AMATH 353
Homework #7

Show your work to earn credit ! Due on Wednesday, May 17, 2023

1. Find and simplify d'Alembert's solution for the wave equation

$$u_{tt} = u_{xx}, \quad 0 < x < \infty, \quad t > 0,$$

for a semi-infinite string with initial conditions

$$u(x,0) = 0, \quad 0 \le x < \infty,$$

$$u_t(x,0) = x\,e^{-x^2}, \quad 0 \le x < \infty,$$

and free boundary condition

$$u_x(0,t) = 0, \quad t \ge 0.$$

2. Use characteristics to construct an xt-diagram for the solution of the wave equation

$$u_{tt} = u_{xx}, \quad 0 < x < \infty, \quad t > 0,$$

for a semi-infinite string with initial conditions

$$u(x,0) = \begin{cases} 1, & 0 \le x < 1, \\ 0, & \text{otherwise}, \end{cases}$$

$$u_t(x,0) = 0, \quad 0 \le x < \infty,$$

and free boundary condition

$$u_x(0,t) = 0, \quad t \ge 0.$$

(What happens to your characteristics at $x = 0$?)

3. If a string of length L has its left end fixed but its right end free, the boundary condition at $x = L$ changes to $u_x(L,t) = 0$. Find the standing-wave solutions of

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < L, \quad t > 0,$$

$$u(0,t) = 0, \quad u_x(L,t) = 0, \quad t \ge 0.$$

4. Find the standing-wave solutions of a finite string with both ends free,

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < L, \quad t > 0,$$

$$u_x(0,t) = 0, \quad u_x(L,t) = 0, \quad t \ge 0.$$