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Throughput analysis for order picking system with multiple pickers and aisle congestion considerations

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ABSTRACT

Most of previous studies in picker-to-parts warehousing systems investigated only single-picker operations and are therefore adequate to evaluate order picking efficiency by travel distance as aisle congestion never takes place in such systems. In real world applications, the congestion inevitably occurs when a system has multiple pickers working together within the same region. This paper presents an approximation method based on a $GI/G/1$ closed queueing network by using self-correcting approximation technique algorithm to evaluate the throughput time of an order picking system with multiple pickers and aisle congestion considerations for different routing policies. The results generated by the proposed method are compared and validated via simulation model using eM-plant simulator for different sizes of warehouses. The results indicate that the approximation method appears to be sufficiently accurate for practical purposes. The sensitivity analysis of the throughput time with respect to order sizes, number of pickers and number of aisles are conducted and the performance of different item storage policies are also evaluated using the proposed approximation model.

1. Introduction

Order picking has been recognized as the most labor intensive and costly activity for manual warehouse operations. According to Coyle et al. [8] and De Koster et al. [9], 50–75% of the total operating costs of a warehouse can be attributed to the order picking operation. Consequently, order picking has been a major research topic and most studies have focused mainly on the evaluation of travel time or travel distance for various operating policies of storage assignment, picking routing and order batching.

The picker-to-parts picking where a picker walks or drives along the aisles to pick items in an order is commonly implemented in practice. Tompkins [27] showed that the order picking time is mainly composed of travel time (50%) and search time (20%) in a typical picker-to-parts warehouse. Although various case studies showed that actives other than travel may substantially contribute to order picking time, travel is often regarded as the dominant component. Bartholdi and Hackman [1] indicated that travel is waste since it costs labor hours but does not add value. Travel time is, therefore, a first target for reduction in the attempt to improve the efficiency of warehouse operations.

As travel time is an increasing function of the travel distance [10,23,15,19], travel distance is often considered as a primary means of measurement in many warehouse operations studies. Thus, the development of a model to analyze accurately the travel distance is crucial before any optimal algorithm can be proposed for operating policies. In the literature, there are numerous ways to evaluate the performance of an order picking system with different operating policies. Caron et al. [5,6] presented an analytical model for traversal and return policies under the cube-per-order (COI)-based ABC curve and then developed a framework for layout design. Hwang et al. [17] developed three analytical models of expected travel distance for three routing policies in the order picking process, i.e., return, traversal and midpoint policies. The performance of the three policies was examined by varying the parameter values of the COI-based ABC curve, number of picks in the list, and ratio of the length to the width of the warehouse.

Hwang and Cho [16] presented a performance evaluation model for the order picking warehousing system in a supply center by using both the mathematical method and the simulation software AutoMod. Gray et al. [12] described and modeled in general terms the composite design and operating problems for a typical order-consolidation warehouse. Detailed simulation employing actual warehousing data was used for validation and fine tuning of the resulting design and operating policies. Petersen II [22] evaluated the impacts of warehouse shape and pick-up/drop-off locations on route performance.

For adopting waiting time as a performance measure for picker-to-parts systems, Chew and Tang [7] considered a travel time model with general item location assignment in a single block rectangular warehouse and modeled the order picking system as an $E_k/G/1$ queueing system with customer batching. The effects of batching and batch sizes on the delay time are discussed with considerations to the picking and sorting times for

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each batch of orders. Based on Chew and Tang's work [7], De Koster and Le-Duc [20] considered a 2-block warehouse and performed a direct analysis on the average throughput time of random orders. Gue et al. [14] developed both analytical and simulation models of order picking systems to investigate system behavior under different levels of activity. They found that congestion among workers can be a significant issue in batch picking areas with high space utilization. Nieuwenhuyse and De Koster [28] provided an analytical approach for approximating the expected system throughput time. They showed that an optimal batching policy which minimizes the expected customer order throughput time can be determined.

Most of previous studies on the evaluation of the throughput time considered only single picker operations and are therefore adequate to measure order picking efficiency by travel distance as congestion never takes place in such systems. However, many pickers frequently work concurrently in the same region of a real world application and aisle congestion inevitably occurs. Although queueing models have been applied to find the waiting time of a picker, those studies merely considered the warehouse as a workstation [7,9] and calculated the waiting time of a picker queueing to enter the warehouse. Since waiting time can be caused by the pickers waiting at each aisle for a multi-picker operation, the research to find both the throughput time and the waiting due to the aisle congestion for the warehouse is important. As the exact analysis of this queueing network is complicated and time-consuming [2], Neuse and Chandy [21] suggested the self-correcting approximation technique SCAT for $M/M/1$ queueing network. The SCAT has the advantages of high accuracy and low storage requirement. Bolch et al. [2] proposed ESCAT, an extension of SCAT, to analyze the $G/G/m$ closed queueing network. Eager et al. [10] further improved ESCAT to get a higher accuracy.

This paper presents an approximation method based on a $GI/G/1$ closed queueing network using ESCAT to evaluate the throughput time of a picker-to-parts order picking system with multiple pickers and aisle congestion considerations for three routing policies. In order to evaluate the performance of the proposed approach, the results generated are compared and validated via simulation approach. Moreover, the sensitivity of results with respect to various levels of input parameters, including number of pickers, number of aisles, order size, storage assignment policy and demand distribution of items, are also discussed in the paper.

2. The warehousing system and picking operations

2.1. Warehousing system

This paper evaluates the throughput time of a multi-picker picker-to-parts system as illustrated in Fig. 1. The warehouse has m aisles, all of which are assumed to have an equal number of racks. Only one item type can be stored in a rack and the number of racks is assumed to be exactly the same as the number of item types. Dummy items with no demand can be created to guarantee this relationship holds if fewer item types exist than racks. The warehouse is rectangular with the stocking aisles running perpendicular to the input point and with the cross-aisles consisting of the front aisle and the back aisle at the respective aisle ends [19,22,7,16]. Moreover, with the platform on the right side of the warehouse, a picker's tour is assumed to commence at the input point and end at the platform. Items are delivered by rider pallet trucks and then unloaded to the platform.

For information support system, a computer aided picking system (CAPS) or electronic paperless pick-to-light system can be utilized in practice in such a picker-to-parts system. The CAPS automatically guides order pickers to the pick locations and ensures that the correct items and amounts are retrieved. The advantages of the CAPS include effectively improving the picking productivity by 50% or more, and reducing the picking task error [4]. Additionally, a CAPS simplifies the training for pickers, thus cuts down the training cost.

In such a system, a picker obtains order picking data from the host computer (HC) at the input point and then the host computer sends the data to a radio frequency (RF) tag for the picker to retrieve items. RF readers are mounted at all aisle entrances and the light-modules are placed on all racks. When a picker arrives at an aisle entrance and uses the RF tags to communicate with the RF reader, the RF reader will display the aisle number that a picker needs to go in and the light-modules will specify the racks containing the items and the amounts of items to be picked in that aisle. Consequently, a picker should only enter the aisles shown on the RF reader and retrieve items from the racks with illuminating modules and so on. Infinite buffers are placed between aisles. If a picker intends to enter an aisle and finds another picker present, he/she would wait in the buffer until the aisle is clear since the whole light-modules in one aisle only display the data of one order to avoid picking errors. Moreover, the buffer in front of the input point is assumed to be infinite;

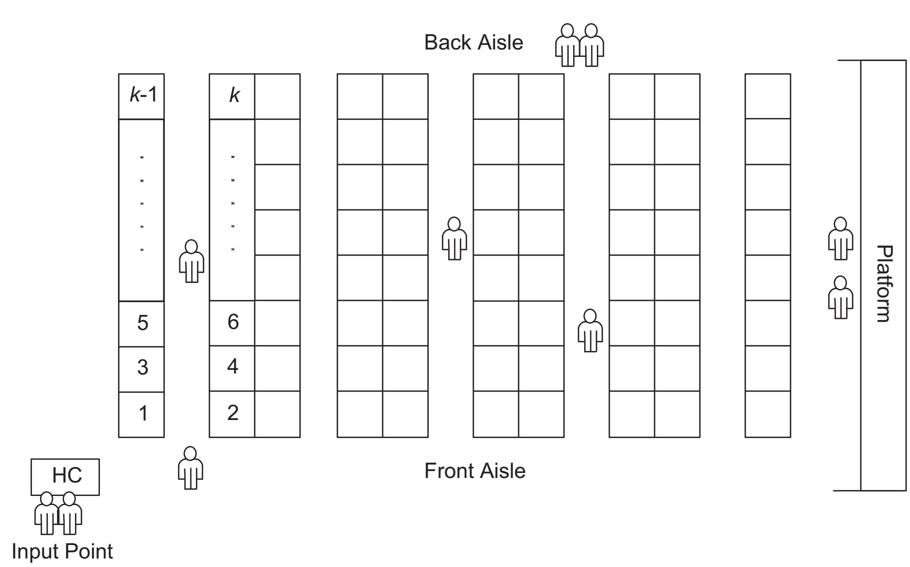

Fig. 1. An order picking system with CAPS.

hence, all pickers can wait in this buffer until their turns to enter the intended aisles.

2.2. Operation assumptions and order attributions

Three routing policies are implemented in this study. Starting from HC, a picker proceeds through the aisles to perform picking operations with the aid of CAPS and then goes back to the HC for next order. Return and traversal policies are prevalent in real world applications due to the convenience of their implementations [25,26]. These three policies are described as follows and shown in Fig. 2:

(a) *Return policy*: A picker enters aisles containing picks from the front cross-aisle only to perform the pick, and then returns to the front cross-aisle.
(b) *Traversal policy*: A picker enters an aisle containing a product in the order from one end and leaving from the opposite end.
(c) *CAPS policy*: A picker enters an aisle containing a product in the order to perform the pick and then always finds the shortest travel distance between the last picking position and the cross-aisle to leave the aisle.

Another consideration is the method by which the customer orders are assigned to order pickers. Strict order picking refers to the case where each customer order is retrieved individually by a single picker. The capacity of a picker is sufficiently large to handle all the items in a picking tour.

The following assumptions on the warehouse under study and the picking operations are made in the paper:

(1) Each item is independent of the other items within an order [7,20].
(2) All the information about orders to be picked is known in advance.
(3) The host computer is located at the front end of the most left aisle in the warehouse and is considered as aisle 0.
(4) The setup time of a picker is constant.
(5) The speed of a picker is constant.
(6) The time needed to unload items to the platform is constant.
(7) The time to pick an item from a rack is constant.
(8) All items of all orders have the same sizes and weights.
(9) No orders can be spread, i.e., strict picking order policy is implemented.

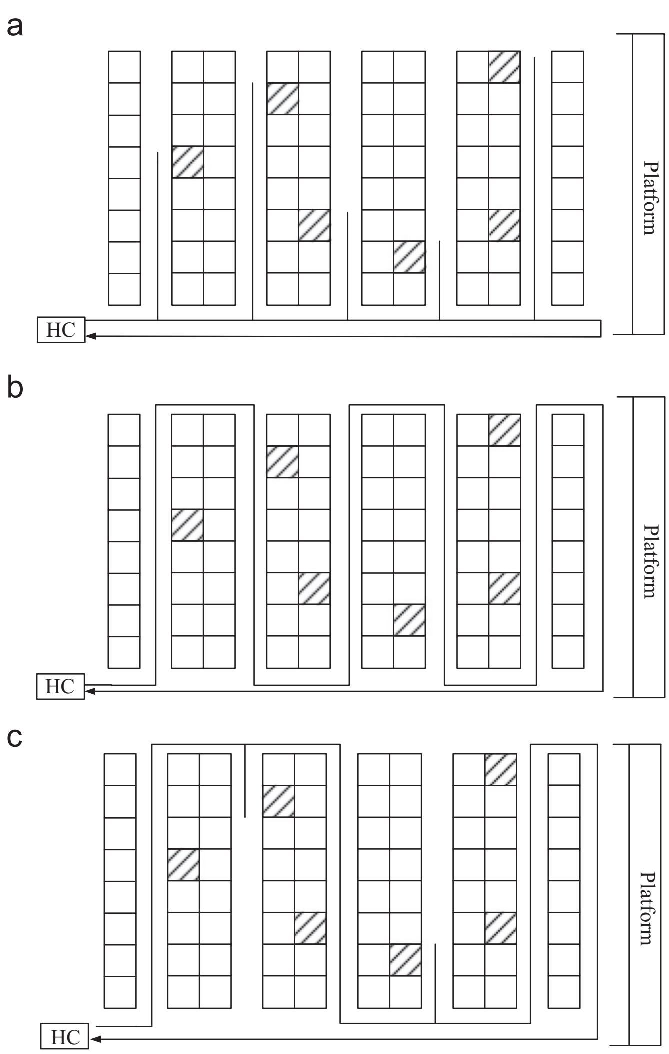

Fig. 2. Routing policies for order picking system where the shaded areas denote pick locations. (a) Return policy (b) Traversal policy (c) CAPS policy.

3. Throughput time evaluation

For the multi-picker warehousing system under investigation, the throughput time measures the duration of a picker fulfilling an order in a tour and it consists of several components which are the setup time at the HC, the unloading time to the platform, the travel time on the cross-aisles and the sum of response time in each aisle.

The following notations are used in the paper:

m	number of aisles in the order picking system;
k	number of racks in an aisle;
N	number of items to be picked in a tour;
n_i	number of items picked in aisle i;
K	number of pickers in the order picking system;
P_{ij}	probability that an item to be picked is stored at rack j of aisle i;
r_{ij}	probability that a picker goes directly to aisle j after completing the picking in aisle i;
d_i	travel distance in aisle i;
t_s	setup time per order;
t_p	picking time per item;
t_u	unloading time of items in an order;
S_i	service time, consisting of picking time and travel time, in aisle i;
T_i	response time in aisle i, including the service time and waiting time;
μ_i	expected service rate in aisle i;
σ_{si}	variance of the service time in aisle i;
c_i	coefficient of variation of the service time in aisle i;
e_i	visit ratio, or the mean number of visits of pickers to aisle i, also known as the relative arrival rate;
L_r	length of a rack;
w	length of an order picking system; i.e. the distance between HC and platform;
v	speed of a picker;
R	throughput time of an order.

Using these notations, the average throughput time for a picker can be represented by

$$R = t_s + t_u + w/v + \sum_{i=1}^{m} T_i. \tag{1}$$

Since the HC is considered as aisle 0 and the platform as aisle $(m+1)$, the average throughput time R can also be expressed as

$$R = w/v + \sum_{i=0}^{m+1} T_i, \tag{2}$$

where T_i includes the waiting time, picking time and travel time of a picker in aisle i.

3.1. The expected service time in an aisle

Many factors, such as the queueing of the pickers waiting to go into an aisle, might affect the throughput time of an order picking system. This is especially true when many pickers work concurrently in the same region. In order to capture the effect of aisle congestion, this paper adopts queueing analysis to measure the response time of each aisle. The order picking system under study can be regarded as a $GI/G/1$ closed queueing network, and HC, platform and each aisle each as a server station, respectively. The time spent by a picker in one aisle is treated as the service time pending on the contents of the picking list. First of all, it is necessary to find the mean service time of a picker in aisle i, for $i=1, 2, 3, \ldots, m$. The service time includes the picking time and the travel time; hence, the expect service time for a picker in aisle i can be represented by

$$E[S_i] = E[n_i] \times t_p + E[d_i]/v \quad \text{for } i = 1,2,\ldots,m, \tag{3}$$

where n_i is the number of items to be picked in aisle i and follows a binomial distribution as

$$f(n_i) = \binom{N}{n_i}\left(\sum_{j=1}^{k} P_{ij}\right)^{n_i}\left(1-\sum_{j=1}^{k} P_{ij}\right)^{1-n_i} \quad \text{for } n_i = 0,1,2,\ldots,N.$$

Thus, the expected number of items to be picked in aisle i is

$$E[n_i] = N \times \sum_{j=1}^{k} P_{ij} \quad \text{for } i = 1,2,\ldots,m. \tag{4}$$

And, the expected travel distance in aisle i is

$$E[d_i] = \sum_{n_i=1}^{N}\left(E[d_i|n_i] \times \binom{N}{n_i}\left(\sum_{j=1}^{k} P_{ij}\right)^{n_i}\left(1-\sum_{j=1}^{k} P_{ij}\right)^{1-n_i}\right) \quad \text{for } i = 1,2,\ldots,m. \tag{5}$$

Next, the expected travel distance for a picker in aisle i given n_i items, $E[d_i|n_i]$, for the three policies are described as follows:

(a) *Return policy*: Based on the model assumptions, an order picker has to travel twice the longitudinal distance from the front cross-aisle to the rack where the last item is picked and then returns to the front cross-aisle. It is shown in Appendix A that

$$E[d_i|n_i] = 2 \times L_r \times \left(k/2 - \sum_{a=1}^{k/2-1}\left(\sum_{b=1}^{a} P_b\right)^{n_i}\right) \quad \text{for } i = 1,2,\ldots,m. \tag{6}$$

(b) *Traversal policy*: With this policy, the expected travel distance always equals to the length of the aisle and can be represented by

$$E[d_i|n_i] = \begin{cases} L_r \times k/2 & n_i \ge 1 \\ 0 & n_i = 0 \end{cases} \quad \text{for } i = 1,2,\ldots,m. \tag{7}$$

(c) *CAPS policy*: The ways that a picker travels in an aisle can be described by the four different situations listed in Table 1 where Y_i is the random variable denoting how a picker enters and leaves aisle i. The probability distribution of random variable Y_i given n_i items is

Table 1
The random variable Y_i denoting the way a picker entering and leaving aisle i.

Y_i	Description
1	The picker enters from the front aisle and leaves at the back aisle
2	The picker enters from the front aisle and leaves at the front aisle
3	The picker enters from the back aisle and leaves at the front aisle
4	The picker enters from the back aisle and leaves at the back aisle

$$P(Y_i|n_i) = \begin{cases} \left(1-\prod_{j=k'}^{k}(1-p_{ij})^{n_i}\right)(P(Y_{i-1}=2)+P(Y_{i-1}=3)), & Y_i = 1 \\ \prod_{j=k'}^{k}(1-p_{ij})^{n_i}(P(Y_{i-1}=2)+P(Y_{i-1}=3)), & Y_i = 2 \end{cases} \quad \text{for } i = 1,2,\ldots,m, \tag{8}$$

where $k'=k/2$ when $k/2$ is odd, and $k'=k/2+1$ when $k/2$ is even.
Or,

$$P(Y_i|n_i) = \begin{cases} (1-\prod_{j=1}^{k'}(1-p_{ij})^{n_i})(P(Y_{i-1}=1)+P(Y_{i-1}=4)), & Y_i = 3 \\ \prod_{j=1}^{k'}(1-p_{ij})^{n_i}(P(Y_{i-1}=1)+P(Y_{i-1}=4)), & Y_i = 4 \end{cases} \quad \text{for } i = 1,2,\ldots,m, \tag{9}$$

where $k'=k/2+1$ when $k/2$ is odd, and $k'=k/2$ when $k/2$ is even.

Consequently, the expected travel distance given random variable Y_i and n_i items is

$$E[d_i|n_i,Y_i] = \begin{cases} L_r \times k/2, & Y_i = 1,3 \\ 2 \times L_r \times (\lceil k/4 \rceil) - \sum_{a=1}^{\lceil k/4 \rceil -1}\left(\sum_{b=1}^{a} P'_b)^{n_i}\right), & Y_i = 2 \\ 2 \times L_r \times (\lceil k/4 \rceil - \sum_{a=1}^{\lceil k/4 \rceil -1}\left(\sum_{b=0}^{a-1} P''_b\right)^{n_i} & Y_i = 4 \end{cases} \quad \text{for } i = 1,2,\ldots,m, \tag{10}$$

where $P'_b = ((P_{i(2b)}+P_{i(2b-1)})/\sum_{s=1}^{k/2} P_{is})$ and $P''_b = ((P_{i(k-2b)}+P_{i(k-2b-1)})/\sum_{s=k/2+1}^{k} P_{is})$

Finally, the expected distance in aisle i given n_i items for this policy is

$$E[d_i|n_i] = \sum_{y=1}^{4} E[d_i|n_i, Y_i = y] \times P(Y_i = y|n_i) \quad \text{for } i = 1,2,\ldots,m. \tag{11}$$

3.2. The variance and second moment of service time in an aisle

Consider a $GI/G/1$ closed queueing system with service rate μ_i, and the coefficients of variation c_i of service times. The service rate μ_i can be obtained as

$$\mu_i = 1/E[S_i] \quad \text{for } i = 1,2,3,\ldots,m. \tag{12}$$

The coefficient of variation c_i of service time is

$$c_i = \frac{\sigma_{Si}}{E[S_i]} \quad \text{for } i = 1,2,3,\ldots,m. \tag{13}$$

The variance of service time can be calculated by

$$\sigma_{si}^2 = E[S_i^2] - E[S_i]^2 \quad \text{for } i = 1,2,\ldots,m. \tag{14}$$

Hence, the second moment of service times can be expressed as

$$E[S_i^2] = E[n_i^2] \times t_p^2 + 2 \times E[n_i] \times t_p \times E[d_i]/v + E[d_i^2]/v^2 \quad \text{for } i = 1,2,\ldots,m, \tag{15}$$

where the second moment of the number of items to be picked in aisle i is

$$E[n_i^2]=\left(N\sum_{j=1}^{k}P_{ij}\right)^2-N\left(\sum_{j=1}^{k}P_{ij}\right)^2+N\left(\sum_{j=1}^{k}P_{ij}\right)\quad \text{for } i=1,2,\ldots,m. \tag{16}$$

Table 2
Related data of the warehouse under experiment.

Parameter	Specification
Number of aisles (m)	10, 15, 20
Number of racks in an aisle (k)	20
Length of a rack (L_r)	2 m
Length of an order picking system (w)	40, 60, 80 m
Speed of picker (v)	1 m/s
Unloading time of items (t_u)	5 s
Setup time per order (t_s)	5 s
Picking time per item (t_p)	5 s

And, the second moment of travel distance is

$$E[d_i^2]=\sum_{n_i=1}^{N}\left(E[d_i^2|n_i]\times\binom{N}{n_i}\left(\sum_{j=1}^{k}P_{ij}\right)^{n_i}\left(1-\sum_{j=1}^{k}P_{ij}\right)^{1-n_i}\right)$$
$$\text{for } i=1,2,\ldots,m. \tag{17}$$

Similarly, the second moment of travel distance for a picker in aisle i given n_i items can be developed for the three policies as follows:

(a) *Return policy*

$$E[d_i^2|n_i]=(2\times L_r)^2\times\left((k/2)^2-\sum_{a=1}^{k/2-1}(2a+1)\left(\sum_{b=1}^{a}P_b\right)^{n_i}\right)$$
$$\text{for } i=1,2,\ldots,m. \tag{18}$$

Table 3
Percentage differences between the proposed model and simulation for routing policy with CAPS.

No. of aisles (m)	Order Size(N)	No of pickers (K)	Simulation model			Proposed model	
			Mean throughput time (s)	Std. dev.	Confidence interval width (at 1%)	Mean throughput time (s)	Percentage difference
10	15	6	279.70	2.36	1.3550	270.81	3.18
		7	287.10	2.51	1.4440	281.36	2.00
		8	294.70	2.75	1.5796	295.09	0.13
		9	308.60	3.47	1.9930	309.61	0.33
		10	322.10	5.09	2.9212	325.90	1.18
	20	6	321.70	2.58	1.4840	314.02	2.39
		7	329.50	2.32	1.3330	326.37	0.95
		8	340.20	3.49	2.0040	340.64	0.13
		9	352.70	3.80	2.1834	355.59	0.82
		10	368.00	3.46	1.9894	373.19	1.41
	25	6	358.00	1.15	0.6632	353.66	1.21
		7	365.00	2.49	1.4324	366.59	0.44
		8	378.20	4.16	2.3878	382.01	1.01
		9	392.50	4.14	2.3794	398.06	1.42
		10	413.40	3.57	2.0474	415.54	0.52
15	20	6	374.70	3.92	2.2494	361.13	3.76
		7	379.80	4.52	2.5938	372.72	1.90
		8	388.90	5.40	3.1038	385.05	1.00
		9	394.80	5.29	3.0362	398.44	0.91
		10	404.00	4.29	2.4662	412.39	2.03
	25	6	420.60	3.37	1.9370	408.25	3.02
		7	427.90	3.18	1.8250	421.06	1.62
		8	437.80	2.53	1.4528	434.43	0.78
		9	444.50	4.45	2.5574	448.81	0.96
		10	455.80	3.49	2.0040	463.10	1.58
	30	6	462.40	2.63	1.5122	452.29	2.23
		7	471.00	2.00	1.1486	465.32	1.22
		8	478.60	3.47	1.9930	479.07	0.10
		9	487.30	3.20	1.8370	495.03	1.56
		10	496.30	3.40	1.9530	511.74	3.02
20	25	6	477.60	3.66	2.1004	453.02	5.43
		7	483.30	6.07	3.4884	463.44	4.28
		8	488.60	4.45	2.5568	474.77	2.91
		9	497.80	4.05	2.3256	486.58	2.30
		10	503.90	4.79	2.7534	498.82	1.02
	30	6	526.20	2.90	1.6644	500.88	5.06
		7	532.90	3.96	2.2722	512.26	4.03
		8	538.00	4.27	2.4514	523.80	2.71
		9	546.60	4.38	2.5134	536.45	1.89
		10	554.60	5.78	3.3178	549.84	0.86
	35	6	570.50	5.30	3.0418	545.38	4.61
		7	578.50	4.74	2.7240	557.31	3.80
		8	583.80	5.69	3.2688	570.02	2.42
		9	589.80	4.59	2.6358	583.52	1.08
		10	597.80	3.68	2.1108	596.84	0.16

Table 4
Percentage differences between the proposed model and simulation for return policy.

No. of aisles (m)	Order size(N)	No. of pickers (K)	Simulation model			Proposed model	
			Mean throughput time (s)	Std. dev.	Confidence interval width (at 1%)	Mean throughput time (s)	Percentage difference
10	15	6	393.60	5.15	2.9556	401.67	2.05
		7	407.10	4.18	2.3978	417.29	2.50
		8	423.50	5.44	3.1250	434.42	2.58
		9	440.20	4.21	2.4182	448.34	1.85
		10	463.80	5.14	2.9506	468.15	0.94
	20	6	452.90	5.00	2.8706	463.14	2.26
		7	466.50	6.24	3.5838	480.08	2.91
		8	483.20	5.92	3.4006	498.16	3.10
		9	502.70	4.60	2.6392	519.25	3.29
		10	524.70	5.19	2.9784	523.79	0.17
	25	6	496.60	3.60	2.0652	509.42	2.58
		7	511.50	4.17	2.3946	525.88	2.81
		8	528.60	4.60	2.6414	544.19	2.95
		9	549.00	7.12	4.0876	563.62	2.66
		10	577.80	6.07	3.4858	573.24	0.79
15	20	6	487.10	6.82	3.9178	504.39	3.43
		7	498.10	4.98	2.8578	516.27	3.52
		8	507.60	7.28	4.1780	531.09	4.42
		9	523.70	7.35	4.2204	546.26	4.13
		10	539.40	4.58	2.6274	563.30	4.24
	25	6	549.30	2.75	1.5796	560.19	1.94
		7	560.50	6.22	3.5734	571.00	1.84
		8	576.50	7.01	4.0266	583.63	1.22
		9	587.20	6.76	3.8836	596.76	1.60
		10	603.50	5.52	3.1714	612.04	1.40
	30	6	600.40	3.69	2.1178	610.09	1.59
		7	612.90	5.69	3.2648	621.16	1.33
		8	627.70	7.01	4.0248	630.23	0.40
		9	638.70	6.22	3.5714	643.80	0.79
		10	655.00	4.59	2.6386	656.32	0.20
20	25	6	619.60	6.67	3.8304	639.23	3.07
		7	629.30	7.36	4.2290	647.23	2.77
		8	644.10	5.28	3.0320	655.01	1.67
		9	655.10	6.44	3.6964	665.99	1.64
		10	663.00	6.88	3.9508	676.59	2.01
	30	6	682.60	7.46	4.2820	694.54	1.72
		7	693.40	8.17	4.6904	699.57	0.88
		8	708.10	6.47	3.7162	704.83	0.46
		9	716.20	6.16	3.5380	710.41	0.81
		10	727.10	7.05	4.0466	716.92	1.42
	35	6	743.50	9.45	5.4294	745.17	0.22
		7	752.20	4.69	2.6908	746.88	0.71
		8	767.30	6.86	3.9420	748.83	2.47
		9	774.50	5.38	3.0896	750.35	3.22
		10	784.40	4.97	2.8546	753.16	4.15

(b) *Traversal policy*

$$E\left[d_i^2 \middle| n_i\right] = \begin{cases} (L_r \times k/2)^2 & n_i \geq 1 \\ 0 & n_i = 0 \end{cases} \quad \text{for } i = 1,2,\ldots,m. \tag{19}$$

(c) *CAPS policy*

$$E[d_i^2 | n_i, Y_i] = \begin{cases} (L_r \times k/2)^2, & Y_i = 1,3 \\ (2 \times L_r)^2 \times \left(\lceil k/4 \rceil^2 - \sum_{a=1}^{\lceil k/4 \rceil - 1} (2a+1) \left(\sum_{b=1}^{a} P'_b \right)^{n_i} \right), & Y_i = 2 \\ (2 \times L_r)^2 \times \left(\lceil k/4 \rceil^2 - \sum_{a=1}^{\lceil k/4 \rceil - 1} (2a+1) \left(\sum_{b=0}^{a-1} P''_b \right)^{n_i} \right), & Y_i = 4 \end{cases}$$

$$\text{for } i = 1,2,\ldots,m. \tag{20}$$

Then, the second moment of travel distance given n_i items can be found by

$$E[d_i^2 | n_i] = \sum_{y=1}^{4} E[d_i^2 | n_i, Y_i = y] \times P(Y_i = y | n_i). \tag{21}$$

3.3. A closed queueing network approximation

Another important queueing network parameter is the relative arrival rate e_i. For a closed network, it follows that:

$$e_i = \sum_{j=1}^{m} e_j r_{ji}, \quad \text{for } i = 1,2,\ldots,m+1. \tag{22}$$

Since there are only m independent equations for the visit ratios in the closed network, these e_i's can only be determined up to a multiplicative constant. Usually it is assumed that $e_0 = 1$, although other possibilities can be used as well. The probability r_{ij} that a picker goes to aisle j directly from aisle i can be discussed

Table 5
Percentage differences between the proposed model and simulation for traversal policy.

No. of aisles (m)	Order Size(N)	No. of pickers (K)	Simulation model			Proposed model	
			Mean throughput time (s)	Std. dev.	Confidence interval width (at 1%)	Mean throughput time (s)	Percentage difference
10	15	6	295.80	2.57	1.4778	311.30	5.24
		7	304.60	2.07	1.1862	321.85	5.66
		8	312.00	2.00	1.1486	332.99	6.73
		9	323.40	1.51	0.8646	344.48	6.52
		10	336.00	3.86	2.2158	350.28	4.25
	20	6	337.90	2.64	1.5182	351.56	4.04
		7	345.40	2.32	1.3318	362.06	4.82
		8	354.10	1.85	1.0640	373.37	5.44
		9	365.90	3.51	2.0158	384.27	5.02
		10	381.10	3.63	2.0872	393.86	3.35
	25	6	372.40	2.37	1.3590	387.18	3.97
		7	379.20	2.53	1.4528	398.06	4.97
		8	389.10	2.81	1.6118	409.73	5.30
		9	401.20	4.39	2.5222	421.47	5.05
		10	419.50	4.38	2.5142	422.15	0.63
15	20	6	403.00	2.54	1.4578	410.59	1.85
		7	410.10	2.81	1.6118	417.03	1.66
		8	416.80	3.36	1.9294	425.81	2.12
		9	424.20	2.66	1.5266	434.74	2.43
		10	432.20	1.69	0.9686	443.43	2.53
	25	6	448.00	3.30	1.8950	452.32	0.95
		7	453.60	3.06	1.7586	459.47	1.28
		8	461.30	4.85	2.7878	465.31	0.86
		9	470.30	4.30	2.4670	473.30	0.63
		10	479.10	4.01	2.3042	481.33	0.46
	30	6	486.20	2.78	1.5970	489.90	0.76
		7	493.80	2.86	1.6422	496.34	0.51
		8	500.90	4.23	2.4280	501.14	0.05
		9	507.90	4.12	2.3670	509.14	0.24
		10	516.60	4.20	2.4092	516.03	0.11
20	25	6	514.00	4.85	2.7872	523.98	1.91
		7	517.90	4.09	2.3514	531.29	2.52
		8	525.40	4.33	2.4840	539.30	2.58
		9	531.30	4.85	2.7878	547.98	3.04
		10	536.10	3.78	2.1732	554.38	3.30
	30	6	558.50	3.78	2.1700	569.26	1.89
		7	566.20	3.91	2.2454	575.81	1.67
		8	573.10	4.53	2.6028	582.10	1.55
		9	577.60	4.74	2.7232	590.01	2.10
		10	584.10	4.15	2.3824	598.09	2.34
	35	6	603.40	4.01	2.3002	610.26	1.12
		7	607.70	6.43	3.6924	616.15	1.37
		8	614.60	4.25	2.4394	622.98	1.35
		9	622.20	3.97	2.2778	629.15	1.10
		10	628.80	4.76	2.7314	637.11	1.30

according to the following three situations based on the procedure of the order picking operation:

(1) If a picker sets out to pick an order from HC, the probability that he/she directly goes to aisle i can be found by

$$r_{0i} = P(a_1=0, a_2=0, \ldots, a_{i-1}=0, a_i>0)$$

$$= P(a_1=0, a_2=0, \ldots, a_{i-1}=0) - P(a_1=0, a_2=0, \ldots, a_i=0)$$

$$= \left(\sum_{a=i}^{m}\sum_{b=1}^{k} P_{ab}\right)^N - \left(\sum_{a=i+1}^{m}\sum_{b=1}^{k} P_{ab}\right)^N, \tag{23}$$

where $\sum_{j=0}^{m} r_{0i} = 1$ and a_i is the number of items picked in aisle i.

(2) The probability that a picker travels next to aisle j from aisle i can be derived by

$$\begin{aligned} r_{ij} &= P(a_{i+1}=0, a_{i+2}=0, \ldots, a_j>0 \mid a_i>0) \\ &= P(a_{i+1}=0, a_{i+2}=0, \ldots, a_{j-1}=0 \mid a_i>0) - P(a_{i+1}=0, a_{i+2}=0, \ldots, a_j=0 \mid a_i>0) \\ &= \frac{(1-\sum_{a=i+1}^{j-1}\sum_{b=1}^{k} P_{ab})^N - (1-\sum_{a=i}^{j-1}\sum_{b=1}^{k} P_{ab})^N - (1-\sum_{a=i+1}^{j}\sum_{a=i+1}^{k} (1-\sum_{a=i}^{j}\sum_{b=1}^{k} P_{ab})^N}{1-(1-\sum_{b=1}^{k} P_{ib})^N}, \end{aligned} \tag{24}$$

where $\sum_{j=0}^{m} r_{ij} = 1$, $i = 1, 2, 3, \ldots, m$, $j = i+1, i+2, \ldots, m+1$ and $r_{i(m+1)} = 1 - \sum_{j=1}^{m} r_{ij}$

(3) When a picker enters the furthest aisle and finishes all the pickings, he/she must go to the platform and then return to

the HC. Thus, the probability $r_{(m+1),0}$ equals to one, or,

$$r_{(m+1),0} = 1. \tag{25}$$

Several algorithms for the exact solution of closed queueing network have been proposed [18,13,3,2]. However, the memory requirements and computation times of these algorithms grow exponentially with the number of pickers in the system. Therefore, a method that obtains approximate results for such a problem needs to be developed. Since ESCAT is derived to solve the $G/G/m$ close queueing network, it can also be used to find the average response time of a picker for each aisle for the system under study.

This paper derives the first and second moments of the order-picker's service time for the return, traversal and CAPS routing polices, respectively, and evaluates the probability models that a picker travels a certain aisle. The moments and models can be induced into ESCAT to obtain the average throughput time. According to the aforementioned approach, the algorithm of evaluation performance can be expressed as

Step 1: Use Eqs. (3)–(11) to calculate the expected service time for each aisle.
Step 2: Use Eqs. (12)–(21) to calculate the coefficient of variation of the service times for each aisle.
Step 3: Determine the probability r_{ij} using Eqs. (23)–(25).
Step 4: Solve Eq. (22) to obtain e_i.
Step 5: Introduce the parameters obtained from Step 1 to Step 3 into the ESCAT algorithm for queueing network provided by Bolch et al. [2] and calculate the average response time in aisle i, $i=0, 1, \ldots, m+1$.
Step 6: Estimate the average throughput time by Eq. (2).

4. Model validation and sensitivity analysis

4.1. Warehouse configurations

In order to validate the accuracy of the provided model, this paper consider three warehouse instances of 10, 15 and 25 aisles, respectively, with other parameters given in Table 2. Moreover, the demand for the items is based on an 80/20 distribution; that is, 20% of the items accounts for 80% of the picking activities. The travel rate of one meter per second and picking time per item of 5 s of a picker are assumed to be constant. The picking time includes all item handling and administration activities.

4.2. Validation with simulation

A simulation model of the order picking system based on the derived analytical algorithm is implemented in eM-plant [11]. The simulation software eM-plant has been used in various environments, such as manufacturing, warehousing, material handling processes and semiconductor manufacturing, and has proven to be a highly effective simulation program [11].

The simulation model calculates the throughput time of each randomly generated picking location according to the probabilities that a picker travels to racks. The model was run 100 times with 300 orders for each order size and number of pickers. The mean throughput time and standard deviation for each picker are recorded, and the width of the confidence interval for the travel distance was assessed at a significance level of 1%.

The percentage differences between the values predicted analytically and the simulation results for the three policies are displayed in Table 3–5, where the percentage difference is expressed as (the throughput time calculated by the proposed algorithm − the throughput time generated by simulation)/(the throughput time calculated by simulation) × 100%. These three tables indicate that the proposed analytical model provides a relatively good approximation of the throughput time for the order picking system under study since except for very few cases, the percentage differences in general are less than 5% for the three routing policies. Despite the large number of simulation runs, the width of the confidence interval stays within 0.6632 to 5.4294 of the mean throughput time at a significance level of 1%.

4.3. Sensitivity analysis

4.3.1. Order sizes and the numbers of pickers and aisles

A warehouse which has 15 picking aisles is implemented for cases with various order sizes, and different numbers of pickers and aisles. The computational results for different order sizes with $K=8$ from the analytical model are shown in Fig. 3. As the order size increases, the throughput time of an order also increases for the three routing policies as shown in Fig. 3. This outcome is predictable since the waiting time of pickers in the aisle rises as the density of items picked from each aisle increases. Fig. 4 indicates that the throughput time of an order increases with

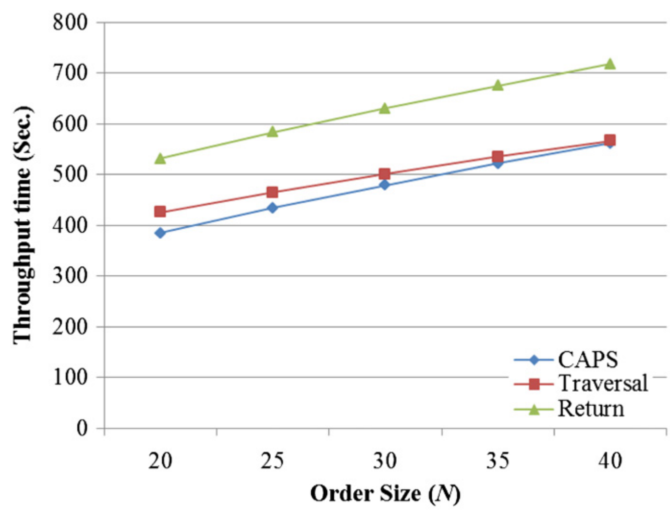

Fig. 3. The throughput time for the three routing policies on different order sizes with $m=15$ and $K=8$.

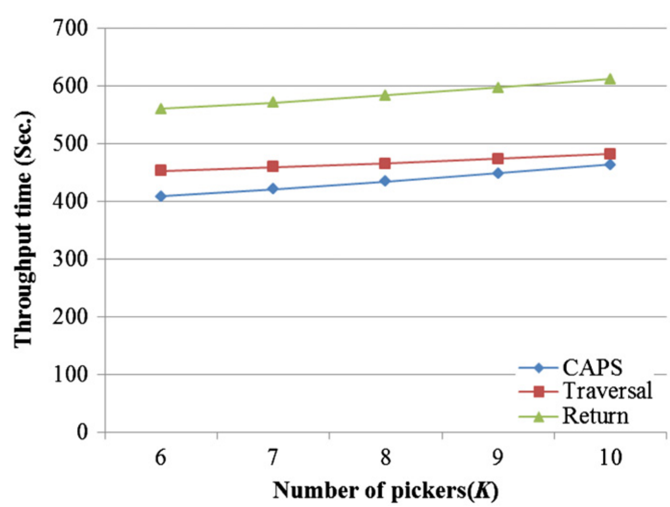

Fig. 4. The throughput time for the three routing policies on different number of pickers with $m=15$ and $N=25$.

the escalation of the number of pickers for a case with $N=25$. As the level of labor is raised in warehouse, the probability of a picker waiting to go into an aisle is thus increased. Fig. 5 shows the performance evaluation for different number of aisles under $N=25$ and $K=8$. The return policy is the worst of the three routing strategies tested. But the performance of the traversal policy with $m=20$ is closer to that of return policy because the traversal policy is sensitive to the number of aisles. However, the CAPS is obviously better than the other two.

4.3.2. Storage assignment policy

This study further examines the impact of the following four storage assignment policies based on the picking frequency of items [24] and these policies are described below and illustrated in Fig. 6.

(a) *Random policy*: The random storage policy is widely used in many warehouses because it is simple to use, often requires less space than other storage methods, and results in a better level utilization of all picking aisles [24].
(b) *Within-aisle policy*: The highest frequency item is stocked in the first storage location of the first aisle and the second highest one is stocked in the second storage location of the first aisle and so on. After the first aisle is filled, the next highest frequency item is stocked in the first location of the second aisle and so on.
(c) *Across-aisle policy*: The highest frequency item is assigned to the first location of the first aisle. The next highest frequency one is assigned to the first location of the second aisle and so on. Once the first locations of all the aisles are assigned, the second location of each aisle is then assigned an item. That is, the area that is close to the front aisle contains the high frequency items and the area close to the back aisle contains the low frequency items.
(d) *Diagonal policy*: Diagonal storage involves having the items stored in the warehouse in a diagonal pattern, with the highest frequency item locating closest to HC.

Fig. 7 indicates that the across-aisle is clearly the best one among the four storage policies. Since the across-aisle policy distributes all items into each aisle equally, the workload of all aisles under it is more balanced than the other three and the blocking effect is thus reduced.

Table 6 lists the expected queue length (excluding the picker being picking) and the utilization rate of each aisle for CAPS routing policies. Under across-aisle and random policies, the expected queue lengths of each aisle are not more than one and the utilization of each aisle is very low because of balanced workload. There is no congestion occurred in the warehouse. On the other hand, the expected queue lengths of aisle 2 and aisle 3 denote that pickers are blocked in several aisles with high service level (high picking frequency) for within-aisle policy and diagonal policy.

The proposed model can also be used to estimate the maximum capacity of an order picking system. This paper investigates the marginal benefit of picking time under the CAPS policy. Fig. 8 presents the throughput time per picking item for $N=25$. As the number of order pickers goes up, the throughput time decreases for all the storage policies examined. However, the throughput time line is flat when the number of picker reaches a certain level. This means that a maximum capacity exists. When the number of

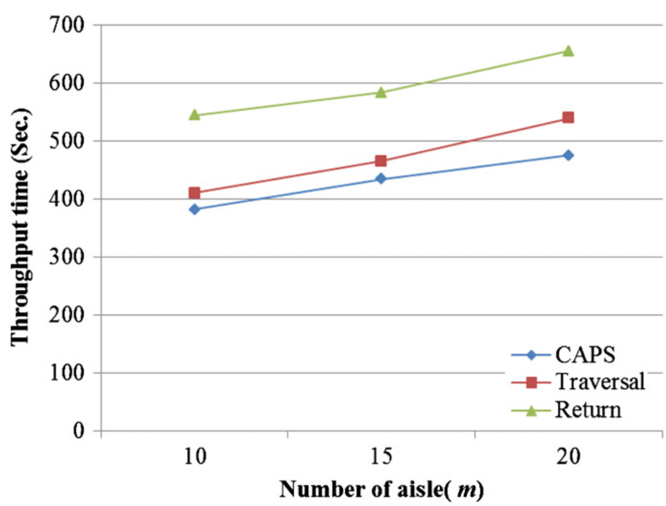

Fig. 5. The throughput time for the three routing policies on different number of aisles with $N=25$ and $K=8$.

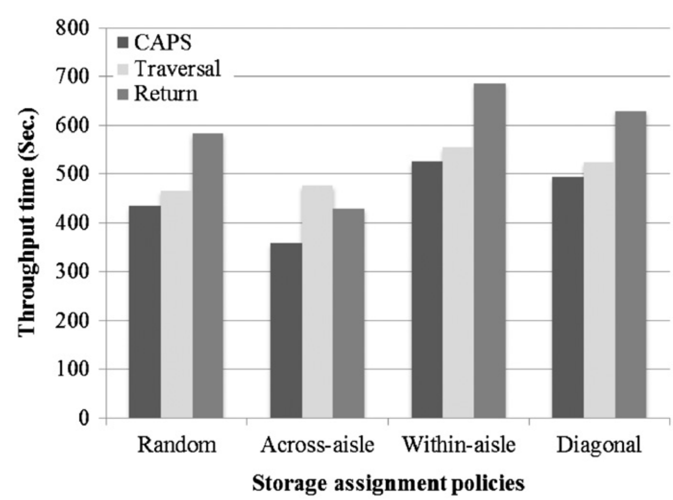

Fig. 7. The throughput time of the four storage assignment policies with $N=25$, $K=8$ and $m=15$.

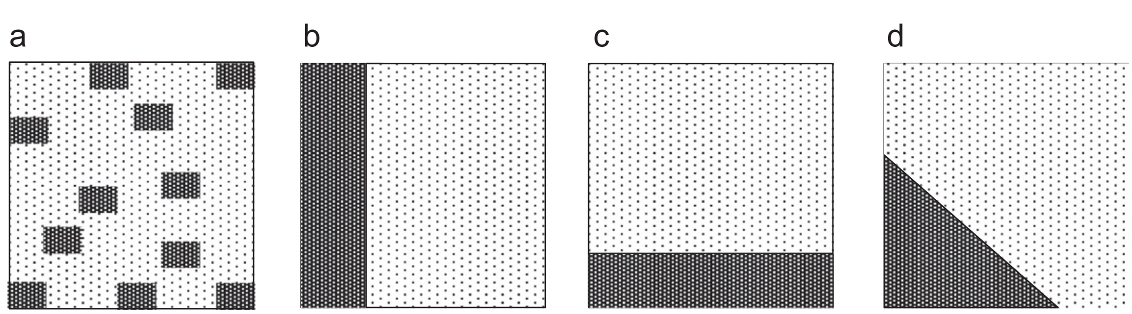

Fig. 6. The four storage assignment policies under experiment. (a) Random. (b) Within-aisle. (c) Across-aisle. (d) Diagonal.

Table 6
Queue lengths and utilizations (in brackets) of each aisle under CAPS policy with $N=25$.

Storage policy	No. of pickers	Aisle 1	Aisle 2	Aisle 3	Aisle 4	Aisle 5	Aisle 6	Aisle 7	Aisle 8	Aisle 9	Aisle 10	Aisle 11	Aisle 12	Aisle 13	Aisle 14	Aisle 15
Random	6	0.004 (0.096)	0.047 (0.261)	0.093 (0.367)	0.078 (0.340)	0.010 (0.115)	0.134 (0.436)	0.016 (0.147)	0.109 (0.392)	0.045 (0.255)	0.039 (0.238)	0.098 (0.375)	0.019 (0.162)	0.162 (0.475)	0.043 (0.254)	0.197 (0.522)
	7	0.006 (0.108)	0.063 (0.291)	0.128 (0.410)	0.107 (0.380)	0.013 (0.128)	0.187 (0.486)	0.021 (0.164)	0.151 (0.438)	0.061 (0.285)	0.052 (0.265)	0.134 (0.418)	0.025 (0.181)	0.228 (0.530)	0.058 (0.283)	0.280 (0.583)
	8	0.007 (0.118)	0.080 (0.318)	0.167 (0.448)	0.139 (0.415)	0.017 (0.140)	0.249 (0.531)	0.026 (0.179)	0.198 (0.478)	0.077 (0.311)	0.066 (0.290)	0.175 (0.457)	0.031 (0.197)	0.305 (0.579)	0.073 (0.309)	0.377 (0.637)
	9	0.008 (0.126)	0.098 (0.342)	0.209 (0.482)	0.173 (0.446)	0.020 (0.151)	0.318 (0.571)	0.031 (0.192)	0.250 (0.514)	0.094 (0.335)	0.081 (0.312)	0.220 (0.491)	0.037 (0.212)	0.393 (0.623)	0.089 (0.333)	0.491 (0.685)
Within-aisle	6	0.003 (0.082)	0.934 (0.860)	0.934 (0.860)	0.934 (0.860)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)	0.002 (0.036)
	7	0.004 (0.084)	1.233 (0.887)	1.233 (0.887)	1.233 (0.887)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)	0.002 (0.037)
	8	0.004 (0.086)	1.542 (0.906)	1.542 (0.906)	1.542 (0.906)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)
	9	0.004 (0.087)	1.859 (0.920)	1.859 (0.920)	1.859 (0.920)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)	0.002 (0.039)
Across-aisle	6	0.007 (0.122)	0.066 (0.306)	0.067 (0.308)	0.068 (0.310)	0.068 (0.311)	0.069 (0.312)	0.069 (0.313)	0.070 (0.314)	0.070 (0.315)	0.070 (0.315)	0.071 (0.316)	0.071 (0.316)	0.071 (0.317)	0.071 (0.317)	0.072 (0.317)
	7	0.009 (0.137)	0.090 (0.344)	0.091 (0.346)	0.093 (0.348)	0.093 (0.349)	0.094 (0.351)	0.095 (0.352)	0.096 (0.353)	0.096 (0.354)	0.097 (0.354)	0.097 (0.355)	0.097 (0.355)	0.097 (0.356)	0.098 (0.356)	0.098 (0.357)
	8	0.012 (0.151)	0.117 (0.379)	0.119 (0.381)	0.121 (0.383)	0.122 (0.385)	0.123 (0.386)	0.124 (0.387)	0.125 (0.388)	0.125 (0.389)	0.126 (0.390)	0.127 (0.391)	0.127 (0.391)	0.127 (0.392)	0.128 (0.392)	0.128 (0.392)
	9	0.014 (0.164)	0.147 (0.410)	0.150 (0.413)	0.152 (0.415)	0.153 (0.417)	0.155 (0.418)	0.156 (0.420)	0.157 (0.421)	0.158 (0.422)	0.159 (0.423)	0.160 (0.423)	0.160 (0.424)	0.161 (0.424)	0.161 (0.425)	0.162 (0.425)
Diagonal	6	0.004 (0.086)	0.945 (0.905)	0.924 (0.877)	0.389 (0.630)	0.138 (0.415)	0.024 (0.179)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)	0.002 (0.038)
	7	0.004 (0.089)	1.314 (0.942)	1.286 (0.913)	0.485 (0.655)	0.161 (0.432)	0.027 (0.186)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)	0.002 (0.040)
	8	0.004 (0.092)	1.721 (0.965)	1.686 (0.935)	0.566 (0.671)	0.179 (0.443)	0.029 (0.191)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)
	9	0.005 (0.093)	2.165 (0.980)	2.110 (0.949)	0.627 (0.681)	0.191 (0.449)	0.031 (0.194)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)	0.002 (0.041)

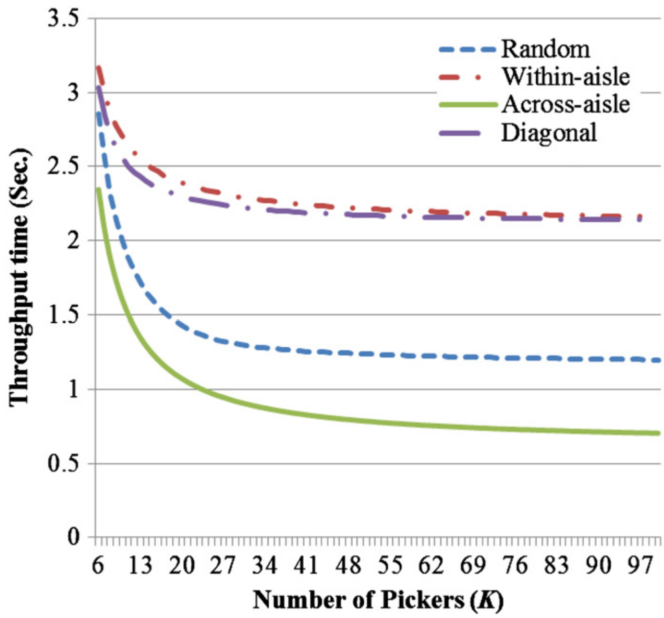

Fig. 8. Average throughput time per item under CAPS policy with $m=15$.

pickers exceeds the maximum capacity of the system, pickers are always queued to enter an aisle for picking items. For example, the maximum capacity of warehousing system under experiment is 30 for the across-aisle policy since its throughput time appears to remain the same as the number of pickers approaches 30. Fig. 8 also reveals that the maximum capacity of the across-aisle policy under CAPS is the largest and its throughput time per item is the lowest among all the polices tested. This implies that by improving the balance of workload among all aisles, the capacity of an order picking system can be improved.

5. Conclusions

This paper proposes an analytical approach for the throughput time of a picker-to-parts system with multiple pickers. In some order picking systems, narrow aisles prohibit pickers from passing one another when in the same aisle, and this leads to congestion [14]. Since the effect of aisle congestion is rather difficult to capture [20], the order picking system under study is regarded as a closed queueing network and an approximation method is derived to calculate the queue length of each aisle and an algorithm is developed to find the throughput time of the system. The proposed model can provide quantifiable estimates on the performance of an order picking system in relation to the service time of an order picker. The results of the simulation experiment conducted validate the accuracy of the proposed analytical algorithm. Moreover, the sensitive analysis of various order sizes and the number of pickers indicates that the level of congestion is higher with larger order sizes and number of pickers. Most of previous studies on storage assignment policies considered only single-picker operations and are therefore adequate to measure order picking efficiency by travel distance. In order to reduce the travel distance, the items with higher demand are assigned to the locations closer to the input/output (I/O) point [19], such as the within-aisle policy [24]. However, the concentration of high demand items in certain aisles will inevitably cause congestions for the multiple-picker environment studied in this paper and may significantly affect the efficiency of the picking process. This study shows that the across-aisle storage policy which balances the workloads among all pickers can reduce the level of congestion and is found to be superior to the random, within-aisle and diagonal policies tested.

Finally, the results of the numerical example indicate that for the cases of medium size ($m=15$) and large size ($m=20$) problems where the utilizations of most aisles are low, the accuracy of the proposed method are very high. Hence, the proposed algorithm may be more applicable to warehouses of medium and large sizes in practice.

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Appendix A

The expected travel distance in aisle i given n_i items for return policy, $E[d_i|n_i]$, can be expressed as

$$E[d_i|n_i]=2\times L_r\times E\,[\text{the furthest picking rack in aisle } i \text{ given } n_i \text{ item}]. \tag{A.1}$$

Let Y denote the farthest rack in an aisle, and define

$$X_j=\begin{cases}1, & \text{if picking rack } (2j-1) \text{ or rack } 2j \text{ is visited} \quad \text{for } j=1,2,\ldots,k/2\\ 0, & \text{otherwise.}\end{cases}$$

The term E [the furthest picking rack in aisle i given n_i item] can be written as

$$\begin{aligned}\sum_{y=1}^{k/2} yP\{Y=y|n_i\} &= \sum_{y=1}^{k/2} yP\{X_1\geq 0,X_2\geq 0,\ldots,X_{y-1}\geq 0,\\ &\quad X_y>0,X_{y+1}=0,\ldots,X_{(k/2)}=0|n_i\}\\ &=\sum_{y=1}^{k/2} y(P\{X_1\geq 0,X_2\geq 0,\ldots,X_{y-1}\geq 0,\\ &\quad X_y\geq 0,X_{y+1}=0,\ldots,X_{(k/2)}=0|n_i\}\\ &\quad -P\{X_1\geq 0,X_2\geq 0,\ldots,X_{y-1}\geq 0,X_y=0,\\ &\quad X_{y+1}=0,\ldots,X_{(k/2)}=0|n_i\})\\ &=\sum_{y=1}^{k/2} y\left(\left(\sum_{b=1}^{y} P_b\right)^{n_i}-\left(\sum_{b=1}^{y-1} P_b\right)^{n_i}\right)\\ &=(k/2)-\sum_{a=1}^{k/2-1} P_b\left(\sum_{b=1}^{a} P_b\right)^{n_i},\end{aligned} \tag{A.2}$$

where $P_b=(P_{i(2b)}+P_{i(2b-1)})/\sum_{s=1}^{k} P_{is}$ is the normalized probability. Thus, Eq. (A.1) can be written as

$$\mathrm{E}[d_i|n_i]=2\times L_r\times (k/2-\sum_{a=1}^{k/2-1}\left(\sum_{b=1}^{a} P_b)^{n_i}\right). \tag{A.3}$$

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