AMATH 353
Homework #3

Show your work to earn credit ! Due on Wednesday, April 19, 2023

1. Consider the Klein–Gordon equation,

$$u_{tt} = a^2 u_{xx} - b\,u\ ,$$

where $b > 0$. Assume that this equation has traveling-wave solutions

$$u(x,t) = f(z) = f(x - ct)\ .$$

What ODE does $f(z)$ satisfy ? Solve this ODE and find the traveling-wave solutions of the Klein–Gordon equation. Are these solutions wave fronts, pulses, wave trains, or none of the above ?

2. For each of the following partial differential equations, find the dispersion relation and the phase speed using wave-train solutions of the form

$$u(x,t) = A\cos(kx - \omega t)\ .$$

Then determine if each equation is dispersive or not.

(a) $u_{tt} = a^2 u_{xx}$

(b) $u_t + u_x + u_{xxx} = 0$

3. For each of the following partial differential equations, find the dispersion relation and the phase speed using the complex wave train

$$u(x,t) = e^{i(kx - \omega t)}\ .$$

Then determine if each equation is dispersive or not.

(a) $u_t + u_x = u_{xx}$

(b) $i\,u_t + 2\,u_{xx} = 0$

4. Consider a medium in which $c = A\,\omega^n$, where c is the phase velocity and A and n are constants, with A positive.

(a) When is the medium nondispersive ? For which values of n is dispersion normal ? Anomalous ?

(b) Show that

$$c_g = \frac{c}{(1-n)}\ ,$$

where c_g is the group velocity.