Computation at the Edge of Chaos

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1 Introduction

For the second assignment, the Langton's λ parameter was analysed. Langton (1990) defined the λ parameter as how random the cellular automata (CA) ruleset is. Hereby a low λ is associated with no randomness at all and the activity in de CA will die out quickly. On the other hand a high λ is associated with being totally random after a single step (Langton, 1990).

Langton (1990) compares the λ parameter with the qualitative classes of CA behaviour created by Wolfram (1984). Wolfram (1984) mentioned the following classes and their behaviour:

I : The CA evolves to limit points

II : The CA evolves to limit cycles

III : The CA evolves to chaotic behaviour of the kind associated with strange attractors.

IV : The CA evolves effectively very long transients

For this assignment an analysis is made between the Wolfram rules and the λ parameter. Hereby is the relations of the classes and λ shown in Figure 1

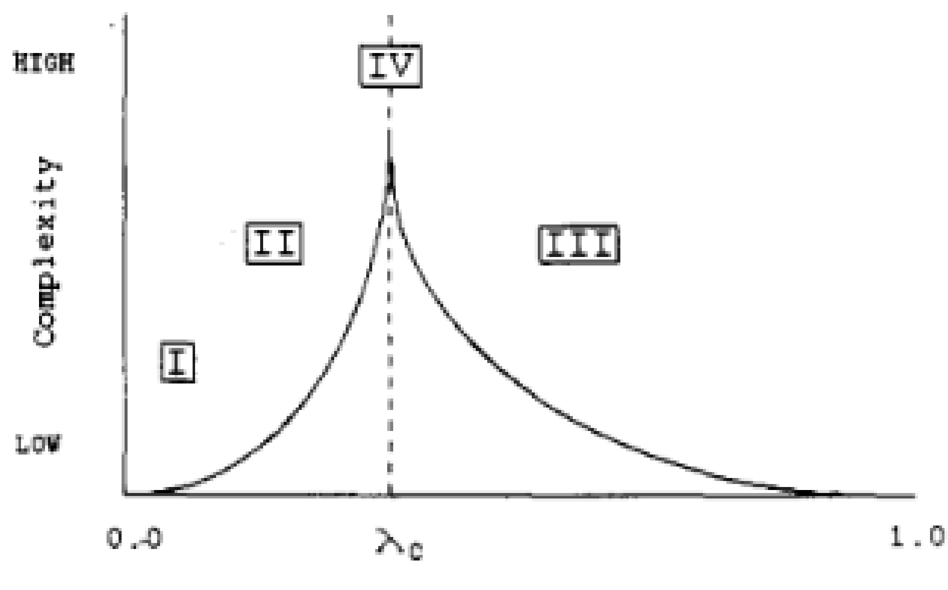

Figure 1: Relation of the Wolfram classes to λ.

2 Methode

For this assignment the following parameters are used: $k = 2, r = 1$. To run the code the program needs to have the rule_class_wolfram.csv file and the PyICS library included. Besides that plotly, numpy and pandas must be installed.

3 Lambda rules

To calculate the λ parameter the following formula is used:

$$\lambda(\Delta) = \frac{k^N - n}{k^N}$$

Hereby is k^N all the states in the ruleset, and n the number of rules that produce a state. The solutions are shown in Figure 2. As seen is that the Figure the λ parameter decreases based on the rule. Based on the literature we expected that the Wolfram class would show a similar relation as mentioned in Figure 1 (Langton, 1990; Wolfram, 1984). A possible reason for this is the way how n is calculated.

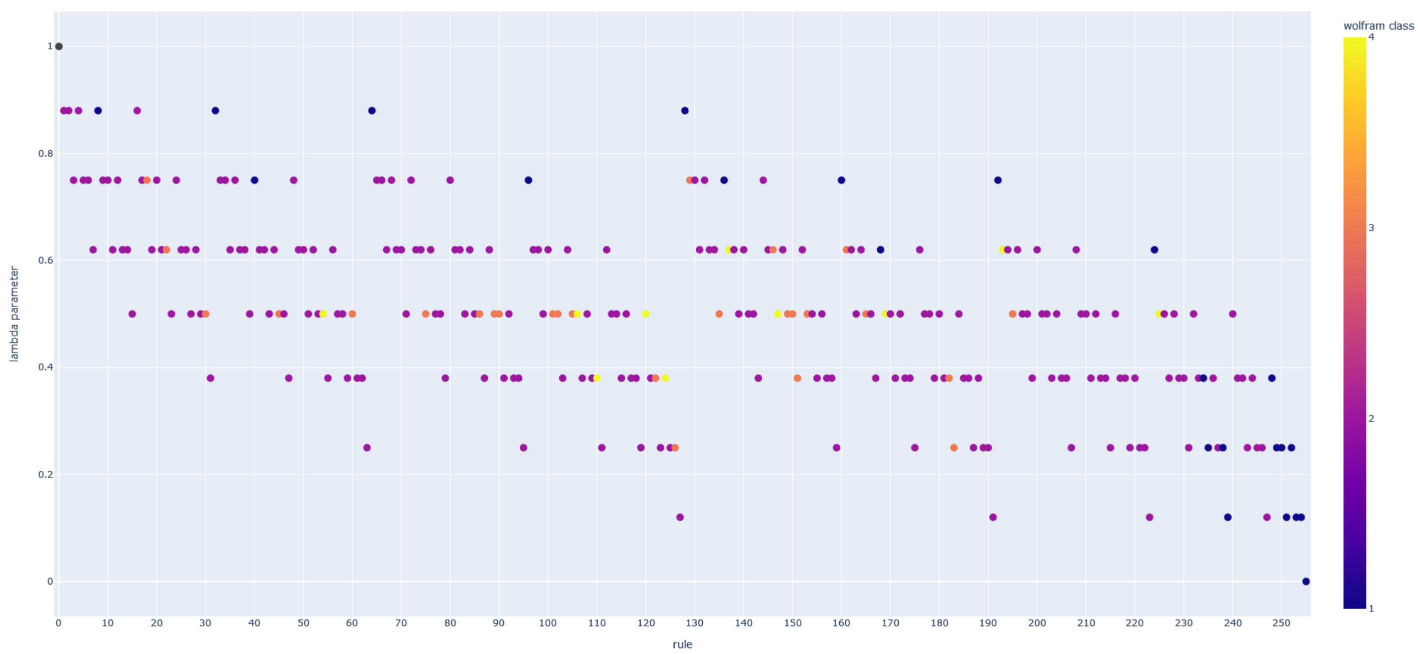

Figure 2: Relation of the Wolfram rules (x-axis) to λ (y-axis) based on the complexity (colour).

4 Tables and Experiment

In the scientific article of Langton (1990) there are two ways to reconstruct a ruleset from the lambda variable. In the first method random-table, lambda is interpreted as a bias on the random selection of states from all possible states as we sequentially fill in the transitions that make up a delta function. In the second method the table-walk-through method, the lambda gets reconstructed to the n variable from the lambda formula from section 3. A ruleset is created with n number of k's and those are in random positions.

The experiment entailed the average single-cell Shannon entropy for lambda with the table-walk-through method for the ruleset and the parameters lambda 0.1 until 0.9 with steps of 0.02, k=2, r=1, matrix shape 64x64 excluding the initial state and 10 different random initial states. The Shannon entropy formula:

$$H(X) = -\sum_i P(X = x_i) \cdot \log_b(P(X = x_i)) \tag{1}$$

Figure 3 shows a sparsely populated gap between 0.02 < H < 0.8.

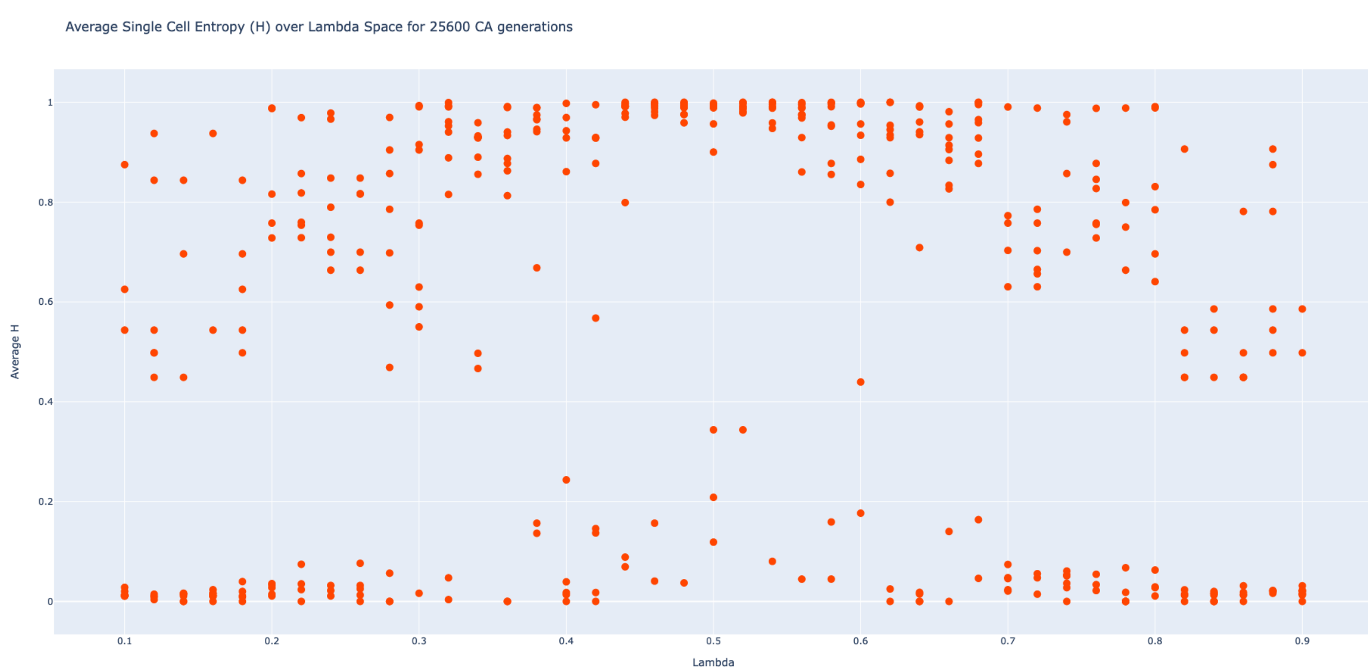

Figure 3: Average Single Cell Entropy (H) over λ Space for 25600 CA generations.

References

Langton, C. G. (1990). Computation at the edge of chaos: Phase transitions and emergent computation. *Physica D: nonlinear phenomena*, 42(1-3):12–37.

Wolfram, S. (1984). Universality and complexity in cellular automata. *Physica D: Nonlinear Phenomena*, 10(1-2):1–35.