

## AMATH 353 Homework #2

Show your work to earn credit ! Due on Wednesday, April 12, 2023

1. Suppose that, at time  $t$ , the value of  $u$  at position  $x$  is given by

$$u = e^{-(x-t)^2}.$$

This function describes a wave that moves to the right with constant speed. Modify  $u(x, t)$  so that

- (a) the wave moves to the left,
- (b) the wave moves to the left with increasing speed,
- (c) the amplitude (height) of the wave decreases as the wave moves to the right.

Illustrate one of the above cases, using two (or more) of the techniques that we discussed in class.

2. In class, I gave six criteria that can be used to classify PDEs. These criteria were order, number of independent variables, linearity, and, for linear equations, kinds of coefficients, homogeneity, and basic type. Use these criteria to classify the equations

- (a)  $u_t + u_{tt} = u_{xx} + 2u_x + u$ ,
- (b)  $u_{xx} + 2u_{xy} + u_{yy} = \cos x$ ,
- (c)  $u_{xx} + u_{yy} = \cos u$ ,
- (d)  $u_{tt} = u u_{xxx} + e^{-t}$ .

3. Give an example of

- (a) a linear, nonhomogeneous PDE, for  $u(x, t)$ , that is second-order in time, and fourth-order in space ( $x$ ),
- (b) a nonlinear PDE, for  $u(x, t)$ , that is first-order in time and third-order in space ( $x$ ).

4. Suppose a string is stretched horizontally and then plucked. Let  $u(x, t)$  represent the vertical displacement of the string at position  $x$  and time  $t$ .

- (a) Give physical and/or graphical interpretations of the partial derivatives  $u_t(x, t)$  and  $u_{tt}(x, t)$ . Give graphical interpretations of  $u_x(x, t)$  and  $u_{xx}(x, t)$ .
- (b) Suppose  $u(x, t)$  satisfies the wave equation  $u_{tt} = a^2 u_{xx}$ . What is an interpretation of the wave equation in terms of acceleration and concavity?