

CE5308QB - Exercise 3 – Modelling of suspended transport in steady current

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This exercise is continuation of **Part 1, 2** in **Exercise 2**.

In this exercise you will create a Matlab model which, for a given sediment size d , uniform channel slope S , and steady discharge per unit width q , solves the flow resistance problem and computes both the bedload and suspended sediment transport rates. For this problem you are to consider a wide river with $d = 1$ mm, slope $S = 0.001$, and the range of steady discharge $q = 0.5\text{--}5$ m²/s.

Part 3 – Using the suspended sediment model (with bedforms present!) discussed in class, compute and plot the variation of the suspended sediment transport qS versus q . Use the Zyserman-Fredsoe relation for determining the reference suspended sediment concentration c_b . **HINT 1:** For each q you should evaluate your analytical velocity and concentration profiles on a vertically stretched grid, with points that are equidistant on a log scale in the interval $b^* \leq z \leq D$ (use the logspace command). **HINT 2:** For the calculation of qS use the trapz command to perform the integral.

Part 4 – Finally plot the Rouse parameter $Z = ws/(\kappa Uf)$ and the ratio qS/qB (use the MPM for the bedload transport) versus q for $d = 1$ mm. Now repeat this plot after changing to $d = 0.3$ mm. For the two specific cases with $q = 5$ m²/s plot the vertical profiles of u , c , and uc versus z . Discuss the observed differences due to changing grain size.

In theory, bedload is transported primarily near the bed and is in contact with the channel bed for a considerable fraction of the time. In contrast, the suspended load consists of particles, that are supported by the turbulence of the mean flow reside primarily in the water column and spend little time in contact with the bed.

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1. Part3 – Solution: Suspended-load distribution

In the general case (where bedforms are present) the flow velocity profile should be generalized to:

$$u(z) = \frac{U_f}{\kappa} \ln \left(\frac{30z}{k_*} \right)$$

where k^* is an effective roughness length.

$$k_* = \frac{D}{\exp(\kappa(V/U_f - 6))}$$

And the sediment concentration profile is given from the Vanoni distribution:

$$c = c_b \left(\frac{D-z}{z} \frac{b}{D-b} \right)^{w_s/(\kappa U_f)}, \quad c_b = c(z=b) \quad (8.9) \quad c_b = \frac{0.331(\theta^* - 0.045)^{1.75}}{1 + \frac{0.331}{0.46}(\theta^* - 0.045)^{1.75}}$$

where c_b is from the Zyserman & Fredsøe empirical formula, and typically $b = 2d$.

The settling velocity w_s is calculated by the method in exel, and then we also calculate the term of Rouse parameter, $Z = w_s/(U_f^* \kappa)$.

Thus, the modifications for **suspended sediment with bedforms present**:

$$q_s = \int_{b_*}^D u c dz, \quad \Phi_s = \frac{q_s}{\sqrt{(s-1)gd^3}}$$

where: $b_* = \max \left\{ b, \frac{k_*}{30} \right\}$

The specific consideration is the range of z , that starts from b^* to ensure positive value u , instead of starting from the boundary layer.

Thus, the results are following. To compare the bedform impact, we extract the following 5 scenario as $q=0.5, 2$ and 4 [m^2/s] in the lower regime, and $q=4.5$ and 5 [m^2/s] in the upper regime (Fig 1& Fig 2). It is notable that in these 3 vertical profiles in Fig 4, the colormap corresponds to different flow discharges, and the specific discharge index can be seen in Fig 2 and Fig 3.

Considering the concentration data ranges over several orders of magnitude and consists of very small values, it is advisable to utilize a logarithmic scale in x-axis (by 'semilogx()'), that is easier to spread out the data and provide more details.

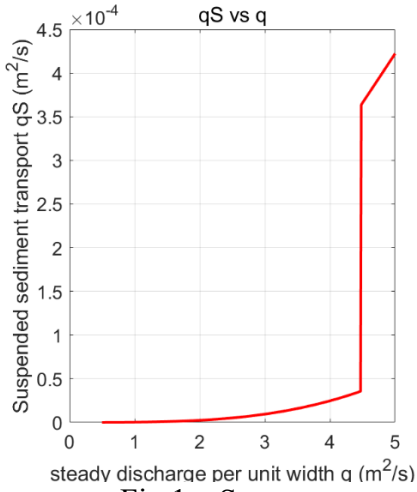


Fig 1. qS vs q

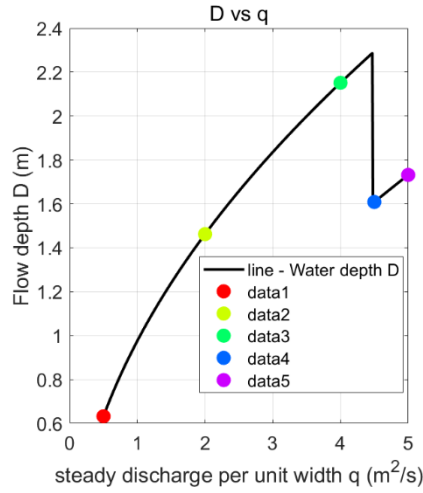


Fig 2. Water Depth and index points

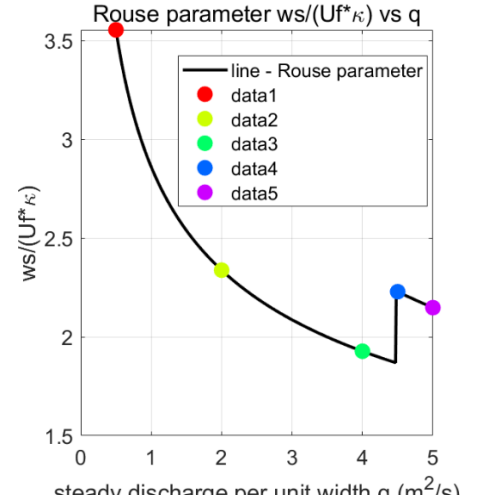


Fig 3. Rouse number and index points

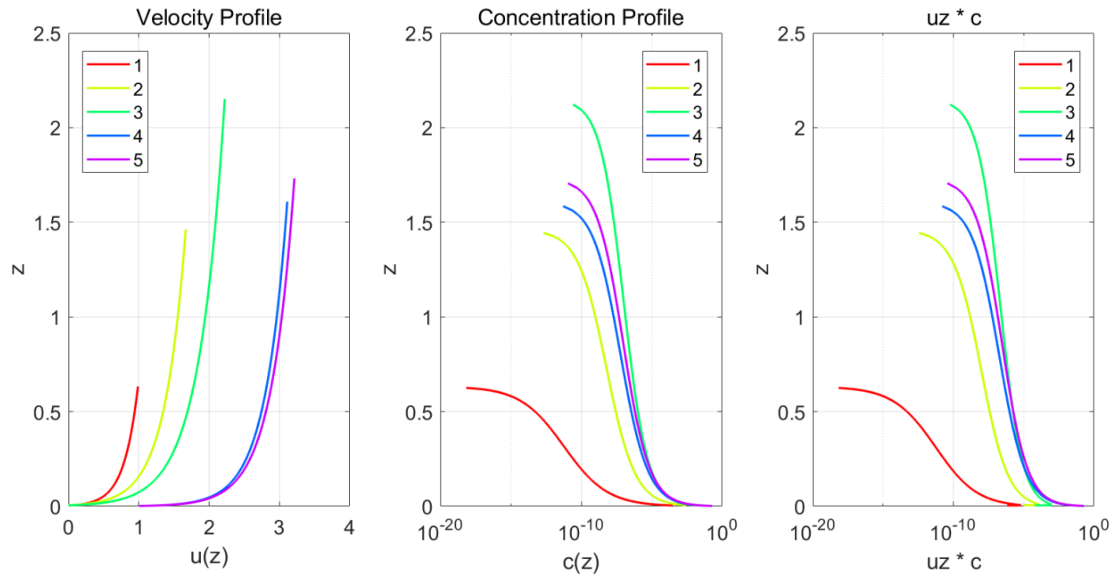


Fig 4. Vertical Profiles

Based on the analysis of different regimes and bedforms in the exe2, the following main conclusions are used in this analysis:

- 1) There is a significant shift in hydro-dynamic properties as the bedform changing under different flow conditions with q increasing, **where the bedform changes in the transition from dunes(ripples) to plane bed.**
- 2) With q increasing the bedload transport rises, while the **total shear stress** drops in the transition and then continues increasing.

From the Fig 1, the suspended load transport occurs the same increasing trend as q developing. From these 3 vertical profiles in Fig 4, unlike the continuous increase trend in the flow velocity as q , the suspended-load concentration distribution shows different variation tendencies as bedform changing. Therefore, instead of the bedform, qS is more related to Rouse number, more discussion is seen in 2.1 Rouse parameter versus q .

1.1. Velocity Profiles

For the velocity profile, from the equation:

$$u(z) = \frac{U_f}{\kappa} \ln\left(\frac{30z}{k_*}\right)$$

There are two parameters influencing $u(z)$ for the single size particle: 1) U_f (Fig 5), and 2) the effective roughness (Fig 6).

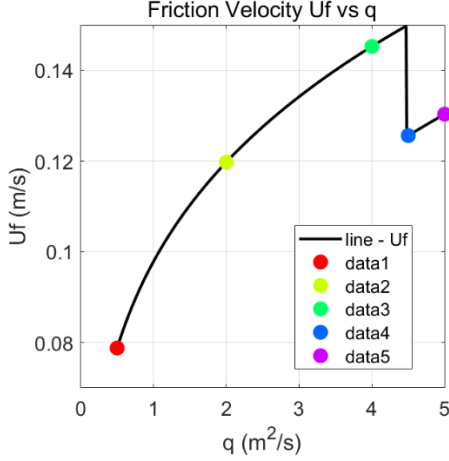


Fig 5. $U_f \sim q$

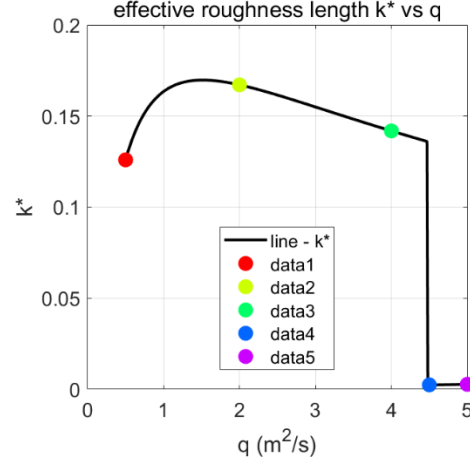


Fig 6. $k^* \sim q$

We can see that at the same depth, the velocity increases, that is mainly caused by the increasing friction velocity U_f , and the fluctuating value of k^* plays a more important role while in the upper regime.

1.2. Suspended-load transport and Concentration Distribution

Considering the bedform, when dunes or ripples exist within a lower discharge, qS remains a lower value, and the suspended-load concentrates near the bed showing a larger value and reduces significantly to the surface with a variable distribution. It can be seen that these 3 index points in the lower regime, before bedform changes, the suspended-load distribution $c(z)$ becomes more uniform and gradual.

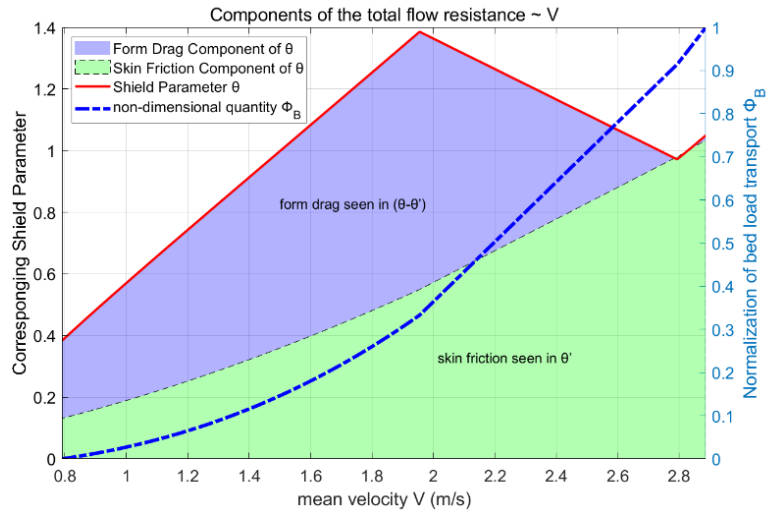


Fig 7. Shields Parameter (Shear stress) and its components

As the bedform transition appears and shifts to plane bed, qS jumps to a higher value meaning more suspended sediment, leading to a sudden derivation from a relatively uniform state in suspended-load distribution. That may relate to more turbidity and turbulence.

The large production of turbulent energy due to shear. [1] For the single particle size, we can just visualize the nondimensional shear parameter – Shield Parameter in

Fig 7 to compare how the shear impacts concentration. Before the transition occurring, shear stress increases and concentration rises at the same depth with q increasing. After the transition, the shear stress drops and restarts rising, the concentration shows the similar changes in Fig 4.

Since we only discuss the Rouse formula in sediment transport in this exercise, the significant property and limitation of this theory is that **the sediment concentration is calculated as zero at the water surface and infinity at the bed**. Vanoni also stated in his classical manual, that the Rouse formula can only represent the shape of the distribution not the actual values in a predictive sense. [2]

2. Part4 – Solution: Impact on grain size

2.1. Rouse parameter versus q

In the concentration profile calculated by the Vanoni distribution:

$$c = c_b \left(\frac{D-z}{z} \frac{b}{D-b} \right)^{w_s / (\kappa U_f)}, \quad c_b = c(z=b) \quad (8.9)$$

where the parameter $Z = w_s / (\kappa U_f)$ is called as Rouse parameter.

1) for $d = 1 \text{ mm}$

The semi-empirical Meyer-Peter & Müller (MPM) bed load formula:

$$\Phi_B = \frac{q_B}{\sqrt{(s-1)gd^3}} \quad \Phi_B = 8(\theta' - \theta_c)^{3/2} \quad (7.60)$$

The total suspended load transport is:

$$q_s = \int_{b_s}^D ucdz, \quad \Phi_s = \frac{q_s}{\sqrt{(s-1)gd^3}}$$

where $w_s = 0.1120$ in Rouse number.

2) for $d=0.3\text{mm}$

Repeat the above calculation, and $w_s = 0.0375$ in Rouse number.

As the range of Rouse parameter in $d=0.3\text{mm}$ case is more representative, showing that it changes more continuous, we normalize and visualize the Vanoni distribution profile to show the impact of Rouse number.

In this exercise, we use the Rouse formula without any modified parameter β , that β is a factor that accounts for differences between turbulent diffusivity of sediment and momentum. [1]

$$Z^* = \frac{w_s}{\beta U_f \kappa}$$

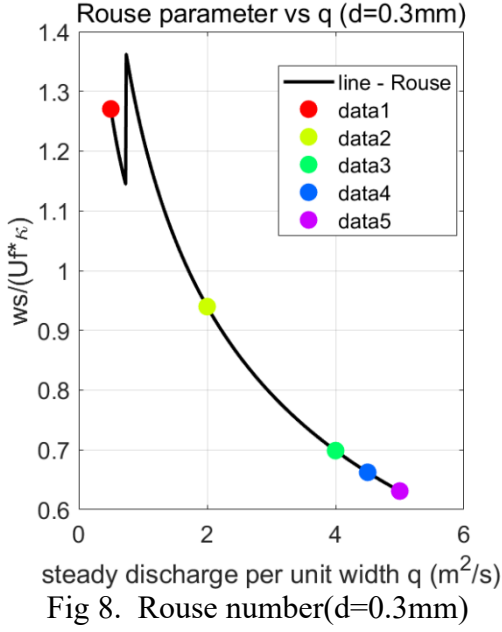


Fig 8. Rouse number($d=0.3$ mm)

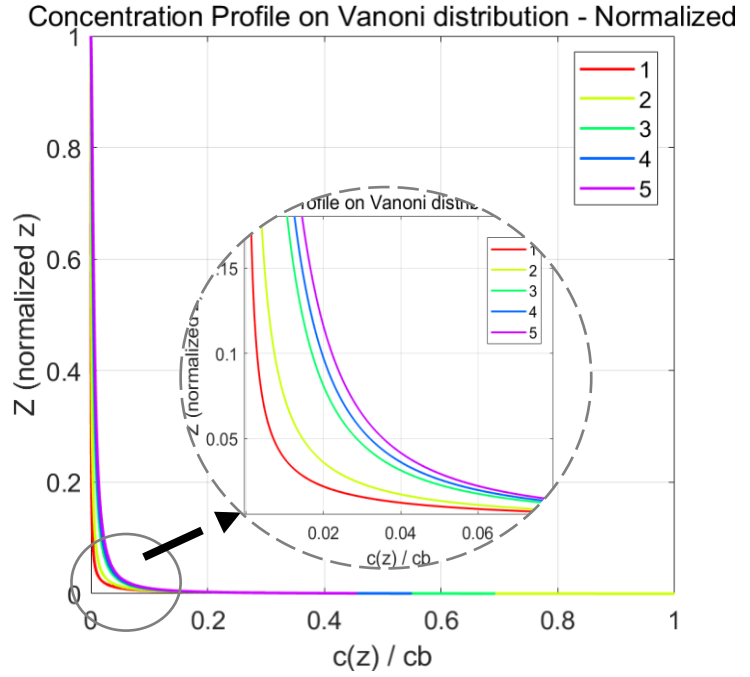


Fig 9. Vanoni distribution-Normalization Scale

Rouse parameter and equation takes into account turbulent diffusion effects and sediment-concentration gradients and shows a general distribution of suspended-sediment concentration in fully turbulent flow. [3] Rouse parameter, that expresses the relative magnitudes of downward particle transfer by settling and upward particle transfer by turbulent mixing.

The purple point in Fig 8 shows a larger discharge corresponding to a lower Rouse number, and the corresponding purple line in Fig 9 shows a more smooth and state trend of suspended-load distribution in the dimensionless plot, indicating that **a lower Rouse number leads to a more uniform suspended-load concentration distribution.**

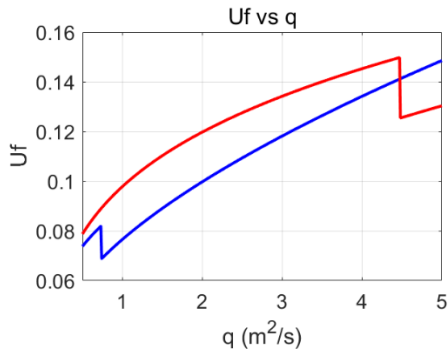


Fig 10. Friction velocity

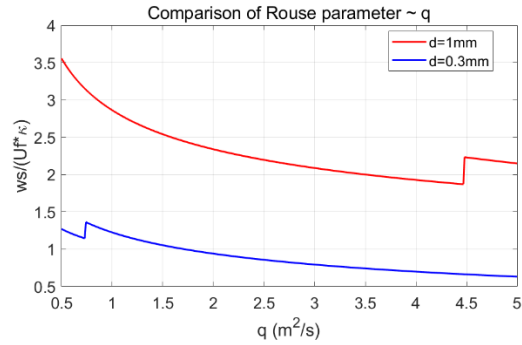


Fig 11. Comparison of Rouse number

In the formula of Rouse parameter $Z = ws/(Uf^* \kappa)$, ws reflects a force balance between the submerged weight of a particle and the drag force associated with its settling motion. And Uf shows similar values within 2 kinds of particle.

Hence, it can be concluded that shear velocity plays a noncritical role in determining the vertical concentration distribution when compared with the other factors. And ws defines the value of Z . [2]

Thus, ws depends on particle size, the Rouse number does as well. **The smaller grain size leads to a lower settling velocity ws , and then its Rouse number is generally**

lower than that of larger particles.

Besides, a lower Rouse number causes more uniform suspended-load distribution, and the details and comparisons are seen in 2.3 Comparisons of u , c , and u^*c versus z .

2.2. Ratio qS/qB versus q

The corresponding plots are following, it is noted that `semilogx()` is used to better visualize the wide range.

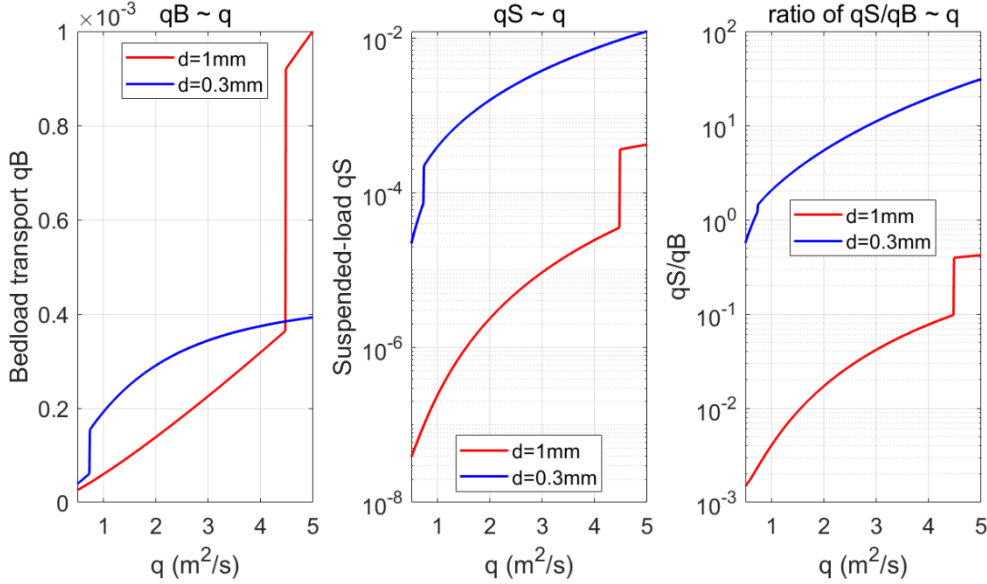


Fig 12. Comparison of qS , qB and its ratio

- 1) For different grain sizes, the order of magnitude of bedload transport qB remains consistent and small. If divide into different regimes, with a lower discharge and in the lower regime, the bedload of 2 scenario approximates, indicating that **the bedload transport qB is largely independent of grain size.** [4] Then, as q increases and bedform shifts into upper regime with larger grain sizes exhibiting more qB .
- 2) In contrast, for suspended-load transport, there is a considerable difference in qS magnitude, showing qS within a smaller particle size far surpasses that of a larger grain size by several orders of magnitude. It can be concluded that **the suspended-load transport rate is inversely related to grain size.**
- 3) The ratio of qS/qB for smaller particles is significantly greater than 1, indicating that suspended load dominates in the total sediment transport, while larger grain sizes primarily accumulate on the bed, forming more bedload.
- 4) Besides, in the Shields regime diagram (Fig 13), it can be verified that fine-grained sediment is more prone to surpassing the critical Shields parameter under identical discharges, thus maintaining suspension easily.

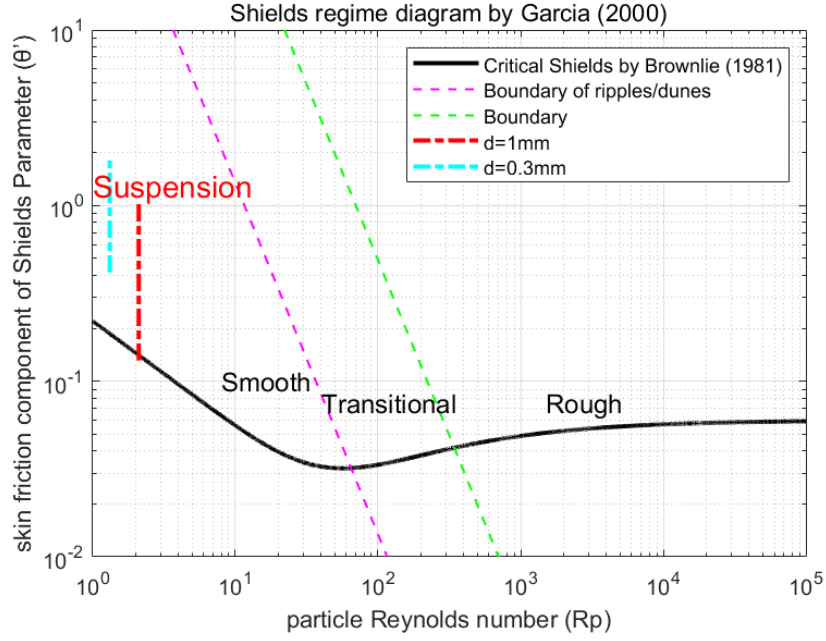


Fig 13. Suspension regime diagram and check

2.3. Comparisons of u , c , and $u \cdot c$ versus z

For these two specific cases with $q = 5 \text{ m}^2/\text{s}$, comparisons of their vertical profiles are following:

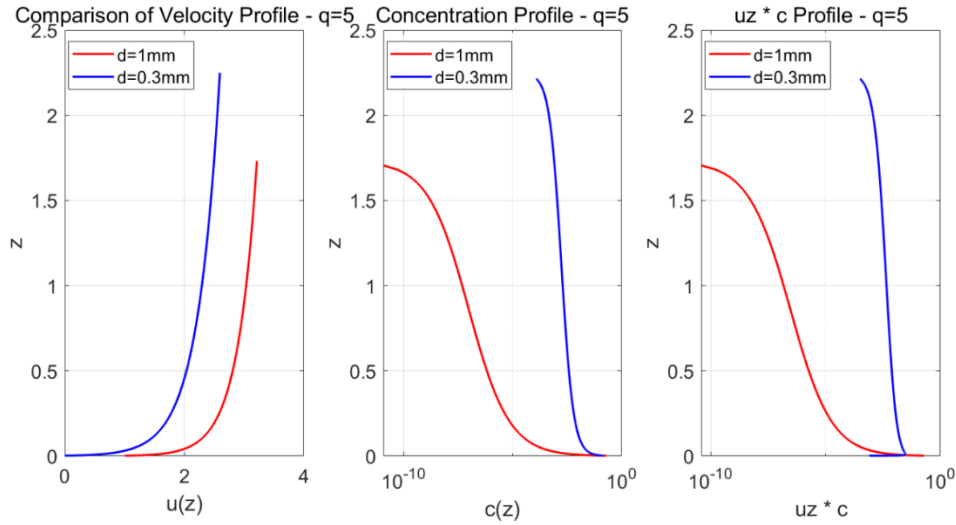


Fig 14. Comparisons of Profiles in 2 grain sizes – normally vertical water depth axis

In a math perspective, as qS is the result of integrating of $u(z) \cdot c$ term over the valid range of water depth, the area of curve in $u(z) \cdot c$ profile is obviously larger in fine particles than that of coarse particles, corresponding to the larger qS for $d=0.3\text{mm}$. Although the coarse particles' $u(z)$ is faster, the concentration c dominates the $u(z) \cdot c$ term. More discussions are seen in 3.2 Sharp increase in $u(z) \cdot c$ profile. To mitigate the difference between water depth, a nondimensional vertical axis ' z/D ' is used to visualize the relative location in the water volume in Fig 15.

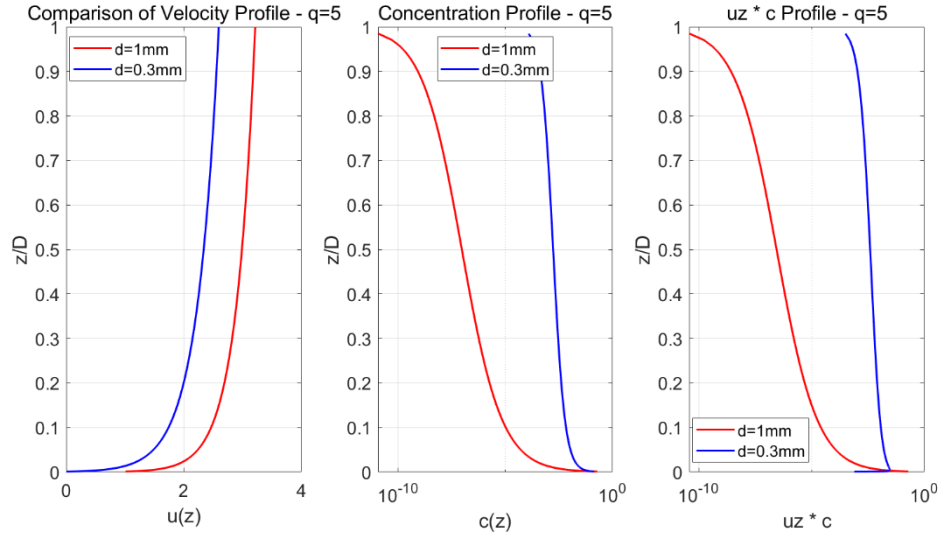


Fig 15. Comparisons of Profiles – y-axis: relative water depth ('1' means the surface)

Thus, we can see the same lognormality distribution of horizontal velocity $u(z)$. Besides, the concentration distribution reflects that with fine particles it distributed more uniformly that coarse material, as in the Fig 16 the more uniform concentration distribution within similar order of magnitude in $d=0.3\text{mm}$, compared to the wider range of concentration in $d=1\text{mm}$.

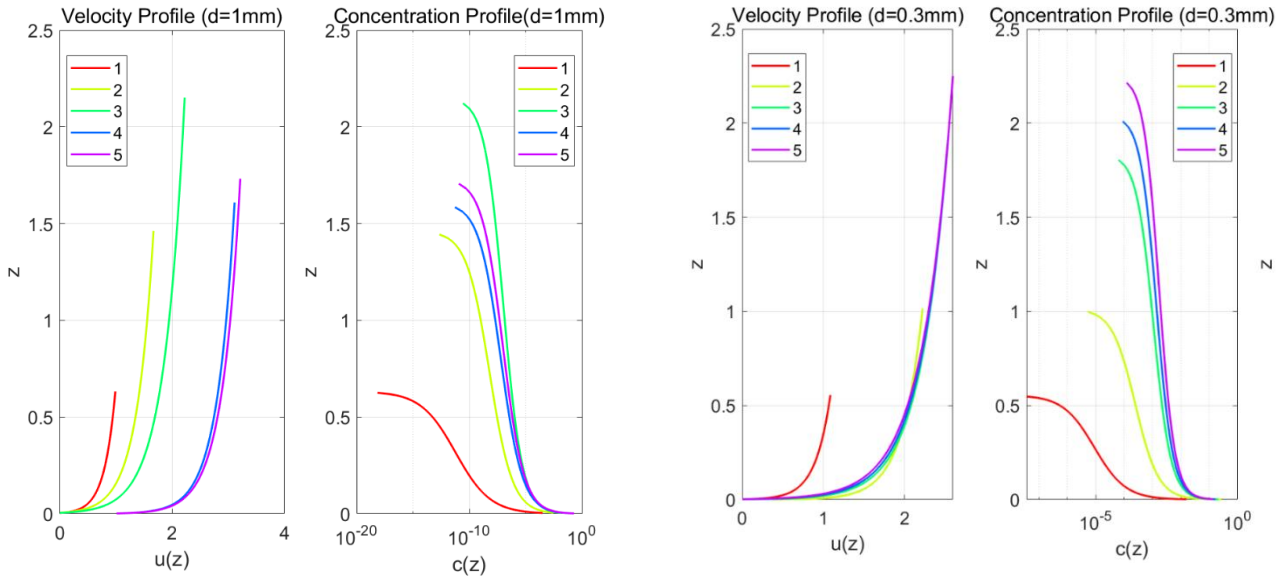


Fig 16. Comparison of Vertical Profiles with 2 grain sizes

Overall, we can see the more uniform concentration distribution, more suspended-load transport and slower velocity with fine particles, that is related to its **larger shear stress** (in 3.1 Relationship of Grain Size to Fall Velocity and Shear Stress[4]).

3. More Discussions

3.1. Relationship of Grain Size to Fall Velocity and Shear Stress[4]

Many sediment transport processes (particularly suspended load relationships) contain a fall velocity w_s . Fall velocity increases with grain diameter.

Additionally, another important parameter in sediment transport is the bed shear stress τ_b , which is the tangential force per unit area that a moving fluid exerts on the sediment bed, which causes transport:

$$\tau_b = \theta \rho g(s - 1)d$$

Thus, we simply calculate shear stress in Fig 17. It shows the τ_b of fine particles is slightly larger than coarse particles in most cases.

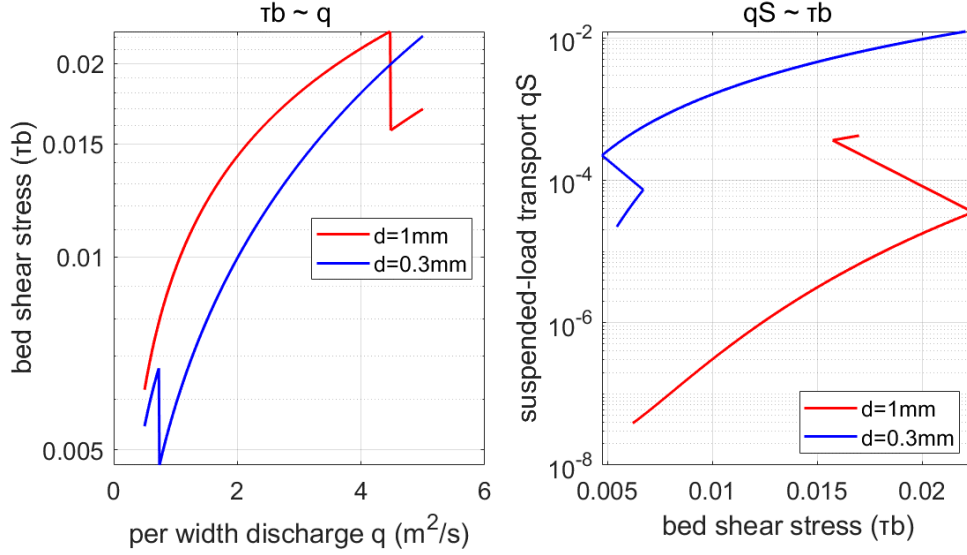


Fig 17. Bed shear stress and qS

From above results in Fig 12, we know that the transport strongly increases with decreasing grain size.

Therefore, the obvious increase in the sediment grain size just causes a small increase in the shear stress on the bed, which leads directly to a significantly greater sediment transport rate for the same free-stream flow conditions. [5]

3.2. Sharp increase in $u(z)*c$ profile

There is a notable difference in the shape of $u(z)*c$ profile in Fig 18, showing that of smaller particles abruptly rises to a peak close to the bed and then commences its descent towards the water surface, that is relative to math, instead of any hydraulic factors.

We can do a derivation calculation for the $u(z)*c$ term:

$$\frac{\partial}{\partial z} \left(\frac{c U \left(\frac{(D-z)b}{z(D-b)} \right)^w \log\left(\frac{30z}{k}\right)}{\kappa} \right) = \frac{c U \left(\frac{b(D-z)}{z(D-b)} \right)^w (D w \log\left(\frac{30z}{k}\right) - D + z)}{\kappa z (z - D)}$$

And the results show the signs of the derivatives under the two scenarios are opposite near the bed, with the derivative of $u(z)*c$ increasing for fine particles and decreasing for coarse particles. However, as z moves away from the bed and close the surface, both derivatives become negative, indicating a decrease in $u(z)*c$.

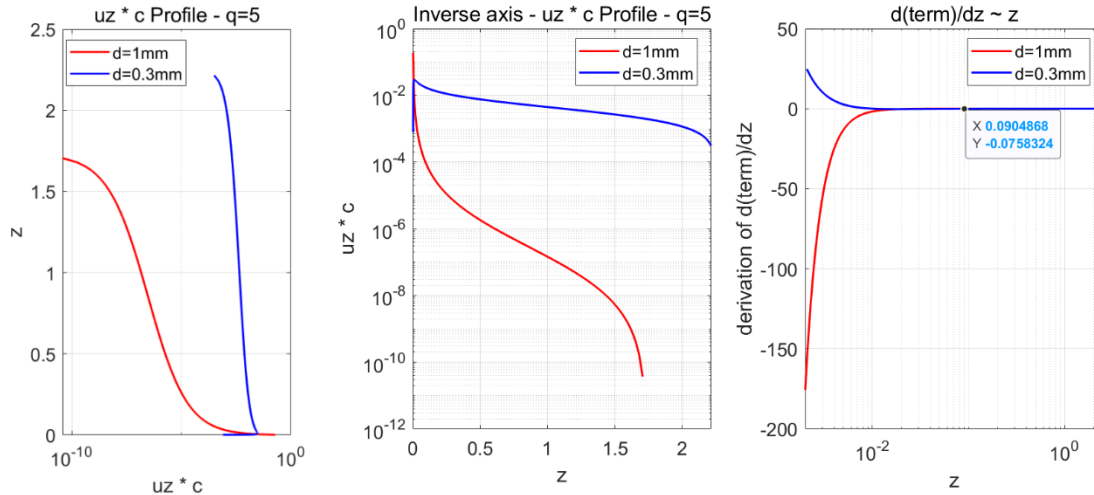


Fig 18. $u(z)*c$ profile and derivation results

4. Conclusion

- 1) In Rouse formula, the sediment concentration is calculated as zero at the water surface and infinity at the bed.
- 2) For the single size of particles, the velocity profile mostly depends on friction velocity, showing the increasing friction velocity Uf causes a higher velocity.
- 3) A lower Rouse number leads to a more uniform suspended-load concentration distribution.
- 4) The bedload transport qB is largely independent of grain size in the lower regime, and the suspended load dominates in the total sediment transport for smaller particles.
- 5) The larger bed shear stress leads to more uniform concentration distribution, slower velocity profile, and more suspended-load transport with fine particles.

5. References

- [1] J. De Leeuw *et al.*, “Entrainment and suspension of sand and gravel.” Dec. 17, 2019. doi: 10.5194/esurf-2019-67.
- [2] S. Nie *et al.*, “Vertical Distribution of Suspended Sediment under Steady Flow: Existing Theories and Fractional Derivative Model,” *Discrete Dyn. Nat. Soc.*, vol. 2017, p. e5481531, Jun. 2017, doi: 10.1155/2017/5481531.
- [3] R. Ettema, “Hunter Rouse—His Work in Retrospect,” *J. Hydraul. Eng.*, vol. 132, no. 12, pp. 1248–1258, Dec. 2006, doi: 10.1061/(ASCE)0733-9429(2006)132:12(1248).
- [4] D. B. King, “Influence of Grain Size on Sediment Transport Rates With Emphasis on the Total Longshore Rate:,” Defense Technical Information Center, Fort Belvoir, VA, Nov. 2005. doi: 10.21236/ADA440672.
- [5] J. Dufek and G. W. Bergantz, “Suspended load and bed-load transport of particle-laden gravity currents: the role of particle–bed interaction,” *Theor. Comput. Fluid Dyn.*, vol. 21, no. 2, pp. 119–145, Mar. 2007, doi: 10.1007/s00162-007-0041-6.

6. Appendix: Script in MATLAB

6.1. Function –uz & c & qS.m

```
function [uz, b_star, c, z_log, qS, uz_mul_c] = log_uz_c_qS(Uf, D, V, cb,
ws, b)
    kappa = 0.4;
    % k* is an effective roughness length.
    num_pts = length(Uf);
    num_var_q = length(Uf);
    uz = zeros(num_pts, num_var_q);
    c = zeros(num_pts, num_var_q);
    ws_val = ws; % constant ws - varies as grain size, instead of Re

    for i = 1:num_pts
        D_val = D(i);
        Uf_val = Uf(i);
        V_val = V(i);
        cb_val = cb(i);
        %ws_val = ws(i);
        k_star_val = D_val/exp(kappa*(V_val/Uf_val - 6));
        b_star_val = max(b, k_star_val/30); % interval value, not save
        z = logspace(log10(b_star_val), log10(D_val), num_pts);

        for j = 1:num_var_q
            if z(j) <= D_val
                uz(j,i) = Uf_val / kappa * log(30 * z(j) / k_star_val);
```

```

        c(j,i) = cb_val*((D_val-z(j))/z(j)*b/(D_val-
b))^ (ws_val/kappa/Uf_val);
        uz_mul_c(j,i) = uz(j,i) * c(j,i);
    end
    z_log(j, i) = z(j);

end
b_star(i) = b_star_val;
qS(i) = trapz(z, uz_mul_c(:,i));
end
end

```

6.2. Part3&4 – one case- Call function() for the solution

% part3: modifications for suspended sediment with bedforms present

```

clc;
clear;
% Read the matrix results from part1;
% theta_prime calculated by upper and lower regime separation method
data= readtable('d1_451_regime.csv');
% Extract [q V D D_prime Uf theta theta_prime]
q_tr = data.Var1;
V = data.Var2;
D = data.Var3;
D_prime = data.Var4;
Uf_prime = data.Var5;
theta = data.Var6;
theta_prime = data.Var7;

% prerequisite calculation
Uf = sqrt(theta./theta_prime).* Uf_prime;
d = 1e-3; % d = 1 mm =1e-3 m
vi = 1e-6; % viscosity
Re = Uf.*d./vi;
theta_cri = 0.165*(Re+0.6).^(-0.8)+0.045*exp(-40.*Re.^-1.3); % explicit
formular of Shield Paramter
kappa = 0.4;

% prepare for concentration
ws = 0.1120; % from 'fsolve_ws.m'; cannot use constant?
g = 9.81;
s = 2.65;
%cD = 1.4 + 36./Re;
%ws = sqrt(4*(s-1)*g*d/3./cD);

```

```

cb = 0.331.*(theta_prime - 0.045).^1.75 ./ (1+0.331/0.46 .*(theta_prime -
0.045).^1.75);
b = 2*d; % typically

% num_pts: calculated points inserted into one section -> correspond to
D(i)
% num_var_matrix: for each q, the number of V, with q increasing [0.5,5]
[uz, b_star, c, z, qS, multip_uzc] = log_uz_c_qS(Uf, D, V, cb, ws, b);
% Integral calculation of qS - function
% calculation of qS use the trapz command to perform the integral
% Calculate qS for each column
%%%% col_u corresponds to row_D or row_q !! (multiple u ~ single D or q)

q_range = transpose(q_tr);
%%% plot qS ~ each q
figure(1)
plot(q_range, qS, 'r', 'LineWidth', 2)
xlabel('steady discharge per unit width q (m^2/s)')
ylabel('Suspended sediment transport qS (m^2/s)')
title('qS vs q')
grid on
set(gca, 'FontSize', 14)

%%% plot ^.term = ws/(Uf*kappa) ~ each q
figure(2)
ws_term = ws./(Uf.*kappa);
plot(q_range, ws_term, 'black', 'LineWidth', 2)
xlabel('steady discharge per unit width q (m^2/s)')
ylabel('ws/(Uf*\kappa)')
title('Rouse parameter ws/(Uf*\kappa) vs q')
legend('line - Rouse parameter', '1: q=0.5', '2: q=2.0', 'Location',
'best'); % use index to mark line in legend
grid on
set(gca, 'FontSize', 14)

% correspond to D; water depth and bed form
figure(3)
plot(q_range, D, 'black', 'LineWidth', 2)
xlabel('steady discharge per unit width q (m^2/s)')
ylabel('Flow depth D (m)')
title('D vs q')
legend('line - Water depth D', 'Location', 'best'); % use index to mark
line in legend
grid on

```

```

set(gca, 'FontSize', 14)

%% Logarithmically spaced vertical coordinates: 1) velocity; 2)
concentration
test_pts = 5; % for q[0.5, 5]
num_pts = length(q_tr);

% customize: generated number must be an integer multiple of 5
% randomly call a predefined color map by lines(), parula(), jet, hsv, hot,
cool, spring, summer, autumn, winter
colormap_lines = hsv(test_pts);

Index = [1, 151, 351, 401, 451];
for i = 1:test_pts
    index = Index(i);
    D_cus(i) = D(index);
    q_cus(i) = q_tr(index);

    % CONT figure(2)
    figure(2)
    hold on;
    set(gca, 'FontSize', 14);
    plot(q_range(index), ws_term(index), 'o', 'Color',
colormap_lines(i, :), ...
        'MarkerFaceColor', colormap_lines(i, :), 'MarkerSize', 10);
    %legend(cellstr(num2str((1:test_pts))), 'Location', 'best'); % use
index to mark line in legend

    figure(3)
    hold on;
    set(gca, 'FontSize', 14);
    plot(q_range(index), D(index), 'o', 'Color', colormap_lines(i, :), ...
        'MarkerFaceColor', colormap_lines(i, :), 'MarkerSize', 10);

    % Velocity Profile
    figure(4)
    subplot(1,3,1)
    plot(uz(:,index), z(:,index), 'LineWidth', 1.5, 'Color',
colormap_lines(i, :));
    %xtickformat('%1e');
    ylabel('z');
    xlabel('u(z)');
    title('Velocity Profile');
    grid on;

```



```

hold on;
set(gca, 'FontSize', 14);
legend(cellstr(num2str((1:test_pts))), 'Location', 'best'); % use
index to mark line in legend

% Concentration Profile
subplot(1,3,2)
semilogx(c(:,index), z(:,index), 'LineWidth', 1.5, 'Color',
colormap_lines(i, :)); % more obvious interval: semilogx()
xtickformat('%1e');
ylabel('z');
xlabel('c(z)');
title('Concentration Profile');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend(cellstr(num2str((1:test_pts))), 'Location', 'best'); % use
index to mark line in legend

% U * C Profile
subplot(1,3,3)
semilogx(multip_uzc(:,index), z(:,index), 'LineWidth', 1.5, 'Color',
colormap_lines(i, :));
xtickformat('%1e');
ylabel('z');
xlabel('uz * c');
title('uz * c');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend(cellstr(num2str((1:test_pts))), 'Location', 'best'); % use
index to mark line in legend

% expansion
% normalize ZZ for Vanoni distribution
ZZ(:,i) = (z(:,index) - b) / (D_cus(i) - b);
delta = (b/D_cus(i));
ZZ_norm(:,i) = (delta - delta.*ZZ(:,i))./(ZZ(:,i) + delta -
delta.*ZZ(:,i));
c_norm(:,i) = c(:,index)./cb(index,1);

figure(5)
plot(c_norm(:,i), ZZ(:,i), 'LineWidth', 1.5, 'Color',
colormap_lines(i, :));

```

```

        ylabel('Z (normalized z)');
        xlabel('c(z) / cb');
        title('Concentration Profile on Vanoni distribution - Normalized');
        grid on;
        hold on;
        legend(cellstr(num2str((1:test_pts)')), 'Location', 'best');
        set(gca, 'FontSize', 14);

end
hold off;

%% results saved in .csv for q=5 & d=1mm
% [z, uz, c, uz*c, qS/qB, ws_term, qB, q]
d1_q5_matrix = [z(:,451), uz(:,451), c(:,451), multip_uzc(:,451),
ratio_qSqB_d1, ws_term, qB, q_tr];
writematrix(d1_q5_matrix, 'd1_res_q5.csv');
% save for Rouse plot - multiple Z; delta = 0.0012

% For d=0.3mm case, firstly calculate hydraulic properties by exe.2 script,
and then run this script after modifying d.

```

6.3. Part4 – Plot the 2 specific cases with q=5

```

% read d=1mm and d=0.3mm results
clc;
clear;
% Read the matrix results from part1;
data_1= readtable('d1_res_q5.csv');
% [z, uz, c, uz*c, qS/qB, ws_term, qB, q]
z_comp = data_1.Var1;
uz_comp = data_1.Var2;
c_comp = data_1.Var3;
uzc_comp = data_1.Var4;
ratio_comp = data_1.Var5;
ws_comp = data_1.Var6;
qB_comp = data_1.Var7;
q_comp = data_1.Var8;

% Read the matrix results: d=0.3mm; saved in 2nd column
data_03= readtable('d03_res_q5.csv');
% [z, uz, c, uz*c, qS/qB, q]
z_comp(:,2) = data_03.Var1;
uz_comp(:,2) = data_03.Var2;
c_comp(:,2) = data_03.Var3;
uzc_comp(:,2) = data_03.Var4;

```

```

ratio_comp(:,2) = data_03.Var5;
ws_comp(:,2) = data_03.Var6;
qB_comp(:,2) = data_03.Var7;
q_comp(:,2) = data_03.Var8;

% Velocity Profile
figure(1)
subplot(1,2,1)
plot(uz_comp(:,1), z_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
plot(uz_comp(:,2), z_comp(:,2), 'LineWidth', 1.5, 'Color', 'b');
ylabel('z');
xlabel('u(z)');
title('Comparison of Velocity Profile - q=5');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best');

% Concentration Profile
subplot(1,2,2)
semilogx(c_comp(:,1), z_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
semilogx(c_comp(:,2), z_comp(:,2), 'LineWidth', 1.5, 'Color', 'b');
xtickformat('%.1e');
ylabel('z');
xlabel('c(z)');
title('Concentration Profile - q=5');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best');

% U * C Profile - zoom in and corresponds to the b_star
figure(2)
semilogx(uzc_comp(:,1), z_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
semilogx(uzc_comp(:,2), z_comp(:,2), 'LineWidth', 1.5, 'Color', 'b');
xtickformat('%.1e');
ylabel('z');
xlabel('uz * c');
title('uz * c Profile - q=5');
grid on;
hold on;

```

```

set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best');

% Rouse parameter  $Z=ws/(\kappa U_f)$ 
figure(3)
plot(q_comp(:,1), ws_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
plot(q_comp(:,2), ws_comp(:,2), 'LineWidth', 1.5, 'Color', 'b');
ylabel('ws/(Uf*\kappa)');
xlabel('q (m^2/s)');
title('Comparison of Rouse parameter ~ q');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best');

% Comparison of ratio
figure(4)
subplot(1,3,1)
plot(q_comp(:,1), qB_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
plot(q_comp(:,2), qB_comp(:,2), 'LineWidth', 1.5, 'Color', 'b');
ylabel('Bedload transport qB');
ylim([0,1e-3]);
xlabel('q (m^2/s)');
xlim([0.5, 5]);
title('qB ~ q');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best'); % use index to mark
line in legend

subplot(1,3,2)
semilogy(q_comp(:,1), ratio_comp(:,1).*qB_comp(:,1), 'LineWidth', 1.5,
'Color', 'r');
hold on;
semilogy(q_comp(:,2), ratio_comp(:,2).*qB_comp(:,2), 'LineWidth', 1.5,
'Color', 'b');
ylabel('Suspended-load qS');
%ylim([0,1e-3]);
xlabel('q (m^2/s)');
xlim([0.5, 5]);
title('qS ~ q');

```

```

grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best'); % use index to mark
line in legend

subplot(1,3,3)
semilogy(q_comp(:,1), ratio_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
semilogy(q_comp(:,2), ratio_comp(:,2), 'LineWidth', 1.5, 'Color',
'b');
ytickformat('%1e');
ylabel('qS/qB');
xlabel('q (m^2/s)');
xlim([0.5, 5]);
title('ratio of qS/qB ~ q');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best');

% Comparison of bedload
figure(5)
semilogy(q_comp(:,1), ratio_comp(:,1), 'LineWidth', 1.5, 'Color', 'r');
hold on;
semilogy(q_comp(:,2), ratio_comp(:,2), 'LineWidth', 1.5, 'Color',
'b');
ytickformat('%1e');
ylabel('qS/qB');
xlabel('q');
title('ratio of qS/qB ~ q');
grid on;
hold on;
set(gca, 'FontSize', 14);
legend('d=1mm', 'd=0.3mm', 'Location', 'best');

```