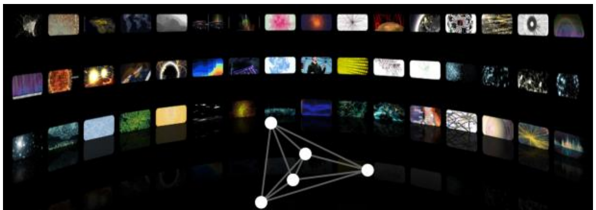


Network Science

Preferential Attachment and Network Measures

Jaap Kaandorp
Slides adapted from: Prof. Albert-László Barabási
Dr. Baruch Barzel, Dr. Mauro Martino

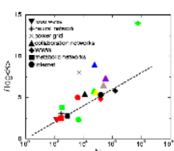
1

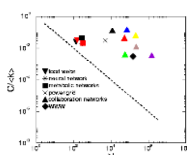
Todays Lecture

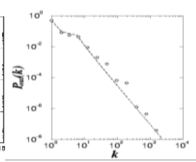
1. **Barabasi – Albert** Model of preferential attachment – how do we get scale free networks?
2. **Robustness** of networks – why is it important?
3. **Centrality Measures** – How important are certain nodes?

2

Empirical findings for real networks







$$l_{rand} = \frac{\log N}{\log\langle k\rangle}$$

$$C_{rand} = \frac{\langle k\rangle}{N}$$

P(*k*) ~ $k^{-\gamma}$

Small World: distances scale logarithmically with the network size

Clustered: clustering coefficient does not depend on network size.

Scale-free: The degrees follow a power-laws distribution.

3

BENCHMARK 2: Random Network Model

Erdös-Rényi Model- Publ. Math. Debrecen 6, 290 (1959)

p = 0 **p = 0.1** **p = 0.15**

- fixed node number *N*
- connecting pairs of nodes with probability *p*

Degree distribution: $P_{rand}(k) \cong C^{k}_{N-1} p^{k} (1-p)^{N-1-k}$

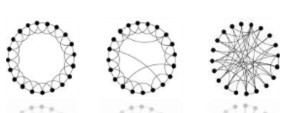
Path length: $l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$

Clustering coefficient: $C_{rand} = p = \frac{\langle k\rangle}{N}$

4

BENCHMARK 3: Small World Model

Watts-Strogatz algorithm – Nature 2008



- fixed node number *N*
- connecting pairs of nodes with probability *p*

Degree distribution: Exponential

Path length: $l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$

Clustering coefficient: $C(b) = C(0)(1- b)$

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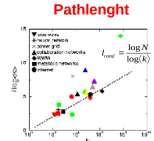
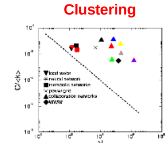
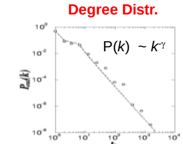
EMPIRICAL DATA FOR REAL NETWORKS







	Pathlenght	Clustering	Degree Distr.
Regular network	$l \gg N^{1/D}$	$C \sim const$	P(k)=δ(k-k$_d$)
Erdos-Renyi	$l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$	$C_{rand} = p = \frac{\langle k\rangle}{N}$	$P(k) = e^{-<k>}\frac{<k>^k}{k!}$
Watts-Strogatz	$l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$	$C \sim const$	*Exponential*

6

SCALE-FREE MODEL
(BA model)

7

BA MODEL: Growth

ER, WS models: the number of nodes, N, is fixed (static models)

Real networks continuously expand by the addition of new nodes

Barabási & Albert, *Science* **286,** 509 (1999)

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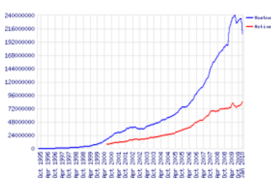
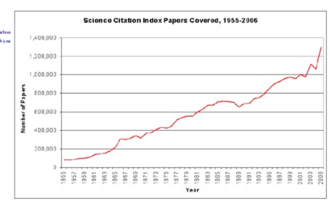
BA MODEL: Growth (www/Pubs)

WWW



http://website101.com/define-ecommerce-web-terms-definitions/

Scientific Publications



http://www.kk.org/thetechnium/archives/2008/10/the_expansion_o.php

Barabási & Albert, *Science* **286,** 509 (1999)

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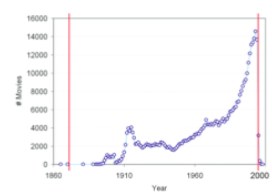
BA MODEL: Growth (Actors/Internet)

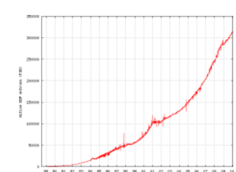
Actor network



Number of movies in IMDB

Herr II, Bruce W., Ke, Weimao, Hardy, Elisha, and Börner, Katy. (2007) Movies and Actors: Mapping the Internet Movie Database. In Conference Proceedings of 11th Annual Information Visualization International Conference (IV 2007), Zurich, Switzerland, July 4-6, pp. 465-469.

Internet



Growth of the Internet routing table

http://www.trainsignaltraining.com/ccna-ipv6

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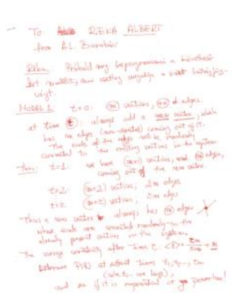
BA MODEL



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BA MODEL: Preferential Attachment

(A) Growth: At each timestep we add a new node with $m (\leq m_0)$ links that connects the new node to m nodes already in the network.

(B) Preferential attachment: The probability $\Pi(k_i)$ that a link of the new node connects to node i depends on the degree k_i of node i as:

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$

we need both!

After t timesteps the Barabási-Albert model generates a network with $N = t + m_0$ nodes and $m_0 + mt$ links

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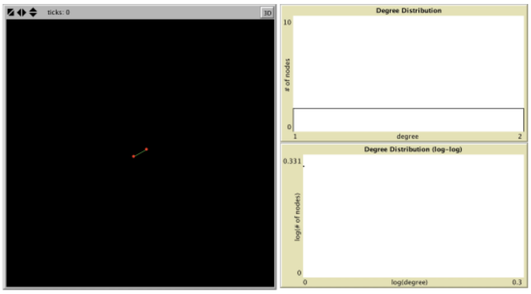
BA MODEL

(1) Networks continuously expand by the addition of new nodes

(2) New node has m links added

Barabási & Albert, *Science* **286,** 509 (1999)



m=1

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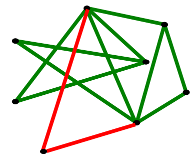
BA MODEL: Growth

(1) Networks continuously expand by the addition of new nodes

Add a new node with m links



Barabási & Albert, *Science* **286,** 509 (1999)

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BA MODEL: Preferential Attachment

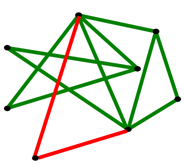
Where will the new node link to?
ER, WS models: choose randomly.

New nodes prefer to link to highly connected nodes (www, citations, IMDB).

PREFERENTIAL ATTACHMENT:

the probability that a node connects to a node with *k* links is proportional to *k*.

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$



Barabási & Albert, *Science* **286,** 509 (1999)

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Origin of SF networks: Growth and preferential attachment

(1) Networks continuously expand by the addition of new nodes

WWW : addition of new documents

(2) New nodes prefer to link to highly connected nodes.

WWW : linking to well known sites

Barabási & Albert, *Science* **286,** 509 (1999)

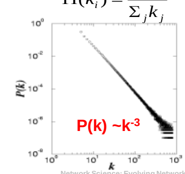
GROWTH:

add a new node with m links

PREFERENTIAL ATTACHMENT:

the probability that a node connects to a node with *k* links is proportional to *k*.

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$



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BA model I

GROWTH:

add a new node with m links

PREFERENTIAL ATTACHMENT:

the probability that a node connects to a node with *k* links is proportional to *k*.

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$

After t time steps in BA model

$N = t + m_0$ *nodes and* $m_0 + mt$ *links*

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BA model II

Assume ki is a time dependent continuous variable, the rate at which each node I acquires links follws:

$$\frac{\partial k_i}{\partial t} = m\Pi(k_i) = m\frac{k_i}{\Sigma_j k_j}$$

m describes that every node arrives with m links, hence node j has m chances to be Choosen. The sum above goes over all nodes in the network except the newly added Nodes

$$\Sigma_j k_j = 2mt - m$$

The top equation becomes

$$\frac{\partial k_i}{\partial t} = \frac{k_i}{2t-1}$$

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BA model III

$$\frac{\partial k_i}{\partial t} = \frac{k_i}{2t-1}$$

We neglect -1 for t>>-1

$$\frac{\partial k_i}{\partial t} \cong \frac{k_i}{2t} \quad or \quad \frac{\partial k_i}{k_i} = \frac{1}{2}\frac{\partial t}{t} \qquad k_i$$

We can integrate

$$k_i(t) = m\left(\frac{t}{t_i}\right)^{\beta}, \beta = \frac{1}{2}$$

Meaning node I joins the network at time ti with m links

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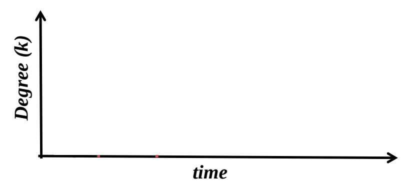
Fitness Model: Can Latecomers Make It?

SF model: $k(t) \sim t^{1/2}$ (first mover advantage)



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Degree distribution I

The probability that degree ki(t) of node I is smaller than a value of k:

$$P(k_i(t) < k) = P\left(t_i > \frac{m^{\frac{1}{\beta}}t}{k^{\frac{1}{\beta}}}\right)$$

In the model we add nodes at equal time intervals, we write the probability that a node arrives at time ti as a random variable with a constant probability density

$$P(t_i) = \frac{1}{m_0 + t}$$

Substitute this in top equation we obtain a cumulative distribution function:

$$P(k) = P\left(t_i \le \frac{m^{\frac{1}{\beta}}t}{k^{\frac{1}{\beta}}}\right) = 1 - \frac{m^{\frac{1}{\beta}}t}{(t+m_0)k^{\frac{1}{\beta}}}$$

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Degree distribution I

We obtain the degree distribution function p(k) by taking the derivative of the cumulative Function P(k):

$$p(k) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{2m^{\frac{1}{\beta}}t}{m_0 t}\frac{1}{k^{\frac{1}{\beta}+1}}$$

Which for t>>m0 reduces to

$$p(k) \approx 2m^{\frac{1}{\beta}}k^{-\gamma}$$

With

$$\gamma = \frac{1}{\beta} + 1 = 3$$

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Degree distribution

$$k_i(t) = m\left(\frac{t}{t_i}\right)^{\beta} \qquad \beta = \frac{1}{2} \qquad P(k) = \frac{2m^2 t}{m_o + t}\frac{1}{k^3} \sim k^{-\gamma} \qquad \boxed{\gamma = 3}$$

(i) The degree exponent is independent of m.

(ii) As the power-law describes systems of rather different ages and sizes, it is expected that a correct model should provide a time-independent degree distribution. Indeed, asymptotically the degree distribution of the BA model is independent of time (and of the system size N)
→ the network reaches a stationary scale-free state.

(iii) The coefficient of the power-law distribution is proportional to m^2.

A.-L.Barabási, R. Albert and H. Jeong, *Physica* A **272,** 173 (1999)

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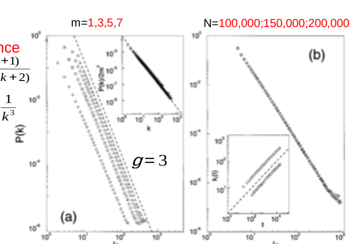
NUMERICAL SIMULATION OF THE BA MODEL

m-dependence

$$P(k) = \frac{2m(m+1)}{k(k+1)(k+2)}$$

$$P(k) = \frac{2m^2 t}{m_o + t}\frac{1}{k^3}$$



Stationarity:
P(k) independent of *N*

Insert:
degree dynamics

$$k_i(t) = m\left(\frac{t}{t_i}\right)^{\beta} \qquad \beta = \frac{1}{2}$$

FIG. 21. Numerical simulations of network evolution: (a) Degree distribution of the Barabási-Albert model, with $N = m_0 + t = 300\,000$ and ○, $m_0 = m = 1$; □, $m_0 = m = 3$; ◇, $m_0 = m = 5$; and △, $m_0 = m = 7$. The slope of the dashed line is $\gamma = 2.9$, providing the best fit to the data. The inset shows the rescaled distribution (see text) $P(k)/2m^2$ for the same values of m, the slope of the dashed line being $\gamma = 3$; (b) $P(k)$ for $m_0 = m = 5$ and various system sizes, ○, $N = 100\,000$; □, $N = 150\,000$; ◇, $N = 200\,000$. The inset shows the time evolution for the degree of two vertices, added to the system at $t_1 = 5$ and $t_2 = 95$. Here $m_0 = m = 5$, and the dashed line has slope 0.5, as predicted by Eq. (81). After Barabási, Albert, and Jeong (1999).

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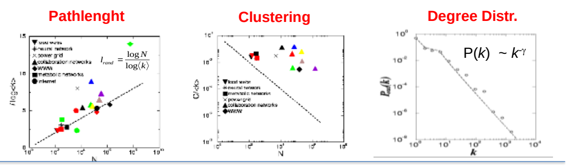
EMPIRICAL DATA FOR REAL NETWORKS



	Pathlenght	Clustering	Degree Distr.
Regular network	$l \gg N^{1/D}$	$C \sim const$	$P(k)=\delta(k-k_d)$
Erdos-Renyi	$l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$	$C_{rand} = p = \frac{\langle k\rangle}{N}$	$P(k) = e^{-\langle k\rangle}\frac{\langle k\rangle^k}{k!}$
Watts-Strogatz	$l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$	$C \sim const$	Exponential
Barabasi-Albert			P(k) ~ $k^{-\gamma}$

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DISTANCES IN SCALE-FREE NETWORKS

$$K_{\max} = K_{\min} N^{\frac{1}{\gamma-1}}$$

Ultra Small World $\langle l\rangle \sim$	$const.$	$\gamma = 2$	Size of the biggest hub is of order O(N). Most nodes can be connected within two layers of it, thus the average path length will be independent of the system size.
	$\frac{\ln\ln N}{\ln(\gamma-1)}$	$2 < \gamma < 3$	The average path length increases slower than logarithmically. In a random network all nodes have comparable degree, thus most paths will have comparable length. In a scale-free network the vast majority of the path go through the few high degree hubs, reducing the distances between nodes.
	$\frac{\ln N}{\ln\ln N}$	$\gamma = 3$	Some key models produce γ=3, so the result is of particular importance for them. This was first derived by Bollobas and collaborators for the network diameter in the context of a dynamical model, but it holds for the average path length as well.
Small World	$\ln N$	$\gamma > 3$	The second moment of the distribution is finite, thus in many ways the network behaves as a random network. Hence the average path length follows the result that we derived for the random network model earlier.

Cohen, Havlin Phys. Rev. Lett. 90, 58701(2003); Cohen, Havlin and ben-Avraham, in Handbook of Graphs and Networks, Eds. Bornholdt and Shuster (Willy-VCH, NY, 2002) Chap. 4; Confirmed also by: Dorogovtsev et al (2002), Chung and Lu (2002); (Bollobas, Riordan, 2002; Bollobas, 1985; Newman, 2001

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PATH LENGTHS IN THE BA MODEL

$$l \approx \frac{\ln N}{\ln\ln N}$$

Bollobas, Riordan, 2002

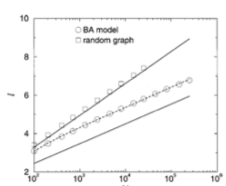

FIG. 23. Characteristic path length ℓ versus network size N in a Barabási-Albert (BA) network with ⟨k⟩=4 (○), compared with a random graph of the same size and average degree generated with the algorithm described in Sec. III.A (□). The dashed line follows Eq. (94), and the solid lines represent Eq. (60) with $z_1=\langle k\rangle$ and z_2 the numerically obtained number of next-nearest neighbors in the respective networks.

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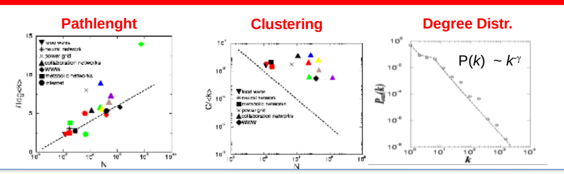
EMPIRICAL DATA FOR REAL NETWORKS



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Watts-Strogatz	$l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$	$C \sim const$	Exponential
Barabasi-Albert	$l \approx \frac{\ln N}{\ln\ln N}$		P(k) ~ $k^{-\gamma}$

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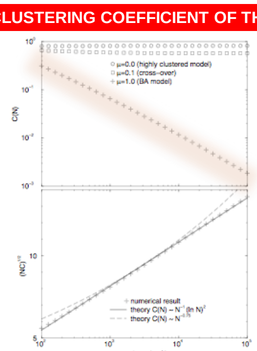
CLUSTERING COEFFICIENT OF THE BA MODEL



$$C = \frac{m}{8}\frac{(\ln N)^2}{N}$$

Konstantin Klemm, Victor M. Eguiluz, Phys. Rev. E 65, 057102 (2002)

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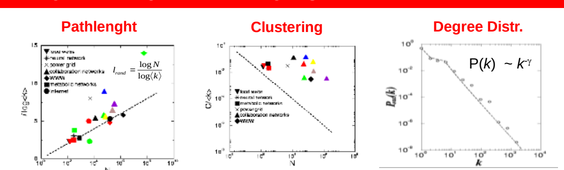
EMPIRICAL DATA FOR REAL NETWORKS



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Watts-Strogatz	$l_{rand} \approx \frac{\log N}{\log\langle k\rangle}$	$C \sim const$	Exponential
Barabasi-Albert	$l \approx \frac{\ln N}{\ln\ln N}$	$C \sim \frac{(\ln N)^2}{N}$	P(k) ~ $k^{-\gamma}$

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The origins of preferential attachment.

MECHANISMS RESPONSIBLE FOR PREFERENTIAL ATTACHMENT

There are two philosophically different takes on this question.

LOCAL
The first approach views preferential attachment as an interplay between random events and some structural network property. These mechanisms do not require global knowledge of the network. Hence we will call them ***local*** *or random mechanisms*.

GLOBAL
The second approach assumes that the addition of each new node or link is preceded by a cost-benefit analysis, balancing various needs with the available resources. This assumes familiarity with the whole network and relies on optimization principles, prompting us to call them ***global*** *or optimized mechanisms*.

LOCAL MECHANISMS

Local mechanisms. The *link selection model* offers perhaps the simplest example of a local or random mechanism capable of generating preferential attachment.

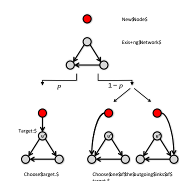
Growth: at each time step we add a new node to the network.

Link selection: we select a link at random and connect the new node to one of nodes at the two ends of the selected link

- The higher is the degree of a node, the higher is the chance that it will be located at the end of the chosen link.
- The more degree-*k* nodes are in the network (i.e., the higher is *pk*), the more likely that a degree *k* node will be at the end of the link.

LOCAL MECHANISMS

Copying model.
When building a new webpage, authors tend to borrow links from webpages covering similar topics, a process captured by the copying model [17, 18]. In the model in each time step a new node with a single link is added to the network. To choose the target node we randomly select a node *u* and follow a two-step procedure:
(a) *Random Connection:* with probability *p* the new node links to *u*.
(b) *Copying:* with probability 1 − *p* we randomly choose an outgoing link of node *u* and connect the new node to the selected link's target. Hence the new node "copies" one of the links of an earlier node.

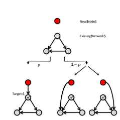


LOCAL MECHANISMS

Copying model – Examples:

Social networks:
the more acquaintances an individual has, the higher is the chance that she will be introduced to new individuals by her existing acquaintances. Without friends, it is difficult to make new friends.

Citation Networks:
no scientist can be familiar with all papers published on a certain topic. If we assume that authors decide what to cite by randomly selecting references from the papers they have already read, then papers with more citations are more likely to be cited again.



GLOBAL MECHANISMS

Optimization. A longstanding assumption of economics is that humans make rational decisions, balancing cost against benefits. In other words, each individual aims to maximize its personal advantage.

Consider the Internet, whose nodes are routers or autonomous systems, connected via cables. Establishing a new Internet connection between two routers requires laying down a cable between them. As this can be costly, each new link is preceded by careful cost-benefit analysis.

Requires **global** information about network and node degrees.

SUMMARY: PROPERTIES OF THE BA MODEL

- **Nr. of nodes:** $N = t$
- **Nr. of links:** $L = m\,t$
- **Average degree:** $\langle k \rangle = \frac{2L}{N} \to 2m$
- **Degree dynamics** $k_i(t) = m\left(\frac{t}{t_i}\right)^{\beta}$ $\beta = \frac{1}{2}$ β: dynamical exponent
- **Degree distribution:** $P(k) \sim k^{-\gamma}$ $\gamma = 3$ γ: degree exponent
- **Average Path Length:** $l \approx \frac{\ln N}{\ln \ln N}$
- **Clustering Coefficient:** $C \sim \frac{(\ln N)^2}{N}$

The network grows, but the degree distribution is stationary.

EVOLVING NETWORK MODELS

The BA model is only a minimal model.

Makes the simplest assumptions:

- linear growth $\langle k \rangle = 2m$
- linear preferential attachment $\Pi(k_i) \propto k_i$

Does not capture

variations in the shape of the degree distribution
variations in the degree exponent
the size-independent clustering coefficient

The BA model can be adapted to describe most features of real networks.

We need to incorporate mechanisms that are known to take place in real networks: addition of links without new nodes, link rewiring, link removal; node removal, constraints or optimization (see Barabasi book)

LESSONS LEARNED: evolving network models

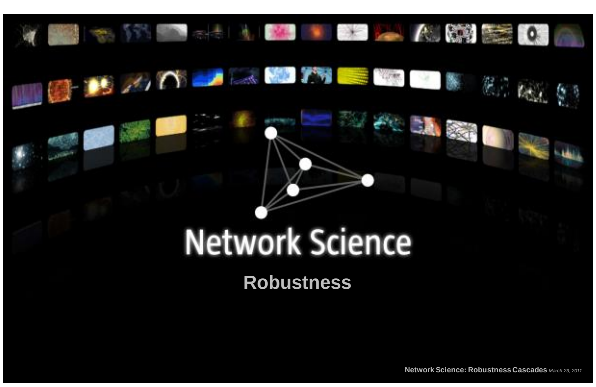
1. There is no universal exponent characterizing all networks.
2. Growth and preferential attachment are responsible for the emergence of the scale-free property.
3. The origins of the preferential attachment is system-dependent.

LESSONS LEARNED: evolving network models

Philosophical change in network modeling:

ER, WS models are static models – the role of the network modeler it to cleverly place the links between a fixed number of nodes to that the network topology mimic the networks seen in real systems.

BA and evolving network models are dynamical models: they aim to reproduce how the network was built and evolved.
Thus their goal is to capture the network dynamics, not the structure.
→ as a byproduct, you get the topology correctly



ROBUSTNESS IN COMPLEX SYSTEMS

Complex systems maintain their basic functions even under errors and failures

cell → mutations

There are uncountable number of mutations and other errors in our cells, yet, we do not notice their consequences.

Internet → router breakdowns

At any moment hundreds of routers on the internet are broken, yet, the internet as a whole does not loose its functionality.

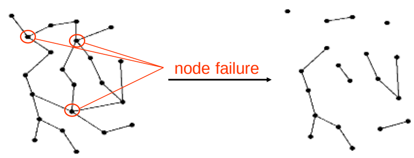
Where does robustness come from?

There are feedback loops in most complex systems that keep tab on the component's and the system's 'health'.

Could the network structure affect a system's robustness?

ROBUSTNESS

Could the network structure contribute to robustness?



How do we describe in quantitave terms the breakdown of a network under node removal?
~percolation theory~

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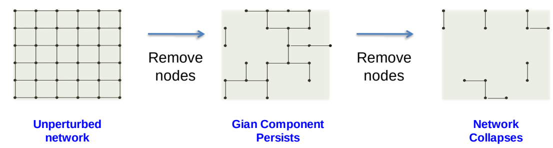
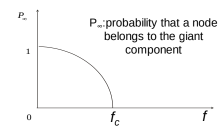
ROBUSTNESS: INVERSE PERCOLATION TRANSITION





f: fraction of removed nodes.

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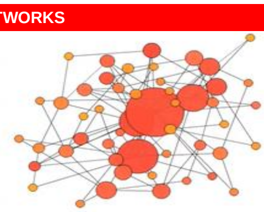
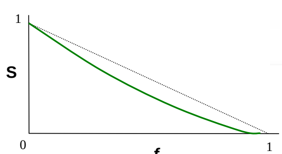
ROBUSTNESS OF SCALE-FREE NETWORKS

Scale-free networks do not appear to break apart under random failures.
Reason: the hubs.
The likelihood of removing a hub is small.





Albert, Jeong, Barabási, *Nature* **406** 378 (2000)

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ROBUSTNESS OF SCALE-FREE NETWORKS

$$f_c = 1 - \frac{1}{\kappa - 1} \qquad \kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} = \left|\frac{2-\gamma}{3-\gamma}\right| \begin{cases} K_{\min} & \gamma > 3 \\ K_{\max}^{3-\gamma} K_{\min}^{\gamma-2} & 3 > \gamma > 2 \\ K_{\max} & 2 > \gamma > 1 \end{cases}$$

$$K_{\max} = K_{\min} N^{\frac{1}{\gamma - 1}}$$

γ>3: κ is finite, so the network will break apart at a finite f_c that depens on K_{min}

γ<3: κ diverges in the N→ ∞ limit, so f_c → 1 !!!
for an infinite system one needs to remove all the nodes to break the system.

For a finite system, there is a finite but large f_c that scales with the system size

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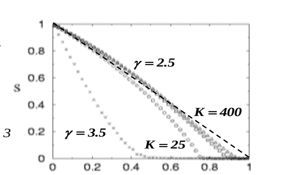
NUMERICAL EVIDENCE

Scale-free random graph with
$$\boldsymbol{P(k) = Ak^{-\gamma},\ \text{with}\ k = m, \ldots K}$$

$$f_c = 1 - \frac{1}{\frac{\gamma-2}{\gamma-3} m - 1} \quad \text{if} \quad \gamma > 3$$

$$f_c = 1 - \frac{1}{\frac{\gamma-2}{3-\gamma} m^{\gamma-2} K^{3-\gamma} - 1} \quad \text{if} \quad 2 < \gamma < 3$$

Cohen et al., Phys. Rev. Lett. 85, 4626 (2000).



Infinite scale-free networks with $\gamma < 3$ do not break down under random node failures.

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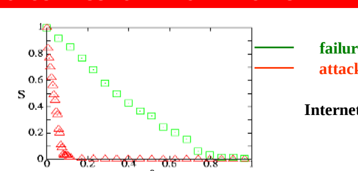
INTERNET'S ROBUSTNESS TO RANDOM FAILURES



R. Albert, H. Jeong, A.L. Barabasi, *Nature* **406** 378 (2000)

$$f_c = 1 - \frac{1}{\kappa - 1}$$

Internet: Router level map, N=228,263; γ=2.1±0.1; *κ=28* → ***f_c=0.962***

AS level map, N= 11,164; γ=2.1±0.1; *κ=264* → ***f_c=0.996***

Internet parameters: Pastor-Satorras & Vespignani, *Evolution and Structure of the Internet:* Table 4.1 & 4.4

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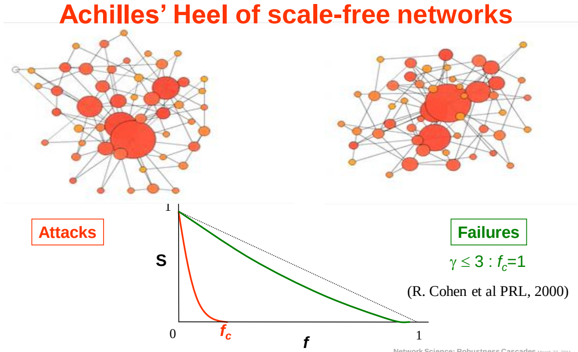
Achilles' Heel of scale-free networks



Albert, Jeong, Barabási, *Nature* **406** 378 (2000)

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Numerical simulations of network resilience

Two networks with equal number of nodes and edges

- ER random graph
- scale-free network (BA model)

Study the properties of the network as an increasing fraction *f* of the nodes are removed.

Node selection: random (errors)

the node with the largest number of edges (attack)

Measures:

S: the fraction of nodes in the largest connected cluster

l: the average distance between nodes in the largest cluster

R. Albert, H. Jeong, A.-L. Barabási, Nature 406, 378 (2000)

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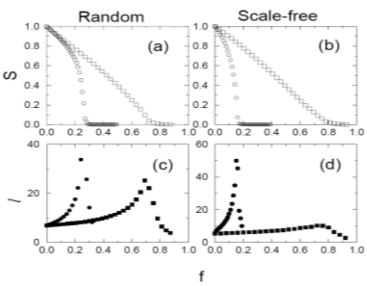
Scale-free networks are more error tolerant, but also more vulnerable to attacks



- squares: random failure
- circles: targeted attack

Failures: little effect on the integrity of the network.
Attacks: fast breakdown

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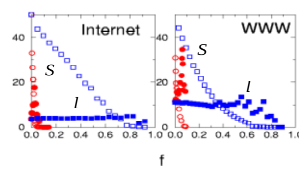
Real scale-free networks show the same dual behavior



- blue squares: random failure
- red circles: targeted attack
- open symbols: *S*
- filled symbols: *l*

- break down if 5% of the nodes are eliminated selectively (always the highest degree node)
- resilient to the random failure of 50% of the nodes.

Similar results have been obtained for metabolic networks and food webs.

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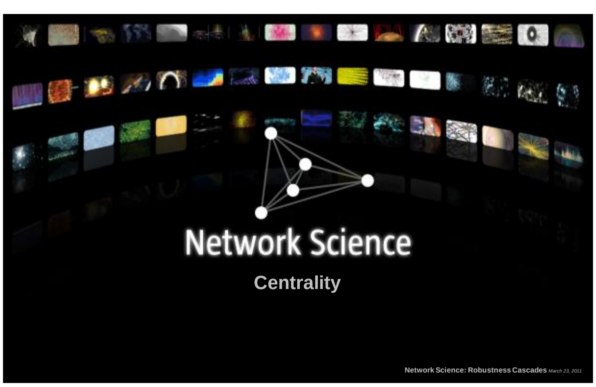

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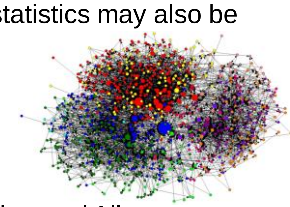
Centrality Measures

- Describing how central a *node or edge* is.
- Network-level statistics may also be derived:
  - Minimum
  - Maximum
  - Average,
  - Etc.
- Indegree / Outdegree / All



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Degree Centrality (Freeman,`79)

- Number of links: d_i
- Normalized: $\dfrac{d_i}{(N - 1)}$
- "Popularity", "Gregariousness".
  - Indicates the *probability* of getting the information / infection spreading in the network.

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Closeness Centrality (Freeman, `79)

- The sum of the shortest paths leading to the other nodes:
  - Distance ~ as inverse measure of centrality.

$$\sum_{i \neq j} l_{ij}$$

- Normalized:
  - A value in [0,1] + "inverted"

$$\frac{N-1}{\sum_{i \neq j} l_{ij}}$$

- Indicates the *speed* of getting the information / the infection.

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Betweenness Centrality (Freeman, `79)

- The number of crossing shortest paths: $\sum_{j,k} \frac{\vec{p}_{jik}}{\vec{p}_{jk}}$
- Normalized (divided by max value): $\sum_{j,k} \frac{\vec{p}_{jik}}{\vec{p}_{jk}} \Big/ \max_i \sum_{j,k} \frac{\vec{p}_{jik}}{\vec{p}_{jk}}$
- Ability to *control* information / infection, or "Brokerage" – *linking* far regions, or Ability to *maintain connectivity*.

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Eigenvector Centrality (Bonacich `72)

- The main Eigen vector of the (possibly weighted) adjacency matrix.
- Recursive definition:
  - Start with centrality 1 at each node.
  - Re-calculate centralities as the weighted sum of the neighbors' centrality
  - Normalize (divide by max(c_i)).
  - Repeat until it reaches a fix point.
- A measure of "being linked to central nodes". (Notice the recursion.)
  - Corresponds to the *probability* of getting the information / infection.

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The end

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