— QUIZ 2 —

AMATH 352:
Applied Linear Algebra & Numerical Analysis

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Due to Gradescope **February 6, 2023** at 11:59pm PST

This quiz should take $\sim$ 1 hour to complete, but you have all day to turn it in. **Follow all directions carefully**. Credit will not be given for work that does not follow directions. You are **not** allowed to work with other students or use your notes, homework assignments, MATLAB, or external sources. Please submit your work to Quiz 2 on Gradescope. Only .pdf files will be accepted.

Good luck!

Name: ____

Student ID: ____

1. (5 points total) Answer the following true-false questions.

For (a)-(b), consider the following matrices:

$$E_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}.$$

(a) (1 point) $E_1 E_2 E_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}.$

True ☐ False ☐

(b) (1 point) $(E_1 E_2 E_3)^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -4 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$

True ☐ False ☐

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For (c)-(e), consider the following system:

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & -1 & 0 \\ -1 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

(c) (1 point) This system has a unique solution.

True ☐ False ☐

(d) (1 point) The coefficient matrix of this system has a LU factorization.

True ☐ False ☐

(e) (1 point) The dimension of Span $\left\{ \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} \right\}$ is 3.

True ☐ False ☐

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2. (5 points total) Consider the following matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 4 & 1 \\ -1 & 1 & 2 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

(a) (4 points) Obtain the LU factorization of $\mathbf{A}$.

$$L = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}, \quad U = \begin{pmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{pmatrix}$$

(b) (1 point) Based on this LU factorization, is $\mathbf{A}$ a full rank matrix? Why or why not? **This should be a one sentence answer.**

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3. (5 points total) Consider the following matrix:

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 2 & 4 & 4 \\ 0 & 1 & -7 & 2 & 3 \\ 1 & 4 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 & 2 \end{pmatrix}.$$

(a) (4 points) Obtain the PLU factorization of $\mathbf{A}$. Use partial pivoting to determine whether to exchange rows.

$$P = \begin{pmatrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{pmatrix}, \quad L = \begin{pmatrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{pmatrix}, \quad U = \begin{pmatrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{pmatrix}$$

(b) (1 point) Based on this PLU factorization, what is the dimension of the span of $\mathbf{A}$'s columns?

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4. (5 points total) Consider the system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(a) (1 point) Reduce $\mathbf{A}$ to an upper-triangular matrix. Is $\mathbf{A}$ invertible?

(b) (2 points) Compute $\mathbf{A}^{-1}$ by Gauss-Jordan elimination. Show each elimination step!

$$\boxed{\mathbf{A}^{-1} = \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}}$$

(c) (1 point) Use your inverse from (b) to solve the linear system.

$$\boxed{\mathbf{x} = \begin{pmatrix} \square \\ \square \\ \square \end{pmatrix}}$$

(d) (1 point) For large $n \times n$ matrices, Gauss-Jordan elimination is more expensive than computing the LU decomposition.

True ☐ False ☐

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5. (10 points total) Consider the following set:

$$\mathcal{S} = \left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} \right\}.$$

(a) (5 points) Is $\begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix} \in \text{Span}\{\mathcal{S}\}$? If so, what linear combination of the vectors in $\mathcal{S}$ results in $\begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$?

(b) (4 points) Are the vectors in $\mathcal{S}$ linearly independent? If not, which vector(s) are linearly dependent and which are linearly independent according to row-echelon form?

(c) (1 point) Which of the following describes $\text{Span}\{\mathcal{S}\}$ geometrically?

- $\text{Span}\{\mathcal{S}\}$ is $\mathbb{R}^3$.
- $\text{Span}\{\mathcal{S}\}$ is a plane through the origin in $\mathbb{R}^3$.
- $\text{Span}\{\mathcal{S}\}$ is a line through the origin in $\mathbb{R}^3$.

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