AMATH 353
Homework #9

Show your work to earn credit ! Due on Wednesday, May 31, 2023.

1. In class, we considered a model for traffic velocity (as a function of traffic density u) that took the form $v(u) = v_1[1 - (u/u_1)]$. A second, different model of traffic velocity is $v(u) = k \ln(u_1/u)$, where k is a positive constant.

   (a) Draw a graph of both functions and describe how the driver-behavior described by the second model differs from the driver-behavior of the first (linear) model.

   (b) For the second velocity function, what is the flux ϕ and the conservation law for traffic density $u(x,t)$? (For the conservation law, assume that there are no sources or sinks, i.e., $f = 0$.)

2. Consider the initial-value problem

$$u_t + t\,x\,u_x = 0\,, \quad -\infty < x < \infty\,, \quad t > 0\,,$$

$$u(x,0) = \frac{1}{1+x^2}\,.$$

   (a) Use the method of characteristics to rewrite your PDE as a system of three first-order ordinary differential equations for t, x, and u. What are the initial conditions for these equations ?

   (b) Solve this system of ODEs to get an exact solution of your PDE.

   (c) What are the characteristics for this problem ? Draw the characteristics in the (x, t) plane.

3. Consider the initial-value problem

$$u_t + x\,u_x + u = 0\,, \quad -\infty < x < \infty\,, \quad t > 0\,,$$

$$u(x,0) = x\,.$$

   (a) Use the method of characteristics to rewrite your PDE as a system of three first-order ordinary differential equations for t, x, and u. What are the intial conditions for these equations ?

   (b) Solve this system of ODEs to get an exact solution of your PDE.

   (c) What are the characteristics for this problem ? Draw the characteristics in the (x, t) plane.