# AMATH 353 HW # 1

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April 5, 2023

1. Evaluate

$$y = \frac{\partial}{\partial x}\cos(e^{tx}) + \frac{\partial}{\partial t}[e^{2t}\sin(x - 3t)] \tag{1}$$

Chain Rule $\frac{d}{dx}f(g(x)) = f'(g(x)) * g'(x)$
Product Rule $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

Solution:
$y = -\sin(e^{tx})(te^{tx}) + [2e^{2t}\sin(x - 3t) + 3e^{2t}\cos(x - 3t)]$

2. Find the general solution of the linear, first-order differential equation

$$\frac{dx}{dt} + -\frac{2}{t}x = t^2\cos(t) \tag{2}$$

using an integrating factor

$f'(t) + p(t)f(t) = h(t)$
Integration Factor = $e^{\int p(t)dt} = e^{\int -2t^{-1}dt} = e^{-2ln(t)} = \frac{1}{t^2}$

General Solution Formula:
$x(t) = \frac{1}{g(t)}\int g(t)h(t)$
$x(t) = \frac{1}{\frac{1}{t^2}}\int \frac{1}{t^2}t^2\cos(t)dx$
$x(t) = t^2(sin(t) + C)$
General Solution:
$x(t) = t^2 sin(t) + Ct^2$

3. Solve the initial-value problem

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0 \quad y(0) = 2, y'(0) = 1. \tag{3}$$

Constant Coefficients Second Order Homogeneous.
$y(x) = e^{\lambda x} y'(x) = \lambda e^{\lambda x} y''(x) = \lambda^2 e^{\lambda x}$
$(\lambda^2 e^{\lambda x} + -1\lambda e^{\lambda x} - 12e^{\lambda x}) = 0$
$e^{\lambda x}(\lambda^2 - \lambda - 12) = 0$
$(\lambda - 4)(\lambda + 3) = 0$
$\lambda = 4, -3$
$y(x) = C_1 e^{4x} + C_2 e^{-3x}$
Two Unknowns Two Condition
$y(0) = 2 = C_1 e^{4(0)} + C_2 e^{-3(0)}$
$2 = C_1 + C_2$
$y'(x) = 4C_1 e^{4x} - 3C_2 e^{-3x}$
$y'(0) = 1 = 4C_1 e^{4(0)} - 3C_2 e^{-3(0)}$
$1 = 4C_1 - 3C_2$
$1 = 4C_1 - 3(2 - C_1)$
$1 = 4C_1 - 6 + 3C_1$
$7 = 7C_1$
$C_1 = 1, C_2 = 2 - 1 = 1$
Solution:
$y(x) = 1e^{4x} + 1e^{-3x}$

4. Find the general solution of the linear, nonhomogeneous, second-order differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dx}{dy} + 5y = 5x + 6 \tag{4}$$

Write your general solution in terms of real-valued (not complex-valued) functions.

$\frac{d^2y}{dx^2} - 4\frac{dx}{dy} + 5y = 5x + 6$
Homogeneous Solution $y_h$
$y'' - 4y' + 5y = 0$
$(\lambda^2 e^{\lambda x} + -4\lambda e^{\lambda x} + 5e^{\lambda x})$
$\lambda^2 - 4\lambda + 5 = 0$ Not Easily Factorable
$\lambda = \frac{(4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)} = \frac{4\pm\sqrt{4}}{2} = 2 \pm i$
$\lambda = 2 + i$ and $2 - i$
Plug into Equation:
$C_1 e^{2x} sin(1x) + C_2 e^{2x} cos(1x)$

Nonhomogenous Solution $y_h + y_c$
Forcing Term $= 5x + 6$
$y(x) = Ax + B, y'(x) = A, y''(x) = 0$
$(5)Ax + (5)B + (-4)A = 5x + 6$
x terms $5A = 5 \quad A = 1$
constant terms $5B - 4A = 6 \quad 5B = 10 \quad B = 2$

Solution:
$y(x) = C_1 e^{2x} sin(x) + C_2 e^{2x} cos(x) + x + 2$

5. Find the general solution of the linear, nonhomogeneous, second-order differential equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t \tag{5}$$

Homogenous Part $x_h$ $x'' - 2x' + x = 0$
$(\lambda^2 - 2\lambda + 1) = 0$ $(\lambda - 1)^2 = 0$
$\lambda = 1 \pm 0i$
General Homogeneous Second Order Solution:
$x_h = C_1 e^t t + C_2 e^t$

Nonhomogenous Part $x_c$
Since repeated Value Multiply Homogenous part by t again
$x_h + x_c = C_1 e^t t^2 + C_2 e^t t + Ae^t$
$x(t) = C_1 e^t t^2 + C_2 e^t t + Ae^t$
$x'(t) = C_1 e^t t^2 + 2C_1 e^t t + C_2 e^t t + C_2 e^t + Ae^t$
$x''(t) = C_1 e^t t^2 + 2C_1 e^t t + 2C_1 e^t t + 2C_1 e^t + C_2 e^t t + C_2 e^t + C_2 e^t + Ae^t$
$((1)x''(t) - 2x'(t) + (1)x(t)) = e^t$
Plug in!
$C_1 e^t t^2 + 2C_1 e^t t + 2C_1 e^t t + 2C_1 e^t + C_2 e^t t + C_2 e^t + C_2 e^t + Ae^t (-2)C_1 e^t t^2 + (-2)2C_1 e^t t + (-2)C_2 e^t t + (-2)C_2 e^t + (-2)Ae^t + C_1 e^t t^2 + C_2 e^t t + Ae^t = e^t$
Simplifies to
$2 * C_1 e^t = e^t$
$C_1 = \frac{1}{2}$ Thats all that can be done with information provided

General Solution: $y(x) = \frac{1}{2} e^t t^2 + C_2 e^t t + C_3 e^t$