+q} & \cdots & x_{m+1} & u_{m+q} & \cdots & u_{m+1}. \end{bmatrix}$. Then we sample from the posterior distribution of β_2 and σ_2^2 such that $Y_2|X_2, \beta_2, \sigma_2^2 \sim MVN(X_2\beta_2, \sigma_2^2 I)$.

8: For each sampled β_2, $-\beta_2(1:p)$ are the AR(p) parameters and $\beta_2(p+1:p+q)$ are the MA(q) parameters.

6.4 Trend and Seasonal DLM

An additive DLM with local trend and 3 seasonal components:

$$y_t = F_t'\theta_t + v_t,\ v_t \sim N(0, \nu)$$

$$\theta_t = G_t\theta_{t-1} + w_t,\ w_t \sim N(0, \frac{\nu W_t}{s_{t-1}})$$

$$F = [1, 0, 1, 0, 1, 0, 1, 0]^T$$

$$G = blockdiag\{I_2, H(3), H(6), H(12)\},\ H(k) = \begin{bmatrix} cos(\frac{2k\pi}{p}) & sin(\frac{2k\pi}{p}) \\ -sin(\frac{2k\pi}{p}) & cos(\frac{2k\pi}{p}) \end{bmatrix}, p = 24$$

$$discount\ factor = (0.99, 0.99, 0.95, 0.95).$$

Use Kalman filter from lectures to compute the posterior distribution of parameters. Simulate predictions with sequential filtering and sampling.

References

[1] William E. Wecker. *Predicting the Turning Points of a Time Series*. The Journal of Business, Vol. 52, No. 1, The University of Chicago Press, 1979, pp. 35-50.

[2] John L. Kling. *Predicting the Turning Points of Business and Economic Time Series*. The Journal of Business, Vol. 60, No. 2, The University of Chicago Press, 1987, pp. 201-238.

[3] Thorsten Poddig, Claus Huber. *Data Mining for the Detection of Turning Points in Financial Time Series* University of Bremen, Germany, 1999.