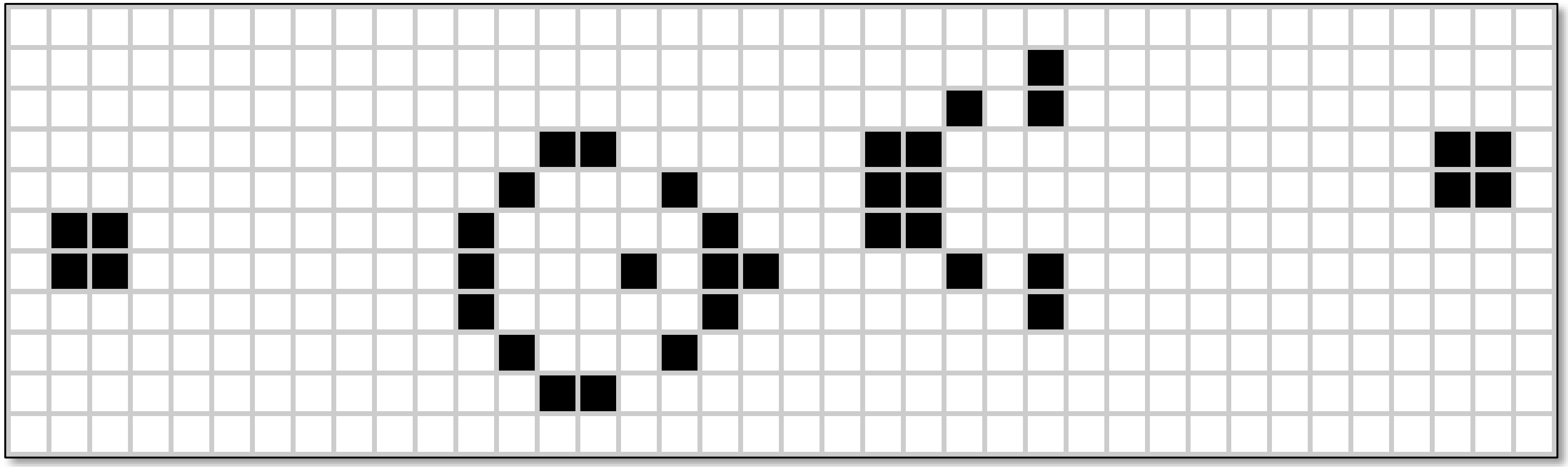
# Modelling with Cellular Automata:

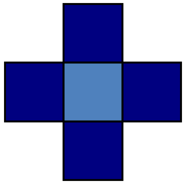
## 2 Dimensional CAs – Game of Life, Universal Computation



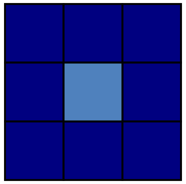
Michael Lees: Computational Science, University of Amsterdam, The Netherlands.

# Two-dimensional CA

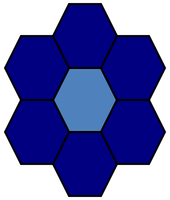
- Lattice is a 2D grid.
- Commonly-used neighborhoods:



von Neumann

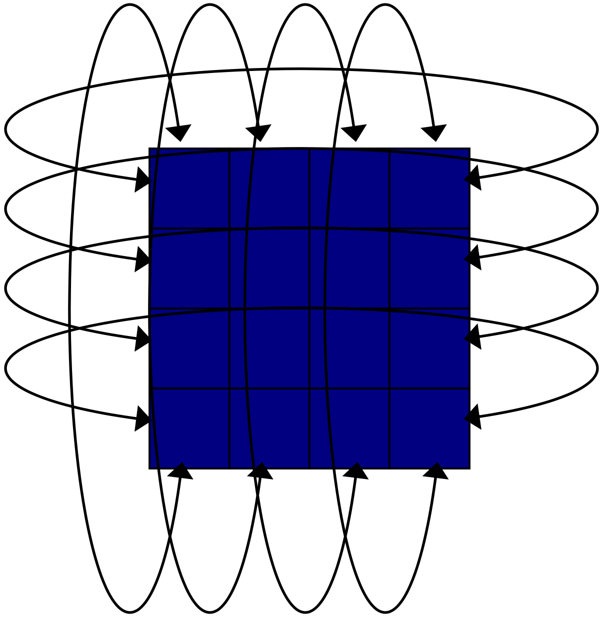


Moore

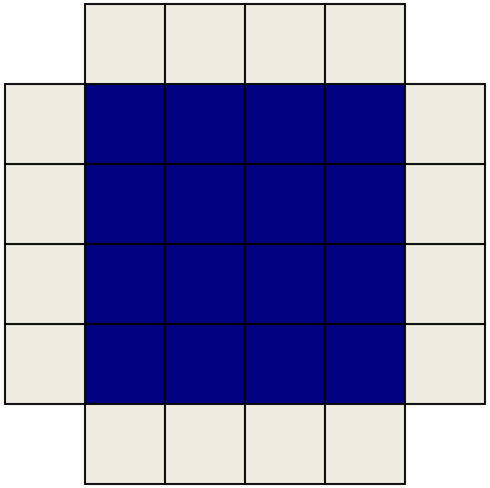


Hexagonal

# Boundary conditions



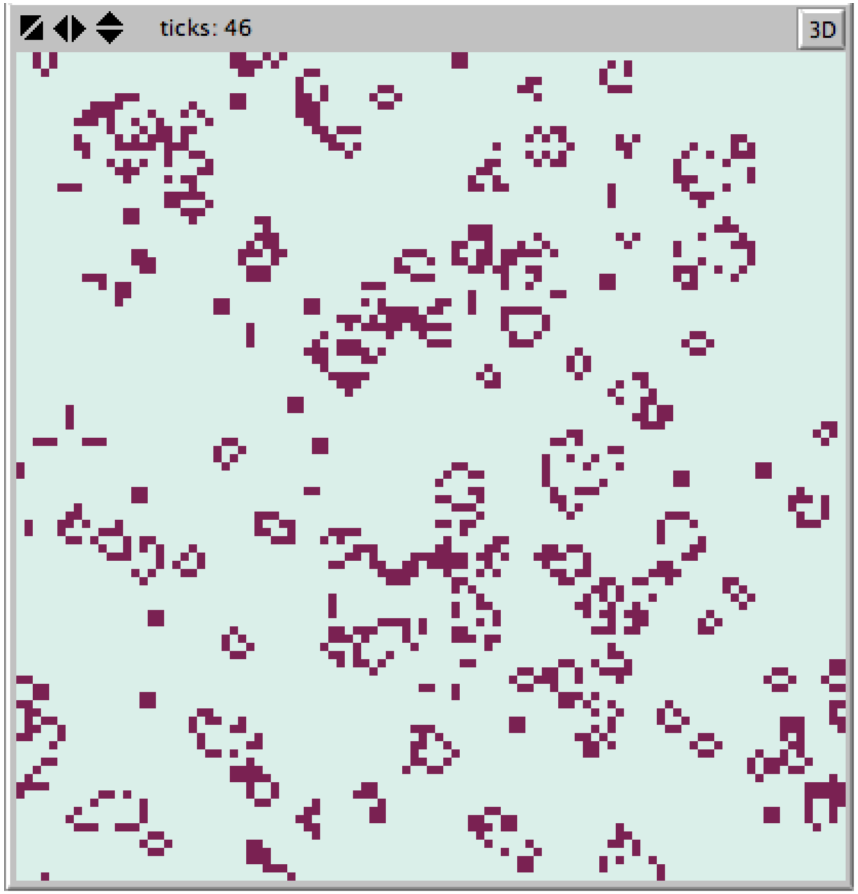
Periodic



Blocking

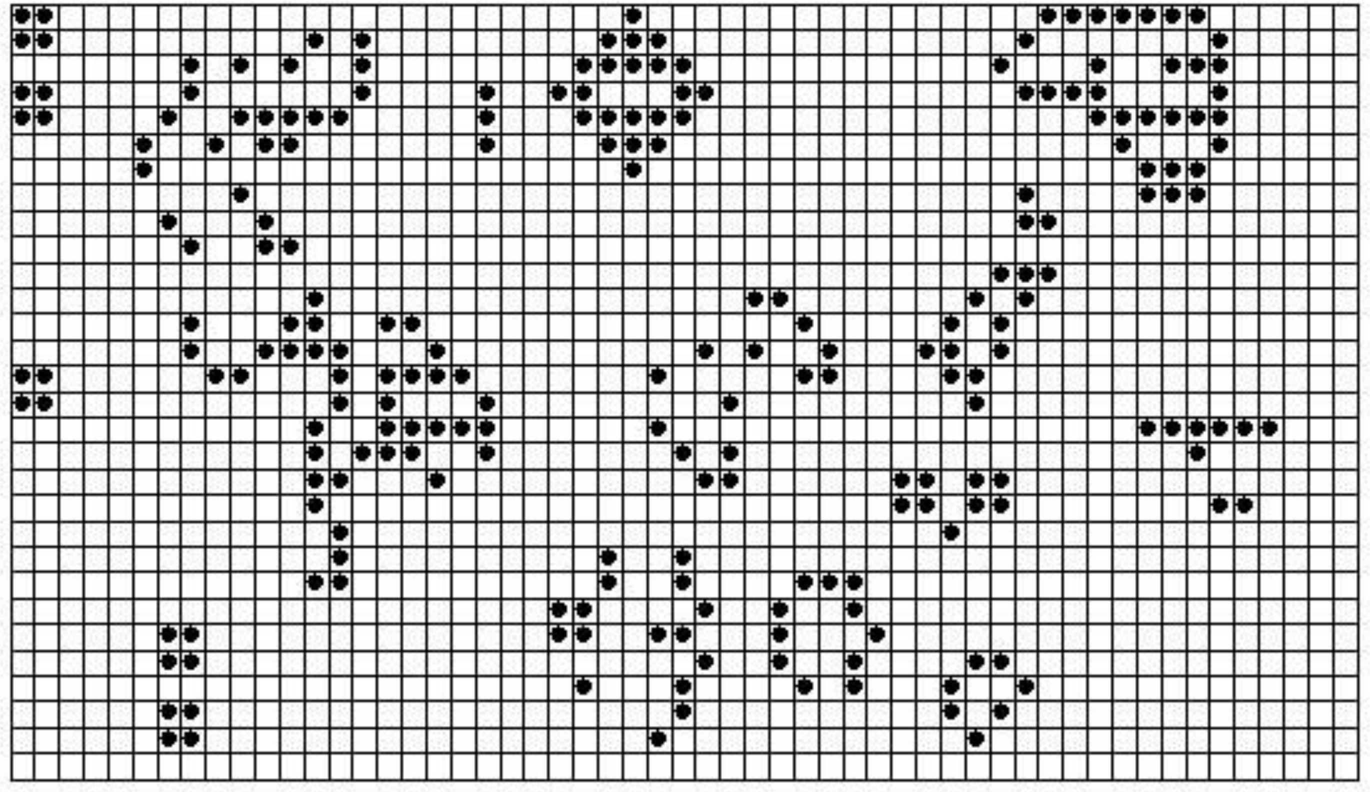
# Conway's Game of Life

John H. Conway



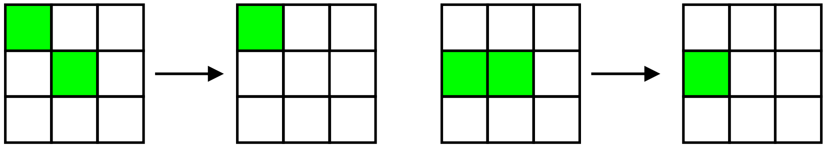
# Conway’s Game of Life

Cells are alive or dead, and transition rule considers Moore neighborhood.

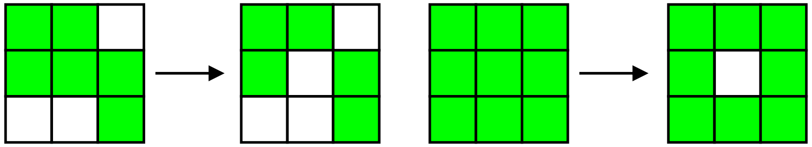


# The Rules

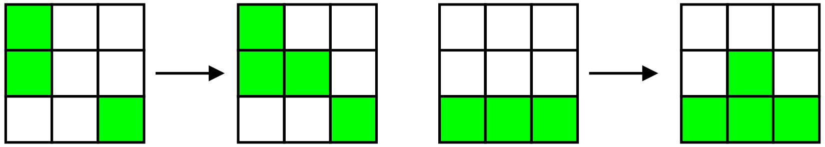
Loneliness: < 2 neighbors alive



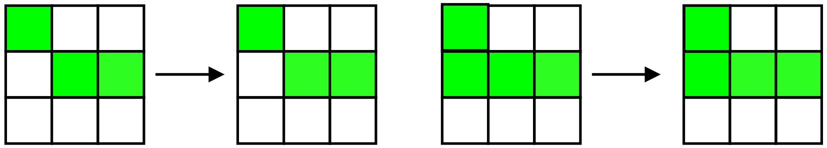
Over crowding: > 3 neighbors alive



Birth: = 3 neighbors alive



Survival: = 2 or 3 neighbors alive



# Why is Life so interesting?

- Early example of emergence, self organization
- Elaborate patterns/dynamics come from simple rules
- “Unlike most computer games, the rules themselves create the patterns, rather than programmers creating a complex set of game situations.”

http://conwaylife.com/

http://www.conwaylife.com/wiki/Conway's_Game_of_Life

# Where did Life come from?

- Ideas from John Von Neuman – interested in machines that could replicate themselves.
- Colonize mars (planets) by first sending machines that farm iron, build replicas of themselves, get more iron etc.
- Is this possible, or do you need more sophisticated machines to build simpler machines, etc.

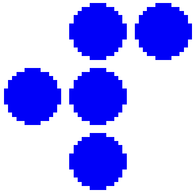
- Tackled this as a mathematician – not an engineer.

The man who finds Game of Life Boring…

# JOHN H. CONWAY

# Complex Emergence

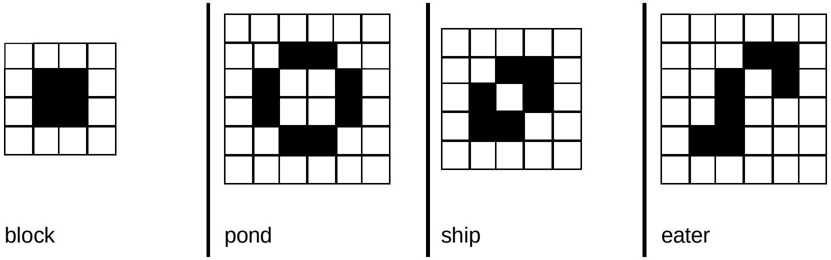
- Hard to predict behavior mentally:



- R-Pentomino – how will this evolve?
- Conway simulated this by hand… does it become stable?

# Patterns of Game of Life

- You can observe patterns in GOL:
  - Static/Still lifes



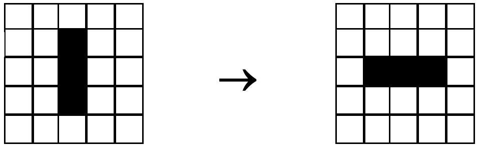
block

pond

ship

eater

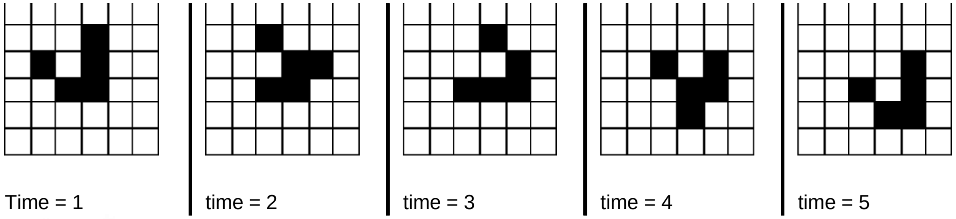
  - Periodic/Oscillators



time = 1

time = 2

  - Moving



Time = 1

time = 2

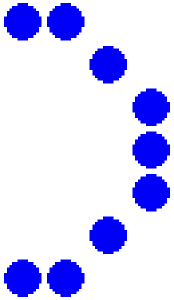
time = 3

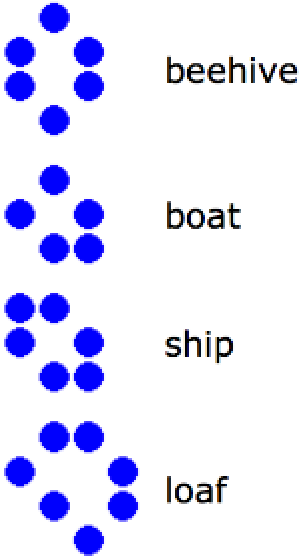
time = 4

time = 5

# The Queen Bee Shuttle

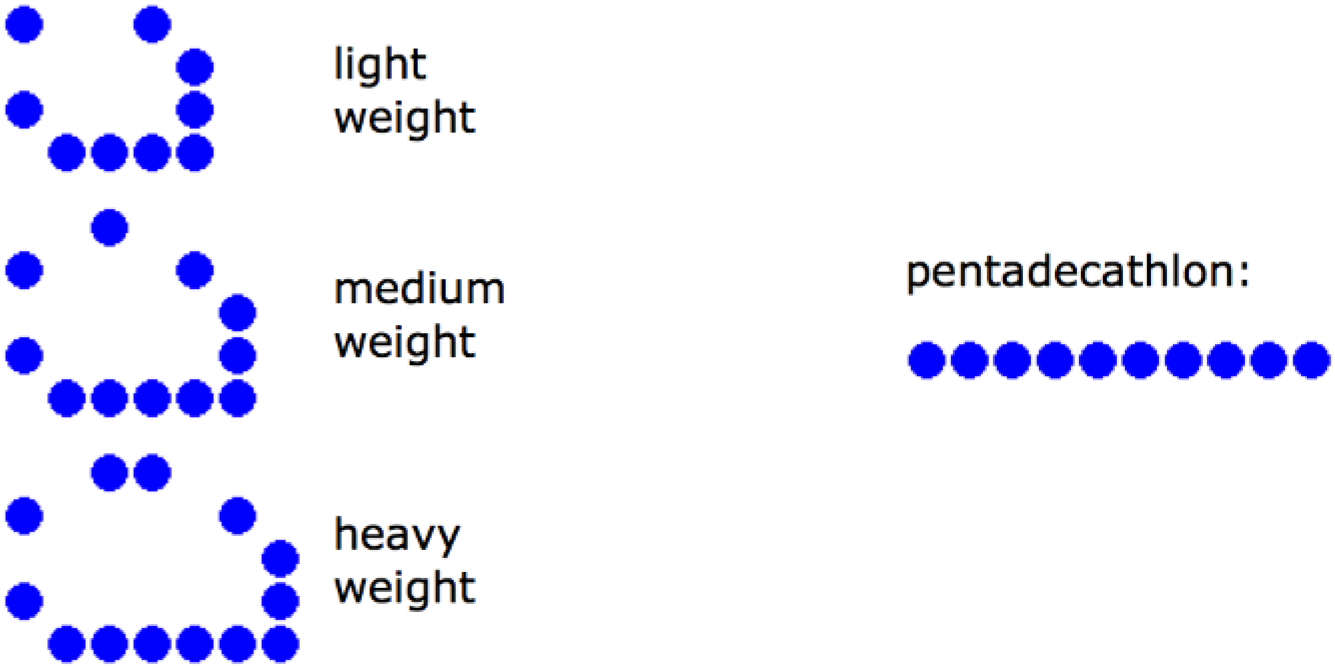
- Hard to predict behavior mentally:





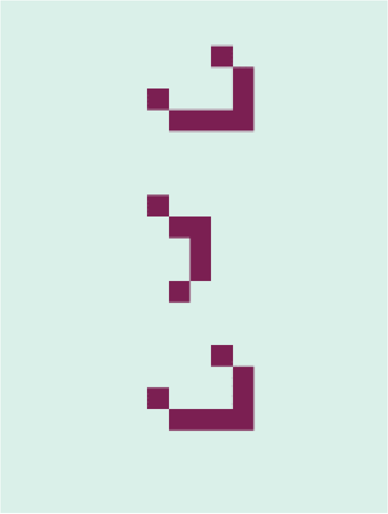
- Conway called this queen bee shuttle

# More interesting early discoveries

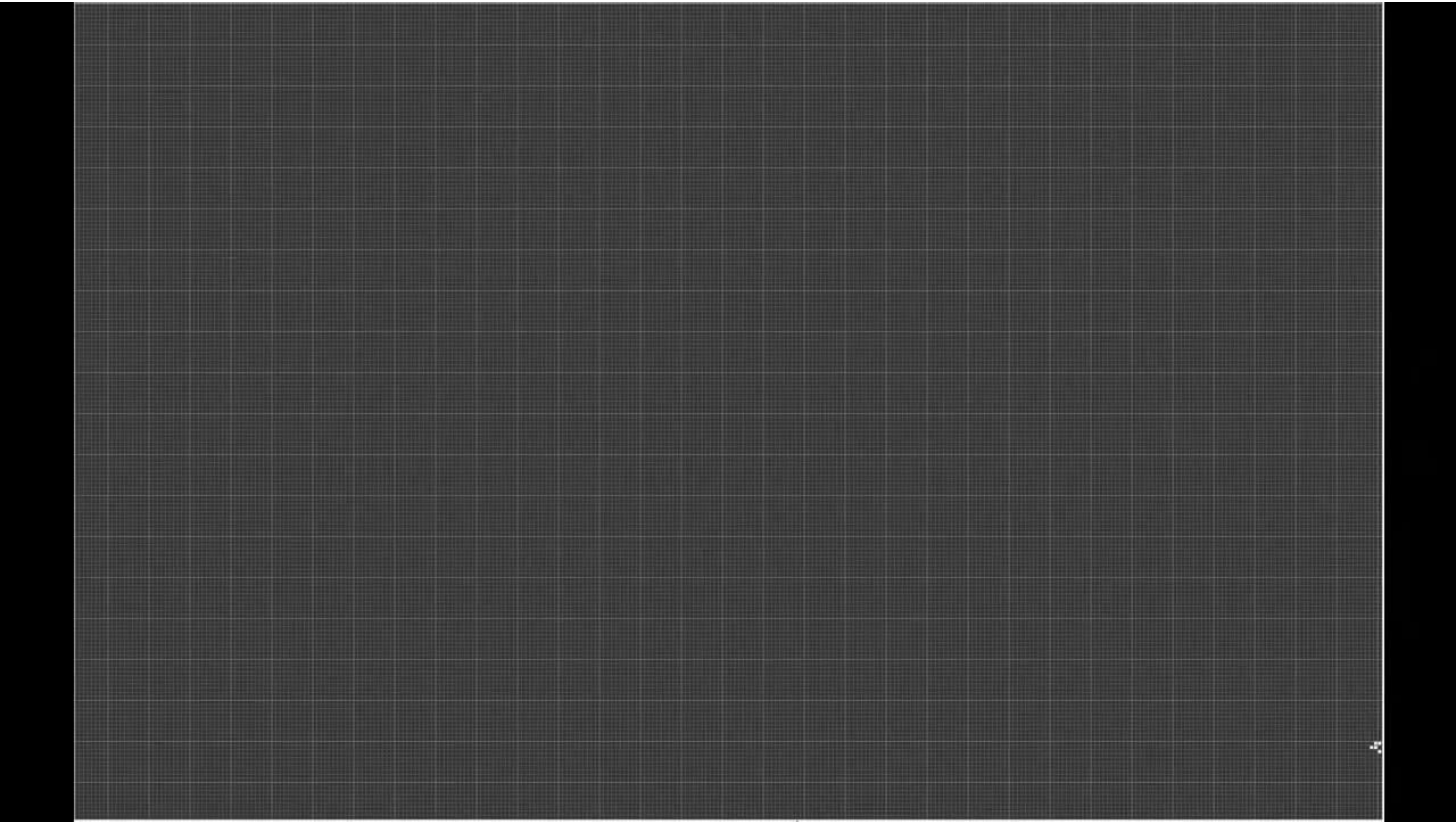


# All stabilize?

- Conway offered a prize for example patterns that go on forever.
- The Puf Train



# This is how some people spend their weekends…



source: https://www.youtube.com/watch?v=C2vgICfQawE

Game of Life and Turing Machines

# UNIVERSAL COMPUTATION

# Universal Computation?

- Show that Life can emulate a Universal Turing machine -  then life can calculate all algorithms!
- **Show that life can create:**
  - A finite-state control (with clock)
  - A tape (with memory)
  - A tape head

# Building parts…

- Need configurations that can be used as clocks, memory, etc.

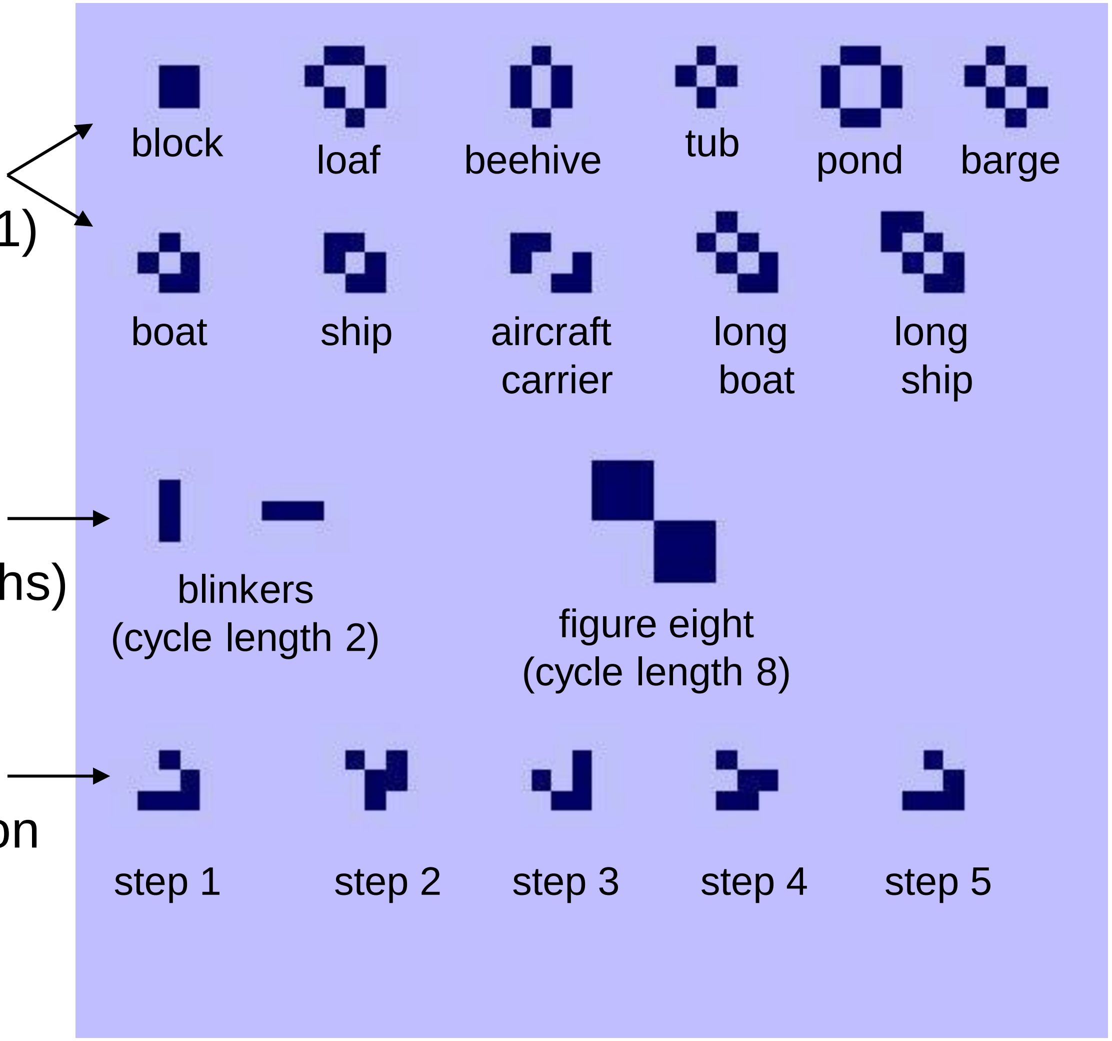
- **We choose a few configurations to help us…**

# Stable and Glider…

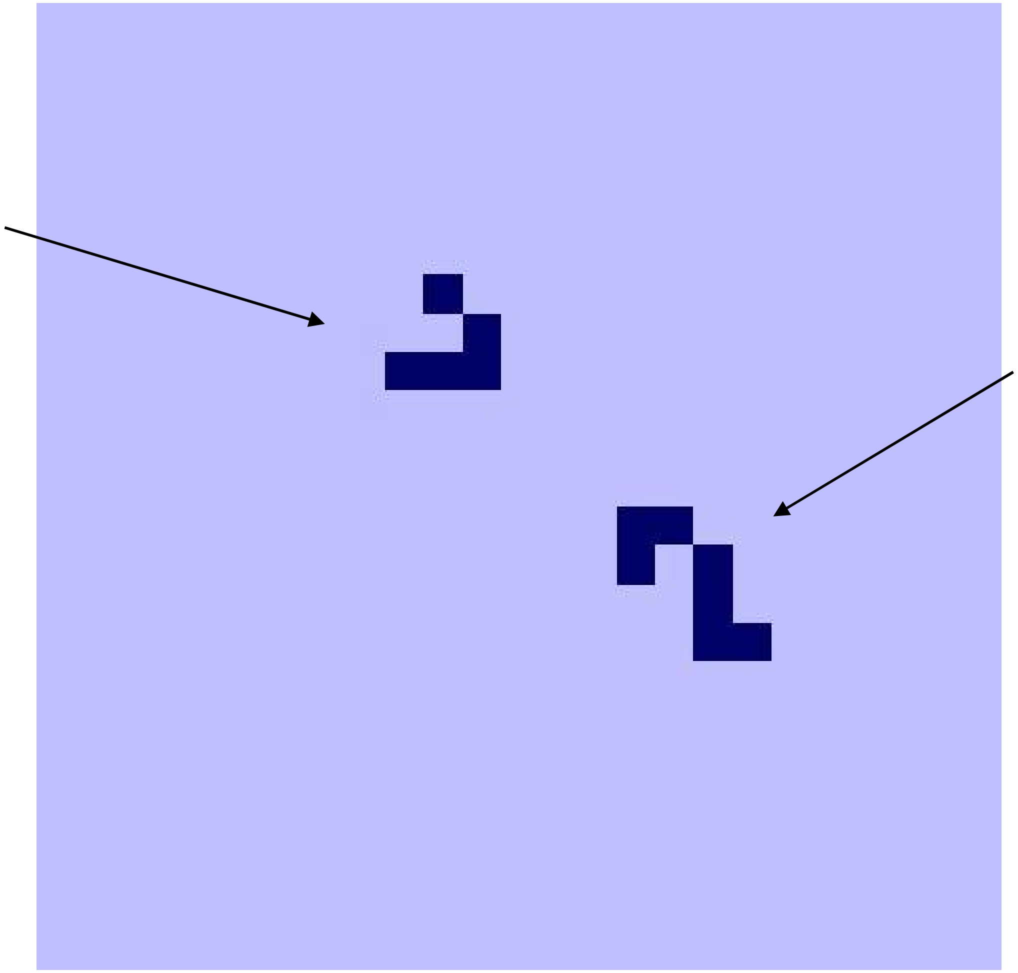
Stable Life
(cycle length of 1)

Periodic Life
(other cycle lengths)

Gliders
(translates position
down and to the
right every
4 steps)



# Glider Eater



Glider which will move southeast towards the “eater”.

A glider “eater”. A stable configuration that swallows the glider and then reconstructs itself.

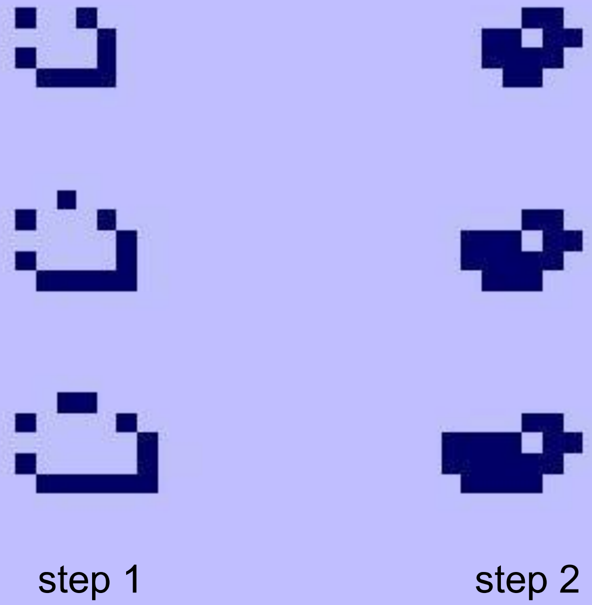
# Spaceships

**Spaceships move to the right!**

lightweight spaceship

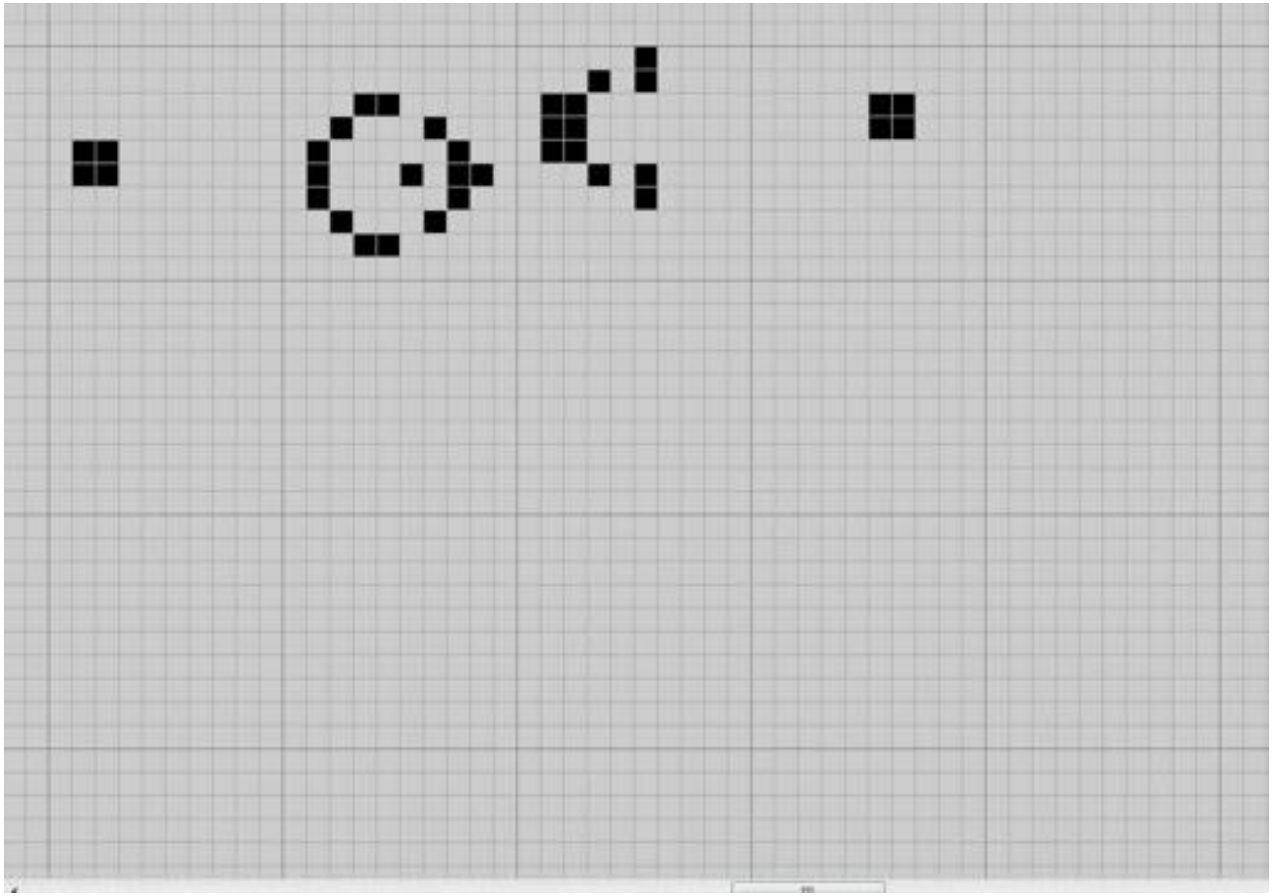
middleweight spaceship

heavyweight spaceship



All have cycle length of 4.

# Glider Gun



This produces an endless stream of gliders, one every 30 steps

# Building Blocks

- – A finite-state control (with clock)
- – A tape (with memory)
- – A tape head

# Building Blocks (to build a TM)

- – A finite-state control (with clock)
- – A tape (with memory)
- – A tape head

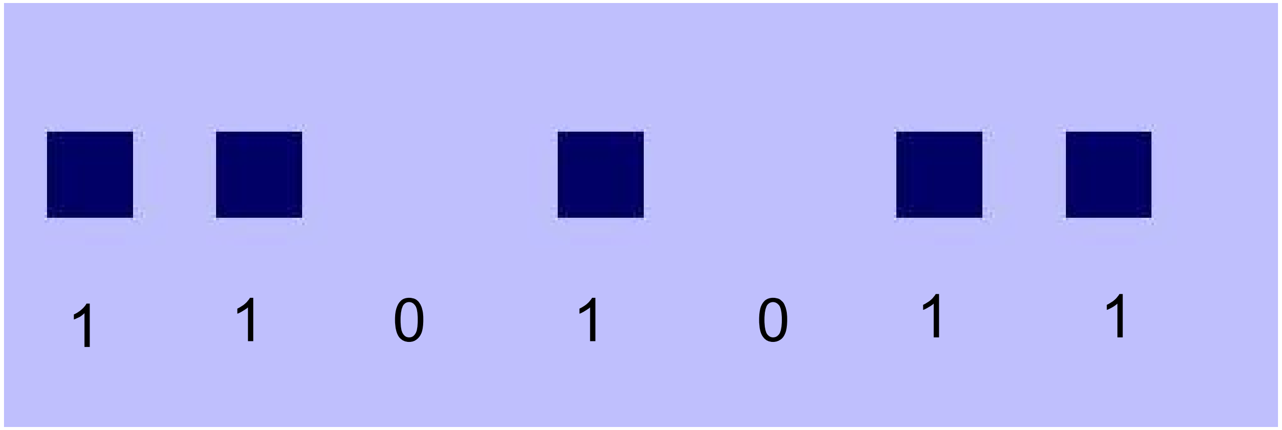
## Functional requirements (in general)

- – Arbitrary information **storage**
  - (whichever information, however long, at any time)
- – Arbitrary information **transfer**
  - (from/to anywhere, at any time)
- – Universal information **modification** (such as NAND-gate)
  - Combines with storage and transfer to construct any Boolean function

# Memory…

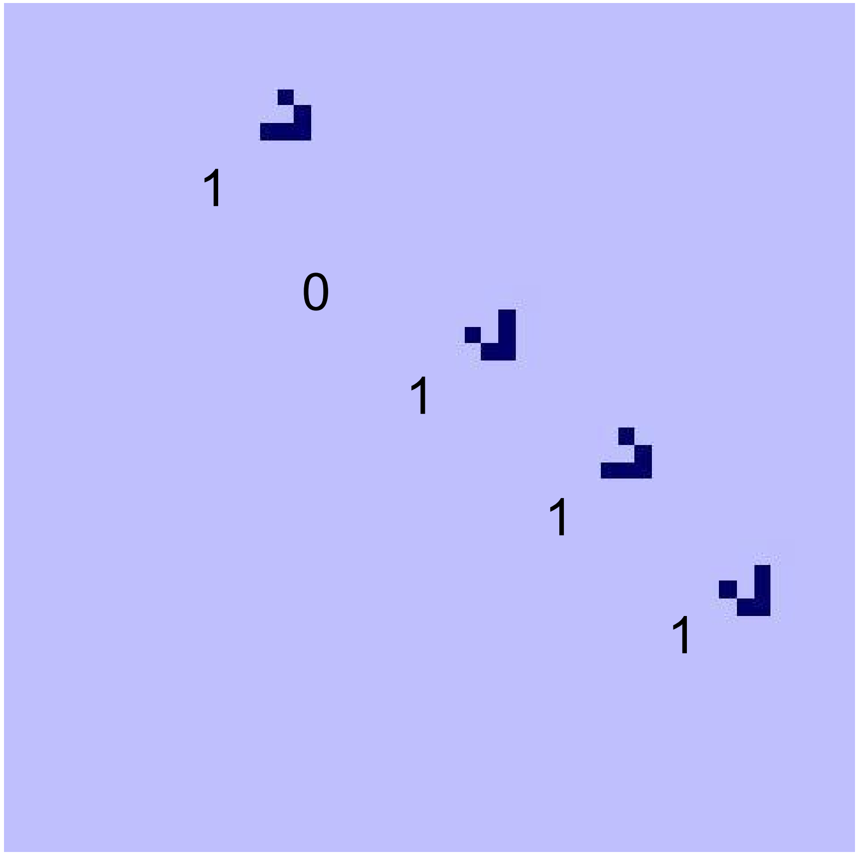
- Arrange stable blocks or stable patterns to store/remember things

- Obvious step to binary storage…



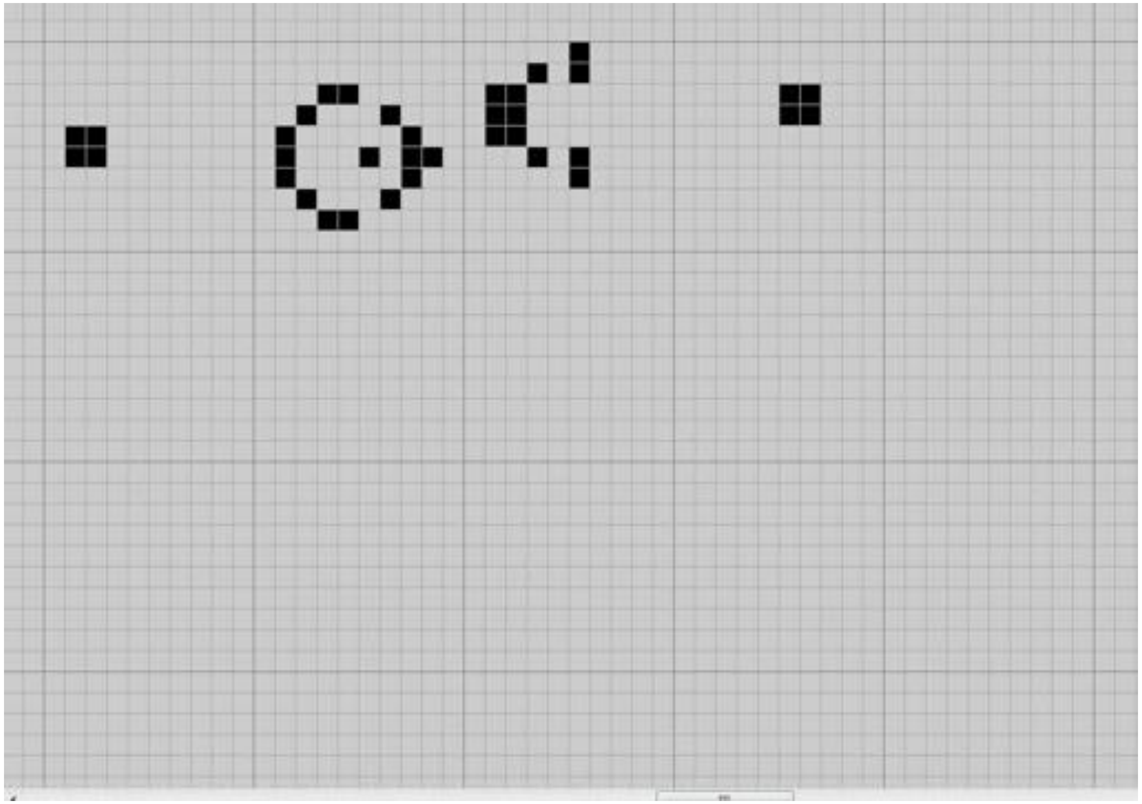
# Memory…more

- Or we can represent memory with gliders, they also transmit information



# Clock

- We have the time of the CA (steps)
- Need to be able to move objects at a set rate… so can move the tape head and transmit information
- Gliders, spaceships, both can move. We need someway to generate them
- **Glider Gun**



# Finite State Control

- The effect of finite state control in a Turing Machine is to take an input string and generate an output string on the tape
- We can consider binary strings without loss of generality
- So, the effect of finite state control is a Boolean function, maps binary to binary

# Finite State Control

- Create Boolean functions from the Game of Life – show we can create any Boolean function.
- If we can prove that we can generate any Boolean function then we can create the function that gives a Universal TM finite-state control

# Boolean Functions

- A function from a string of 0's and 1's to another of 0's and 1's
  - $f : \{0,1\}^n \rightarrow \{0,1\}^m$

- Possible to build a function
  - $f: \{0,1\}^n \rightarrow \{0,1\}^m$, from a set of m functions $f_i:\{0,1\}^n \rightarrow \{0,1\}$
  - $f(x_1,x_2,\ldots,x_n) = (f_1(x_1,x_2,\ldots,x_n)\ ,\ f_2(x_1,x_2,\ldots,x_n)\ ,\ldots,\ f_m(x_1,x_2,\ldots,x_n))$

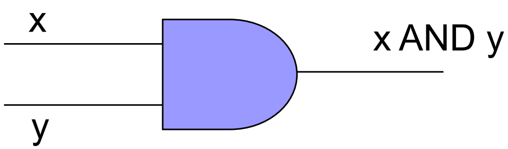
# Boolean Functions: Logic gates

- Real computer chips are built using Logic gate:
  - AND, OR, NOT, XOR, NAND, NOR, etc.
    - All map $\{0,1\}^2 \rightarrow \{0,1\}$

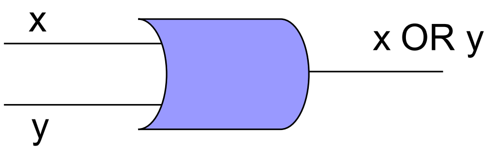
- Most important ones:
  - AND, OR, NOT

# Boolean Functions: Logic gates

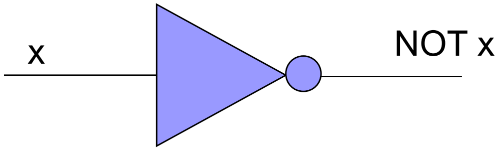
| x | y | x AND y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



| x | y | x AND y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |



| x | NOT x |
|---|---|
| 0 | 1 |
| 1 | 0 |

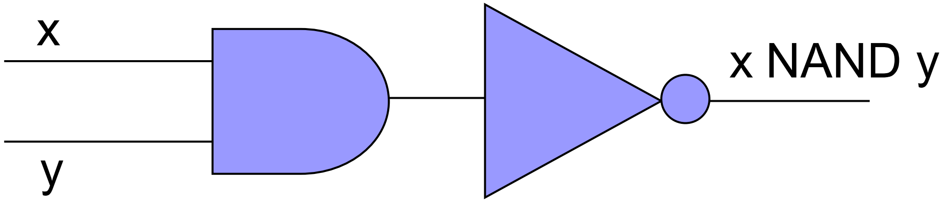


# All possible Boolean Functions?

- Can combine…

- How to create NAND? (NOT AND)

| **x** | **y** | **x NAND y**<br>**= NOT(x AND y)** |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



# Universal gates: AND, OR, NOT

- **Theorem**: AND, OR and NOT are sufficient for calculating all Boolean functions

- **Proof:** Start with example, consider a Boolean function *f* as shown in table of next slide.

# Universal gates: Proof

| $x_1$ | $x_2$ | $x_3$ | f |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- For each line where the output is a 1, we can construct a Boolean function that mimics *f*
- Line 2 would be:

  **(NOT $x_1$) AND (NOT $x_2$) AND $x_3$**
- This will have value 1 when $x_1$= 0, $x_2$= 0, and $x_3$= 1.
- Line ??

  **(NOT $x_1$) AND $x_2$ AND $x_3$**

  **$x_1$ AND $x_2$ AND $x_3$**

# Universal gates: Proof

| $x_1$ | $x_2$ | $x_3$ | f |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

- We can OR all true rules together to get *f*. *Disjunctive Normal Form* of *f*

**((NOT $x_1$) AND (NOT $x_2$) AND $x_3$) OR**
**((NOT $x_1$) AND $x_2$ AND $x_3$) OR**
**($x_1$ AND $x_2$ AND $x_3$)**

We can generalize to an arbitrary Boolean function with n inputs. Now recall f : $\{0,1\}_n$ □ $\{0,1\}_m$ can be constructed from a set of m functions $f_i$ : $\{0,1\}_n$ □ {0,1} (n=3 here). And each $f_i$ can be constructed in disjunctive normal form (as shown above). Therefore, the entire function can be built from a collection of disjunctive normal forms which are just a collection of NOT, AND, and OR gates.

# Universal gates: NAND

- **Theorem**: NAND is a universal gate by itself. We actually need only 1 gate: NAND, then we just need to show GOL can build a NAND!
- **Proof:** We can build AND, OR, NOT from NAND.

x AND y = (x NAND y) NAND (x NAND y)

x OR y = (x NAND x) NAND (y NAND y)

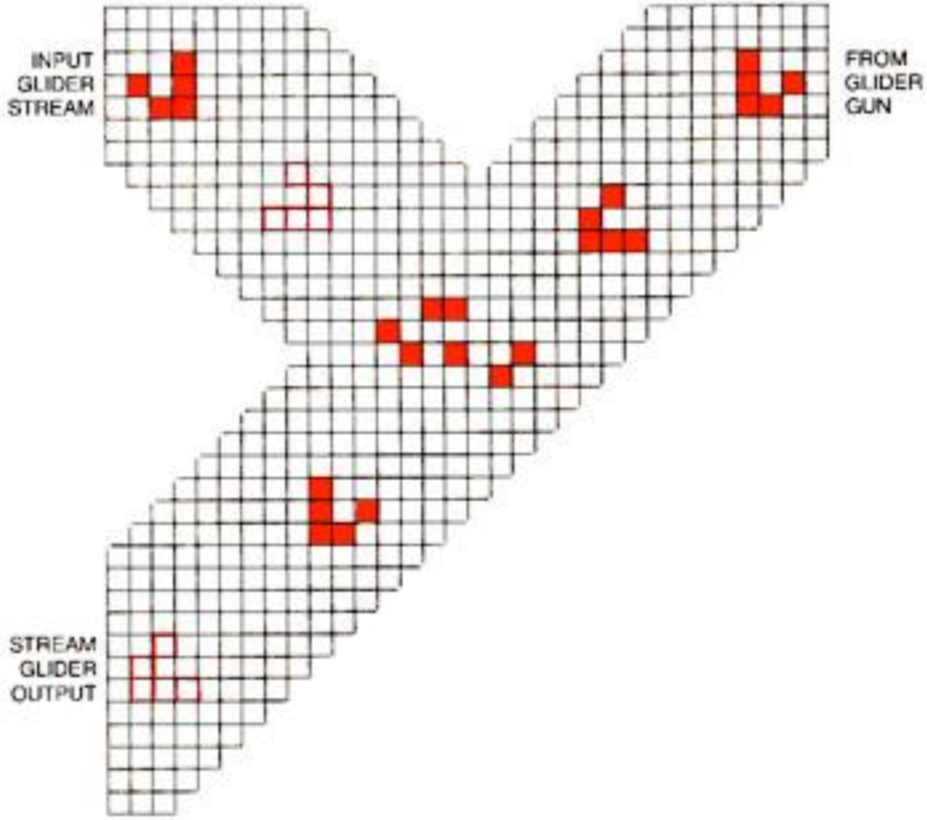
NOT x = x NAND x

# Next steps?

- We now know that a universal turing machine’s finite state control can implemented as a series of NAND gates.
- In order to prove GOL is equivalent to a UTM:
  - Clock (periodic states)
  - Memory (Stable or moving)
  - Finite State Control (?)
- This means we only now need to show that GOL can create NAND gates (NOT + AND).

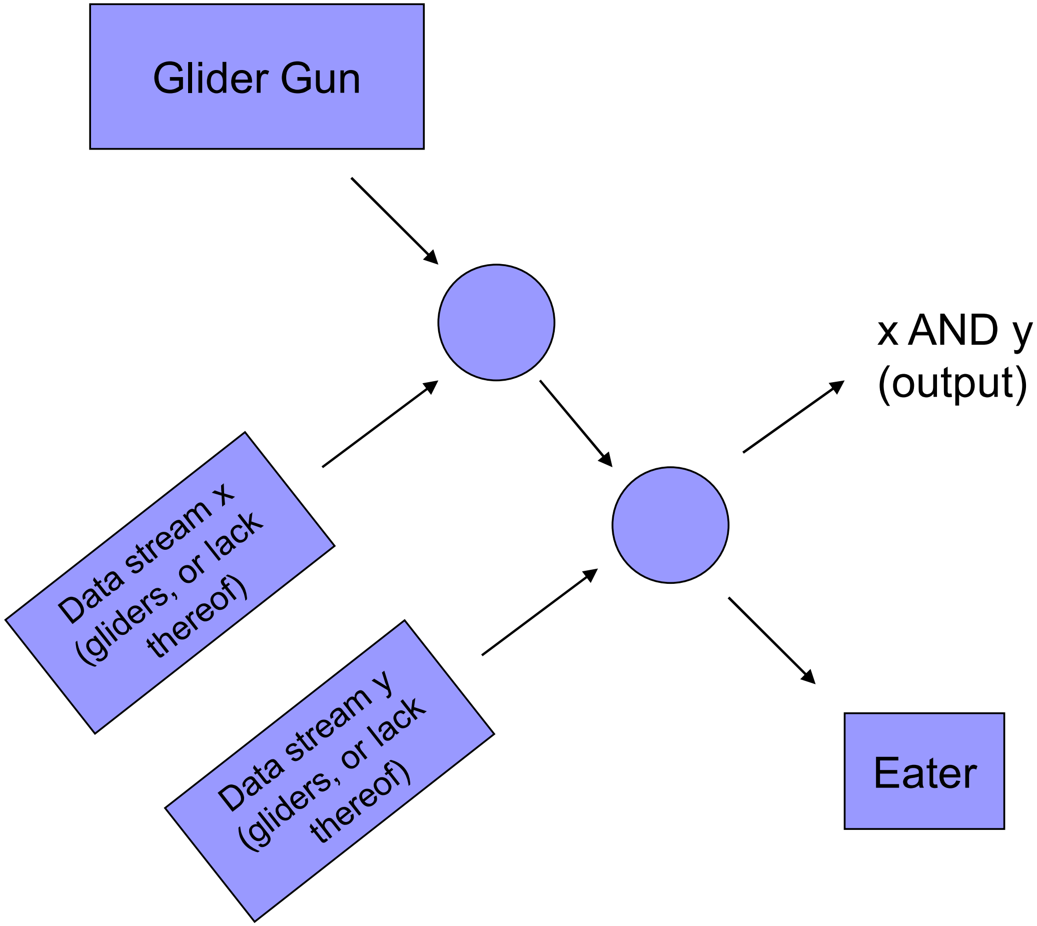
# GOL NOT gate

- Consider a stream (s1) of data constructed by gliders
  - glider =1, no glider = 0
- Create another stream (s2) of gliders that is all 1's and orient at right angles.
- Two streams collide and annihilate each other
- Second stream (s2) will kill 1's and 0's of s1 will go through as 1's



# GOL AND gate

- Two input glider streams x and y collide with all 1 glider gun.
- Circles are location of collisions
- Eater collects extras


Glider Gun
x AND y (output)
Data stream x (gliders, or lack thereof)
Data stream y (gliders, or lack thereof)
Eater

# GOL is a Universal Computer!

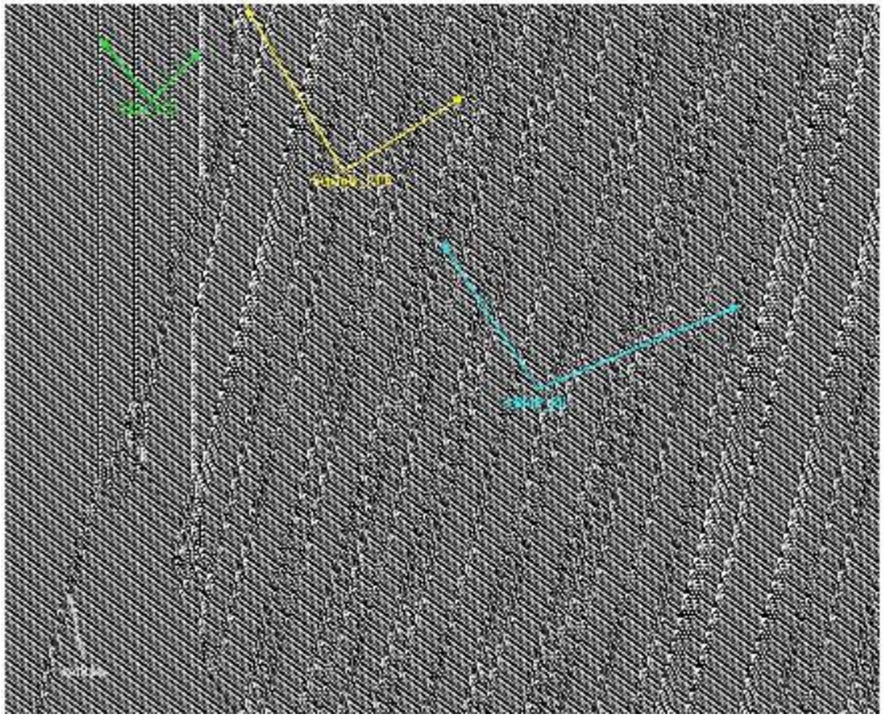
- Proof Outline:
  - Have memory (stable/periodic blocks).
  - Have AND and NOT gates which can be combined to create a NAND gate.
  - NAND gates can build any Boolean function.
  - In particular can build the Boolean function for the Universal Turing Machine's (UTM) finite state control.
  - A UTM can run any other TM.
  - All algorithms are TM's.
  - Therefore, Life can run any algorithm – it's a universal computer!

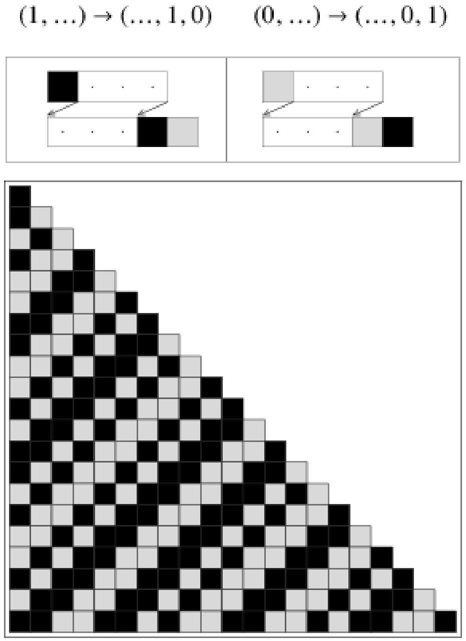
# GOL is a Universal Computer!

- This proof is abstract, but implementations have been done!
  - Paul Rendell, 2001. “A Turing Machine in Conway’s Game Life.”
    - http://rendell-attic.org/gol/tm.htm
    - The implementation is huge.
  - Chapman, 2002, constructed a full blown Universal TM.
    - http://www.igblan.free-online.co.uk/igblan/ca/
    - Requires 268,096 initially active cells on a grid roughly 2,600 by 21,500.

# Simplest Universal CA?

- ECA rule 110!
- Map R110 → Cyclic tag system → UTM





adapted from Wolfram, S. *A New Kind of Science*. Wolfram Media, p. 93, 2002.

Tag system (not cyclic!)