Amelia Cotter- Homework 6

1

$$A\mathbf{x} = \mathbf{b} \quad A = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Normal Equation: $\quad A^T = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}, \quad A^T A = 6, \quad \mathbf{b} = 2 \quad \rightarrow \quad 6\mathbf{x} = 2 \quad \mathbf{x} = 1/3$

QR: $\quad Q = \dfrac{A}{||A||} = \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}, \quad Q^T = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix}, \quad R = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \sqrt{6}$

$$R\mathbf{x} = Q^T\mathbf{b} \quad \rightarrow \quad \sqrt{6}\mathbf{x} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\sqrt{6}\mathbf{x} = \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}} \;=\; 6x = 3 \;\rightarrow\; \mathbf{x} = 1/3$$

2

a. MATLAB shows a warning that says the dimensions of the matrices do not agree

b. $x = \begin{pmatrix} 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0 \end{pmatrix}^T$

3

Let x=2, y=3, c=4

a.

$$F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix}$$

$$4F\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 4\begin{pmatrix} 2-3 \\ 2+3 \end{pmatrix} \qquad F\begin{pmatrix} 8 \\ 12 \end{pmatrix} = \begin{pmatrix} 8-12 \\ 8+12 \end{pmatrix} \rightarrow F\begin{pmatrix} 8 \\ 12 \end{pmatrix} = 4\begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ 20 \end{pmatrix}$$

The transformation is linear

b.

$$F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y + 1 \\ x - y - 1 \end{pmatrix}$$

$$4F\begin{pmatrix}x\\y\end{pmatrix} = 4\begin{pmatrix}x+y+1\\x-y-1\end{pmatrix} \qquad F\begin{pmatrix}4x\\4y\end{pmatrix} = \begin{pmatrix}x+y+1\\x-y-1\end{pmatrix}$$

$$4F\begin{pmatrix}2\\3\end{pmatrix} = 4\begin{pmatrix}6\\-2\end{pmatrix} \qquad F\begin{pmatrix}8\\12\end{pmatrix} = \begin{pmatrix}8+12+1\\8-12-1\end{pmatrix}$$

$$4F\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}24\\-8\end{pmatrix} \qquad F\begin{pmatrix}8\\12\end{pmatrix} = \begin{pmatrix}21\\-5\end{pmatrix}$$

The transformation is not linear

c.

$$F\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}xy\\x-y\end{pmatrix}$$

$$4F\begin{pmatrix}x\\y\end{pmatrix} = 4\begin{pmatrix}xy\\x-y\end{pmatrix} \qquad F\begin{pmatrix}4x\\4y\end{pmatrix} = \begin{pmatrix}4xy\\4x-4y\end{pmatrix}$$

$$4F\begin{pmatrix}2\\3\end{pmatrix} = 4\begin{pmatrix}6\\-1\end{pmatrix} \qquad F\begin{pmatrix}8\\12\end{pmatrix} = \begin{pmatrix}8*12*4\\4*8-4*12\end{pmatrix}$$

$$4F\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}24\\-4\end{pmatrix} \qquad F\begin{pmatrix}8\\12\end{pmatrix} = \begin{pmatrix}384\\-16\end{pmatrix}$$

The transformation is not linear

4

$$f\left(\begin{pmatrix}1\\2\\3\end{pmatrix}\right) = \begin{pmatrix}1\\2\end{pmatrix}, \quad f\left(\begin{pmatrix}1\\0\\1\end{pmatrix}\right) = \begin{pmatrix}2\\2\end{pmatrix}, \quad f\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = \begin{pmatrix}0\\-2\end{pmatrix} \quad f\left(\begin{pmatrix}2\\4\\3\end{pmatrix}\right) = ?$$

$$\left(\begin{array}{ccc|c}1 & 1 & 0 & 2\\2 & 0 & 0 & 4\\3 & 1 & 1 & 3\end{array}\right) \rightarrow \begin{array}{cc}2+x_2=2 & x_2=0\\2x_1=4 & x_1=2\\6+x_3=3 & x_3=-3\end{array}$$

$$2f\left(\begin{pmatrix}1\\2\\3\end{pmatrix}\right) - 3f\left(\begin{pmatrix}0\\0\\1\end{pmatrix}\right) = f\left(\begin{pmatrix}2\\4\\3\end{pmatrix}\right) \qquad 2\begin{pmatrix}1\\2\end{pmatrix} - 3\begin{pmatrix}0\\-2\end{pmatrix} = \begin{pmatrix}2\\10\end{pmatrix}$$

5

a. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then the zero vector in $\mathbb{R}^n$ always gets mapped to the zero vector in $\mathbb{R}^m$.

b. The two-norm of a vector in $\mathbb{R}^n$ is a linear function from $\mathbb{R}^n$ to $\mathbb{R}$.

c. Let $\mathbf{v}$ be some fixed vector in $\mathbb{R}^n$. The function $f(\mathbf{x}) = \mathbf{v}^T\mathbf{x}$ is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}$.

d. Every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ has a $m \times n$ canonical matrix representation.
T; 1st page of Lecture 18 notes

6

a. $\begin{pmatrix} \cos\pi/3 & -\sin\pi/3 \\ \cos\pi/3 & \sin\pi/3 \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$

b. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

c. $\begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$

Multi-step Problem

b. Coefficients are -4.8, 297.7, 127.2
c. The rocket was launched 4.8m below sea level with initial velocity 297.7m/s and acceleration due to gravity of 127.2m/s^2
d. The rocket reached the pinnacle of its trajectory at 29.7s