CONTENTS

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Data Analysis for

Surfzone Hydrodynamics

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1. Comparison of the measured value and theoretical value

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1.0 Intro & Global settings

a) To compare of the measured value and theoretical value, define a deviation equation to estimate the fitting degree:

$$Deviation(\%) = \frac{\text{data[theoretical] - data[measurement]}}{\text{data[measurement]}}$$

b) Based on the given measured value *h*, *fbar*, *Hs*, *θ*, and the density of sea water ρ_sea=1025 kg/m³, calculate the wave parameters for the following calculation.

$$u = a\omega \frac{\cosh(k(z+h))}{\sinh(kh)} \cos(kx - \omega t)$$

$$E = \frac{1}{16}\rho g \gamma^2 h^2 = \frac{1}{16}\rho g Hs^2$$

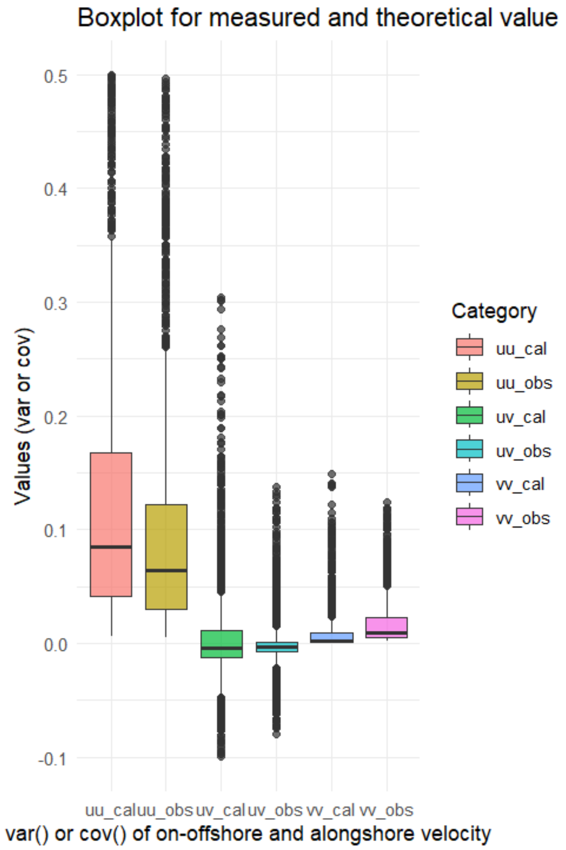
$$\overline{a\cos(kx-\omega t)}^2 = \frac{1}{2}a^2$$

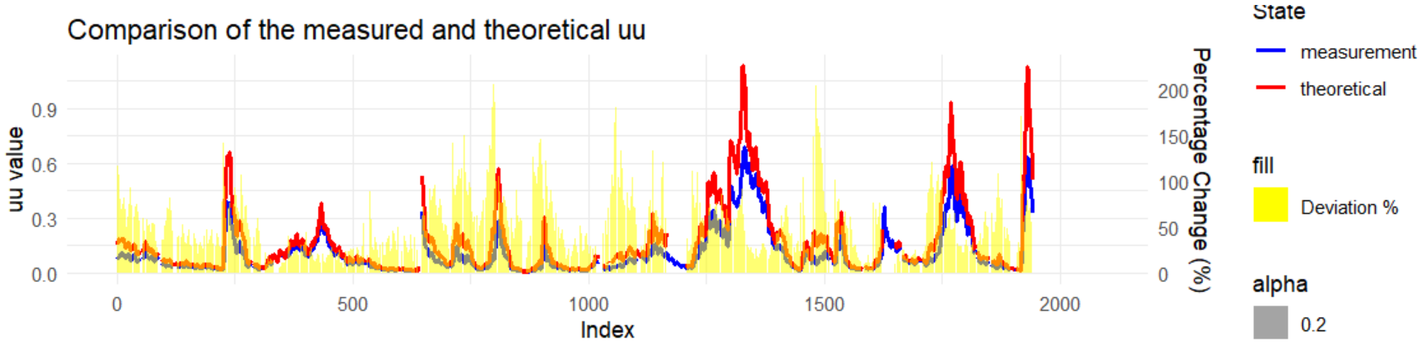
$$\overline{\eta^2} = \left(\frac{Hs}{4}\right)^2$$

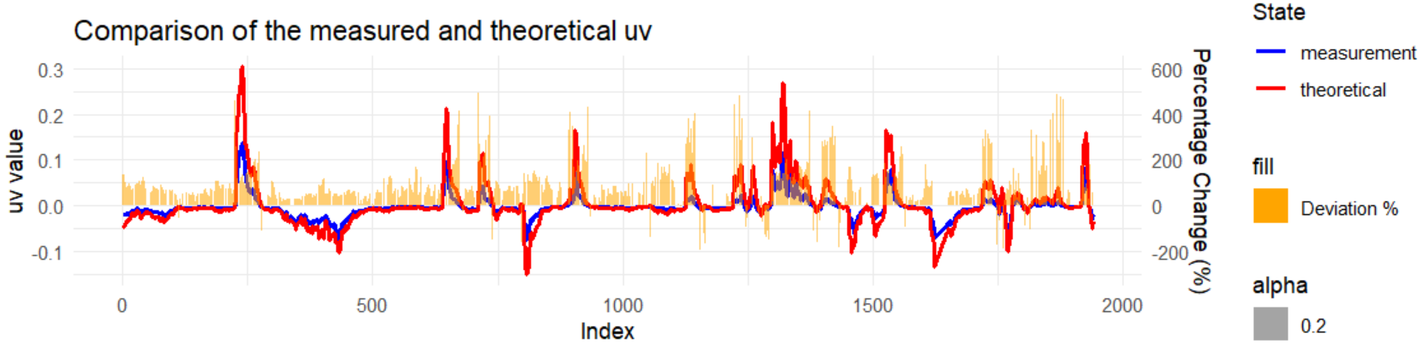
c) All data analysis and plot is made with R scripts.

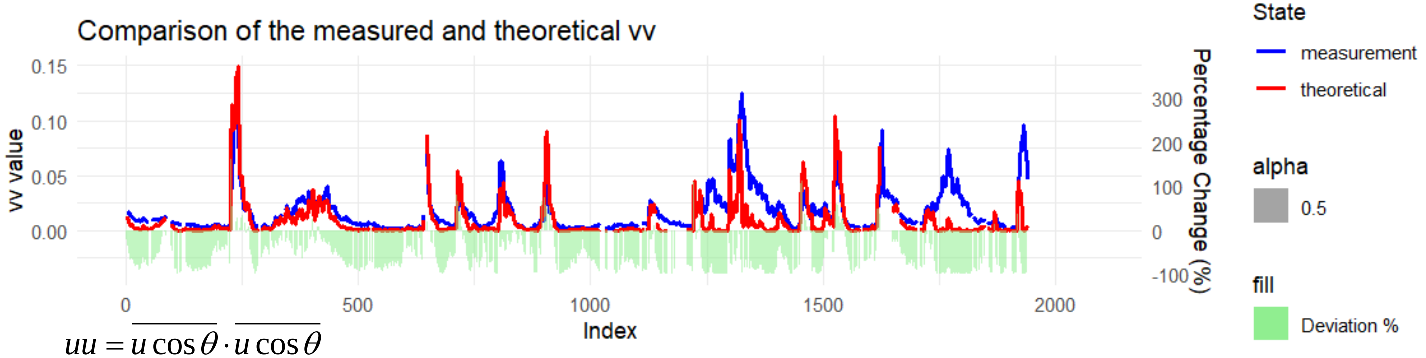
1. Comparison of measured value and theoretical value

1.1 uu, uv, vv









- Theoretical *uu*, *uv* and *vv* represent better agreement with around 50% deviation.
- Most deviation of *uu* and *vv* show the reverse trends, referring to the theoretical value of *vv* is smaller than measurement. And the deviation of *uv* shows an obvious fluctuation.
- From the statistical indicators (one−hour averages), the median of *vv*≈0 indicates most measurement value of V is stable.

- For the boxplot, the better fitted distribution occurs at vv, inferring to the smaller median and variability, and less outliers.

$$uu = \overline{u\cos\theta}\cdot\overline{u\cos\theta} = \overline{\eta^2}\left[\frac{\omega\cosh(k(z+h))}{\sinh(kh)}\right]^2\cos^2\theta = \frac{1}{16}Hs^2\left[\frac{\omega\cosh(k(z+h))}{\sinh(kh)}\right]^2\cos^2\theta$$

$$uv = \overline{u\cos\theta}\cdot\overline{u\sin\theta} = \overline{\eta^2}\left[\frac{\omega\cosh(k(z+h))}{\sinh(kh)}\right]^2\cos\theta\sin\theta$$

$$vv = \overline{u\sin\theta}\cdot\overline{u\sin\theta} = \overline{\eta^2}\left[\frac{\omega\cosh(k(z+h))}{\sinh(kh)}\right]^2\sin^2\theta$$

1. Comparison of measured value and theoretical value: wave energy flux

1.2 Fx, Fy

For wave energy flux F:

$$E = \rho g \overline{\eta^2}$$

$$F = E c_g$$

where c_g is the group velocity magnitude given by

$$c_g = \frac{c}{2}\left[1 + \frac{2kh}{\sinh(2kh)}\right]$$

with $c = \frac{\omega}{k}$ the phase speed.

In the theory:

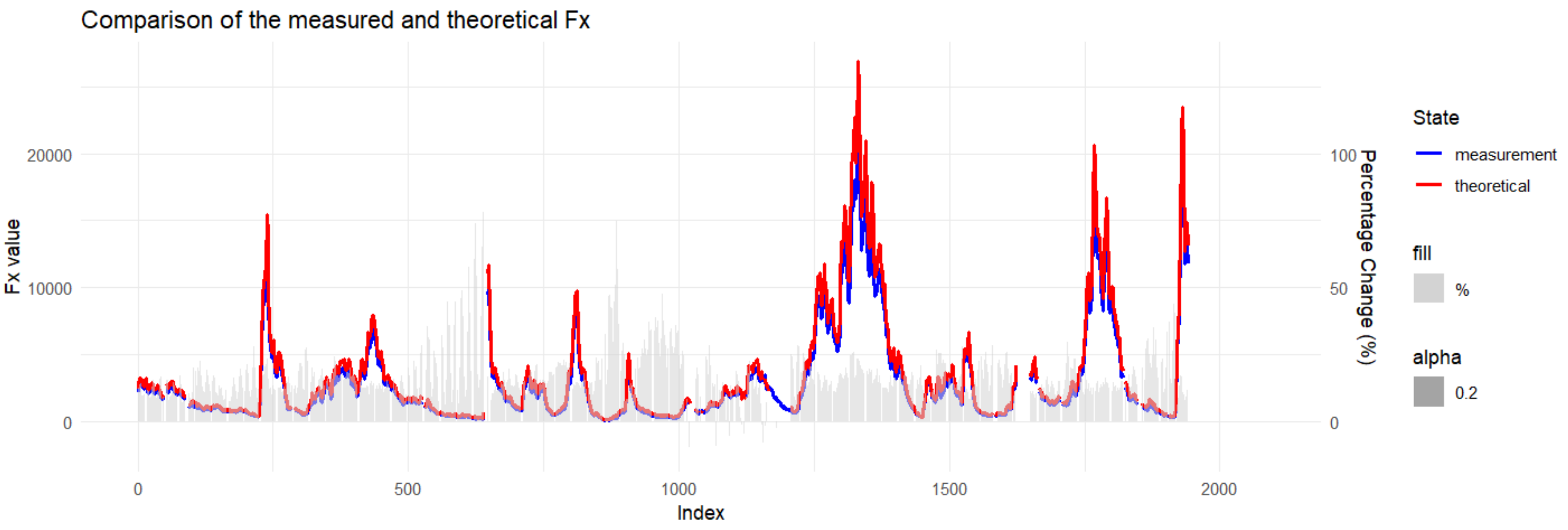
a) At the x direction:

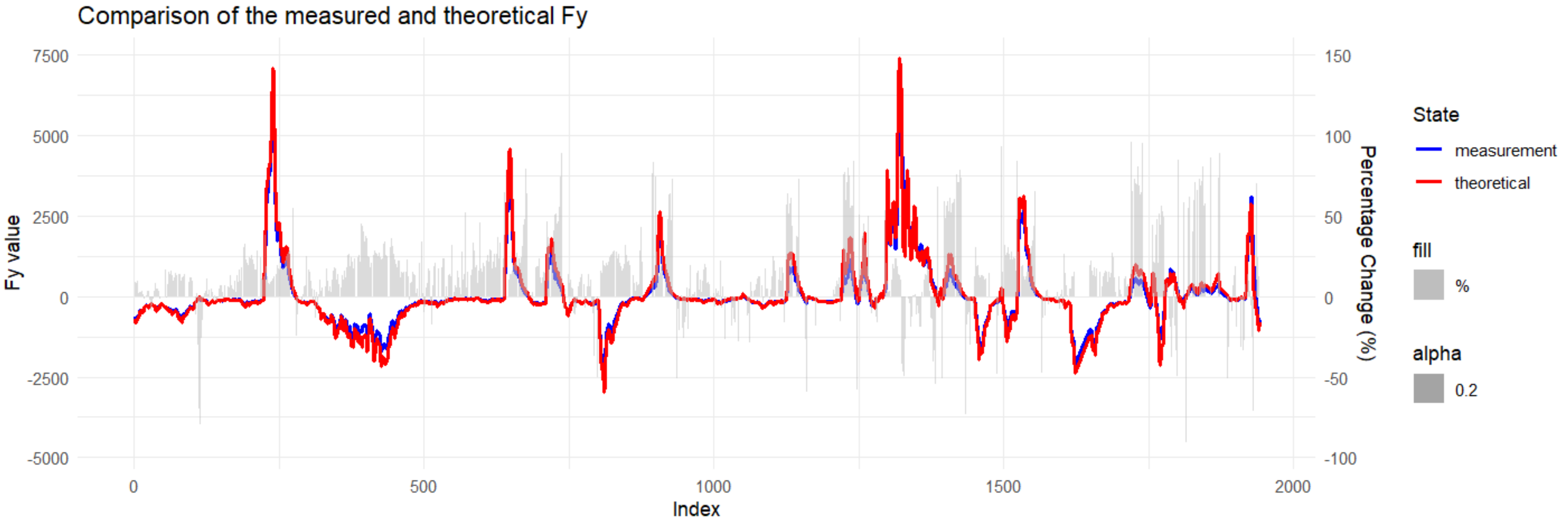
Fx = F * cosθ

b) At the y direction:

Fy = F * sinθ

- The better agreement occurs in Fx with most deviation<25%.
- For Fy, it shows a more obvious deviation, that may be related to the main wave propagation at the x direction, and it can be seen that Fx is significantly greater than the Fy by **an order of magnitude**.





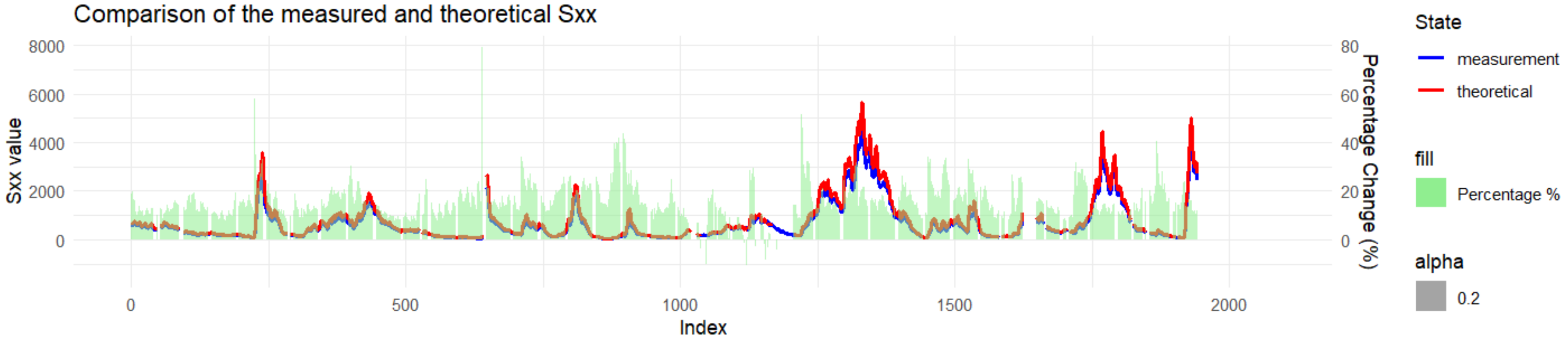
1. Comparison of measured value and theoretical value: Radiation stress

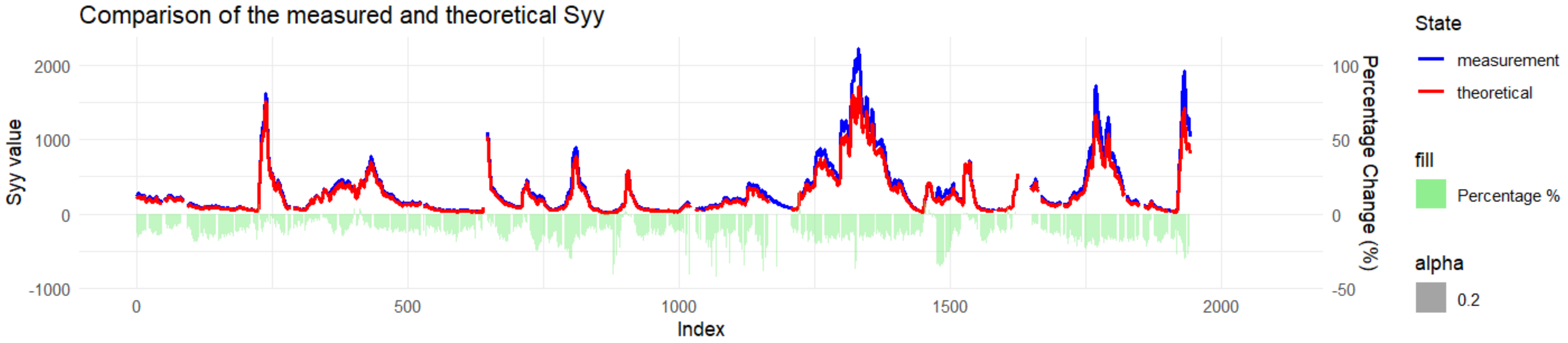
1.3 Sxx, Sxy, Syy

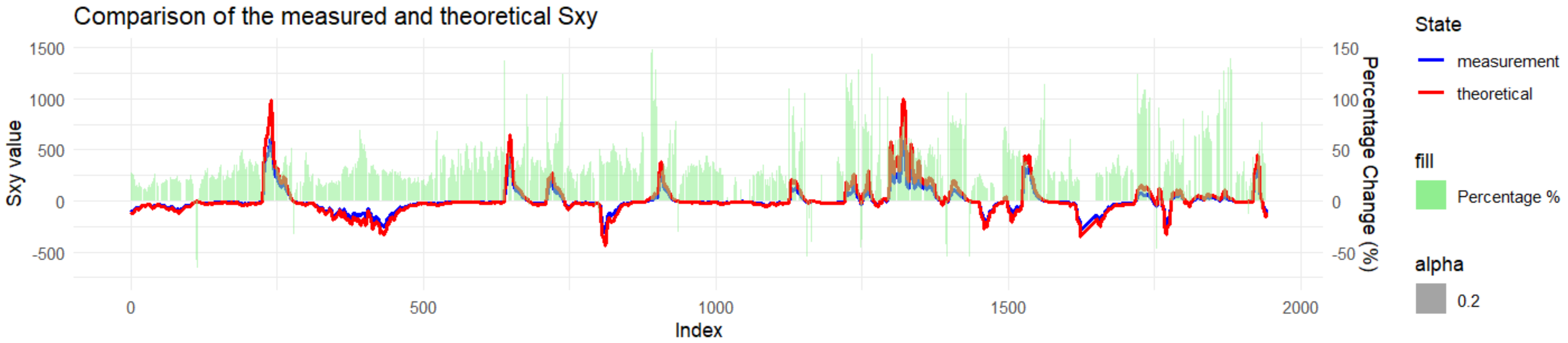
For the radiation stress:

- $S_{xx} = \frac{E}{2}[2\frac{c_g}{c}\cos\theta^2 + (2\frac{c_g}{c} - 1)]$
- $S_{yy} = \frac{E}{2}[2\frac{c_g}{c}\sin\theta^2 + (2\frac{c_g}{c} - 1)]$
- $S_{xy} = E\frac{c_g}{c}\sin\theta\cos\theta$

- For **Sxx and Syy,** the theoretical and observed values show a better agreement, with 20% for Sxx and -5% for Syy.
- For **Sxy**, the theoretical calculations exhibit a significant deviation from the measured values, more than 50% deviation.



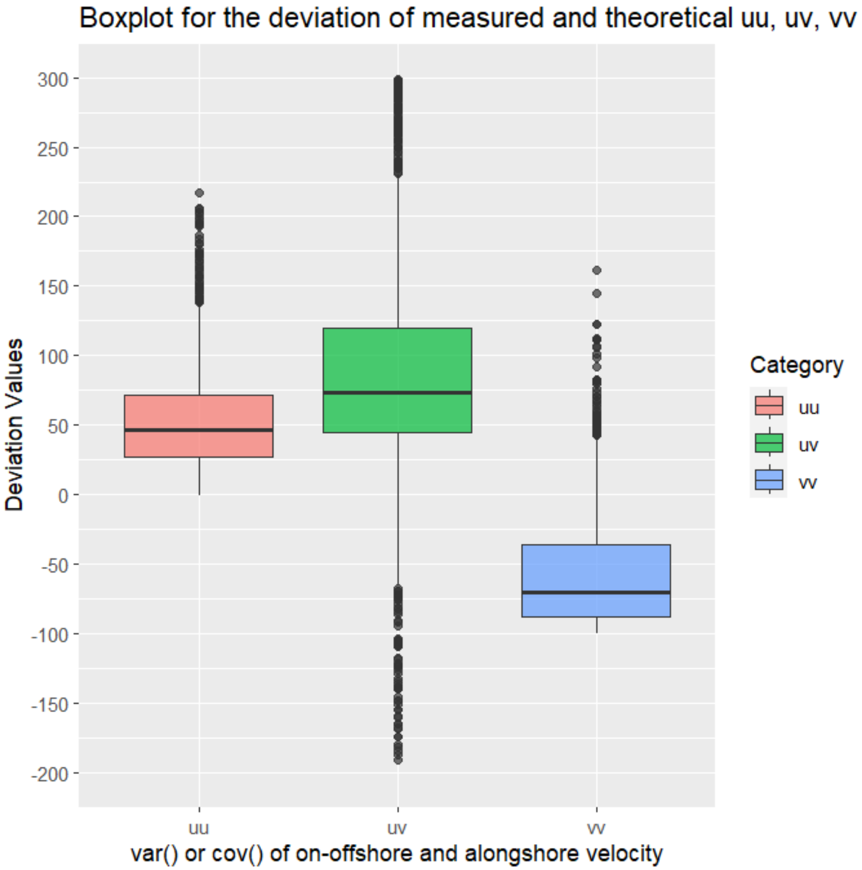


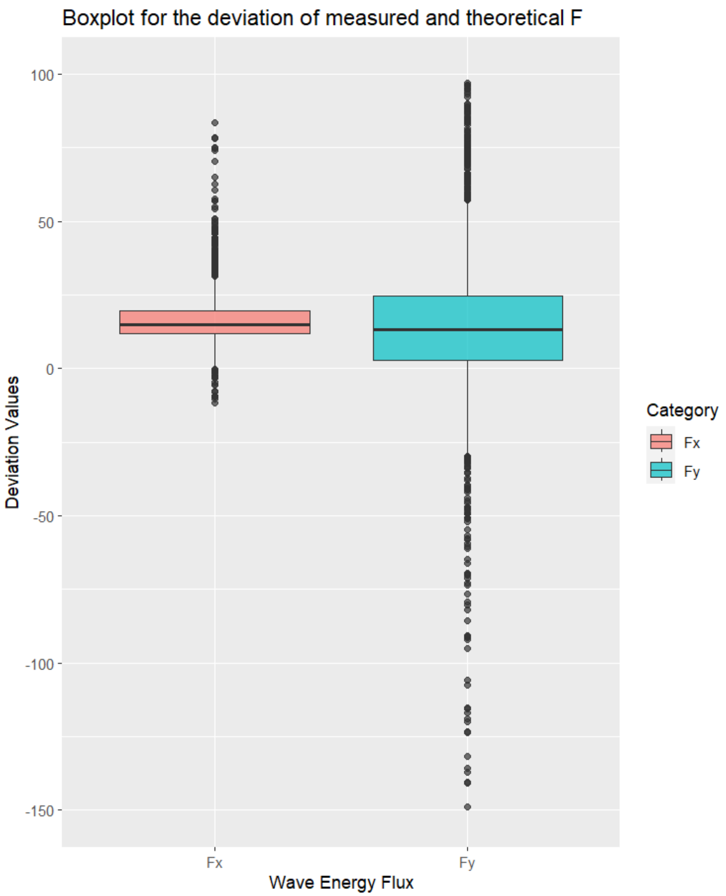


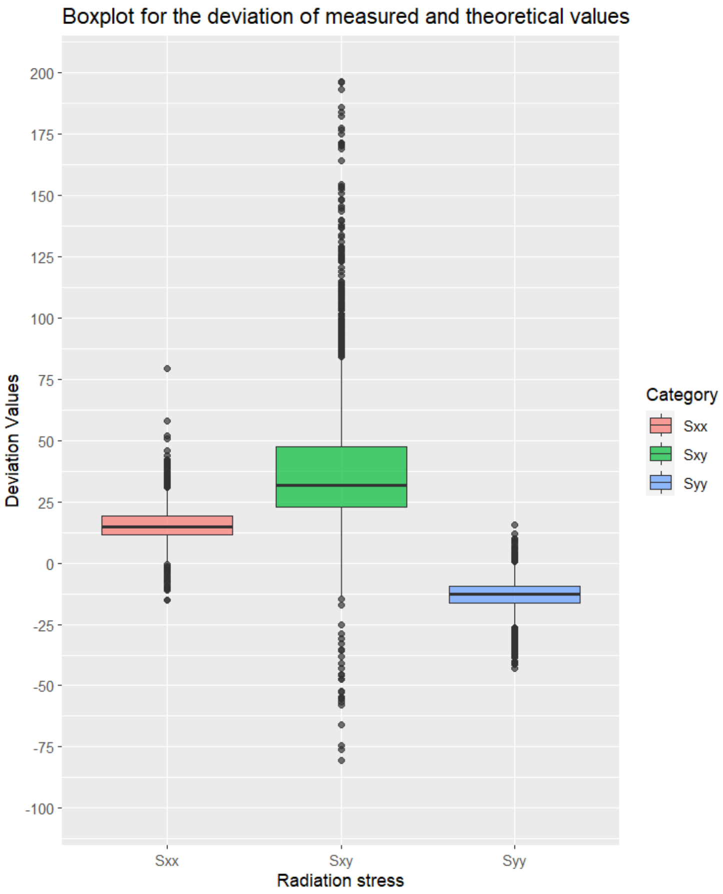
1. Comparison of measured value and theoretical value

1.4 Summary

$$Deviation(\%) = \frac{data[theoretical] - data[measurement]}{data[measurement]}$$



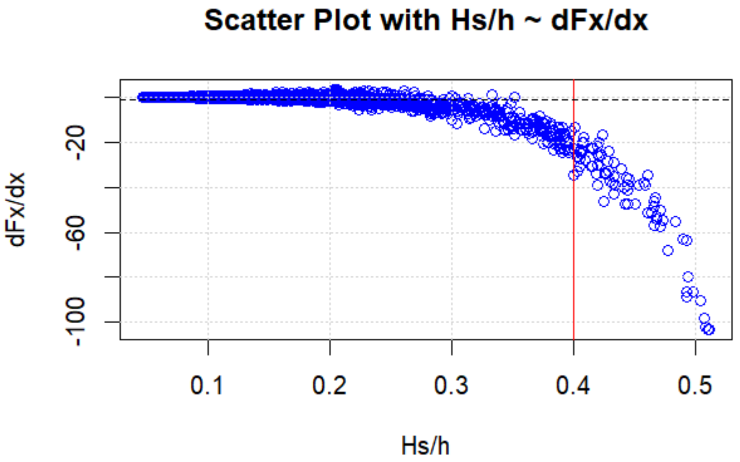




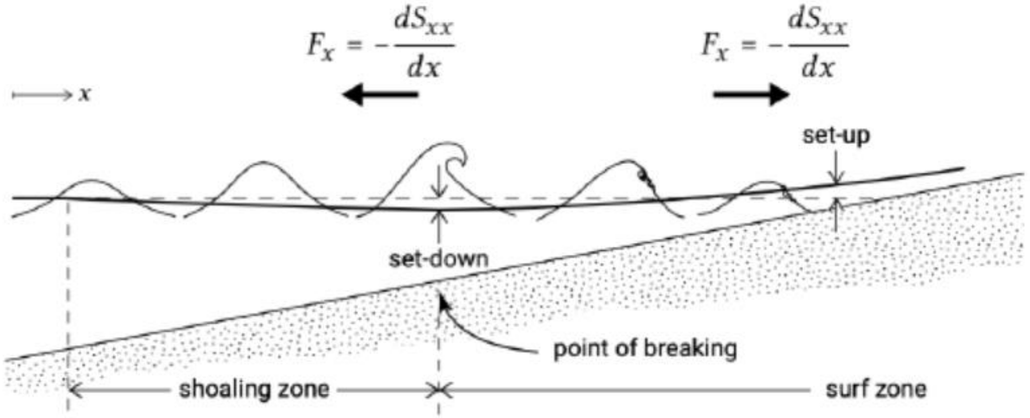
- The boxplots show the **deviation degree** of theoretical value comparing the measurement.
- As the best match occurs in uu, Fx and Sxx at the x direction, it is inferred that wave components can be accurately calculated in the on-offshore direction.
- There are many outliers with the huge deviation in vv, Fy and Syy, which are related to the alongshore direction.

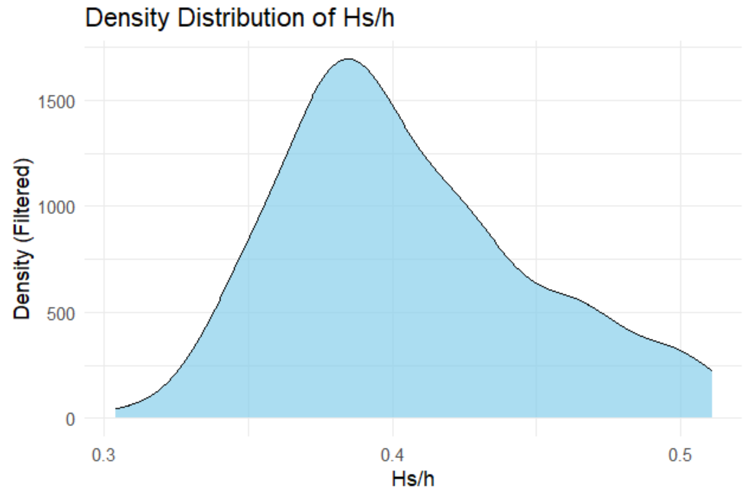
2. Wave breaking in the surf zone

- The view that breaking extracts energy flux from the shoreward propagating waves (seen in Fig), shows the force *Fx* changes (i.e. $\mathbf{d}F\mathbf{x}\ /\mathbf{dx} \neq \mathbf{0}$) at the breaker point in the surf zone.
- In the surf zone and in theory, Hs/h=γ=0.4.
- In the scatter plot for *Hs/h* versue *dFx/dx*, when the wave breaks obviously, referring to the dFx/dx<-10 or less, and after filtered, the value of Hs/h can be approximated as 0.4.



Filter
dFx/dx < -10





- In the surf zone, *Sxx* decreases in the positive x-direction; the resulting negative gradient in is equivalent to a force *Fx* directed in the onshore direction.

3. Onshore volume transport and Offshore return current

3.1 The onshore component

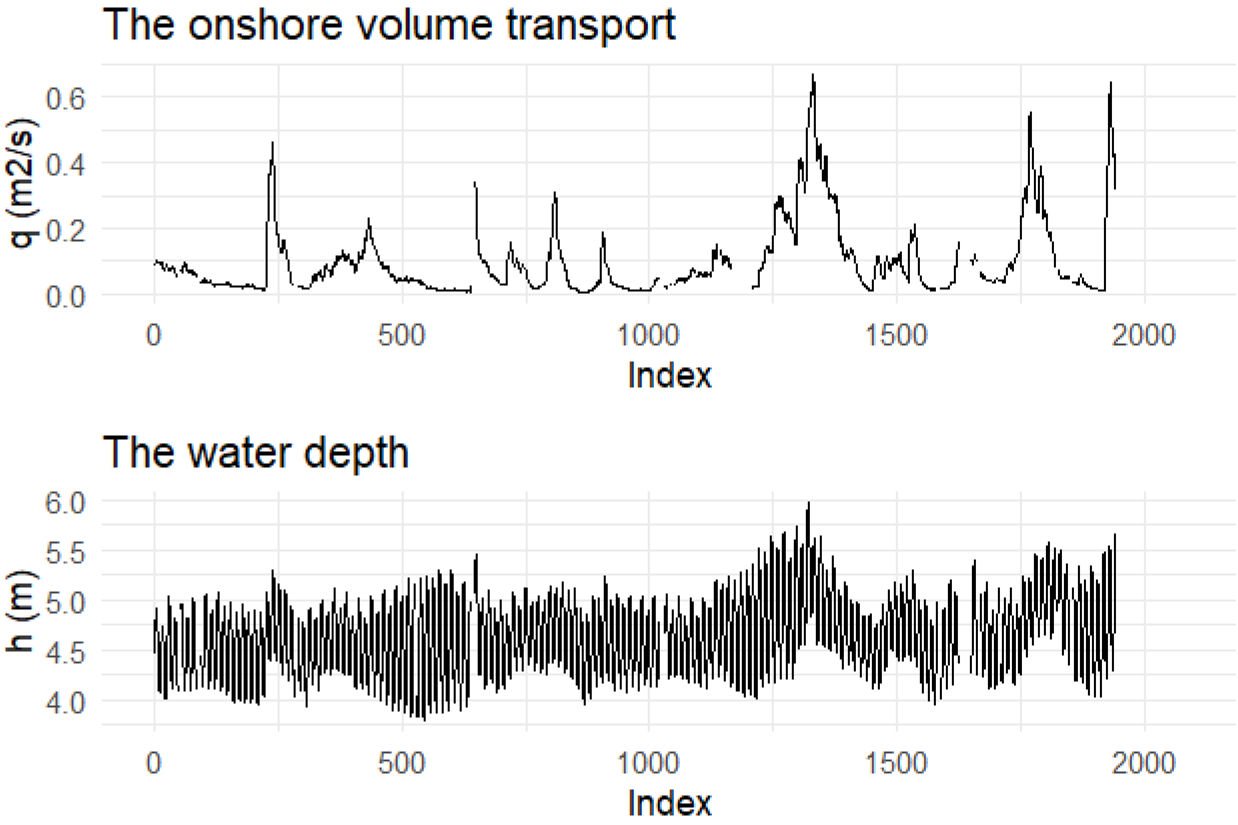
The time-averaged depth-integrated volume transport is independent of x: $\overline{\int_{-h}^{\eta} u dz} = \int_{-h}^{0} \overline{u} dz + \overline{\int_{0}^{\eta} u dz}$

Where the second term is the wave-induced Eulerian volume transport in the onshore component.

$$q_{onshore} = \overline{\int_0^\eta u dz} = \overline{u(z=0)\eta} - 0$$

$$= a\omega \frac{\cosh(k(z+h))}{\sinh(kh)} \cos(kx-\omega t) * a\cos(kx-\omega t)$$

$$\xrightarrow{z=0} \frac{1}{2} a^2 \left[\frac{\omega \cosh(kh)}{\sinh(kh)}\right] = \frac{1}{16} Hs^2 \left[\frac{\omega \cosh(kh)}{\sinh(kh)}\right]$$



- For the wave-induced Eulerian volume transport, **q**.
- Visualizing the water depth *h*, it shows that when wave increases, the volume transport q also increases.

3. Onshore volume transport and Offshore return current

3.2 The offshore return current velocity - Undertow

a) **Undertow**: In breaking waves, the mass transport towards the coast between wave crest and wave trough may be quite large, resulting in rather large seaward directed velocities under the wave trough level (see Fig. 3.1). The large return current is called undertow.

b) To calculate the depth-averaged velocity U below the trough, considering the net transport equilibrium between onshore component and undertow (return-flow) component, the absolute value of volume is same: *q_onshore = - q_return*

$$U = \frac{1}{h}\int_{-h}^{0} u dz = \frac{1}{h} * q_{return}$$

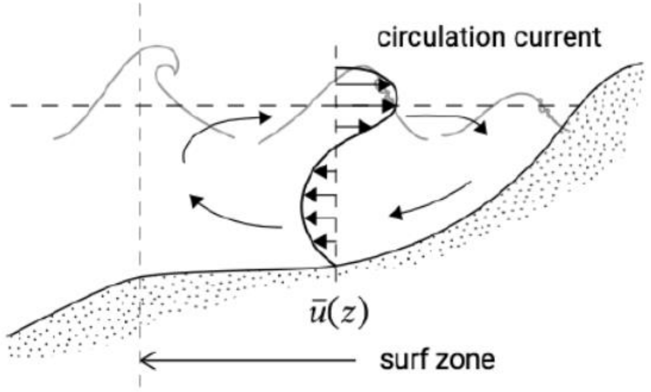
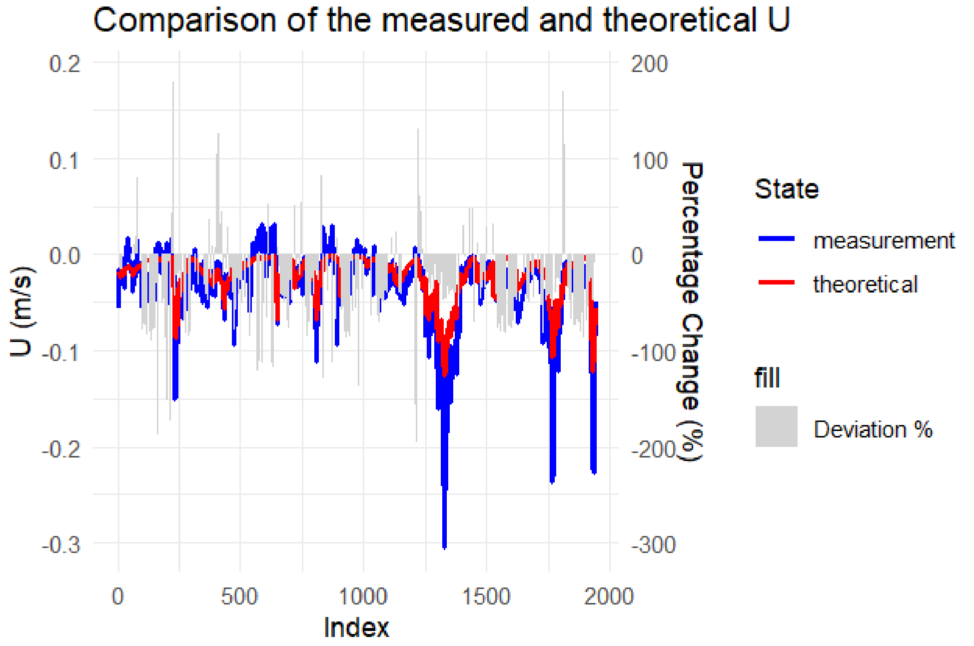
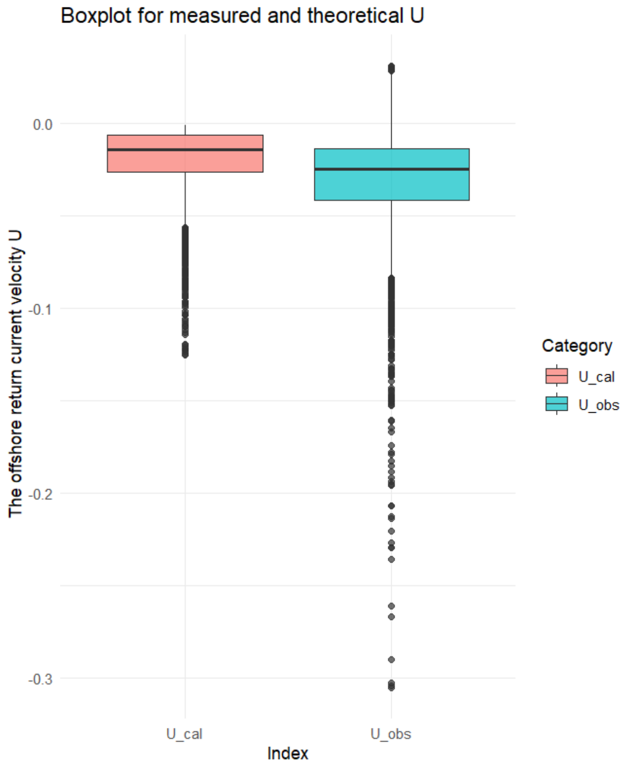


Fig 3.1. The **undertow** is a return current below the wave trough level to compensate for the onshore mass flux in the surf zone.





- The calculation can match the measurement better in the return current velocity U, with most deviation equal to 50%.