Homework 4.

Amath 352
Applied Linear Algebra and Numerical Analysis

Due: 2/3/23 at 11:59pm to Gradescope

Directions:

Complete all component skills exercises as neatly as possible. Up to 2 points may be deducted for homework that is illegible and/or poorly organized. You are encouraged to type homework solutions, and a half bonus point will be awarded to students who use LaTeX. (Check out my LaTeX beginner document and overleaf.com if you are new to LaTeX.) If you prefer not to type homeworks, I ask that **homeworks be scanned.** (I will not accept physical copies.) In addition, **homeworks must be in .pdf format.**

Pro-Tips:

- You have access to some solutions of the textbook exercises and are encouraged to use them. Note that these solutions are not always correct, so double check your work just in case.
- Teamwork makes the dream work, but please indicate at the top of your assignment who your collaborators are.
- Don't wait until the last minute. ☺

Component Skills Exercises

Exercise 1. (CS2.5)

a. Obtain the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ by Gauss-Jordan elimination. Use this inverse to solve $\mathbf{Ax} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$.

b. Show by Gauss-Jordan elimination that the inverse of a 2×2 matrix $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\mathbf{A}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, provided $ad - bc \neq 0$.

Remark: You're going to have to consider the case when $a = 0$ separately.

Exercise 2. (CS3.1)

The set $\mathbb{R}^n$ together with the usual vector addition and scalar multiplication operations is one of the most important examples of a vector space, but there are other examples! Complete Exercise 2.1.2 in Olver and Shakiban. Be sure to show that this set, together with its vector addition and scalar multiplication operations, satisfy all eight properties of a vector space.

Exercise 3. (CS3.1)

From Olver and Shakiban, complete Exercise 2.2.2(a)-(e).

Hint: For 2.2.2(c), think how one could write this single vector as a linear combination of two vectors that don't depend on r or s.

Exercise 4. (CS3.1)

From Olver and Shakiban, complete Exercise 2.3.5(a)-(c).

Note: Feel free to use plotting software of your choice. I recommend GeoGebra. You can also try sketching these subspaces by hand, but that's tricky business.

Exercise 5. (CS3.2)

From Olver and Shakiban, complete Exercise 2.3.3(a)-(b).

Exercise 6. (CS3.2)

From Olver and Shakiban, complete Exercise 2.4.2(b)-(d).

Exercise 7. (CS3.2)

From Olver and Shakiban, complete Exercise 2.4.5(a),(c). What is the dimension of each of these planar subspaces?

NO MULTI-STEP PROBLEM OR CODING THIS WEEK! ☺