— QUIZ 1 —

AMATH 352:
Applied Linear Algebra & Numerical Analysis

Due to Gradescope **January 23, 2022** at 11:59pm PST

This quiz should take ∼ 1 hour to complete, but you have all day to turn it in. **Follow all directions carefully**. Credit will not be given for work that does not follow directions. You are **not** allowed to work with other students or use your notes, homework assignments, MATLAB, or external sources. Please submit your work to Quiz 1 on Canvas. Only .pdf files will be accepted.

"We don't make mistakes, just happy little accidents." – Bob Ross

Name: ____

Student ID: ____________________________________

1. (5 points total) Consider the following matrices:

$$A = \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 \\ -\frac{3}{2} & 1 \end{pmatrix} \quad C = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad D = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad E = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Use these matrices to answer the following true-false questions. You do not have to justify your choices.

(a) (1 point) $\frac{1}{2}A^2 - 3A + I = 0$, where I and 0 are the 2×2 identity and zero matrices, respectively.

True ☐ False ☐

(b) (1 point) B is the inverse matrix of A.

True ☐ False ☐

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(c) (1 point) C is an idempotent matrix, meaning $C^2 = C$.

True ☐ False ☐

(d) (1 point) The matrix product CD is not commutable.

True ☐ False ☐

(e) (1 point) Using MATLAB notation, $E = \Big[\, [\,0\ ;\ 0\,]\ ,\ \frac{1}{6}A + \frac{1}{3}B\, \Big]$.

True ☐ False ☐

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2. (5 points total) Determine the output of each of the following MATLAB codes. Put your answers in the boxes provided.

(a) (1 point)

```
x = [-3 6 -7 -12 -11 1 2 8 0 -5];
M = x(1);
for j = 1:length(x)
   if x(j)<M
      M = x(j);
   end
end
M % What is M?
```

$$\boxed{M = \square}$$

(b) (1 point)

```
x = [-3 6 -7 8 0];
y = [3 -4 -3 8 -9];
z = [0 0 0 0 0];
for j = 1:length(x)
   if x(j) < y(j)
      z(j) = x(j)+y(j);
   else % If the previous condition is not satisfied, do the following instead.
      z(j) = x(j)*y(j);
   end
end
z % What is z?
```

$$\boxed{z = \begin{pmatrix} \square & \square & \square & \square & \square \end{pmatrix}}$$

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(c) (2 points)

```
A = [0 1 1 1 1; 0 0 1 1 1; 0 0 0 1 1; 0 0 0 0 1; 0 0 0 0 0];
j = 0;
B = A;
while B(1,5) > 0
   j = j+1;
   B = B*A;
end
B % What is B?
j % What is j?
```

$$B = \begin{pmatrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{pmatrix}, \quad j = \square$$

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(d) (1 point)

```
A = [2 -2; 0 1];
B = [1 2; 0 2];
C = A*B;
D = [B;B(2,:)];
E = (D*C).^2;
E % What is E?
```

$$E = \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix}$$

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3. (5 points total) The following code computes the average *diagonal* sum of a $n \times n$ upper-triangular matrix U. (The diagonals below the main diagonal are not included in this computation since their entries all equal zero.)

```
[n,~] = size(U); % U is a given n x n upper-triangular matrix.
diagSum = zeros(n,1); % diagSum is a n x 1 column vector of zeros.
totDiagSum = 0;

for k = 1:n
   for i = 1:n-k+1 % Don't include the flops to compute n-k+1.
      diagSum(k) = diagSum(k) + U(i,k+i-1); % Don't include the flops to compute k+i-1.
   end
   totDiagSum = totDiagSum + diagSum(k);
end

avgDiagSum = totDiagSum/n; % avgDiagSum is the average diagonal sum of U.
```

(a) (3 points) Compute the flop count of the code above. Be sure to exclude the arithmetic operations mentioned in the comments above.

Hint: $\displaystyle\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$.

(b) (2 points) What is the asymptotic flop count of the code above? For large matrices U, is this code more or less expensive than back substitution?

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4. (15 points total) In fluid mechanics, the Cauchy stress tensor τ is a 3×3 matrix that can predict the force of fluid pushing against a flat surface. In particular, if $\mathbf{x} = [x_1 \ ; \ x_2 \ ; \ x_3]$ represents the normal vector to a flat surface in 3-dimensional space such that

$$||\mathbf{x}|| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

is the area of the surface, then the matrix multiplication $\tau\mathbf{x}$ represents the force vector on that surface.

Suppose for a given fluid configuration that the Cauchy stress tensor is

$$\tau = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

Use this stress tensor to answer the following questions.

(a) (3 points) Compute the force vector on a surface with normal vector $\mathbf{x} = [1 \ ; \ 1 \ ; \ 1]$.

(b) (4 points) Suppose the force vector on an unknown surface is $\mathbf{f} = [0 \ ; \ 0 \ ; \ 1]$. Derive a 3×3 system of equations for the components x_j of the normal vector to the unknown surface. Express your equations in linear system notation, in matrix-vector form, and as an augmented matrix.

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(c) (3 points) Reduce your linear system in (b) to triangular form by Gaussian elimination. (You can use the augmented matrix of the system if you prefer.)

(d) (3 points) Solve the triangular system from (c) by back substitution.

(e) (2 points) What is the normal vector of the unknown surface from (b)? What is its surface area?

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