Homework 7.

Amath 383
Introduction to Continuous Mathematical Modeling

Due: 11/29/22 at 11:59pm to Gradescope

Directions:

Complete all exercises as neatly as possible. Up to 3 points may be deducted for homework that is illegible and/or poorly organized. You are encouraged to type homework solutions, and a half bonus point will be awarded to students who use LaTeX. (Check out my LaTeX beginner document and overleaf.com if you are new to LaTeX.) If you prefer not to type homeworks, I ask that **homeworks be scanned.** (I will not accept physical copies.) In addition, **homeworks must be in .pdf format.**

Pro-Tips:

- You have access to the textbook, which inspired some of these exercises. You may find that the textbook offers alternative explanations that can help you solve these exercises.
- If a result was already derived in class, no need to rederive it here. Just make sure you cite where in the lecture notes the result your using can be found.
- Teamwork makes the dream work, but please indicate at the top of your assignment who your collaborators are.
- Don't wait until the last minute. ☺

Exercise 1. (Unit 3.1)

Suppose you borrow $\$120,000$ from a bank to cover your UW tuition. The bank charges a 5% interest rate that will be compounded daily.

a. Suppose you pay the bank $\$1,500$ per month. How many years will it take you to pay off your loan?

120,000 5 % interest.

$P_n = P_0(1+r)^{\frac{365n}{12}} - \frac{M(1-(1+r)^{\frac{365n}{12}})}{1-(1+r)^{\frac{365}{12}}}$

$0 = 120,000(1+\frac{0.05}{365})^{\frac{365n}{12}} - \frac{1500(1-(1+\frac{0.05}{365})^{\frac{365n}{12}})}{1-(1+\frac{0.05}{365})^{\frac{365}{12}}}$

$\boxed{n = 8.13 \text{ years}}$

b. What should your monthly payment be if you intend to pay off your loan in 4 years? How much money did you you pay to the bank in addition to the $\$120,000$ you borrowed because of the compounding interest over these 4 years?

Your monthly payment to be intended if you plan to pay off your loan in 4 years is

$M = \frac{P_0(1+r)^{\frac{365n}{12}}(1-(1+r))^{\frac{365}{12}}}{1-(1+r)^{\frac{365n}{12}}}$

$M = \frac{120,000(1+(\frac{0.05}{365})^{\frac{365*48}{12}}(1-(1+(\frac{0.05}{365}))^{\frac{365}{12}}}{1-(1+(\frac{0.05}{365}))^{\frac{365*48}{12}}}$

$\boxed{M = \$2,764.06}$

The total amount of money paid to the bank. Assuming there are 365 days in every year and no leap years for the four years.

$M * 48 \text{ months} = \$2764.06 * 48 = \$132,672$

$\$146,566.32 - \$120,000 = \boxed{\$12,672}$

Exercise 2. (Units 3.1)

Let p_n represent the change in price of a brand new Play Station 6 (PS6) n months after its launch day [1]. Let D_n represent the corresponding demand for a PS6 also measured n months after launch day[2]. Assume D_n is negatively proportional to the current change in price:

$$D_n = -\alpha p_n,$$

where α is a positive constant. Thus, if the price of a PS6 increases, the demand decreases.

Let S_n represent the supply of PS6 units n months after launch day[3]. Assume S_n is positively proportional to the change in price of the PS6 one month prior plus a constant rate of production of new PS6 units:

$$S_n = \beta p_{n-1} + \gamma,$$

where β and γ are positive constants. In other words, if the price of the PS6 increases in the previous month, manufacturers of the PS6 are inclined to produce more units during the current month to make more profit.

From these two laws, we can derive dynamics for the fluctuations in price of the PS6 over time.

a. In an ideal world, supply and demand are equal to each other. Assuming this is true, derive a first-order difference equation for p_n. Write your difference equation in the form $p_{n+1} = f(p_n)$. You may have to shift your n variable.

> Supply and Demand are equal to each other.
> $S_n = D_n$ Substitute in for S_n and D_n.
> $\beta p_{n-1} + \gamma = -\alpha p_n \rightarrow \beta p_n + \gamma = -\alpha p_{n+1} \rightarrow p_{n+1} = -\frac{1}{\alpha}(\beta p_n + \gamma)$
> $\boxed{P_n = -\frac{\beta}{\alpha} p_n + -\frac{\gamma}{\alpha}}$

b. Solve your difference equation from (a) subject to the initial condition $p_0 = 0$.

[1] Thus, if $p_n < 0$, the price on month n is less than the price on launch day, and conversely, if $p_n > 0$, the price on month n is greater than the price on launch day.

[2] In other words, D_n is the change in the number of people interested in purchasing a PS6 n months after launch day. If $D_n < 0$, the demand on month n is less than the demand on launch day, and conversely, if $D_n > 0$, the demand on month n is greater than the demand on launch day.

[3] In other words, S_n is the change in the number of PS6 units available to be purchased n months after launch day. If $S_n < 0$, the supply on month n is less than the supply on launch day, and conversely, if $S_n > 0$, the supply on month n is greater than the supply on launch day.

$p_{n+1} = \frac{1}{\alpha}(-\beta p_n - \gamma) \rightarrow p_{n+1} + \frac{\beta}{\alpha} p_n = -\frac{\gamma}{\alpha} \rightarrow p_{n+2} + \frac{\beta}{\alpha} p_{n+1} = -\frac{\gamma}{\alpha}$
Subtract Equations to get second order homogeneous
$-p_{n+2} + -\frac{\beta}{\alpha} p_{n+1} + p_{n+1} + \frac{\beta}{\alpha} p_n = 0 \rightarrow -p_{n+2} + (-\frac{\beta}{\alpha} + 1)p_{n+1} + \frac{\beta}{\alpha} p_n = 0 \rightarrow$ Multiply by -1?
$p_{n+2} + (\frac{\beta}{\alpha} - 1)p_{n+1} - \frac{\beta}{\alpha} p_n = 0 \rightarrow \lambda^2 + (\frac{\beta}{\alpha} - 1)\lambda - \frac{\beta}{\alpha} = 0$
Quadratic Formula
$\frac{1}{2}\left(-(\frac{\beta}{\alpha} - 1) \pm \sqrt{(\frac{\beta}{\alpha} - 1)^2 - 4(1)(-\frac{\beta}{\alpha}}\right) \rightarrow \frac{1}{2}\left(-\frac{\beta}{\alpha} + 1 \pm \sqrt{\frac{\beta^2}{\alpha^2} - 2\frac{\beta}{\alpha} + 1 + 4\frac{\beta}{\alpha}}\right) \rightarrow$
$\frac{1}{2}\left(-\frac{\beta}{\alpha} + 1 \pm \sqrt{\frac{\beta^2}{\alpha^2} + 2\frac{\beta}{\alpha} + 1}\right) \rightarrow \frac{1}{2}\left(-\frac{\beta}{\alpha} + 1 \pm \sqrt{(\frac{\beta}{\alpha} + 1)^2}\right) \rightarrow \frac{1}{2}\left(-\frac{\beta}{\alpha} + 1 \pm (\frac{\beta}{\alpha} + 1)\right)$
Plus solution to quadratic
$\lambda_1 = \frac{1}{2}\left(-\frac{\beta}{\alpha} + 1 + (\frac{\beta}{\alpha} + 1)\right) \rightarrow \frac{1}{2}(1+)) \rightarrow \frac{1}{2}(2) \rightarrow (1)$
Minus solution to quadratic
$\lambda_2 = \frac{1}{2}\left(-\frac{\beta}{\alpha} + 1 - (\frac{\beta}{\alpha} + 1)\right) \rightarrow \frac{1}{2}\left(-2\frac{\beta}{\alpha}\right) \rightarrow \left(-\frac{\beta}{\alpha}\right)$
General Solution
$P_n = C_1\lambda_1^n + C_2\lambda_2^n \rightarrow P_n = C_1(1)^n + C_2(-\frac{\beta}{\alpha})^n$
Initial Condition $p_0 = 0$
$p_0 = 0 = C_1(1)^0 + C_2(-\frac{\beta}{\alpha})^0 \rightarrow p_0 = 0 = C_1 + C_2 \rightarrow C_1 = -C_2$
Initial Condition $p_1 = -\frac{\gamma}{\alpha}$
$p_1 + \frac{\beta}{\alpha} p_0 = -\frac{\gamma}{\alpha} \rightarrow p_1 + \frac{\beta}{\alpha}(0) = -\frac{\gamma}{\alpha} \rightarrow p_1 = -\frac{\gamma}{\alpha}$
$p_1 = -\frac{\gamma}{\alpha} = C_1(1)^1 + C_2(-\frac{\beta}{\alpha})^1 \rightarrow -\frac{\gamma}{\alpha} = C_1 + C_2(-\frac{\beta}{\alpha}) \rightarrow -\frac{\gamma}{\alpha} = C_1 + (-C_1)(-\frac{\beta}{\alpha}) \rightarrow$
$-\frac{\gamma}{\alpha} = C_1 + C_1\frac{\beta}{\alpha} \rightarrow -\frac{\gamma}{\alpha} = C_1(1 + \frac{\beta}{\alpha}) \rightarrow C_1 = \frac{-\frac{\gamma}{\alpha}}{1+\frac{\beta}{\alpha}} \rightarrow C_1 = \frac{-\gamma}{\alpha+\beta} \rightarrow C_2 = \frac{\gamma}{\alpha+\beta}$

$$\boxed{f_n = -\frac{\gamma}{\alpha+\beta} + \frac{\gamma}{\alpha+\beta}(-\frac{\beta}{\alpha})^n, \alpha \neq 0, \alpha + \beta \neq 0}$$

c. In a stable market, the price of the PS6 fluctuates but eventually settles down to a finite value after enough time has passed. What condition on the parameters α and β is needed to ensure your solution to (b) represents a stable market?

A stable market is when $|f'(p^*)| < 1$
$p_{n+1} = \frac{1}{\alpha}(-\beta p_n - \gamma) \rightarrow f(p_n) = \frac{1}{\alpha}(-\beta p_n - \gamma) \rightarrow f'(p_n) = -\frac{\beta}{\alpha}$
$|f'(p_n)| = |-\frac{\beta}{\alpha}| < 1 \rightarrow -1 < -\frac{\beta}{\alpha} < 1 \rightarrow 1 > \frac{\beta}{\alpha} > -1 \rightarrow \alpha > \beta > -\alpha$
$\boxed{\text{Stable market when } \alpha > \beta > -\alpha, \alpha \neq 0,}$

d. Assuming the condition in (c) is satisfied, what does your solution for p_n converge to as $n \rightarrow \infty$? Is the price of a PS6 after a long time has passed more or less expensive than its launching price?

$P_n = -\frac{\gamma}{\beta+\alpha} + \frac{\gamma}{\beta+\alpha}(-\frac{\alpha}{\beta})^n$
Condition in (c) ensures that $|-\frac{\beta}{\alpha}| < 1$, meaning that $\lim_{n\rightarrow\infty}(-\frac{\beta}{\alpha})^n = 0$
So $\lim_{n\rightarrow\infty} P_n = \lim_{n\rightarrow\infty}(-\frac{\gamma}{\beta+\alpha} + \frac{\gamma}{\beta+\alpha}(-\frac{\beta}{\alpha})^n) \rightarrow \lim_{n\rightarrow\infty} P_n = (-\frac{\gamma}{\beta+\alpha})$
In the long time the PS6 goes to its original launching price.

e. In a fluctuating market, the price of the PS6 never converges to a final value, but the price remains bounded. What condition on the parameters α and β is needed to ensure your solution to (b) represents a semi-stable market? What is the solution for p_n in this special case?

A semi-stable market. $|f'(p^*)| = 1$, $f'(p^*) = 1$ or $f'(p^*) = -1$,
$p_{n+1} = -\frac{\beta}{\alpha}p_n - \frac{\gamma}{\alpha} \to f'(p) = -\frac{\beta}{\alpha} \to |f'(p)| = |-\frac{\beta}{\alpha}| = 1$
Split
$-\frac{\beta}{\alpha} = 1 \to \alpha = -\beta$
$-\frac{\beta}{\alpha} = -1 \to \alpha = \beta$
$-\frac{\beta}{\alpha} = 1 \to f_n = -\frac{\gamma}{\alpha+\beta} + \frac{\gamma}{\alpha+\beta}(-\frac{\beta}{\alpha})^n \to f_n = -\frac{\gamma}{\alpha+\beta} + \frac{\gamma}{\alpha+\beta}(1)^n \to f_n = -\frac{\gamma}{\alpha+\beta} + \frac{\gamma}{\alpha+\beta} = 0$
$-\frac{\beta}{\alpha} = -1 \to f_n = -\frac{\gamma}{\alpha+\beta} + \frac{\gamma}{\alpha+\beta}(-1)^n \to f_n = -\frac{\gamma}{\alpha+\beta} + \frac{\gamma}{\alpha+\beta} = 0$ and $-2\frac{\gamma}{\alpha+\beta}$
This special case has weird behavior.

Exercise 3. (Unit 3.2)

Consider a first-order difference equation of the form

$$p_{n+1} = p_n(A + Bp_n^2), \tag{1}$$

where A and B are real, nonzero constants to be determined.

a. What condition on the constants A and B is needed to guarantee that (1) has three distinct fixed points? What are these fixed points?

Turn $_{n+1}$ and $_n$ to fixed point *
$p_{n+1} = p_n(A + Bp_n^2) \to p_n^* = p_n^*(A + B(p_n^*)^2) \to 1 - A = B(p_n^*)^2 \to \frac{1-A}{B} = (p_n^*)^2 \to p_n^* = \pm\sqrt{\frac{1-A}{B}}, B \neq 0, A < 1$
B must not be zero so there is no divide by zero error. If A is greater than or equal to one the fixed points are imaginary, so A must be less than one. $p_n^* = 0$ is a fixed point from the initial division.

$$\boxed{\text{Three fixed points at } p^* = 0, p^* = \sqrt{\frac{1-A}{B}}, p^* = -\sqrt{\frac{1-A}{B}}, B \neq 0, A < 1}$$

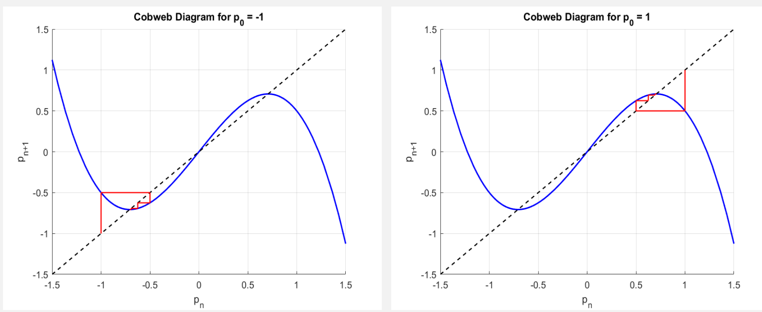
b. Assume A and B satisfy the condition derived in (a). Then, (1) has three distinct fixed points. If we require the nonzero fixed points to be stable, show that $1 < A < 2$.

$f(p_n) = p_n(A + Bp_n^2) \to f'(p_n) = (A + Bp_n^2) + p_n(2Bp_n) \to f'(p_n) = A + 3Bp_n^2$
Fixed point is stable if $|f'(p_n)| < 1$. Plug in nonzero fixed points.
$f'(p_n) = A + 3B(\pm\sqrt{\frac{1-A}{B}})^2 \to f'(p_n) = A + 3B(\frac{1-A}{B}) \to f'(p_n) = A + 3(1-A) \to f'(p_n) = 3 - 2A \to |3 - 2A| < 1 \to -1 < 3 - 2A < 1 \to 1 > -3 + 2A > -1 \to 4 > 2A > 2 \to \boxed{2 > A > 1}$

c. Given $1 < A < 2$, show that your condition in (a) implies $B < 0$. Thus, if $1 < A < 2$ and $B < 0$, (1) has three distinct fixed points, and the nonzero fixed points are stable.

The inequality $\frac{1-A}{B} > 0$ must be true so that the fixed points are real, and not imaginary as the nonzero fixed points are $\pm\sqrt{\frac{1-A}{B}}$. Considering we showed $1 < A < 2$
For $\frac{1-A}{B} > 0$ both numerator and demoninator must have the same sign.
$1 - A > 0? \to 1 > A$ this contradicts $1 < A < 2$ so $1 - A \not> 0$
$1 - A < 0? \to 1 < A \to 1 < A < 2$ this we showed was true. Thus $1 - A < 0$ so $B < 0$ as they have to be matching the same sign for $\frac{1-A}{B}$ to be positive.

d. Suppose $A = 3/2$ and $B = -1$. Using plotting software of your choice, create a cobweb diagram of (1) given the initial conditions $p_0 = 1$ and $p_0 = -1$. Consider at least 10 iterations for both of your initial conditions. Does your cobweb diagram support your conclusion from (c)?



The Cobweb diagram supports my conclusion from (c), which is if $1 < A < 2$ and $B \neq 0$ are true then these solutions are stable and converge to $\sqrt{\frac{1-A}{B}} = \sqrt{\frac{1-\frac{3}{2}}{-1}} = \sqrt{\frac{1}{2}} \approx 0.7071$ for a nonzero initial condition. Therefore the nonzero fixed points are $p_* = \pm\sqrt{\frac{1-A}{B}}$

Exercise 4. (Unit 3.3)

Suppose I flip a coin n times, each time recording whether the coin lands on heads (H) or tails (T). Then, I'll end up with a particular sequence of the letters H and T of length n. Suppose I'm interested in the probability of flipping a coin n times and never having two H's appear consecutively anywhere in my sequence. How do I compute this probability? I'm glad you asked.

Probability is the total number of desired outcomes (in this case, sequences of length n with no consecutive H's in them) divided by the total number of possible outcomes. It's not hard to see that the total number of outcomes is 2^n: there are two possibilities for each entry in my sequence of n letters. The total number of desired outcomes is trickier to compute.

If $n = 1$, then the number of desired outcomes is 2: I can have one H in my list or one T in my list. It's impossible to have two consecutive H's in either case because my list consists of only one letter. If $n = 2$, then the number of desired outcomes is 3: my list can be HT, TH, or TT. In general, for a given n, my total number of desired outcomes is a function of n. Call this function $f(n)$. My goal is to obtain a recursion relation for $f(n)$.

Suppose I want to construct a sequence of length n in which two H's never appear back to back. I'll split my analysis into two cases: (i) when the last letter in the sequence is T and (ii) when the last letter in the sequence is H. If the last letter in my sequence ends in a T, I have to make sure that the first $n-1$ terms of this sequence don't have two H's back to back. According to my definition of $f(n)$ above, there are $f(n-1)$ possibilities for these first $n-1$ terms. Thus, there are $f(n-1)$ number of sequences of type (i). Similarly, if the last letter in my sequence is a H, then the $n-1$ letter has to be a T, otherwise my sequence will have two H's back to back. In addition, I have to make sure the first $n-2$ terms of this sequence don't have two H's back to back. According to my definition of $f(n)$ above, there are $f(n-2)$ possibilities for these first $n-2$ terms. Thus, there are $f(n-2)$ number of sequences of type (ii).

Bringing my results together from these two cases, I obtain the following recursion relation:

$$f(n) = f(n-1) + f(n-2). \tag{2}$$

a. What is the general solution of (2)?

$f(n) = f(n-1) + f(n-2)$

$f(n) - f(n-1) - f(n-2) = 0 \to f(n+2) - f(n+1) - f(n) = 0 \to \lambda^2 - \lambda - 1 = 0$

$\lambda_{1,2} = \frac{1}{2}\left(-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}\right) \to \frac{1}{2}\left(1 \pm \sqrt{1+4}\right) \to \frac{1}{2}\left(1 \pm \sqrt{5}\right)$

$\lambda_1 = \left(\frac{1+\sqrt{5}}{2}\right)$, $\lambda_2 = \left(\frac{1-\sqrt{5}}{2}\right)$

$$\boxed{\text{General Solution: } f_n = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

b. Suppose your general solution from (a) satisfies the initial conditions $f(1) = 2$ and $f(2) = 3$. What is $f(n)$?

Initial Condition $p_1 = 2$

$p_1 = 2 = C_1\left(\frac{1+\sqrt{5}}{2}\right)^1 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^1 \to 2 = C_1\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right) \to$

$4 = C_1\left(1+\sqrt{5}\right) + C_2\left(1-\sqrt{5}\right) \to C_1 = \frac{4-C_2(1-\sqrt{5})}{1+\sqrt{5}}$

Initial Condition $p_2 = 3$

$3 = C_1\left(\frac{1+\sqrt{5}}{2}\right)^2 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^2 \to p_2 = 3 = \left(\frac{4-C_2(1-\sqrt{5})}{1+\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^2 + C_2\left(\frac{1-\sqrt{5}}{2}\right)^2 \to$

$C_2 = \frac{5-3\sqrt{5}}{10}$

$C_1 = \frac{4-C_2(1-\sqrt{5})}{1+\sqrt{5}} = \frac{4-(\frac{5-3\sqrt{5}}{10})(1-\sqrt{5})}{1+\sqrt{5}} = \frac{5+3\sqrt{5}}{10}$

$$\boxed{P_n = \frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{5-3\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

c. Using your result from (c), what is the probability that I flip a coin 15 times without having two heads appear back to back?

Probability $= \frac{P_n}{2^n}$

$\frac{f(15)}{2^{15}} \to \frac{\frac{5+3\sqrt{5}}{10}\left(\frac{1+\sqrt{5}}{2}\right)^{15} + \frac{5-3\sqrt{5}}{10}\left(\frac{1-\sqrt{5}}{2}\right)^{15}}{2^{15}} \to \frac{1597}{32768} = 0.04874$

$\boxed{\text{The probability of a coin flip without two heads back to back is 4.9\%}}$

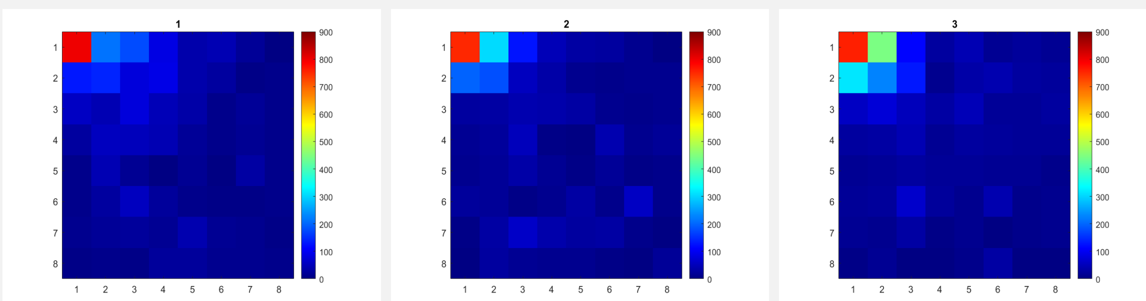
Exercise 5.

A medical imaging detector consists of a large crystal that sits atop a silicon photomultiplier (SiPM). The crystal is a large clear chunk and the photomultiplier consists of an 8x8 grid of sensors. When an xray comes into the crystal, it makes a flash of light and each of the 8x8 grid of sensors picks up some of that light. If the flash is far from the sensor within the crystal, the sensors will show a broad area that is illuminated at low levels. If the flash occurs close to the sensor, the flash will be smaller and more intense.

To collect the signal from the detector, 64 wires from from the SiPM and each must be analyzed. Each wire has supporting electronics that are very expensive. The goal is to reduce the number of output channels while maintaining the position information from the signal.

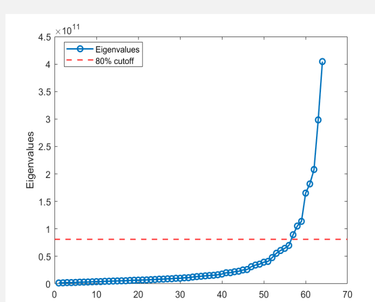
Download the matlab data file *detector_ data.mat* that contains real-world detector data. Each of the 64 rows of the data represents one out the 8x8 sensor outputs and each column is a single flash of light.

a. Visualize three of the photon interaction events to display: make an 8x8 matrix in matlab called A. Select any one column from the data, call this *col* for example *col* = *7* selects the 7th column from the data. To see the light distribution of the flash over the 8x8 array use the commands: *A(:) = data(:,col) ; imagesc(A) ; axis image ; colorbar*



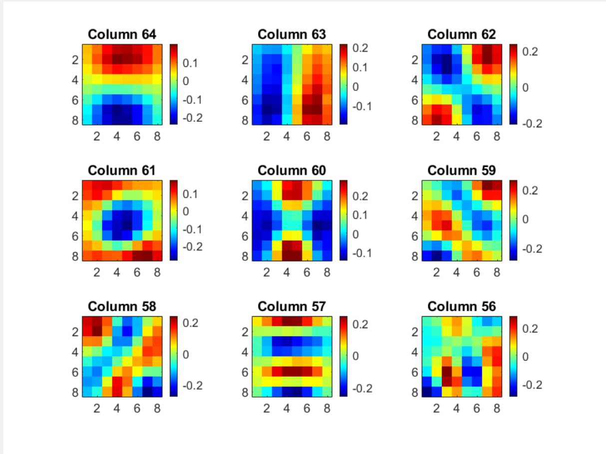
Visualization of the first three plots.

b. Compute the Principal Components of the dataset and determine the number of dimensions that are needed to carry the information contained in the data: this is the number of wires needed to come out from the sensor.



From this figure it shows we need only 9 eigenvalues are required to capture 80% of the data.

c. Visualize the most important Principal Components that you found, these are the design of the resistive circuit required to reduce the number of channels. (If you determined that the data was 7-dimensional, visualize the first 7 primary Principal Components)




```matlab
%% AMATH 383 HW 7 Exercise 5
set(0,'DefaultFigureColorMap',feval('jet'))

load('C:\Users\julie\OneDrive - UW\Desktop\01 AMATH 383\HW 7\all_detector_data-1.mat')

%% Problem 5a
A = zeros(8,8);

for ii = 1:3
    A(:) = all_data(:,ii);
    imagesc(A,[0,900]) ; axis image; colorbar
 title(ii); pause
end

%% Problem 5b
mean_vect = mean(all_data,2);

for ii = 1:64
    all_data(ii,:) = all_data(ii,:) - mean_vect(ii);
end

C = all_data * all_data';
[V,D] = eig(C);
D_diag = diag(D);

plot(diag(D),'o-','LineWidth',1.5,'DisplayName','Eigenvalues')
xlabel('Dimensions');
ylabel('Eigenvalues');
```

```
yline(max(D_diag)*.2, 'Color','red','LineWidth',1.5,'LineStyle','--','DisplayName','80% cutoff')
legend('Location','best');

%% Excercise 5c
figure;
for idx = 1:9
    ii = 64 - (idx - 1);
    subplot(3, 3, idx);
    A(:) = V(:, ii);
    imagesc(A);
    axis image;
    colorbar;
    title(['Column ' num2str(ii)]);
end

%% AMATH 383 Excercise 3d

clc; clear; close all;

% Define parameters
A = 3/2;
B = -1;
N = 10;

% Define function
f = @(p) p.* (A + B* p.^2);

% Generate cobweb diagrams for p0 = 1 and p0 = -1
p0 = [1, -1];
cobweb_plot(f, 1, N, A, B);
cobweb_plot(f, -1, N, A, B);

% Cobweb plot function
function cobweb_plot(f, p0, n_iter, A, B)
    figure; hold on;
    fplot(f, [-1.5, 1.5], 'b', 'LineWidth', 1.5); % Plot f(p)
    fplot(@(p) p, [-1.5, 1.5], 'k--', 'LineWidth', 1.2); % Plot p_n+1 = p_n
    xlabel('p_n'); ylabel('p_{n+1}');
    title(['Cobweb Diagram for p_0 = ', num2str(p0)]);
    grid on;

    % Generate cobweb steps
    p_n = p0;
    for i = 1:n_iter
        p_next = f(p_n);
```

```
        plot([p_n, p_n], [p_n, p_next], 'r', 'LineWidth', 1.2); % Vertical line
        plot([p_n, p_next], [p_next, p_next], 'r', 'LineWidth', 1.2); % Horizontal line
        p_n = p_next;
    end
    %plot(-sqrt((1-A)/B),-sqrt((1-A)/B),'o')
    %plot(sqrt((1-A)/B),sqrt((1-A)/B),'o')

    hold off;
end
```