AMATH 353
Homework #8

Show your work to earn credit ! Due on Wednesday, May 24, 2023

1. (Knobel, 13.1)

   Show that the principle of superposition does not apply to solutions of the boundary value problem

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < L, \quad t > 0,$$

$$u(0,t) = 1, \quad u(L,t) = 0 .$$

2. (Knobel, 13.3)

   Find the solutions of

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \quad t > 0,$$

$$u(0,t) = 0, \quad u(1,t) = 0$$

   that satisfy the following initial conditions:

   (a) $u(x,0) = 10 \sin \pi x + 3 \sin 4\pi x, \quad u_t(x,0) = 0$,

   (b) $u(x,0) = \sin 2\pi x, \quad u_t(x,0) = -3 \sin 2\pi x$.

3. (from Knobel, 14.2)

   Find the Fourier sine expansion for the function

$$f(x) = x(1-x),$$

   on the interval [0, 1]. Be sure to show your work for full credit.

4. (Knobel, 14.6)

   Consider the displacement of a vibrating string with fixed ends given by

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \quad t > 0,$$

$$u(0,t) = 0, \quad u(1,t) = 0 .$$

   Use the Fourier sine series found in exercise 3 (above) to write down the solution of the vibrating string with the following initial conditions

   (a) $u(x,0) = x(1-x), \quad u_t(x,0) = 0$,

   (b) $u(x,0) = 0, \quad u_t(x,0) = x(1-x)$.