AMATH 353 HW # 2

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1. Suppose that, at time t, the value of u at position x is given by

$$u = e^{-(x-t)^2}. \tag{1}$$

This function describes a wav e that moves to the right with constant speed. Modify $u(x,t)$ so that

(a) the wave moves to the left,

$$\boxed{u = e^{-(x+t)^2}.}$$

(b) the wave moves to the left with increasing speed,

$$\boxed{u = e^{-(x+t^2)^2}.}$$

(c) the amplitude (height) of the wave decreases as the wave moves to the right.

$$\boxed{u = \frac{1}{t}e^{-(x-t)^2}.}$$

Illustrate one of the above cases, using two (or more) of the techniques that we discussed in class.
see next page

Figure 1, Graph of $u(x,t) = e^{-(x+t)^2}$at specific time intervals

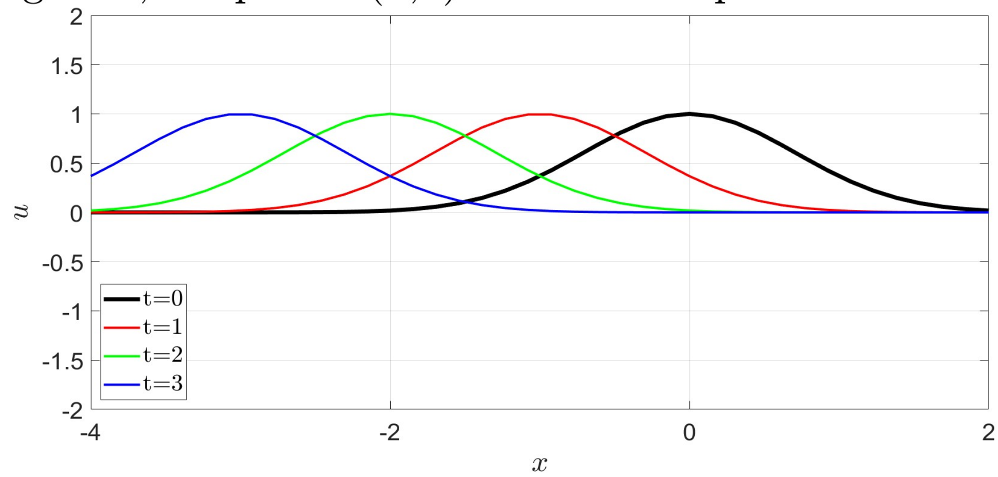


Figure 2, xt-diagram plot of $u(x,t) = e^{-(x+t)^2}$

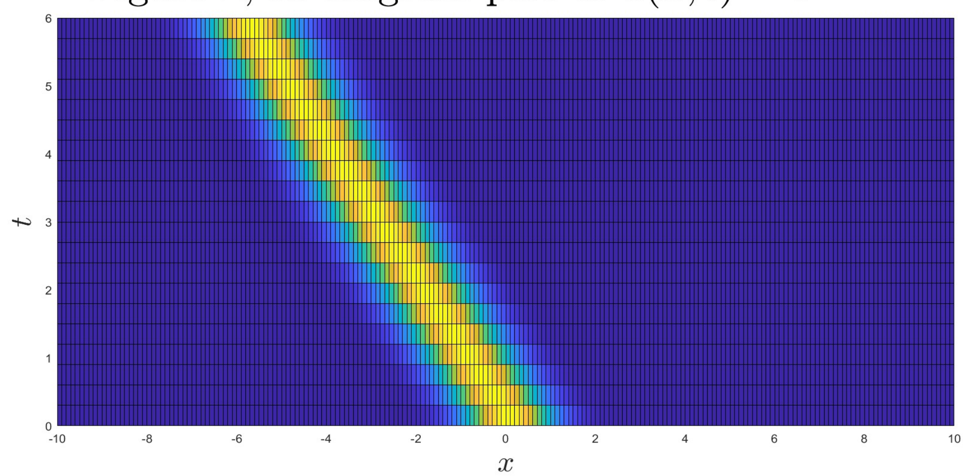
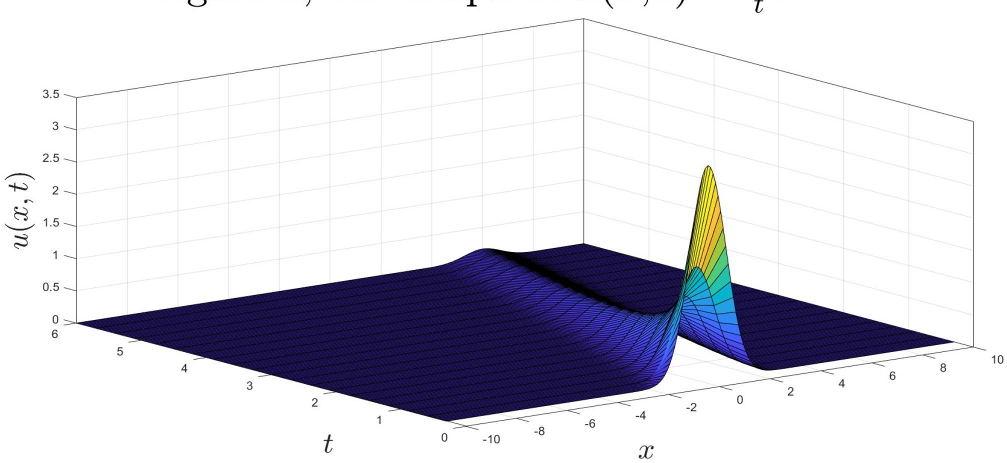


Figure 3, 3D Graph of $u(x,t) = \frac{1}{t}e^{-(x-t)^2}$



2. In class, I gave six criteria that can be used to classify PDEs. These criteria were order, number of independent variables, linearity, and, for linear equations, kinds of coefficients, homogeneity, and basic type. Use these criteria to classify the equations

   (a) $u_t + u_{tt} = u_{xx} + 2u_x + u$,

   > $u_{tt} - u_{xx} + u_t - 2u_x - u = 0$
   > Second Order, Two Independent Variables x and t, Linear, Homogeneous Constant Coefficients. Since the discriminant $b^2 - 4ac = 1^2 - 4 * 1 * 0 = 1 > 0$ which is a hyperbolic equation.

   (b) $u_{xx} + 2u_{xy} + u_{yy} = \cos(x)$,

   > $u_{xx} + 2u_{xy} + u_{yy} = \cos(x)$
   > Second Order, Two Independents Variables x and y, Linear, Non-Homogeneous, Constant Coefficients. Since the discriminant $b^2 - 4ac = 2^2 - 4 * 1 * 1 = 4 - 4 = 0 = 0$ this is a parabolic equation.

   (c) $u_{xx} + u_{yy} = \cos(u)$,

   > $u_{xx} + u_{yy} - \cos(u) = 0$
   > Second Order, 2 Independent Variables, Non-Linear.

   (d) $u_{tt} = u * u_{xxxx} + e^{-t}$.

   > $u_{tt} - u * u_{xxxx} = e^{-t}$
   > Two Independent Variables, Fourth Order with respect to x, Second Order with respect to t, Fourth Order overall, Non-Linear.

3. Give an example of

   (a) a linear, nonhomogeneous PDE, for $u(x, t)$, that is second-order in time, and fourth-order in space (x),

   > $u_{xxxx} + u_{tt} = e^{(x-t)}$

   (b) a nonlinear PDE, for $u(x, t)$, that is first-order in time and third-order in space (x).

   > $u_{xxx} + u * u_t^2 = 0$

4. Suppose a string is stretched horizontally and then plucked. Let $u(x,t)$ represent the vertical displacement of the string at position x and time t.

(a) Give physical and/or graphical interpretations of the partial derivatives $u_t\ (x,t)$ and $u_{tt}(x,t)$. Give graphical interpretations of $u_x\ (x,t)$ and $u_{xx}\ (x,t)$.

$u_t\ (x,t)$ The velocity of a point on the string at position x. For Example if $u_t(1,t) = 2t$ the at point $x = 2$ the wave height of the string will be increasing at $2t$ per unit time.

$u_{tt}\ (x,t)$ The acceleration of a point on the string at position x. Tells us the acceleration of the vertical displacement of the string.

$u_x\ (x,t)$ The slope of the physical string at position x at a specific time t. Basically the slope of the tangent line of point on the string at time t.

$u_{xx}\ (x,t)$ The concavity of the physical string at position x.

(b) Suppose $u(x,t)$ satisfies the wave equation $u_{tt} = a^2 u_{xx}$. What is an interpretation of the wave equation in terms of acceleration and concavity?

$u_{tt} = a^2 u_{xx}$
The wave equation demonstrates that the acceleration of the wave's vertical displacement is proportional to the concavity of the wave. If the wave is concave up it will accelerate proportionally upwards. If the wave is concave down it will accelerate proportionally downwards.