AMATH 353
Homework #6

Show your work to earn credit ! Due on Wednesday, May 10, 2023

1 Use characteristics to construct an xt-diagram for the solution of

$$u_{tt} = u_{xx}, \ -\infty < x < \infty, \ t > 0,$$

$$u(x,0) = \begin{cases} 1, & 0 \le x \le 1, \\ -1, & 1 < x \le 2, \\ 0, & \text{otherwise}, \end{cases}$$

$$u_t(x,0) = 0, \ -\infty < x < \infty.$$

2. Use characteristics and an xt-diagram to solve the initial-value problem

$$u_{tt} = u_{xx}, \ -\infty < x < \infty, \ t > 0,$$

$$u(x,0) = 0, \ -\infty < x < \infty,$$

$$u_t(x,0) = \begin{cases} 0, & x < 0, \\ 1, & x \ge 0. \end{cases}$$

Plot a snapshot of your solution for $t = 1$.

3. Find and simplify d'Alembert's solution for the wave equation

$$u_{tt} = u_{xx}, \ 0 < x < \infty, \ t > 0,$$

for a semi-infinite string with initial conditions

$$u(x,0) = 0, \ 0 \le x < \infty,$$

$$u_t(x,0) = x e^{-x^2}, \ 0 \le x < \infty,$$

and fixed boundary condition

$$u(0,t) = 0, \ t \ge 0.$$

4. Use characteristics and a first-quadrant xt-diagram to solve

$$u_{tt} = u_{xx}, \ 0 < x < \infty, \ t > 0,$$

for a semi-infinite string with the initial conditions

$$u(x,0) = \begin{cases} 0, & 0 \le x \le 1, \\ 1, & 1 < x < 2, \\ 0, & x \ge 2, \end{cases}$$

$$u_t(x,0) = 0, \ 0 \le x < \infty,$$

and the fixed boundary condition

$$u(0,t) = 0, \ t \ge 0.$$

Keep in mind that characteristics bounce and flip (with a sign change) at the boundary $x = 0$.